10 Counting Methods and Probability

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 10: simplifying expressions, multiplying binomials, and finding areas.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. The **coefficient** of x^2 in the expression $3x^3 - 15x^2 + 4$ is __?__.
2. Written as a fraction in lowest terms, the **ratio** of 18 to 45 is __?__.
3. The expressions $x + 3$ and $2x - 1$ are examples of **binomials** because they have __?__ terms.

SKILLS CHECK

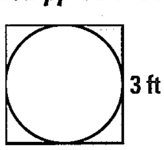
Simplify the expression. ***(Review p. 2 for 10.1.)***

4. $\frac{6 \cdot 5 \cdot 4 \cdot 3}{2 \cdot 1}$ 5. $\frac{13 \cdot 12 \cdot 11}{10 \cdot 9 \cdot 8}$ 6. $\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

Find the product. ***(Review p. 346 for 10.2.)***

7. $(x + y)^3$ 8. $(5x + 1)^3$ 9. $(3x - 2y)^3$

Find the area of the shaded region. Assume all shapes are circles or squares. ***(Review pp. 991–992 for 10.3.)***

10.


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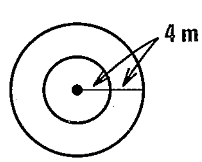

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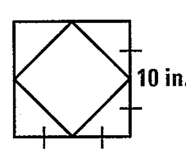


@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 10, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 733. You will also use the key vocabulary listed below.

Big Ideas

1. **Using permutations and combinations**
2. **Finding probabilities**
3. **Constructing binomial distributions**

Key Vocabulary

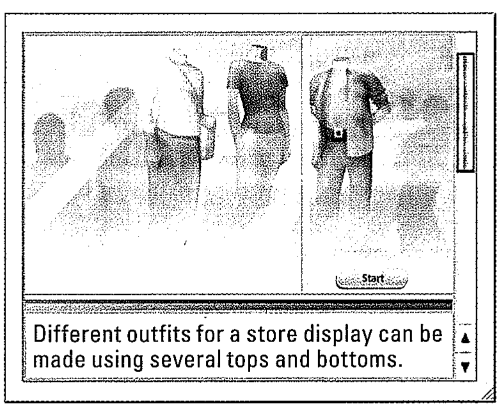
- permutation, *p. 684*
- combination, *p. 690*
- binomial theorem, *p. 693*
- probability, *p. 698*
- compound event, *p. 707*
- overlapping events, *p. 707*
- disjoint events, *p. 707*
- independent events, *p. 717*
- dependent events, *p. 718*
- conditional probability, *p. 718*
- random variable, *p. 724*
- binomial distribution, *p. 725*

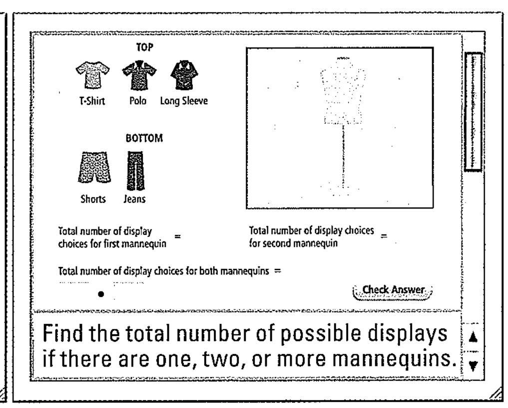
Why?

You can use the fundamental counting principle and permutations to calculate the number of choices for a situation. For example, you can count the number of possible outcomes of an event or the number of ways to complete a task.

Animated Algebra

The animation illustrated below for Exercise 69 on page 689 helps you answer this question: How does the number of clothing choices affect the number of different ways can you dress mannequins in a display?





Animated Algebra at classzone.com

Other animations for Chapter 10: pages 701, 711, 716, 722, and 726

EXAMPLE 6 Find permutations with repetition

Find the number of distinguishable permutations of the letters in (a) MIAMI and (b) TALLAHASSEE.

Solution

a. MIAMI has 5 letters of which M and I are each repeated 2 times. So, the number of distinguishable permutations is $\frac{5!}{2! \cdot 2!} = \frac{120}{2 \cdot 2} = 30$.

b. TALLAHASSEE has 11 letters of which A is repeated 3 times, and L, S, and E are each repeated 2 times. So, the number of distinguishable permutations is $\frac{11!}{3! \cdot 2! \cdot 2! \cdot 2!} = \frac{39{,}916{,}800}{6 \cdot 2 \cdot 2 \cdot 2} = 831{,}600$.

✓ **GUIDED PRACTICE** for Example 6

Find the number of distinguishable permutations of the letters in the word.

8. MALL **9.** KAYAK **10.** CINCINNATI

10.1 EXERCISES

HOMEWORK KEY
◯ = **WORKED-OUT SOLUTIONS** on p. WS18 for Exs. 13, 35, and 65
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 42, 55, 57, and 68

SKILL PRACTICE

1. VOCABULARY What is a permutation of *n* objects?

2. ★ WRITING Simplify the formula for ${}_nP_r$ when $r = 0$. *Explain* why this result makes sense.

EXAMPLE 1 on p. 682 for Exs. 3–6

TREE DIAGRAMS An object has an attribute from each list. Make a tree diagram that shows the number of different objects that can be created.

3.

T-Shirts
Size: M, L, XL
Type: long-sleeved, short-sleeved

4.

Toast
Bread: white, wheat
Spread: jam, margarine

5.

Meal
Entrée: chicken, fish, pasta
Side: corn, green beans, potato

6.

Furniture
Wood: cherry, mahogany, oak, pine
Finish: stained, painted, unfinished

EXAMPLE 2 on p. 683 for Exs. 7–10

FUNDAMENTAL COUNTING PRINCIPLE Each event can occur in the given number of ways. Find the number of ways all of the events can occur.

7. Event A: 2 ways; Event B: 4 ways

8. Event A: 5 ways; Event B: 2 ways

9. Event A: 4 ways; Event B: 3 ways; Event C: 5 ways

10. Event A: 3 ways; Event B: 6 ways; Event C: 5 ways; Event D: 2 ways

EXAMPLE 3
on p. 683
for Exs. 11–17

LICENSE PLATES **For the given configuration, determine how many different license plates are possible if (a) digits and letters can be repeated, and (b) digits and letters cannot be repeated.**

11. 4 letters followed by 3 digits

12. 2 letters followed by 5 digits

13. 4 letters followed by 2 digits

14. 5 digits followed by 3 letters

15. 1 digit followed by 5 letters

16. 6 letters

17. ★ **MULTIPLE CHOICE** How many different license plates with 2 letters followed by 4 digits are possible if digits and letters cannot be repeated?

(A) 3,276,000 (B) 6,760,000 (C) 32,292,000 (D) 45,697,600

EXAMPLES 4 and 5
on pp. 684–685
for Exs. 18–41

FACTORIALS **Evaluate the expression.**

18. 7! **19.** 11! **20.** 1! **21.** 8!

22. 4! **23.** 0! **24.** 12! **25.** 6!

26. $3! \cdot 4!$ **27.** $3(4!)$ **28.** $\frac{8!}{(8-5)!}$ **29.** $\frac{9!}{4! \cdot 4!}$

PERMUTATIONS **Find the number of permutations.**

30. ${}_4P_4$ **31.** ${}_6P_2$ **32.** ${}_{10}P_1$ **33.** ${}_8P_7$

34. ${}_7P_4$ **35.** ${}_9P_2$ **36.** ${}_{13}P_8$ **37.** ${}_7P_7$

38. ${}_5P_0$ **39.** ${}_9P_4$ **40.** ${}_{11}P_4$ **41.** ${}_{15}P_0$

42. ★ **SHORT RESPONSE** Let n be a positive integer. Find the number of permutations of n objects taken $n - 1$ at a time. *Compare* your answer with the number of permutations of all n objects. Does this make sense? *Explain.*

EXAMPLE 6
on p. 686
for Exs. 43–55

PERMUTATIONS WITH REPETITION **Find the number of distinguishable permutations of the letters in the word.**

43. OFF **44.** TREE **45.** SKILL **46.** YELLOW

47. GRAVEL **48.** PANAMA **49.** ARKANSAS **50.** FACTORIAL

51. MAGNETIC **52.** HONOLULU **53.** CLEVELAND **54.** MISSISSIPPI

55. ★ **MULTIPLE CHOICE** What is the number of distinguishable permutations of the letters in the word HAWAII?

(A) 24 (B) 180 (C) 360 (D) 720

56. **ERROR ANALYSIS** In bingo, balls labeled from 1 to 75 are drawn from a container without being replaced. *Describe* and correct the error in finding the number of ways the first 4 numbers can be chosen for a game of bingo.

$$75 \cdot 75 \cdot 75 \cdot 75 = 31{,}640{,}625$$ ✗

57. ★ **SHORT RESPONSE** *Explain* how the fundamental counting principle can be used to justify the formula for the number of permutations of n distinct objects.

SOLVING EQUATIONS **Solve for n.**

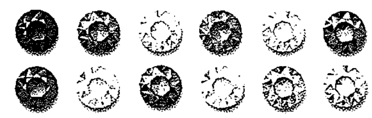
58. ${}_nP_4 = 8({}_nP_3)$ **59.** ${}_nP_6 = 5({}_nP_5)$ **60.** ${}_nP_5 = 9({}_nP_4)$

61. **CHALLENGE** Find the number of distinguishable permutations of 6 letters that are chosen from the letters in the word MANATEE.

PROBLEM SOLVING

EXAMPLE 2
on p. 683
for Exs. 62–63

62. **CLASS RINGS** You want to purchase a class ring. The ring can be made from 3 different metals. You can choose from 6 different side designs and 12 different stones. How many different class rings are possible?

Metal	Side Design		Stone
Auralite	Academics	Literature	
Gold	Art	Music	
Silver	Athletics	Technology	

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63. **ENVIRONMENT** Since 1990, the Goldman Environmental Prize has been awarded annually to 6 grassroots environmentalists, one from each of 6 regions. The regions consist of 52 countries in Africa, 47 in Europe, 45 in Asia, 36 in island nations, 19 in South and Central America, and 3 in North America. How many different sets of 6 countries can be represented by the prize winners in a given year?

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EXAMPLES 4, 5, and 6
on pp. 684–686
for Exs. 64–66

64. **PHOTOGRAPHY** A photographer lines up the 15 members of a family in a single line in order to take a photograph. How many different ways can the photographer arrange the family members for the picture?

65. **SCHOOL CLUBS** A Spanish club is electing a president, vice president, and secretary. The club has 9 members who are eligible for these offices. How many different ways can the 3 offices be filled?

66. **MUSIC** The window of a music store has 8 stands in fixed positions where instruments can be displayed. In how many ways can 3 identical guitars, 2 identical keyboards, and 3 identical violins be displayed?

67. **MULTI-STEP PROBLEM** You are designing an entertainment center. You want to include three audio components and three video components.

   a. You want one of each audio component listed at the right. How many selections of audio components are possible?

   b. You want one of each video component listed at the right. How many selections of video components are possible?

   c. How many selections of all six audio and video components are possible?

Entertainment Center	
Audio Components	**Video Components**
5 receivers	7 TV sets
8 CD players	9 DVD players
6 speakers	4 game systems

68. ★ **EXTENDED RESPONSE** To keep computer files secure, many programs require the user to enter a password. The shortest allowable passwords are typically 6 characters long and can contain both letters and digits.

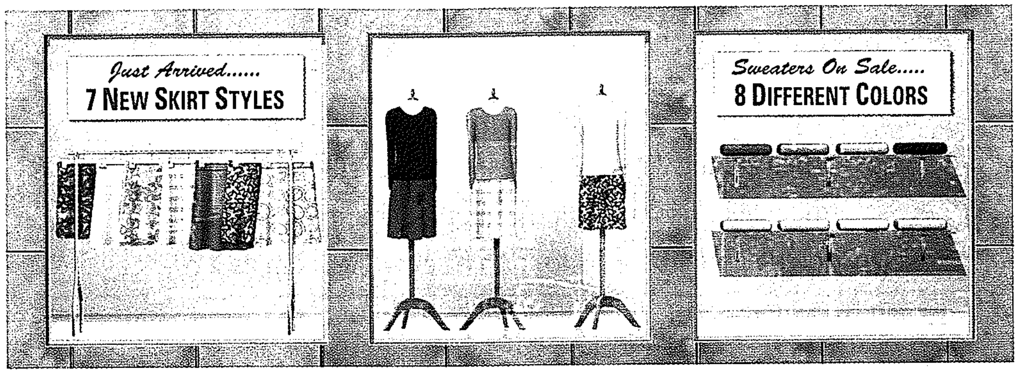
   a. **Calculate** How many 6-character passwords are possible if characters can be repeated?

   b. **Calculate** How many 6-character passwords are possible if characters cannot be repeated?

   c. **Draw Conclusions** Which type of password is more secure? *Explain.*

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE**

69. **CLOTHING DISPLAY** An employee at a clothing store is creating a display. The display has 3 different mannequins. Each mannequin is to wear a different sweater and a different skirt. How many different displays can be created?



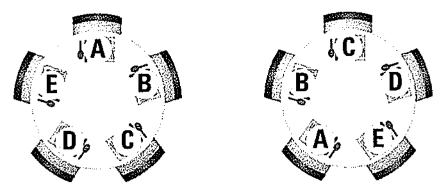
Animated Algebra at classzone.com

70. **CROSS COUNTRY** Three schools are competing in a cross country race. School A has 6 runners, school B has 5 runners, and school C has 4 runners. For scoring purposes, the finishing order of the race only considers the school of each runner. How many different finishing orders are there for the 15 runners?

71. **CHALLENGE** You have learned that $n!$ represents the number of ways that n objects can be placed in a *linear* order, where it matters which object is placed first. Now consider *circular* permutations in which objects are placed in a circle, so that it does *not* matter which object is placed first.

a. Suppose you are seating 5 people at a circular table. How many different ways can you arrange the people around the table?

b. Find a formula for the number of permutations of n objects placed in clockwise order around a circle when only the relative order of the objects matters. *Explain* how you derived your formula.



The two arrangements shown represent the same permutation.

MIXED REVIEW

PREVIEW
Prepare for Lesson 10.2 in Exs. 72–77.

Find the product. *(p. 346)*

72. $(x-8)(x+8)$ 73. $(4x-5)(4x+5)$ 74. $(x+7)^2$

75. $(5x-6y)^2$ 76. $(3x-2)^3$ 77. $(4x+3y)^3$

Find the inverse of the function. *(p. 438)*

78. $f(x)=4x-9$ 79. $f(x)=-x+6$ 80. $f(x)=4x^5$

81. $f(x)=x^2, x\geq 0$ 82. $f(x)=x^3+5$ 83. $f(x)=3x^5-1$

Graph the equation.

84. $y^2=-24x$ *(p. 620)* 85. $x^2+y^2=20$ *(p. 626)* 86. $\frac{x^2}{9}+\frac{y^2}{36}=1$ *(p. 634)*

87. $\frac{x^2}{81}-\frac{y^2}{121}=1$ *(p. 642)* 88. $(x+3)^2+y^2=16$ *(p. 650)* 89. $\frac{(y-1)^2}{16-x^2}=1$ *(p. 650)*

EXAMPLE 7 Find a coefficient in an expansion

Find the coefficient of x^4 in the expansion of $(3x + 2)^{10}$.

Solution

From the binomial theorem, you know the following:

$$(3x + 2)^{10} = {}_{10}C_0(3x)^{10}(2)^0 + {}_{10}C_1(3x)^9(2)^1 + \cdots + {}_{10}C_{10}(3x)^0(2)^{10}$$

Each term in the expansion has the form ${}_{10}C_r(3x)^{10-r}(2)^r$. The term containing x^4 occurs when $r = 6$:

$${}_{10}C_6(3x)^4(2)^6 = (210)(81x^4)(64) = 1{,}088{,}640x^4$$

▶ The coefficient of x^4 is 1,088,640.

✓ **GUIDED PRACTICE for Example 7**

11. Find the coefficient of x^5 in the expansion of $(x - 3)^7$.
12. Find the coefficient of x^3 in the expansion of $(2x + 5)^8$.

10.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS18 for Exs. 17, 29, and 49

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 35, 40, 41, and 52

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The binomial expansion of $(a + b)^n$ is given by the _?_.
2. ★ **WRITING** *Explain* the difference between permutations and combinations.

EXAMPLES 1, 2, and 3 on pp. 690–691 for Exs. 3–18

COMBINATIONS **Find the number of combinations.**

3. ${}_5C_2$
4. ${}_{10}C_3$
5. ${}_9C_6$
6. ${}_8C_2$
7. ${}_{11}C_{11}$
8. ${}_{12}C_4$
9. ${}_7C_5$
10. ${}_{14}C_6$

ERROR ANALYSIS ***Describe*** **and correct the error in finding the number of combinations.**

11. $${}_6C_2 = \frac{6!}{(6-2)!} = \frac{720}{24} = 30$$ ✗

12. $${}_8C_3 = \frac{8!}{3!} = \frac{40{,}320}{6} = 6720$$ ✗

CARD HANDS **Find the number of possible 5-card hands that contain the cards specified. The cards are taken from a standard 52-card deck.**

13. 5 face cards (kings, queens, or jacks)
14. 4 kings and 1 other card
15. 1 ace and 4 cards that are not aces
16. 5 hearts or 5 diamonds
17. At most 1 queen
18. At least 1 spade

EXAMPLE 4
on p. 692
for Exs. 19–23

19. **USING PATTERNS** Copy Pascal's triangle on page 692 and add rows for $n = 6, 7, 8, 9,$ and 10.

PASCAL'S TRIANGLE Use the rows of Pascal's triangle from Exercise 19 to write the binomial expansion.

20. $(x + 3)^6$ **21.** $(y - 3z)^{10}$ **22.** $(a + b^2)^8$ **23.** $(2s - t^4)^7$

EXAMPLES 5 and 6
on p. 693
for Exs. 24–31

BINOMIAL THEOREM Use the binomial theorem to write the binomial expansion.

24. $(x + 2)^3$ **25.** $(c - 4)^5$ **26.** $(a + 3b)^4$ **27.** $(4p - q)^6$

28. $(w^3 - 3)^4$ **29.** $(2s^4 + 5)^5$ **30.** $(3u + v^2)^6$ **31.** $(x^3 - y^2)^4$

EXAMPLE 7
on p. 694
for Exs. 32–35

32. Find the coefficient of x^5 in the expansion of $(x - 2)^{10}$.

33. Find the coefficient of x^3 in the expansion of $(3x + 2)^5$.

34. Find the coefficient of x^6 in the expansion of $(x^2 - 3)^8$.

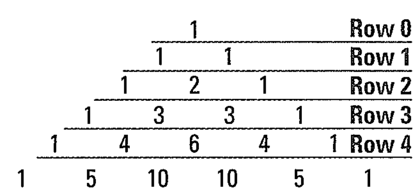
35. ★ **MULTIPLE CHOICE** Which is the coefficient of x^4 in the expansion of $(x - 3)^7$?

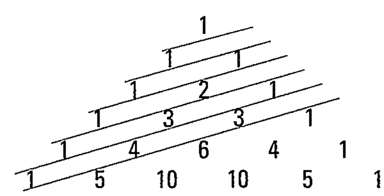
Ⓐ −945 Ⓑ −35 Ⓒ −27 Ⓓ 2835

PASCAL'S TRIANGLE In Exercises 36 and 37, use the diagrams shown.

36. What is the sum of the numbers in each of rows 0–4 of Pascal's triangle? What is the sum in row n?

37. *Describe* the pattern formed by the sums of the numbers along the diagonal segments of Pascal's triangle.





REASONING In Exercises 38 and 39, decide whether the problem requires *combinations* or *permutations* to find the answer. Then solve the problem.

38. **NEWSPAPER** Your school newspaper has an editor-in-chief and an assistant editor-in-chief. The staff of the newspaper has 12 students. In how many ways can students be chosen for these two positions?

39. **STUDENT COUNCIL** Five representatives from a senior class of 280 students are to be chosen for the student council. In how many ways can students be chosen to represent the senior class on the student council?

40. ★ **MULTIPLE CHOICE** A relay race has a team of 4 runners who run different parts of the race. There are 20 students on your track squad. In how many ways can the coach select students to compete on the relay team?

Ⓐ 4845 Ⓑ 40,000 Ⓒ 116,280 Ⓓ 160,000

41. ★ **SHORT RESPONSE** *Explain* how the formula for ${}_nC_n$ suggests the definition $0! = 1$.

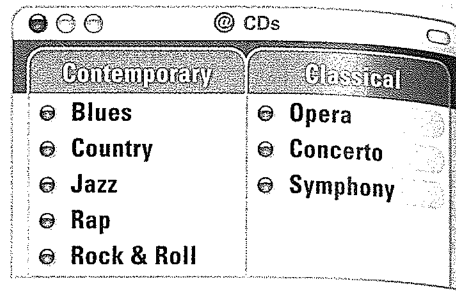
CHALLENGE Verify the identity. *Justify* each of your steps.

42. ${}_nC_0 = 1$ **43.** ${}_nC_n = 1$ **44.** ${}_nC_r \cdot {}_rC_m = {}_nC_m \cdot {}_{n-m}C_{r-m}$

45. ${}_nC_1 = {}_nP_1$ **46.** ${}_nC_r = {}_nC_{n-r}$ **47.** ${}_{n+1}C_r = {}_nC_r + {}_nC_{r-1}$

EXAMPLES 1, 2, and 3 on pp. 690–691 for Exs. 48–50

48. MUSIC You want to purchase 3 CDs from an online collection that contains the types of music shown at the right. You want each CD to contain a different type of music such that 2 CDs are different types of contemporary music and 1 CD is a type of classical music. How many different sets of music types can you choose?



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(49.) FLOWERS You are buying a bouquet. The florist has 18 types of flowers that you can use to make the bouquet. You want to use *exactly* 3 types of flowers. How many different combinations of flower types can you use in your bouquet?

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50. ARCADE GAMES An arcade has 20 different arcade games. You want to play at least 14 of them. How many different combinations of arcade games can you play?

51. MULTI-STEP PROBLEM A televised singing competition picks a winner from 20 original contestants over the course of five episodes. During each of the first, second, and third episodes, 5 singers are eliminated by the end of the episode. The fourth episode eliminates 2 more singers, and the winner is selected at the end of the fifth episode.

a. How many combinations of 5 singers out of the original 20 can be eliminated during the first episode?

b. How many combinations of 5 singers out of the 15 singers who started the second episode can be eliminated during the second episode?

c. How many combinations of singers can be eliminated during the third episode? during the fourth episode? during the fifth episode?

d. Find the total number of ways in which the 20 original contestants can be eliminated to produce a winner.

52. ★ EXTENDED RESPONSE A group of 15 high school students is volunteering at a local fire station. Of these students, 5 will be assigned to wash fire trucks, 7 will be assigned to repaint the station's interior, and 3 will be assigned to do maintenance on the station's exterior.

a. Calculate One way to count the number of possible job assignments is to find the number of permutations of 5 W's (for "wash"), 7 R's (for "repainting"), and 3 M's (for "maintenance"). Use this method to write the number of possible job assignments first as an expression involving factorials and then as a number.

b. Calculate Another way to count the number of possible job assignments is to first choose the 5 W's, then choose the 7 R's, and then choose the 3 M's. Use this method to write the number of possible job assignments first as an expression involving factorials and then as a number.

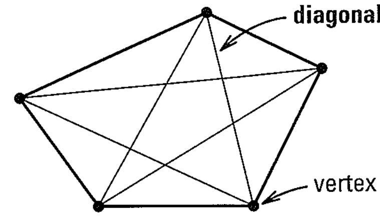
c. Analyze *Compare* your results from parts (a) and (b). *Explain* why they make sense.


Volunteers in Aniak, Alaska

○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

53. **CHALLENGE** A polygon is *convex* if no line that contains a side of the polygon contains a point in the interior of the polygon. Consider a convex polygon with n sides.



a. Use the combinations formula to write an expression for the number of line segments that join pairs of vertices on an n-sided polygon.

b. Use your result from part (a) to write a formula for the number of diagonals of an n-sided convex polygon.

MIXED REVIEW

PREVIEW
Prepare for Lesson 10.3 in Exs. 54–57.

Find the area of the figure. *(pp. 991–992)*

54. Circle with radius 16 inches

55. Rectangle with sides 8.25 feet and 12.1 feet

56. Triangle with base 15 centimeters and height 18 centimeters

57. Trapezoid with bases 12 meters and 16 meters, and height 9 meters

Solve the equation.

58. $8\sqrt{4x} - 5 = 19$ *(p. 452)*

59. $(x - 2)^{3/2} = 216$ *(p. 452)*

60. $\ln(x + 4) = \ln 5$ *(p. 515)*

61. $10^{4x} - 5 = 11$ *(p. 515)*

62. $\frac{x}{x - 2} = \frac{x + 3}{x + 1}$ *(p. 589)*

63. $\frac{1}{x - 3} + 3 = \frac{2x}{x + 3}$ *(p. 589)*

Write an equation of the perpendicular bisector of the line segment joining the two points. *(p. 614)*

64. $(-4, -2), (6, 2)$

65. $(9, -2), (3, 6)$

66. $(-8, -13), (7, 10)$

67. $(6, 9.3), (0, 8.2)$

QUIZ *for Lessons 10.1–10.2*

For the given license plate configuration, find how many plates are possible if letters and digits (a) can be repeated and (b) cannot be repeated. *(p. 682)*

1. 2 letters followed by 3 digits

2. 3 digits followed by 3 letters

Find the number of distinguishable permutations of the letters in the word. *(p. 682)*

3. AWAY

4. IDAHO

5. LETTER

6. TENNESSEE

Find the number of combinations. *(p. 690)*

7. ${}_8C_6$

8. ${}_7C_4$

9. ${}_9C_0$

10. ${}_{12}C_{11}$

Use the binomial theorem to write the binomial expansion. *(p. 690)*

11. $(x + 5)^5$

12. $(2s - 3)^6$

13. $(3u + v)^4$

14. $(2x^3 - 3y)^5$

15. Find the coefficient of x^3 in the expansion of $(x + 2)^9$. *(p. 690)*

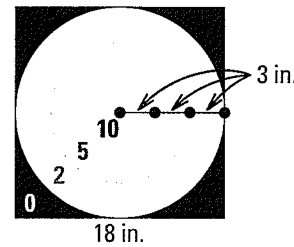
16. **MENU CHOICES** A pizza parlor runs a special where you can buy a large pizza with 1 cheese, 1 vegetable, and 2 meats for $12. You have a choice of 5 cheeses, 10 vegetables, and 6 meats. How many different variations of the pizza special are possible? *(p. 690)*


, Alaska

GEOMETRIC PROBABILITY Some probabilities are found by calculating a ratio of two lengths, areas, or volumes. Such probabilities are **geometric probabilities**.

EXAMPLE 5 Find a geometric probability

DARTS You throw a dart at the square board shown. Your dart is equally likely to hit any point inside the board. Are you more likely to get 10 points or 0 points?



Solution

$$P(10\text{ points}) = \frac{\text{Area of smallest circle}}{\text{Area of entire board}}$$

$$= \frac{\pi \cdot 3^2}{18^2} = \frac{9\pi}{324} = \frac{\pi}{36} \approx 0.0873$$

$$P(0\text{ points}) = \frac{\text{Area outside largest circle}}{\text{Area of entire board}}$$

$$= \frac{18^2 - (\pi \cdot 9^2)}{18^2} = \frac{324 - 81\pi}{324} = \frac{4 - \pi}{4} \approx 0.215$$

▶ Because $0.215 > 0.0873$, you are more likely to get 0 points.

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✓ **GUIDED PRACTICE** for Example 5

7. **WHAT IF?** In Example 5, are you more likely to get 5 points or 0 points?

10.3 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS18 for Exs. 7, 17, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 26, 27, 32, and 42

◆ = **MULTIPLE REPRESENTATIONS** Ex. 40

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A probability that is the ratio of two lengths, areas, or volumes is called a(n) __?__ probability.

2. ★ **WRITING** *Explain* the difference between theoretical probability and experimental probability. Give an example of each.

EXAMPLE 1 on p. 698 for Exs. 3–16

CHOOSING NUMBERS You have an equally likely chance of choosing any integer from 1 through 50. Find the probability of the given event.

3. An even number is chosen.
4. A number less than 35 is chosen.
5. A perfect square is chosen.
6. A prime number is chosen.
7. A factor of 150 is chosen.
8. A multiple of 4 is chosen.
9. A two-digit number is chosen.
10. A perfect cube is chosen.

CHOOSING CARDS A card is randomly drawn from a standard deck of 52 cards. Find the probability of drawing the given card.

11. The king of diamonds

12. A king

13. A spade

14. A black card

15. A card other than a 2

16. A face card (a king, queen, or jack)

EXAMPLE 2
on p. 699
for Exs. 17–19

LOTTERIES In Exercises 17 and 18, find the probability of winning the lottery according to the given rules. Assume numbers are selected at random.

(17.) You must correctly select 6 out of 48 numbers. The order of the numbers is not important.

18. You must correctly select 4 numbers, each an integer from 0 to 9. The order of the numbers is important.

19. ★ **MULTIPLE CHOICE** What is the probability (rounded to three decimal places) that 2 randomly selected months both have 31 days?

(A) 0.159 (B) 0.227 (C) 0.318 (D) 0.340

EXAMPLE 3
on p. 700
for Exs. 20–25

ODDS You randomly choose a marble from a bag. The bag contains 10 black, 8 red, 4 white, and 6 blue marbles. Find the indicated odds.

20. In favor of choosing white

21. In favor of choosing blue

22. Against choosing red

23. Against choosing black

ERROR ANALYSIS ***Describe*** **and correct the error in calculating the odds against getting a 5 or 6 when rolling a six-sided die.**

24. Odds against 5 or 6 $= \frac{4}{6} = \frac{2}{3}$

25. Odds against 5 or 6 $= \frac{2}{4} = \frac{1}{2}$

26. ★ **OPEN-ENDED MATH** Flip a coin 10 times. What is the experimental probability of getting heads?

27. ★ **SHORT RESPONSE** The probability of event A is 0.3. What are the odds in favor of event A? *Explain.*

EXAMPLE 4
on p. 700
for Exs. 28–32

ROLLING A DIE The results of rolling a six-sided die 150 times are shown. Use the table to find the experimental probability of the given event. ***Compare*** **your answer to the theoretical probability of the event.**

28. Rolling a 5

29. Rolling an even number

30. Rolling a number less than 5

31. Rolling any number but a 3

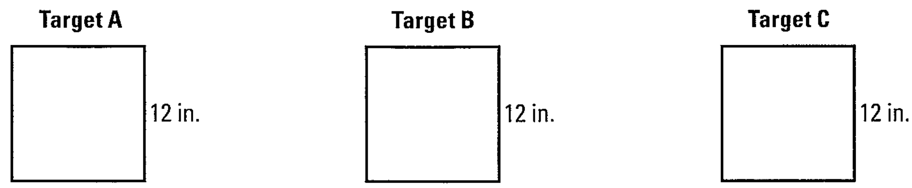
Roll	⚀	⚁	⚂	⚃	⚄	⚅
Number of occurrences	27	22	18	26	27	30

32. ★ **MULTIPLE CHOICE** You flip a coin 80 times. You get heads 37 times and tails 43 times. What is the experimental probability of getting heads?

(A) 0.4625 (B) 0.5 (C) 0.5375 (D) 0.8605

33. **REASONING** Find the probability that the vertex of the graph of $y = x^2 - 6x + c$ is above the x-axis if c is a randomly chosen integer from 1 to 20.

34. CHALLENGE Suppose you throw a dart at each square target below. Assume that the dart is equally likely to hit any point inside the target.



a. Calculate What is the probability that the dart lands inside the circle in target A? inside a circle in target B? inside a circle in target C?

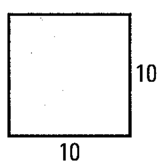
b. Generalize Consider the general case where a square target with sides 12 inches long contains n^2 identical circles arranged in n rows and n columns. Make a conjecture about the probability that a dart lands inside one of the circles. Then prove your conjecture.

PROBLEM SOLVING

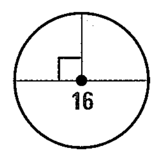
EXAMPLE 5 on p. 701 for Exs. 35–37

GEOMETRIC PROBABILITY Find the probability that a dart thrown at the given target will hit the shaded region. Assume the dart is equally likely to hit any point inside the target.

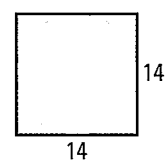
35.



36.



37.



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38. JURY SELECTION A jury of 12 people is selected from a pool of 30 people that includes 12 men and 18 women. What is the probability that the jury will be composed of 12 women?

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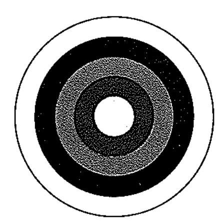
39. ARCHERY The standard archery target used in competition has a diameter of 80 centimeters. Find the probability that an arrow shot at the target will hit the center circle, which has a diameter of 16 centimeters. Assume the arrow is equally likely to hit any point inside the target.



40. MULTIPLE REPRESENTATIONS On a typical weekday, there are 1,181,100 one-way trips taken on the public transportation system operated by the Massachusetts Bay Transit Authority. Of these trips, 376,900 are bus rides. Suppose a one-way trip is selected at random.

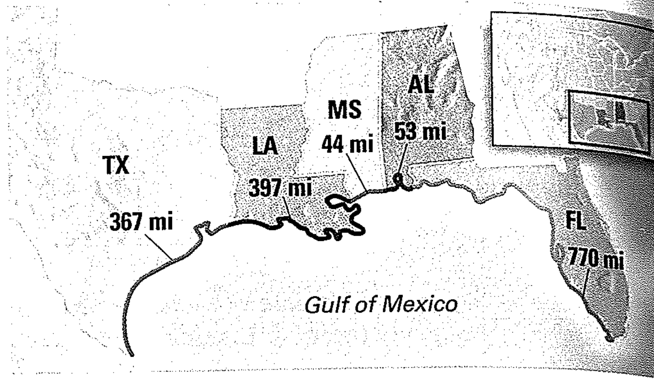
a. Using Fractions What is the probability, expressed as a fraction, that the trip was taken on a bus?

b. Using Decimals What is the probability, expressed as a decimal, that the trip was taken on a bus?

c. Using Percents What is the probability, expressed as a percent, that the trip was taken on a bus?

d. Using Odds What are the odds in favor of the trip having been on a bus?

41. **GULF COAST** The map shows the length of shoreline (in miles) along the Gulf of Mexico for each state that borders the body of water. What is the probability that a ship coming ashore at a random point in the Gulf of Mexico lands in the given state?



a. Texas

b. Florida

c. Alabama

42. ★ **EXTENDED RESPONSE** A magician claims to be able to read minds. To test this claim, five cards numbered 1 through 5 are used. A subject selects two cards from the five cards and concentrates on the numbers.

a. What is the probability that the two numbers chosen are 3 and 4?

b. What is the probability that the magician can correctly identify the two numbers by guessing?

c. Suppose the magician is able to consistently identify the two numbers about half the time. Does this support the magician's claim to be a mind reader? *Explain.*

43. **CHALLENGE** In a guessing game, one player secretly places four different-colored pegs on a board in each of four positions: A, B, C, or D. A second player guesses the configuration of the pegs by placing an identical set of pegs in slots A, B, C, and D on an identical board. The second player is then told how many of the pegs are in the correct slot.

a. What is the probability that the second player has all four pegs correct on the first guess?

b. What is the probability that the second player has exactly one peg correct on the first guess?

c. The second player is told she has placed two pegs in the correct slot. The player then switches two of the pegs. What is the probability that the player now has all four pegs in the correct slot?

MIXED REVIEW

Graph the function.

44. $y = 4(0.75)^x$ *(p. 486)* 45. $f(x) = 3e^{-2x}$ *(p. 492)* 46. $y = \ln x + 2$ *(p. 499)*

47. $y = \left(\frac{3}{2}\right)^x$ *(p. 478)* 48. $g(x) = \frac{-1}{x+1} - 2$ *(p. 558)* 49. $y = \frac{3x+1}{x^2-4}$ *(p. 565)*

Evaluate the expression without using a calculator. *(p. 499)*

50. $\log_4 64$ 51. $\ln e$ 52. $\log_6 36$ 53. $\log_2 512$

54. $\ln e^{2.9}$ 55. $\log_{1/3} 9$ 56. $\log_9 27$ 57. $\log_4 \frac{1}{32}$

PREVIEW
Prepare for Lesson 10.4 in Exs. 58–61.

Find the number of combinations. *(p. 690)*

58. ${}_8C_3$ 59. ${}_{10}C_9$ 60. ${}_7C_4$ 61. ${}_{12}C_5$

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 10.1–10.3

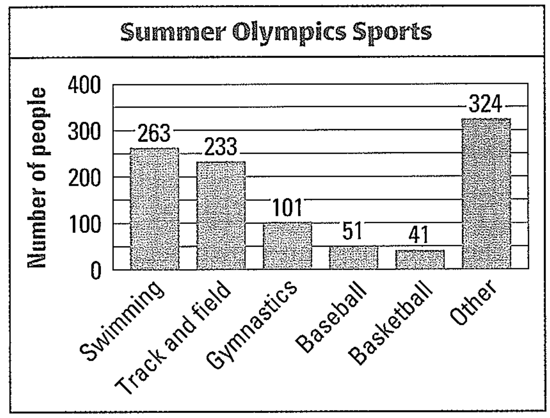
1. **MULTI-STEP PROBLEM** Five people walk into a movie theater and look for empty seats.
   a. Find the number of ways the people can be seated if there are 5 empty seats.
   b. Find the number of ways the people can be seated if there are 8 empty seats.
   c. Generalize your results from parts (a) and (b) by writing an expression involving factorials for the number of ways the people can be seated if there are n empty seats.
   d. What is the minimum value of n such that there are at least 1 million ways the people can be seated?

2. **MULTI-STEP PROBLEM** You and a friend are meeting at the gym to work out. You both agree to arrive between 9:00 A.M. and 9:30 A.M. You will wait for each other for up to 10 minutes.
   a. Let x be your arrival time (in minutes after 9:00 A.M.), and let y be your friend's arrival time (in minutes after 9:00 A.M.). Write inequalities representing the time intervals in which you and your friend arrive.
   b. If you and your friend are to meet, the difference between your arrival times must not exceed 10 minutes. Write two inequalities that show this fact.
   c. Graph your inequalities from parts (a) and (b) in the same coordinate plane.
   d. Using your graph from part (c), find the probability that you and your friend will meet at the gym.

3. **GRIDDED ANSWER** You want to make a fruit smoothie using 3 of the fruits listed. How many different fruit smoothies can you make?

4. **GRIDDED ANSWER** In a high school fashion show, how many ways can 1 freshman, 2 sophomores, 2 juniors, and 3 seniors line up in front of the judges if the contestants in the same class are considered identical?

5. **EXTENDED RESPONSE** The graph shows the results of a survey in 2004 that asked U.S. adults which sport they would most like to participate in at the Summer Olympics.



   a. Find the probability that a randomly selected U.S. adult would like to participate in track and field.
   b. Is your answer from part (a) a *theoretical* or *experimental* probability? *Explain.*
   c. What are the odds in favor of a randomly selected U.S. adult preferring to participate in gymnastics?

6. **SHORT RESPONSE** You must take 18 elective courses to meet your graduation requirements for college. There are 30 courses that you are interested in. Does finding the number of possible course selections involve *permutations* or *combinations*? *Explain.* How many different course selections are possible?

7. **OPEN-ENDED** Give an example of a real-life problem for which the answer is the sum of two combinations. Show how to find the answer.

8. **GRIDDED ANSWER** An ice cream shop offers a choice of 31 flavors. How many different ice cream cones can be made with three scoops of ice cream if each scoop is a different flavor and the order of the scoops is not important?

10.4 Find Probabilities Using Venn Diagrams

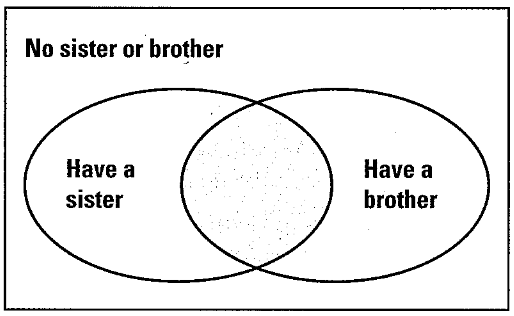
QUESTION **How can you use a Venn diagram to find probabilities involving two events?**

In Lesson 10.3, you learned how to compute the probability of one event. In some situations, however, you might be interested in the probability that two events will occur simultaneously. You also might be interested in the probability that at least one of two events will occur. This activity demonstrates how a Venn diagram is useful for computing such probabilities.

EXPLORE **Use a Venn diagram to collect data**

STEP 1 ***Complete a Venn diagram***

Copy the Venn diagram shown below. Ask the members of your class if they have a sister, have a brother, have both, or have neither. Write their names in the appropriate part of the Venn diagram.



STEP 2 ***Complete a table***

Copy and complete the frequency table. When determining the frequency for a category, be sure to include all the students who are in the category. Note that a student can belong to more than one category.

Category	Number of students
Have a sister	?
Have a brother	?
Have both a sister and brother	?
Do not have a sister or brother	?

DRAW CONCLUSIONS **Use your data to complete these exercises**

1. A student from your class is selected at random. Find the probability of each event. *Explain* how you found your answers.
   - **a.** The student has a sister.
   - **b.** The student has a brother.
   - **c.** The student has a sister and a brother.
   - **d.** The student does not have a sister or a brother.
2. Find the probability that a randomly selected student from your class has either a sister or a brother. *Explain* how you found your answer.
3. How could you calculate the answer to Exercise 2 using your answers from Exercise 1?

10.4 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS18 for Exs. 11, 21, and 45
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 34, 39, 40, 44, and 47

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The union or intersection of two events is called a(n) __?__.

2. ★ **WRITING** Are the events A and $\overline{A}$ disjoint? *Explain.* Then give an example of a real-life event and its complement.

EXAMPLE 1 on p. 707 for Exs. 3–8

DISJOINT EVENTS **Events A and B are disjoint. Find $P(A \text{ or } B)$.**

3. $P(A) = 0.3, P(B) = 0.1$
4. $P(A) = 0.55, P(B) = 0.2$
5. $P(A) = 0.41, P(B) = 0.24$
6. $P(A) = \frac{2}{5}, P(B) = \frac{3}{5}$
7. $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$
8. $P(A) = \frac{2}{3}, P(B) = \frac{1}{5}$

EXAMPLES 2 and 3 on p. 708 for Exs. 9–15

OVERLAPPING EVENTS **Find the indicated probability.**

9. $P(A) = 0.5, P(B) = 0.35$
   $P(A \text{ and } B) = 0.2$
   $P(A \text{ or } B) =$ __?__
10. $P(A) = 0.6, P(B) = 0.2$
    $P(A \text{ or } B) = 0.7$
    $P(A \text{ and } B) =$ __?__
11. $P(A) = 0.28, P(B) = 0.64$
    $P(A \text{ or } B) = 0.71$
    $P(A \text{ and } B) =$ __?__
12. $P(A) = 0.46, P(B) = 0.37$
    $P(A \text{ and } B) = 0.31$
    $P(A \text{ or } B) =$ __?__
13. $P(A) = \frac{2}{7}, P(B) = \frac{4}{7}$
    $P(A \text{ and } B) = \frac{1}{7}$
    $P(A \text{ or } B) =$ __?__
14. $P(A) = \frac{6}{11}, P(B) = \frac{3}{11}$
    $P(A \text{ or } B) = \frac{7}{11}$
    $P(A \text{ and } B) =$ __?__

15. ★ **MULTIPLE CHOICE** What is $P(A \text{ or } B)$ if $P(A) = 0.41$, $P(B) = 0.53$, and $P(A \text{ and } B) = 0.27$?

    Ⓐ 0.12   Ⓑ 0.67   Ⓒ 0.80   Ⓓ 0.94

EXAMPLE 4 on p. 709 for Exs. 16–19

FINDING PROBABILITIES OF COMPLEMENTS **Find $P(\overline{A})$.**

16. $P(A) = 0.5$
17. $P(A) = 0$
18. $P(A) = \frac{1}{3}$
19. $P(A) = \frac{5}{8}$

CHOOSING CARDS **A card is randomly selected from a standard deck of 52 cards. Find the probability of drawing the given card.**

20. A king *and* a diamond
21. A king *or* a diamond
22. A spade *or* a club
23. A 4 *or* a 5
24. A 6 *and* a face card
25. *Not* a heart

ERROR ANALYSIS ***Describe*** **and correct the error in finding the probability of randomly drawing the given card from a standard deck of 52 cards.**

26.
P(heart or face card)
$= P(\text{heart}) + P(\text{face card})$
$= \frac{13}{52} + \frac{12}{52}$
$= \frac{25}{52}$

27.
P(club or 9)
$= P(\text{club}) + P(9) + P(\text{club and } 9)$
$= \frac{13}{52} + \frac{4}{52} + \frac{1}{52}$
$= \frac{9}{26}$

FINDING PROBABILITIES Find the indicated probability. State whether A and B are disjoint events.

28. $P(A) = 0.25$
$P(B) = 0.4$
$P(A \text{ or } B) = 0.50$
$P(A \text{ and } B) = \underline{\ ?\ }$

29. $P(A) = 0.6$
$P(B) = 0.32$
$P(A \text{ or } B) = \underline{\ ?\ }$
$P(A \text{ and } B) = 0.25$

30. $P(A) = \underline{\ ?\ }$
$P(B) = 0.38$
$P(A \text{ or } B) = 0.65$
$P(A \text{ and } B) = 0$

31. $P(A) = \frac{8}{15}$
$P(B) = \underline{\ ?\ }$
$P(A \text{ or } B) = \frac{3}{5}$
$P(A \text{ and } B) = \frac{2}{15}$

32. $P(A) = \frac{1}{2}$
$P(B) = \frac{1}{6}$
$P(A \text{ or } B) = \frac{2}{3}$
$P(A \text{ and } B) = \underline{\ ?\ }$

33. $P(A) = 16\%$
$P(B) = \underline{\ ?\ }$
$P(A \text{ or } B) = 32\%$
$P(A \text{ and } B) = 8\%$

34. ★ **OPEN-ENDED MATH** *Describe* a real-life situation that involves two disjoint events A and B. Then describe a real-life situation that involves two overlapping events C and D.

ROLLING DICE Two six-sided dice are rolled. Find the probability of the given event. (Refer to Example 4 on page 709 for the possible outcomes.)

35. The sum is 3 or 4.

36. The sum is not 7.

37. The sum is greater than or equal to 5.

38. The sum is less than 8 or greater than 11.

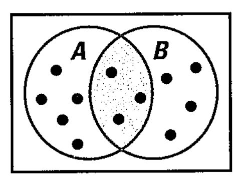
39. ★ **MULTIPLE CHOICE** Two six-sided dice are rolled. What is the probability that the sum is a prime number?

Ⓐ $\frac{13}{36}$ Ⓑ $\frac{7}{18}$ Ⓒ $\frac{5}{12}$ Ⓓ $\frac{5}{11}$

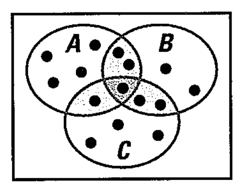
40. ★ **SHORT RESPONSE** Use the first diagram at the right to explain why this equation is true:

$$P(A) + P(B) = P(A \text{ or } B) + P(A \text{ and } B)$$

41. CHALLENGE Use the second diagram at the right to derive a formula for $P(A \text{ or } B \text{ or } C)$.



Ex. 40



Ex. 41

PROBLEM SOLVING

EXAMPLES 1, 2, and 3 on pp. 707–708 for Exs. 42–44

42. CLASS ELECTIONS You and your best friend are among several candidates running for class president. You estimate that there is a 45% chance you will win and a 25% chance your best friend will win. What is the probability that either you or your best friend win the election?

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43. BIOLOGY You are performing an experiment to determine how well plants grow under different light sources. Out of the 30 plants in the experiment, 12 receive visible light, 15 receive ultraviolet light, and 6 receive both visible and ultraviolet light. What is the probability that a plant in the experiment receives either visible light or ultraviolet light?

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EXAMPLES 4 and 5
on p. 709
for Exs. 44–46

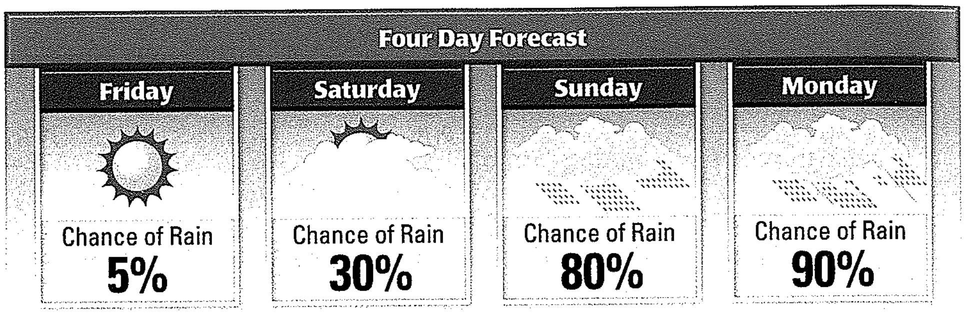
44. ★ **MULTIPLE CHOICE** Refer to the chart below. Which of the following probabilities is greatest?

Ⓐ P(rains on Sunday)

Ⓑ P(does not rain on Saturday)

Ⓒ P(rains on Monday)

Ⓓ P(does not rain on Friday)



(45.) **DRAMA CLUB** The organizer of a cast party for a drama club asks each of 6 cast members to bring one food item from a list of 10 items. What is the probability that at least 2 of the 6 cast members bring the same item?

46. **HOME ELECTRONICS** A development has 6 houses with the same model of garage door opener. Each opener has 4096 possible transmitter codes. What is the probability that at least 2 of the 6 houses have the same code?

47. ★ **EXTENDED RESPONSE** Use the given information about a farmer's tomato crop to complete parts (a)–(c).

a. 40% of the tomatoes are partially rotten, 30% of the tomatoes have been fed on by insects, and 12% are partially rotten *and* have been fed on by insects. What is the probability that a randomly selected tomato is partially rotten *or* has been fed on by insects?

b. 20% of the tomatoes have bite marks from a chipmunk and 7% have bite marks *and* are partially rotten. What is the probability that a randomly selected tomato has bite marks *or* is partially rotten?

c. Suppose the farmer finds out that 6% of the tomatoes have bite marks *and* have been fed on by insects. Do you have enough information to determine the probability that a randomly selected tomato has been fed on by insects *or* is partially rotten *or* has bite marks from a chipmunk? If not, what other information do you require?

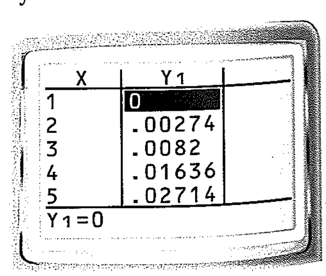
48. **MULTI-STEP PROBLEM** Follow the steps below to explore a famous probability problem called the *birthday problem*. (Assume that there are 365 possible birthdays.)

a. Calculate Suppose that 6 people are chosen at random. Find the probability that at least 2 of the people share the same birthday.

b. Calculate Suppose that 10 people are chosen at random. Find the probability that at least 2 of the people share the same birthday.

c. Model Generalize the results from parts (a) and (b) by writing a formula for the probability $P(x)$ that at least 2 people in a group of x people share the same birthday. (*Hint:* Use ${}_nP_r$ notation in your formula.)

d. Analyze Enter the formula from part (c) into a graphing calculator. Use the *table* feature to make a table of values. For what group size does the probability that at least 2 people share the same birthday first exceed 50%?



○ = **WORKED-OUT SOLUTIONS on p. WS1**

★ = **STANDARDIZED TEST PRACTICE**

49. **PET STORE** A pet store has 8 black Labrador retriever puppies (5 females and 3 males) and 12 yellow Labrador retriever puppies (4 females and 8 males). You randomly choose one of the Labrador retriever puppies. What is the probability that it is a female or a yellow Labrador retriever?

50. **CHALLENGE** You own 50 DVDs consisting of 25 comedies, 15 dramas, and 10 thrillers. You randomly pick 4 movies to watch during a long train ride. What is the probability that you pick at least one DVD of each type of movie?

MIXED REVIEW

Use the given values to write an equation relating x and y. Then find the value of y when $x = 8$.

51. x, y vary directly; $x = -5, y = 20$ *(p. 107)*

52. x, y vary directly; $x = 54, y = -9$ *(p. 107)*

53. x, y vary inversely; $x = 12, y = -4$ *(p. 551)*

54. x, y vary inversely; $x = -2, y = -3$ *(p. 551)*

Find the inverse of the function. *(p. 437)*

55. $f(x) = 3x - 7$

56. $f(x) = -5x + 3$

57. $f(x) = -6x^2, x \leq 0$

58. $f(x) = -2.5x^5$

59. $f(x) = 4x^2 - 12, x \geq 0$

60. $f(x) = 0.2x^3 + 0.5$

PREVIEW
Prepare for Lesson 10.5 in Exs. 61–64.

Each event can occur in the given number of ways. Find the number of ways all of the events can occur. *(p. 682)*

61. Event A: 2 ways, Event B: 4 ways

62. Event A: 13 ways, Event B: 7 ways

63. Event A: 3 ways, Event B: 5 ways, Event C: 6 ways

64. Event A: 12 ways, Event B: 11 ways, Event C: 8 ways, Event D: 10 ways

QUIZ *for Lessons 10.3–10.4*

A card is randomly drawn from a standard deck of 52 cards. Find the probability of drawing the given card. *(p. 698)*

1. The queen of hearts
2. An ace
3. A diamond
4. A red card
5. A card other than a 10
6. The 6 of clubs

You randomly select a marble from a bag. The bag contains 8 black, 13 red, 7 white, and 12 blue marbles. Find the indicated odds. *(p. 698)*

7. In favor of choosing blue
8. In favor of choosing black or white
9. Against choosing red
10. Against choosing red or white

Find the indicated probability. *(p. 707)*

11. $P(A) = 0.6$
$P(B) = 0.35$
$P(A \text{ or } B) = \underline{?}$
$P(A \text{ and } B) = 0.2$

12. $P(A) = \underline{?}$
$P(B) = 0.44$
$P(A \text{ or } B) = 0.56$
$P(A \text{ and } B) = 0.12$

13. $P(A) = 0.75$
$P(B) = \underline{?}$
$P(A \text{ or } B) = 0.83$
$P(A \text{ and } B) = 0.25$

14. $P(A) = 8\%$
$P(B) = 33\%$
$P(A \text{ or } B) = 41\%$
$P(A \text{ and } B) = \underline{?}$

15. **COMPUTERS** A manufacturer of computer chips finds that 1% of the chips produced are defective. What is the probability that out of 8 chips, at least 2 are defective? *(p. 707)*

PROBLEM SOLVING WORKSHOP

LESSON 10.4

Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 709

MULTIPLE REPRESENTATIONS In Example 4 on page 709, you found theoretical probabilities involving the sum of two dice. You can also perform a *simulation* to estimate these probabilities.

PROBLEM

DICE When two six-sided dice are rolled, there are 36 possible outcomes. Find the probability of the given event.

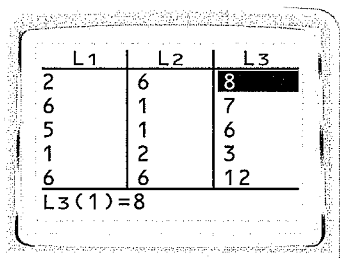
a. The sum is not 6.

b. The sum is less than or equal to 9.

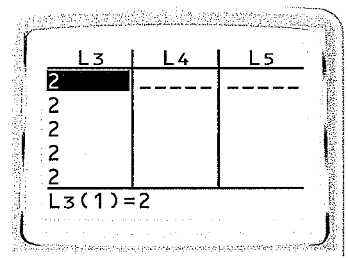
METHOD

Using a Simulation An alternative approach is to use the random number feature of a graphing calculator to simulate rolling two dice. You can then use the results of the simulation to find the experimental probabilities for the problem.

STEP 1 **Generate** two lists of 120 random integers from 1 to 6 by entering randInt(1,6,120) into lists L_1 and L_2. Define list L_3 to be the sum of lists L_1 and L_2.



STEP 2 **Sort** the sums in list L_3 in ascending order using the command SortA(L_3). Scroll through the list and count the frequency of each sum.



STEP 3 **Find** the probabilities.

a. Divide the number of times the sum was 6 by the total number of simulated rolls, then subtract the result from 1.

b. Divide the number of times the sum was greater than 9 by the total number of simulated rolls, then subtract the result from 1.

PRACTICE

1. **WRITING** *Compare* the probabilities found in the simulation above with the theoretical probabilities found in Example 4 on page 709.

2. **SIMULATIONS** Use the results of the simulation above to find the experimental probability that the sum is greater than or equal to 4. *Compare* this to the theoretical probability of the event.

3. **SIMULATIONS** Use the results of the simulation above to find the experimental probability that the sum is not 8 or 9. *Compare* this to the theoretical probability of the event.

4. **REASONING** How could you change the simulation above so that the results would be closer to the theoretical probabilities of the events? *Explain.*

Extension: Apply Set Theory

Use after Lesson 10.4

GOAL Define the concepts of sets, operations on sets, and subsets.

Key Vocabulary
- set
- union
- intersection
- complement
- subset

A **set** is a collection of distinct objects. Each object in a set is called an **element** or **member** of the set. A set is denoted by enclosing its elements in braces. For example, if A is the set of positive integers less than 5, then $A = \{1, 2, 3, 4\}$.

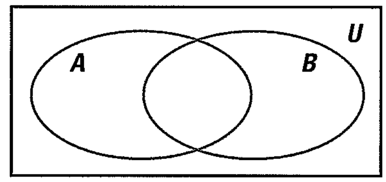
There are two special sets that are often used. The set with no elements is called the **empty set** and is denoted by $\emptyset$. The set of all elements under consideration is called the **universal set** and is denoted by U.

KEY CONCEPT *For Your Notebook*

Operations on Sets

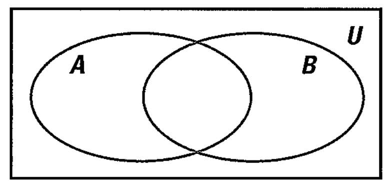
The **union** of two sets A and B is written as $A \cup B$ and is the set of all elements in *either* A or B.

$A \cup B$



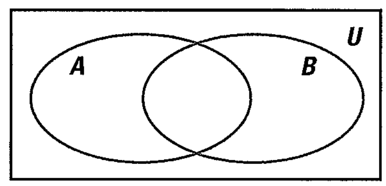
The **intersection** of two sets A and B is written as $A \cap B$ and is the set of all elements in *both* A and B.

$A \cap B$



The **complement** of a set A is written as $\overline{A}$ and is the set of all elements in the universal set U that are *not* in A.

$\overline{A}$



EXAMPLE 1 Perform operations on sets

Let U be the set of all integers from 1 to 10. Let $A = \{1, 2, 4, 8\}$ and let $B = \{2, 4, 6, 8, 10\}$. Find the indicated set.

a. $A \cup B$ **b.** $A \cap B$ **c.** $\overline{A}$ **d.** $\overline{A \cup B}$

Solution

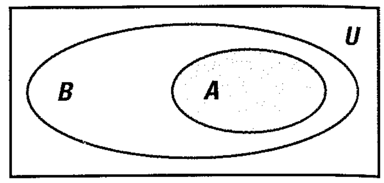
a. $A \cup B = \{1, 2, 4, 8\} \cup \{2, 4, 6, 8, 10\} = \{1, 2, 4, 6, 8, 10\}$

b. $A \cap B = \{1, 2, 4, 8\} \cap \{2, 4, 6, 8, 10\} = \{2, 4, 8\}$

c. $\overline{A} = \overline{\{1, 2, 4, 8\}} = \{3, 5, 6, 7, 9, 10\}$

d. $\overline{A \cup B} = \overline{\{1, 2, 4, 8\} \cup \{2, 4, 6, 8, 10\}} = \overline{\{1, 2, 4, 6, 8, 10\}} = \{3, 5, 7, 9\}$

SUBSETS If every element of a set A is also an element of a set B, then A is a **subset** of B. This relationship is written as $A \subseteq B$. For any set A, $\emptyset \subseteq A$ and $A \subseteq A$. In the diagram at the right, A is a subset of B.



EXAMPLE 2 Identify subsets

Let $A = \{-2, 1, \sqrt{3}, \pi\}$, $B = \{1, \pi, 5\}$, and $C = \{-2, 1, 3, \pi, 5\}$.

a. Is $B \subseteq A$? **b.** Is $B \subseteq C$? **c.** Is $C \subseteq (A \cup B)$?

Solution

a. Not every element of B is an element of A, because 5 is not an element of A. So, B is *not* a subset of A.

b. Every element of B is an element of C. So, B *is* a subset of C.

c. Note that $A \cup B = \{-2, 1, \sqrt{3}, \pi\} \cup \{1, \pi, 5\} = \{-2, 1, \sqrt{3}, \pi, 5\}$. Not every element of C is an element of $A \cup B$, because 3 is not an element of $A \cup B$. So, C is *not* a subset of $A \cup B$.

PRACTICE

EXAMPLE 1 on p. 715 for Exs. 1–8

OPERATIONS ON SETS Let U be the set of all whole numbers from 1 to 20. Let $A = \{2, 3, 5, 7, 11, 13, 17\}$, $B = \{1, 4, 9, 16\}$, and $C = \{2, 5, 8, 11, 14, 17, 20\}$. Find the indicated set.

1. $A \cup B$
2. $A \cap B$
3. $\overline{A}$
4. $\overline{B}$
5. $A \cup B \cup C$
6. $\overline{A} \cap C$
7. $\overline{C \cup B}$
8. $B \cup (A \cap C)$

EXAMPLE 2 on p. 716 for Exs. 9–12

SUBSETS Let $A = \{-5, \pi, 10\}$, $B = \{-5, 1, \sqrt{5}, 10\}$, and $C = \{-5, 2, \pi, 10\}$.

9. Is $A \subseteq B$?
10. Is $A \subseteq C$?
11. Is $(A \cap B) \subseteq C$?

12. **REASONING** List all the subsets of the set $A = \{-2, 4, 9\}$.

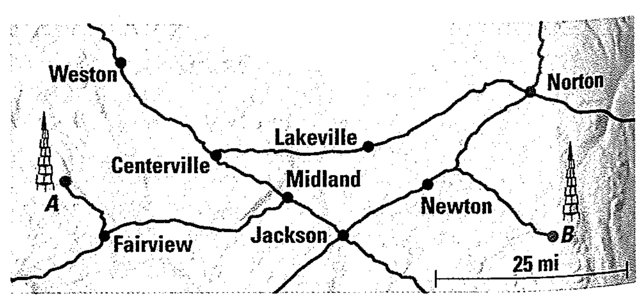
OPERATIONS ON SETS Consider the sets defined below. Find the indicated set.

U = the set of all 12 months
X = the set of all 30 day months
Y = the set of all 31 day months
Z = the set of all months ending with "r"

13. $X \cup Z$
14. $X \cap Y$
15. $\overline{Z}$
16. $\overline{X \cup Y}$

17. **REASONING** Is the set of all irrational numbers a subset of the real numbers? of the integers? *Explain.*

18. **RADIO** Two radio towers are set up at points A and B on the map at the right. Each radio tower has a signal that can reach towns up to 50 miles away. Find the set of all towns that can receive a signal from both of the towers.



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✓ **GUIDED PRACTICE** for Examples 6 and 7

7. **WHAT IF?** In Example 6, what is the probability that you and your friends choose different costumes if the store sells 20 different costumes?

8. **BASKETBALL** A high school basketball team leads at halftime in 60% of the games in a season. The team wins 80% of the time when they have the halftime lead, but only 10% of the time when they do not. What is the probability that the team wins a particular game during the season?

10.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS18 for Exs. 13, 25, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 32, 34, and 41

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The probability that B will occur given that A has occurred is called the _?_ of B given A.

2. ★ **WRITING** *Explain* the difference between dependent events and independent events, and give an example of each.

EXAMPLES 1 and 2 on pp. 717–718 for Exs. 3–15

INDEPENDENT EVENTS **Events A and B are independent. Find the indicated probability.**

3. $P(A) = 0.4$, $P(B) = 0.6$, $P(A \text{ and } B) =$ _?_

4. $P(A) = 0.3$, $P(B) = 0.4$, $P(A \text{ and } B) =$ _?_

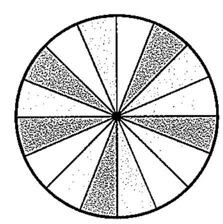
5. $P(A) = 0.25$, $P(B) =$ _?_, $P(A \text{ and } B) = 0.2$

6. $P(A) = 0.5$, $P(B) =$ _?_, $P(A \text{ and } B) = 0.1$

7. $P(A) =$ _?_, $P(B) = 0.8$, $P(A \text{ and } B) = 0.6$

8. $P(A) =$ _?_, $P(B) = 0.9$, $P(A \text{ and } B) = 0.45$

SPINNING A WHEEL **You are playing a game that involves spinning the wheel shown. Find the probability of spinning the given colors.**



9. green, then blue

10. red, then yellow

11. blue, then red

12. yellow, then green

13. blue, then green, then red

14. green, then red, then yellow

15. ★ **MULTIPLE CHOICE** Events A and B are independent. What is $P(A \text{ and } B)$ if $P(A) = 0.3$ and $P(B) = 0.2$?

(A) 0.06 (B) 0.1 (C) 0.5 (D) 0.6

EXAMPLE 4 on p. 719 for Exs. 16–25

DEPENDENT EVENTS **Events A and B are dependent. Find the indicated probability.**

16. $P(A) = 0.3$, $P(B|A) = 0.6$, $P(A \text{ and } B) =$ _?_

17. $P(A) = 0.7$, $P(B|A) = 0.5$, $P(A \text{ and } B) =$ _?_

18. $P(A) = 0.8$, $P(B|A) =$ _?_, $P(A \text{ and } B) = 0.32$

19. $P(A) = 0.6$, $P(B|A) =$ _?_, $P(A \text{ and } B) = 0.45$

20. $P(A) =$ _?_, $P(B|A) = 0.4$, $P(A \text{ and } B) = 0.2$

21. $P(A) = 0.7$, $P(B|A) =$ _?_, $P(A \text{ and } B) = 0.63$

CONDITIONAL PROBABILITY **Let n be a randomly selected integer from 1 to 20. Find the indicated probability.**

22. n is 2 given that it is even

23. n is 5 given that it is less than 8

24. n is prime given that it has 2 digits

25. n is odd given that it is prime

EXAMPLES 5 and 6
on pp. 719–720
for Exs. 26–32

DRAWING CARDS **Find the probability of drawing the given cards from a standard deck of 52 cards (a) with replacement and (b) without replacement.**

26. A club, then a spade

27. A queen, then an ace

28. A face card, then a 6

29. A 10, then a 2

30. A king, then a queen, then a jack

31. A spade, then a club, then another spade

32. ★ **MULTIPLE CHOICE** What is the approximate probability of drawing 3 consecutive hearts from a standard deck of 52 cards without replacement?

(A) 0.0122 (B) 0.0129 (C) 0.0156 (D) 0.0166

33. **ERROR ANALYSIS** Events A and B are independent. *Describe* and correct the error in finding $P(A \text{ and } B)$.

$P(A) = 0.4, P(B) = 0.5$

$P(A \text{ and } B) = 0.4 + 0.5 = 0.9$ ✗

34. ★ **OPEN-ENDED MATH** Flip a set of 3 coins and record the number of coins that come up heads. Repeat until you have a total of 10 trials.

a. What is the experimental probability that a trial results in 2 heads?

b. *Compare* your answer from part (a) with the theoretical probability that a trial results in 2 heads.

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35. **REASONING** Let A and B be independent events. What is the relationship between $P(B)$ and $P(B|A)$? *Explain.*

36. **CHALLENGE** How many times must you roll two six-sided dice for there to be at least a 50% chance that you roll two 6's at least once?

PROBLEM SOLVING

EXAMPLES 3 and 4
on pp. 718–719
for Exs. 37–38

37. **SCHOOL BUS** Angela usually rushes to make it to the bus stop in time to catch the school bus, and will often miss the bus if it is early. The bus comes early to Angela's stop 28% of the time. What is the probability that the bus will come early at least once during a 5 day school week?

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38. **ENVIRONMENT** The table shows the numbers of species in the United States listed as endangered or threatened as of September, 2004. Find **(a)** the probability that a listed animal is a bird and **(b)** the probability that an endangered animal is a bird.

	Endangered	Threatened
Mammals	69	9
Birds	77	14
Reptiles	14	22
Amphibians	11	10
Other	219	74

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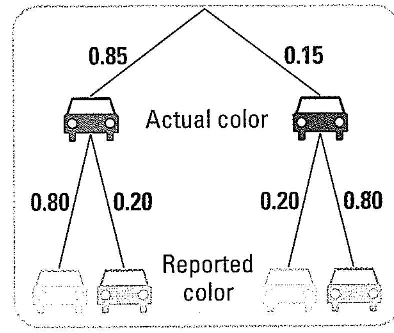
○ = **WORKED-OUT SOLUTIONS on p. WS1**   ★ = **STANDARDIZED TEST PRACTICE**

EXAMPLE 7
on p. 720
for Exs. 39–40

39. **TENNIS** A tennis player wins a match 55% of the time when she serves first and 47% of the time when her opponent serves first. The player who serves first is determined by a coin toss before the match. What is the probability that the player wins a given match?

40. **ACCIDENT REENACTMENT** You are a juror for a trial involving a nighttime car accident in a certain city. Use the tree diagram and the facts below to determine the probability that the car involved in the accident was blue.



- The make of the car is known. Of the cars in the city matching this make, 85% are green and 15% are blue.
- A witness of the accident identified the car as blue.
- In reenactments of the accident, the witness correctly reported the color of the car 80% of the time.

41. ★ **EXTENDED RESPONSE** A football team is losing by 14 points near the end of a game. The team scores two touchdowns (worth 6 points each) before the end of the game. After each touchdown, the coach must decide whether to go for 1 point with a kick (which is successful 99% of the time) or 2 points with a run or pass (which is successful 45% of the time).

   a. **Calculate** If the team goes for 1 point after each touchdown, what is the probability that the coach's team wins? loses? ties?

   b. **Calculate** If the team goes for 2 points after each touchdown, what is the probability that the coach's team wins? loses? ties?

   c. **Reasoning** Can you develop a strategy so that the coach's team has a probability of winning the game that is greater than the probability of losing? If so, explain your strategy and calculate the probabilities of winning and losing using your strategy.

42. **CHALLENGE** It is estimated that 5.9% of Americans have diabetes. Suppose a medical lab uses a test for diabetes that is 98% accurate for people who have the disease and 95% accurate for people who do not have it. Find the conditional probability that a randomly selected person actually has diabetes given that the lab test says they have it.

MIXED REVIEW

PREVIEW
Prepare for Lesson 10.6 in Exs. 43–48.

Use the binomial theorem to write the binomial expansion. *(p. 337)*

43. $(x + 1)^6$
44. $(x - 3)^5$
45. $(3x + 2)^7$
46. $(5x - 1)^5$
47. $(4x + y)^6$
48. $(2x - 3y)^4$

Let $f(x) = x^2 + 2$ and $g(x) = x - 4$. Perform the indicated operation and state the domain. *(p. 428)*

49. $f(x) + g(x)$
50. $f(x) - g(x)$
51. $f(x) \cdot g(x)$
52. $\frac{f(x)}{g(x)}$
53. $f(g(x))$
54. $g(f(x))$
55. $f(f(x))$
56. $g(g(x))$

Solve the equation.

57. $4^{x+1} = 8^{3x}$ *(p. 515)*
58. $4 \ln x = 10$ *(p. 515)*
59. $\frac{2}{x-3} - \frac{1}{x+2} = \frac{x-5}{x+2}$ *(p. 589)*
60. $\frac{x}{x-2} + \frac{1}{x+1} = \frac{2x+1}{x+1}$ *(p. 589)*

CLASSIFY DISTRIBUTIONS
Note that the distribution in Example 1 on p. 724 is symmetric, while the distribution in Example 3 on p. 726 is skewed.

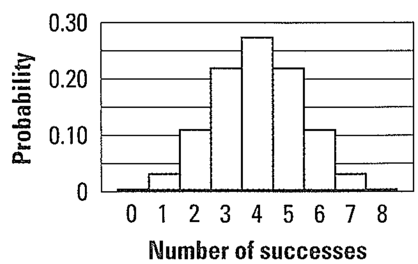
SYMMETRIC AND SKEWED DISTRIBUTIONS Suppose a probability distribution is represented by a histogram. The distribution is **symmetric** if you can draw a vertical line that divides the histogram into two parts that are mirror images. A distribution that is *not* symmetric is called **skewed.**

EXAMPLE 5 Classify distributions as symmetric or skewed

Describe the shape of the binomial distribution that shows the probability of exactly k successes in 8 trials if (a) $p = 0.5$ and (b) $p = 0.9$.

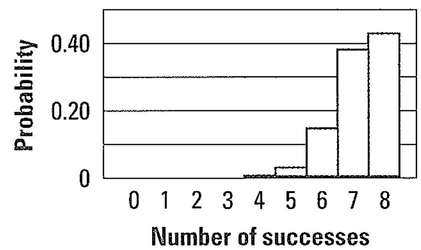
Solution

a.



Symmetric; the left half is a mirror image of the right half.

b.



Skewed; the distribution is not symmetric about any vertical line.

✓ **GUIDED PRACTICE** for Example 5

5. A binomial experiment consists of 5 trials with probability p of success on each trial. Describe the shape of the binomial distribution that shows the probability of exactly k successes if (**a**) $p = 0.4$ and (**b**) $p = 0.5$.

10.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS19 for Exs. 5, 21, and 45

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 32, 39, and 48

◆ = **MULTIPLE REPRESENTATIONS** Ex. 47

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A probability distribution represented by a histogram is __?__ if you can draw a vertical line dividing the histogram into two parts that are mirror images.

2. ★ **WRITING** *Explain* the difference between a binomial experiment and a binomial distribution.

EXAMPLE 1 on p. 724 for Exs. 3–5

CONSTRUCTING PROBABILITY DISTRIBUTIONS Make a table and a histogram showing the probability distribution for the random variable.

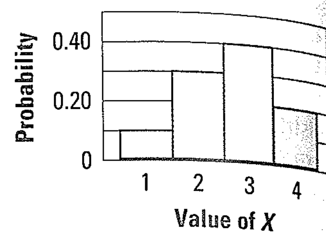
3. X = the number on a table tennis ball randomly chosen from a bag that contains 5 balls labeled "1," 3 balls labeled "2," and 2 balls labeled "3."

4. W = 1 if a randomly chosen letter is A, E, I, O, or U and 2 otherwise.

5. N = the number of digits in a random integer from 0 through 999.

EXAMPLE 2
on p. 725
for Exs. 6–9

INTERPRETING PROBABILITY DISTRIBUTIONS **In Exercises 6–9, use the given histogram of a probability distribution for a random variable X.**



6. What is the probability that X is equal to 1?

7. What is the most likely value for X?

8. What is the probability that X is odd?

9. ★ **MULTIPLE CHOICE** What is the probability that X is at least 3?

Ⓐ 0.2   Ⓑ 0.4   Ⓒ 0.6   Ⓓ 0.8

EXAMPLES 3 and 4
on p. 726
for Exs. 10–32

CALCULATING PROBABILITIES **Calculate the probability of tossing a coin 20 times and getting the given number of heads.**

10. 1   11. 2   12. 4   13. 6

14. 9   15. 12   16. 15   17. 18

BINOMIAL PROBABILITIES **Calculate the probability of randomly guessing the given number of correct answers on a 30-question multiple choice exam that has choices A, B, C, and D for each question.**

18. 0   19. 2   20. 6   (21.) 11

22. 15   23. 21   24. 26   25. 30

ERROR ANALYSIS ***Describe*** **and correct the error in calculating the probability of rolling a 1 exactly 3 times in 5 rolls of a six-sided die.**

26.
$$P(k = 3) = {}_5C_3\left(\frac{1}{6}\right)^{5-3}\left(\frac{5}{6}\right)^3 \approx 0.161$$ ✗

27.
$$P(k = 3) = \left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{5-3} \approx 0.003$$ ✗

BINOMIAL DISTRIBUTIONS **Calculate the probability of k successes for a binomial experiment consisting of n trials with probability p of success on each trial.**

28. $k \le 3, n = 7, p = 0.3$ 29. $k \ge 5, n = 8, p = 0.6$

30. $k \le 2, n = 5, p = 0.12$ 31. $k \ge 10, n = 15, p = 0.75$

32. ★ **MULTIPLE CHOICE** You perform a binomial experiment consisting of 10 trials with a probability of success of 36% on each trial. What is the most likely number of successes?

Ⓐ 3   Ⓑ 4   Ⓒ 6   Ⓓ 7

EXAMPLE 5
on p. 727
for Exs. 33–38

HISTOGRAMS **A binomial experiment consists of n trials with probability p of success on each trial. Draw a histogram of the binomial distribution that shows the probability of exactly k successes. *Describe* the distribution as either *symmetric* or *skewed*. Then find the most likely number of successes.**

33. $n = 3, p = 0.3$ 34. $n = 6, p = 0.5$ 35. $n = 4, p = 0.16$

36. $n = 7, p = 0.85$ 37. $n = 8, p = 0.025$ 38. $n = 12, p = 0.5$

39. ★ **OPEN-ENDED MATH** Construct a symmetric probability distribution for a random variable X and a skewed probability distribution for a random variable Y. Make a table and a histogram for each distribution.

In Exercises 40–42, you will derive the binomial probability formula on page 725. Consider a binomial experiment with n trials and probability p of success on each trial.

40. For any particular sequence of k successes and $n - k$ failures, what is the probability that the sequence occurs? *Explain.*

41. How many sequences of k successes and $n - k$ failures are there? *Explain.*

42. **CHALLENGE** Use your results from Exercises 40 and 41 to justify the binomial probability formula.

PROBLEM SOLVING

EXAMPLES 3 and 4 on p. 726 for Exs. 43–46

43. **HEALTH** About 1% of people are allergic to bee stings. What is the probability that exactly 1 person in a class of 25 is allergic to bee stings?

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44. **BASKETBALL** Predrag Stojakovic of the Sacramento Kings made 92.7% of his free throw attempts in the 2003–2004 NBA regular season. What is the probability that he will make exactly 10 of his next 15 free throw attempts?

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45. **BLOOD TYPE** The chart shows the distribution of blood types (O, A, B, AB) and Rh factor ($^+$ or $^-$) for human blood. If, at random, 10 people donate blood to a blood bank during a certain hour, find the probability of each event.

Percent of Population by Blood Type							
O^+	O^-	A^+	A^-	B^+	B^-	AB^+	AB^-
37%	6%	34%	6%	10%	2%	4%	1%

a. Exactly 5 of the people are type A^+.

b. Exactly 2 of the people are Rh^-.

c. At most 2 of the people are type O.

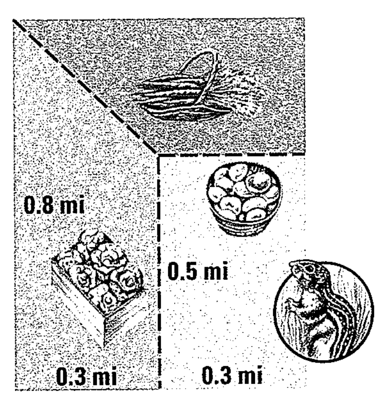
d. At least 5 of the people are Rh^+.

46. **FINE ARTS** A survey states that 35% of people in the United States visited an art museum in a certain year. You randomly select 10 U.S. citizens.

a. Draw a histogram showing the binomial distribution of the number of people who visited an art museum.

b. What is the probability that at most 4 people visited an art museum?

47. **MULTIPLE REPRESENTATIONS** An average of 7 gopher holes appear on the farm shown each week. Let X represent how many of the 7 gopher holes appear in the carrot patch. Assume that a gopher hole has an equal chance of appearing at any point on the farm.



a. **Calculating Probabilities** Find $P(X)$ for $X = 0, 1, 2, \ldots, 7$.

b. **Making a Table** Make a table showing the probability distribution for X.

c. **Making a Histogram** Make a histogram showing the probability distribution for X.

48. ★ **EXTENDED RESPONSE** Assume that having a male child and having a female child are independent events and that the probability of each is 0.5.

a. A couple has 4 male children. Evaluate the validity of this statement: "The first 4 kids were all boys, so the next one will probably be a girl."

b. What is the probability of having 4 male children and then a female child?

c. Let X be a random variable that represents the number of children a couple already has when they have their first female child. Draw a histogram of the distribution of $P(X)$ for $0 \le X \le 10$ and describe its shape.

49. **CHALLENGE** An entertainment system has n speakers. Each speaker will function properly with probability p, independent of whether the other speakers are functioning. The system will operate effectively if at least 50% of its speakers are functioning. For what values of p is a 5-speaker system more likely to operate than a 3-speaker system?

MIXED REVIEW

PREVIEW
Prepare for Lesson 11.1 in Exs. 50–55.

Evaluate the expression. *(p. 2)*

50. $8 + 24 \div 4$

51. $4 \cdot 3 + 28 \div 7$

52. $35 - 3 \cdot 2 \div 8$

53. $6 - (15 \cdot 2)^2 \div 9$

54. $2 + 48 \div 6 \cdot 4 - 5$

55. $14 - 9 \div 3 + 40 \div 8$

Solve the inequality algebraically. Then graph the solution.

56. $5 - 2x \le 12$ *(p. 41)*

57. $1 < 4x - 3 < 7$ *(p. 41)*

58. $-2 < 3x - 5 \le 4$ *(p. 41)*

59. $6x^2 \ge 36$ *(p. 300)*

60. $3x^2 + 11x - 4 < 0$ *(p. 300)*

61. $3x^2 + 9x < x^2 + 4$ *(p. 300)*

Find all zeros of the polynomial function. *(p. 379)*

62. $f(x) = x^3 - 4x^2 - 7x + 10$

63. $g(x) = 3x^3 - 3x^2 + 75x - 75$

64. $h(x) = x^4 - x^3 - 5x^2 - x - 6$

65. $f(x) = 2x^4 + 5x^3 + 29x^2 + 80x - 48$

QUIZ *for Lessons 10.5–10.6*

Find the probability of randomly drawing the given marbles from a bag of 6 red, 9 green, and 5 blue marbles without replacement. *(p. 717)*

1. red, then green

2. blue, then red

3. green, then green

Calculate the probability of getting the given number of 6's when rolling a six-sided die 10 times. *(p. 724)*

4. 0

5. 1

6. 4

7. 8

A binomial experiment consists of n trials with probability p of success on each trial. Draw a histogram of the binomial distribution that shows the probability of exactly k successes. *(p. 724)*

8. $n = 5, p = 0.2$

9. $n = 8, p = 0.5$

10. $n = 6, p = 0.72$

11. **MENU CHOICES** You and 4 friends are in line at lunch and are each selecting a beverage. There are 5 types of beverages available. What is the probability that all of you will select different beverages? *(p. 717)*

Graphing Calculator ACTIVITY *Use after Lesson 10.6*

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Keystrokes

10.6 Create a Binomial Distribution

QUESTION **How can you use a graphing calculator to calculate binomial probabilities?**

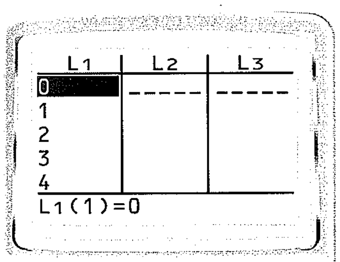
Some calculators have a binomial probability distribution function that you can use to calculate binomial probabilities. You can then use the calculator to draw a histogram of the distribution.

EXAMPLE **Calculate binomial probabilities**

TV NEWS According to a survey, 38% of U.S. adults get their news primarily from television. Suppose you survey 6 adults at random. Draw a histogram of the binomial distribution showing the probability that television is the primary news source for exactly k adults. What is the most likely number of adults in your survey who get their news primarily through television?

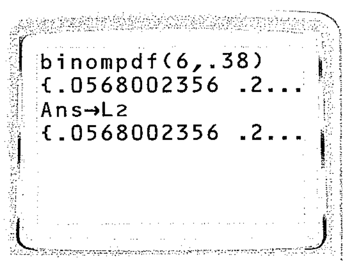
STEP 1 ***Enter values of k***

Let $p = 0.38$ be the probability that television is a person's primary news source. Enter the k-values 0 through 6 into list L_1 on the graphing calculator.



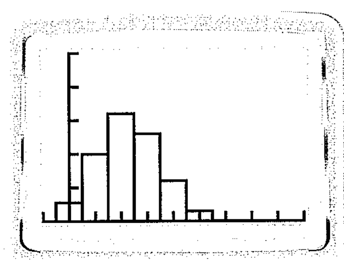
STEP 2 ***Find values of P(k)***

Enter the binomial probability command to generate $P(k)$ for all seven k-values. Store the results in list L_2.



STEP 3 ***Draw histogram***

Set up the histogram to use the numbers in list L_1 as x-values and the numbers in list L_2 as frequencies. Draw the histogram in a suitable viewing window.



From the histogram in Step 3, you can see that $k = 2$ is the most likely number of the 6 adults surveyed who get their news primarily through television.

PRACTICE

A binomial experiment consists of n trials with probability p of success on each trial. Use a graphing calculator to draw a histogram of the binomial distribution that shows the probability of exactly k successes. Then find the most likely number of successes.

1. $n = 12, p = 0.29$
2. $n = 14, p = 0.58$
3. $n = 15, p = 0.805$
4. **WHAT IF?** In the example, how do your histogram and the most likely number of adults change if you survey 14 adults at random?

Lessons 10.4–10.6

1. **MULTI-STEP PROBLEM** You and a friend are playing a word game that involves lettered tiles. The distribution of letters is shown below.

A	9	H	2	O	8	V	2
B	2	I	9	P	2	W	2
C	2	J	1	Q	1	X	1
D	4	K	1	R	6	Y	2
E	12	L	4	S	4	Z	1
F	2	M	2	T	6	Blank	2
G	3	N	6	U	4		

   a. You randomly draw 1 tile. What is the probability of getting a vowel? (Assume that Y is a consonant.)

   b. You randomly draw 2 tiles without replacement. What is the probability of getting 2 vowels?

   c. At the start of the game, you randomly choose 7 tiles without replacement. What is the probability that all of the tiles are vowels?

2. **MULTI-STEP PROBLEM** According to a survey, 62% of U.S. adults consider themselves sports fans. You randomly select 14 adults to survey.

   a. Draw a histogram of the binomial distribution showing the probability that k adults consider themselves sports fans.

   b. What is the most likely number of adults who consider themselves sports fans?

   c. What is the probability that at least 7 adults consider themselves sports fans?

3. **SHORT RESPONSE** A manufacturer makes briefcases with numbered locks. The locks can be set so that any one of 1000 different codes will open the briefcase. Four friends have briefcases from this manufacturer. What is the probability that at least 2 of the 4 briefcases have the same code? If two more friends buy the same briefcase, how does the probability that at least 2 of the briefcases have the same code change?

4. **OPEN-ENDED** Write a real-life problem that you can solve using a tree diagram and conditional probabilities. Draw the tree diagram and show how to solve the problem.

5. **GRIDDED ANSWER** A softball player gets a hit in about 31% of her at-bats. You randomly select 15 of the player's at-bats. What is the most likely number of hits the player will have in those at bats?

6. **EXTENDED RESPONSE** The owner of a lawn mowing business owns three old and unreliable riding mowers. As long as one of the mowers is working, the owner can stay productive. From past experience, one of the mowers is unusable 10% of the time, one is unusable 8% of the time, and one is unusable 18% of the time.

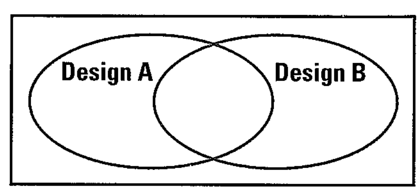
   a. Find the probability that all three mowers are unusable on a given day.

   b. Find the probability that at least one of the mowers is usable on a given day.

   c. Suppose the least reliable mower stops working completely. How does this affect the probability that the lawn mowing business can be productive on a given day?

7. **EXTENDED RESPONSE** A computer software company is performing a market test on two designs, A and B, for its new software program. Out of 250 people who view the designs, 85 like design A, 135 like design B, and 45 like both designs.

   a. Copy and complete the Venn diagram.



   b. What is the probability that a person likes design A or design B?

   c. What is the probability that a person does not like either design?

   d. *Explain* how you can calculate the probability from part (c) if you know the probability from part (b).

10 CHAPTER SUMMARY

BIG IDEAS — For Your Notebook

Big Idea 1

Using Permutations and Combinations

PERMUTATIONS **Order is important**	Permutations of n distinct objects	$n!$	Number of ways to arrange 10 students at 10 desks: $10! = 3{,}628{,}800$
	Permutations of n distinct objects taken r at a time	${}_nP_r = \dfrac{n!}{(n-r)!}$	Number of ways to arrange 8 students at 10 desks: $\dfrac{10!}{2!} = 1{,}814{,}400$
	Permutations of n objects where one object is repeated s_1 times, another is repeated s_2 times, and so on	$\dfrac{n!}{s_1! \cdot s_2! \cdot \ldots \cdot s_k!}$	Number of distinguishable permutations of the letters in STUDENTS: $\dfrac{8!}{2! \cdot 2!} = 10{,}080$
COMBINATIONS **Order is not important**	Combinations of r objects taken from a group of n distinct objects	${}_nC_r = \dfrac{n!}{(n-r)! \cdot r!}$	Number of ways to choose 8 students from a set of 10 students: $\dfrac{10!}{2! \cdot 8!} = 45$

Big Idea 2

Finding Probabilities

The following table shows which formula to use when finding probabilities involving two events A and B.

Overlapping Events	Independent Events	Dependent Events
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$	$P(A \text{ and } B) = P(A) \cdot P(B)$	$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

Big Idea 3

Constructing Binomial Distributions

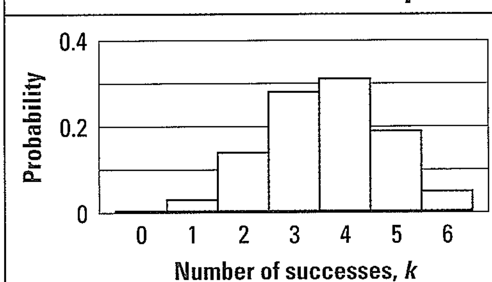
For a binomial experiment, the probability of exactly k successes in n trials is

$$P(k \text{ successes}) = {}_nC_k p^k (1-p)^{n-k}$$

where the probability of success on each trial is p.

A binomial distribution shows the probabilities of all possible outcomes in a binomial experiment. The distribution is skewed if $p \neq 0.5$.

Binomial Distribution for $n = 6, p = 0.6$



10 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- permutation, *p. 684*
- factorial, *p. 684*
- combination, *p. 690*
- Pascal's triangle, *p. 692*
- binomial theorem, *p. 693*
- probability, *p. 698*
- theoretical probability, *p. 698*
- odds, *p. 699*
- experimental probability, *p. 700*
- geometric probability, *p. 701*
- compound event, *p. 707*
- overlapping events, *p. 707*
- disjoint or mutually exclusive events, *p. 707*
- independent events, *p. 717*
- dependent events, *p. 718*
- conditional probability, *p. 718*
- random variable, *p. 724*
- probability distribution, *p. 724*
- binomial distribution, *p. 725*
- binomial experiment, *p. 725*
- symmetric distribution, *p. 727*
- skewed distribution, *p. 727*

VOCABULARY EXERCISES

1. Copy and complete: A(n) __?__ is a selection of r objects from a group of n objects where the order of the objects selected is not important.

2. **WRITING** *Explain* the difference between the probability of an event and the odds in favor of the event.

3. **WRITING** You randomly select 10 cards, one by one, from a standard deck of 52 cards without replacement. You record the number of diamonds you get. Is this a binomial experiment? *Explain.*

4. **WRITING** Let event A be randomly selecting a green marble from a bag that contains red, green, and blue marbles. Let event B be randomly selecting a marble that is not red from the same bag. Are events A and B disjoint events? *Explain.*

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 10.

10.1 Apply the Counting Principle and Permutations *pp. 682–689*

EXAMPLE

An ice skating competition features 8 skaters. How many different ways can the skaters finish the competition? How many different ways can 3 of the skaters finish first, second, and third?

There are 8! ways the skaters can finish the competition.

$$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$$

There are ${}_8P_3$ ways that 3 of the skaters can finish first, second, and third.

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336$$

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Chapter Review Practice

EXAMPLES 4 and 5 on pp. 684–685 for Exs. 5–9

EXERCISES

5. PHOTOGRAPHY You are placing 12 pictures on separate pages in an album. How many different ways can you order the 12 pictures in the album? How many different ways can 4 of the 12 pictures be placed on the first 4 pages?

Find the number of permutations.

6. ${}_9P_1$ **7.** ${}_5P_5$ **8.** ${}_6P_3$ **9.** ${}_{10}P_2$

10.2 Use Combinations and the Binomial Theorem

pp. 690–697

EXAMPLE

Use the binomial theorem to expand $(x + 5y)^4$.

$$(x + 5y)^4 = {}_4C_0x^4(5y)^0 + {}_4C_1x^3(5y)^1 + {}_4C_2x^2(5y)^2 + {}_4C_3x^1(5y)^3 + {}_4C_4x^0(5y)^4$$

$$= (1)(x^4)(1) + (4)(x^3)(5y) + (6)(x^2)(25y^2) + (4)(x)(125y^3) + (1)(1)(625y^4)$$

$$= x^4 + 20x^3y + 150x^2y^2 + 500xy^3 + 625y^4$$

EXAMPLES 3, 5, and 6 on pp. 691–693 for Exs. 10–14

EXERCISES

Use the binomial theorem to write the binomial expansion.

10. $(t + 3)^6$ **11.** $(2a + b^2)^4$ **12.** $(w - 8v)^4$ **13.** $(r^3 - 4s)^5$

14. ICE CREAM An ice cream vendor sells 15 flavors of ice cream. You want to sample *at least* 4 of the flavors. How many different combinations of ice cream flavors can you sample?

10.3 Define and Use Probability

pp. 698–704

EXAMPLE

You roll a standard six-sided die. Find the probability of rolling a number less than 3.

Two outcomes correspond to rolling a number less than 3: rolling a 1 or 2.

$$P(\text{rolling less than 3}) = \frac{\text{Number of ways to roll less than 3}}{\text{Number of ways to roll the die}} = \frac{2}{6} = \frac{1}{3}$$

EXAMPLES 1 and 4 on pp. 698–700 for Exs. 15–19

EXERCISES

You have an equally likely chance of choosing any integer from 1 through 30. Find the probability of the given event.

15. An even number is chosen.

16. A multiple of 5 is chosen.

17. A factor of 60 is chosen.

18. A prime number is chosen.

19. COMMUTING Out of 250 work days, a commuter arrived at work on time 47 times on Mondays, 43 times on Tuesdays, 48 times on Wednesdays, 39 times on Thursdays, and 40 times on Fridays. For a randomly selected work day, what is the probability that the commuter arrived at work on time?

10 CHAPTER REVIEW

10.4 Probabilities of Disjoint and Overlapping Events
pp. 707–713

EXAMPLE

Let A and B be events such that $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{2}$, and $P(A \text{ and } B) = \frac{1}{3}$. Find $P(A \text{ or } B)$.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{2}{3} + \frac{1}{2} - \frac{1}{3} = \frac{5}{6}$$

EXERCISES

EXAMPLES 2 and 4 on pp. 708–709 for Exs. 20–22

Let A and B be events such that $P(A) = 0.32$, $P(B) = 0.48$, and $P(A \text{ and } B) = 0.12$. Find the indicated probability.

20. $P(A \text{ or } B)$ **21.** $P(\overline{A})$ **22.** $P(\overline{B})$

10.5 Probabilities of Independent and Dependent Events
pp. 717–723

EXAMPLE

Find the probability of selecting a club and then another club from a standard deck of 52 cards if (a) you replace the first card before selecting the second, and (b) you do *not* replace the first card.

Let event A be "the first card is a club" and B be "the second card is a club."

a. $P(A \text{ and } B) = P(A) \cdot P(B) = \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{16} = 0.0625$

b. $P(A \text{ and } B) = P(A) \cdot P(B|A) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17} \approx 0.0588$

EXERCISES

EXAMPLE 5 on p. 719 for Exs. 23–25

Find the probability of randomly selecting the given marbles from a bag of 5 red, 8 green, and 3 blue marbles if (a) you replace the first marble before drawing the second and (b) you do *not* replace the first marble.

23. red, then green **24.** blue, then red **25.** green, then green

10.6 Construct and Interpret Binomial Distributions
pp. 724–730

EXAMPLE

Find the probability of tossing a coin 12 times and getting exactly 4 heads.

$$P(k = 4) = {}_nC_k p^k (1 - p)^{n-k} = {}_{12}C_4 (0.5)^4 (1 - 0.5)^8 = 495(0.5)^4(0.5)^8 \approx 0.121$$

EXERCISES

EXAMPLE 3 on p. 726 for Exs. 26–29

Find the probability of tossing a coin 8 times and getting the given number of heads.

26. 6 **27.** 4 **28.** 7 **29.** 0

10 CHAPTER TEST

Find the number of permutations or combinations.

1. ${}_5P_2$
2. ${}_8P_3$
3. ${}_{12}P_7$
4. ${}_{17}P_{10}$
5. ${}_4C_3$
6. ${}_7C_7$
7. ${}_{18}C_4$
8. ${}_9C_5$

Use the binomial theorem to write the binomial expansion.

9. $(x + 5)^3$
10. $(3a - 3)^5$
11. $(s + t^2)^4$
12. $(c^3 - 2d^2)^6$

A card is randomly drawn from a standard deck of 52 cards. Find the probability of drawing the given card.

13. A queen
14. A red king
15. A diamond
16. Not a club

Find the indicated probability.

17. $P(A) = 0.3$
    $P(B) = 0.6$
    $P(A \text{ or } B) = \underline{\ ?\ }$
    $P(A \text{ and } B) = 0.1$

18. $P(A) = 35\%$
    $P(B) = \underline{\ ?\ }$
    $P(A \text{ or } B) = 80\%$
    $P(A \text{ and } B) = 20\%$

19. $P(A) = \underline{\ ?\ }$
    $P(\overline{A}) = \frac{2}{5}$

20. A and B are independent.
    $P(A) = 0.15$
    $P(B) = 0.6$
    $P(A \text{ and } B) = \underline{\ ?\ }$

21. A and B are dependent.
    $P(A) = 60\%$
    $P(B|A) = \underline{\ ?\ }$
    $P(A \text{ and } B) = 25\%$

22. A and B are dependent.
    $P(A) = \underline{\ ?\ }$
    $P(B|A) = 0.4$
    $P(A \text{ and } B) = 0.36$

Calculate the probability of k successes for a binomial experiment consisting of n trials with probability p of success on each trial.

23. $k = 4, n = 11, p = 0.4$
24. $k \le 2, n = 5, p = 0.7$
25. $k \ge 8, n = 9, p = 0.9$

26. **TRUE-OR-FALSE QUIZ** Calculate the probability of randomly guessing at least 8 correct answers on a 10 question true-or-false quiz.

27. **GOVERNMENT** There are 15 members on a city council. On a recent agenda item, 8 of the council members voted in favor of a budget increase for city park improvements. How many combinations of council members could have voted in favor of the budget increase?

28. **PARACHUTING** A parachuter is attempting to land within a square in the middle of a circular landing area. The square has sides 25 feet long, and the diameter of the landing area is 40 feet. If the parachuter is equally likely to first touch the ground at any point within the landing area, what is the probability that the parachuter first touches the ground within the square?

29. **EDUCATION** A high school has an enrollment of 1800 students. There are 1050 females enrolled in the school. The high school has 1200 students who are involved in an after-school activity, 725 of whom are female. What is the probability that a randomly selected student at the school is a female who is not involved in an after-school activity?

30. **FISHING** A study found that 9% of people cite fishing as their favorite leisure-time activity. Suppose you randomly survey 8 people about their leisure-time activities. What is the probability that at least 2 of the people cite fishing as their favorite?