Alg 2

13 Trigonometric Ratios and Functions

Before

In previous courses and in previous chapters, you learned the following skills, which you'll use in Chapter 13: using the Pythagorean theorem, solving equations using inverse functions, and finding angle measures in triangles.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. The **reciprocal** of $\frac{4}{5}$ is _?_.
2. Functions f and g are **inverses** of each other if _?_ and _?_.
3. An equation of the **circle** with center (0, 0) and a radius of 1 unit is _?_.

SKILLS CHECK

A right triangle has legs with lengths a and b and a hypotenuse with length c. Find the unknown side length. ***(Review p. 995 for 13.1.)***

4. $a = 8, b = 10$ **5.** $a = 2.5, c = 6.5$ **6.** $b = 9, c = 11$

Solve the equation. ***(Review p. 515 for 13.4.)***

7. $4^x - 5 = 3$ **8.** $\log_2 x = -1$ **9.** $-5 + 2 \ln 3x = 20$

The measures of the angles of a triangle are given. Find the value of x. ***(Review p. 995 for 13.5, 13.6.)***

10. $x°, 65°, 55°$ **11.** $90°, x°, x°$ **12.** $41°, 107°, x°$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 13, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 897. You will also use the key vocabulary listed below.

Big Ideas

1. **Using trigonometric functions**
2. **Using inverse trigonometric functions**
3. **Applying the law of sines and law of cosines**

KEY VOCABULARY

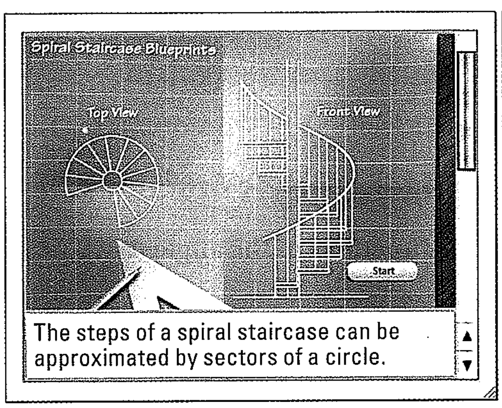
- sine, *p. 852*
- cosine, *p. 852*
- tangent, *p. 852*
- cosecant, *p. 852*
- secant, *p. 852*
- cotangent, *p. 852*
- radian, *p. 860*
- central angle, *p. 861*
- unit circle, *p. 867*
- reference angle, *p. 868*
- inverse sine, *p. 875*
- inverse cosine, *p. 875*
- inverse tangent, *p. 875*
- law of sines, *p. 882*
- law of cosines, *p. 889*

Why?

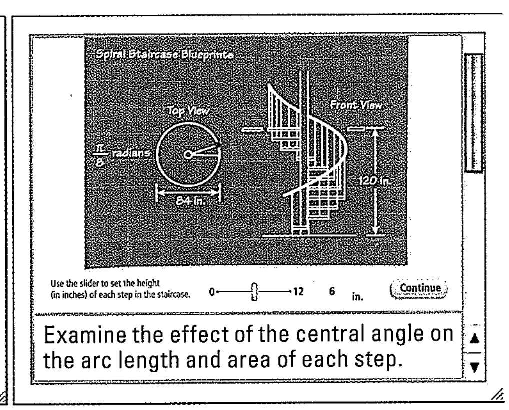
You can use angle measures and trigonometry to find lengths and areas in real life. For example, you can use an angle measure to find the area of each step in a spiral staircase.

Animated Algebra

The animation illustrated below for Exercise 53 on page 864 helps you answer this question: How does the central angle of a step in a spiral staircase affect the step's area?



The steps of a spiral staircase can be approximated by sectors of a circle.



Examine the effect of the central angle on the arc length and area of each step.

Animated Algebra at classzone.com

Other animations for Chapter 13: pages 854, 867, 884, and 897

13.1 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS21 for Exs. 5, 11, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 20, 33, and 36

◆ = **MULTIPLE REPRESENTATIONS** Ex. 34

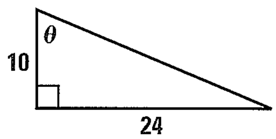
SKILL PRACTICE

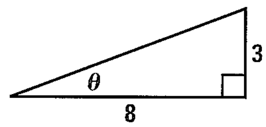
1. **VOCABULARY** What is an angle of elevation?

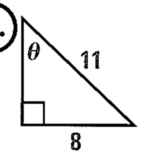
2. ★ **WRITING** *Explain* what it means to solve a right triangle.

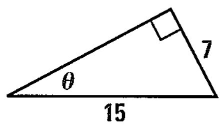
EXAMPLE 1 on p. 852 for Exs. 3–8

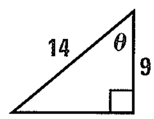
EVALUATING FUNCTIONS Evaluate the six trigonometric functions of the angle θ.

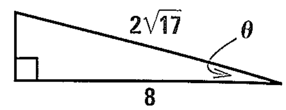
3. 

4. 

5. 

6. 

7. 

8. 

EXAMPLE 2 on p. 853 for Exs. 9–16

FINDING VALUES Let θ be an acute angle of a right triangle. Find the values of the other five trigonometric functions of θ.

9. $\sin\theta = \frac{5}{6}$

10. $\cos\theta = \frac{5}{8}$

11. $\tan\theta = \frac{7}{3}$

12. $\csc\theta = \frac{10}{7}$

13. $\sec\theta = \frac{12}{5}$

14. $\cot\theta = \frac{6}{11}$

15. ★ **MULTIPLE CHOICE** In a right triangle, θ is an acute angle and $\cos\theta = \frac{4}{9}$. What is the value of $\tan\theta$?

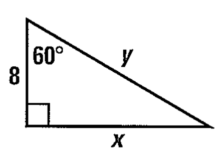
Ⓐ $\frac{4\sqrt{65}}{65}$ Ⓑ $\frac{\sqrt{65}}{9}$ Ⓒ $\frac{\sqrt{65}}{4}$ Ⓓ $\frac{9}{4}$

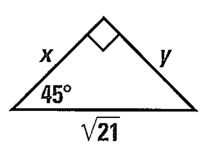
16. **ERROR ANALYSIS** *Describe* and correct the error in finding $\csc\theta$, given that θ is an acute angle of a right triangle and $\cos\theta = \frac{7}{11}$.

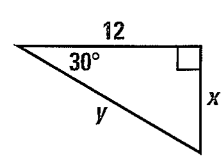
$$\csc\theta = \frac{1}{\cos\theta} = \frac{11}{7}$$ ✗

EXAMPLE 3 on p. 854 for Exs. 17–20

FINDING SIDE LENGTHS Find the exact values of x and y.

17. 

18. 

19. 

20. ★ **MULTIPLE CHOICE** In a 30°-60°-90° triangle, the longer leg has a length of 5. What is the length of the shorter leg?

Ⓐ $\frac{5\sqrt{3}}{3}$ Ⓑ $\frac{5\sqrt{3}}{2}$ Ⓒ $\frac{10\sqrt{3}}{3}$ Ⓓ $5\sqrt{3}$

EXAMPLE 4
on p. 854
for Exs. 21–28

SOLVING TRIANGLES **Solve $\triangle ABC$ using the diagram and the given measurements.**

21. $A = 35°, c = 16$

22. $B = 53°, a = 12$

23. $B = 18°, c = 24$

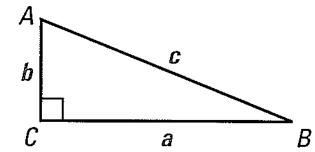
24. $A = 67°, b = 7$

25. $B = 75°, a = 15$

26. $A = 49°, c = 27$

27. $A = 64°, b = 32$

28. $B = 24°, c = 10.8$

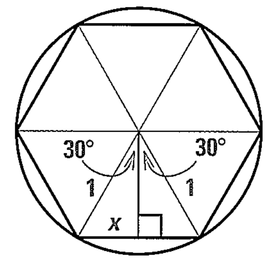


29. CHALLENGE A procedure for approximating π based on the work of Archimedes is to inscribe a regular hexagon in a circle.

a. Use the diagram at the right to solve for x. What is the perimeter of the hexagon?

b. Show that a regular n-sided polygon inscribed in a circle of radius 1 has a perimeter of $2n \cdot \sin\left(\frac{180}{n}\right)^{\circ}$.

c. Use the result from part (b) to find an expression in terms of n that approximates π. Then evaluate the expression when $n = 50$.

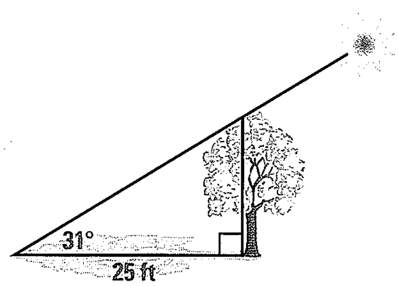


PROBLEM SOLVING

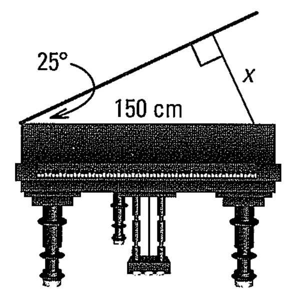
EXAMPLES 5 and 6
on p. 855
for Exs. 30–35

In Exercises 30 and 31, use the information in the diagram to solve the problem.

30. TREE HEIGHT A tree casts the shadow shown. What is the height of the tree?



31. GRAND PIANO Find the length of the prop holding open the piano.



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32. RAILWAY The Falls Incline Railway at Niagara Falls has an angle of elevation of 36°. The railway extends a horizontal distance of about 138 feet. Find the height and length of the railway.

33. ★ **SHORT RESPONSE** A submersible traveling at a depth of 250 feet dives at an angle of 15° with respect to a line parallel to the water's surface. It travels a horizontal distance of 1500 feet during the dive. What is the depth of the submersible after the dive? *Explain* how the angle of the dive affects the final depth.

34. MULTIPLE REPRESENTATIONS You are climbing Mount Massive in Colorado. You are at an altitude of 11,200 feet. You measure the angle of elevation to a ridge above you to be 29.4°. The distance (along the face of the mountain) between you and the ridge is 6315 feet.

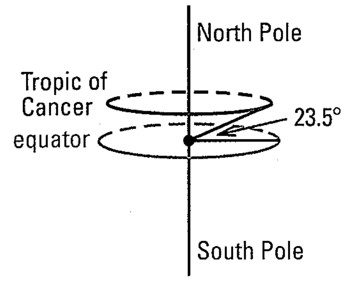
a. Drawing a Diagram Draw a diagram that represents this situation.

b. Writing an Equation Write and solve an equation to find the altitude of the ridge.

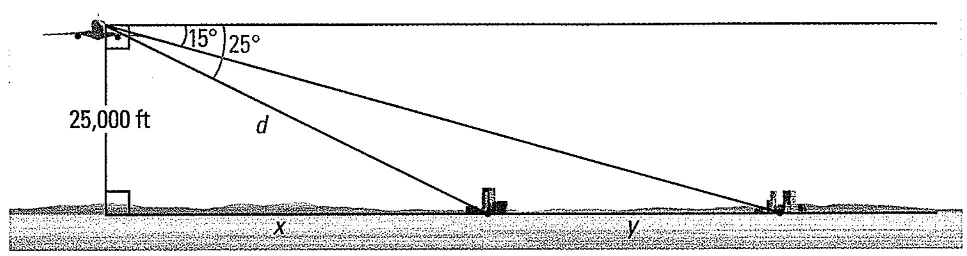
35. **TROPIC OF CANCER** The Tropic of Cancer is the circle of latitude farthest north of the equator where the sun can appear directly overhead. It lies 23.5° north of the equator, as shown.

a. Find the circumference of the Tropic of Cancer using 3960 miles as Earth's approximate radius.

b. What is the distance between two points on the Tropic of Cancer that lie directly across from each other?



36. ★ **EXTENDED RESPONSE** A passenger in an airplane sees two towns directly to the left of the plane.



a. What is the distance d from the airplane to the first town?

b. What is the horizontal distance x from the airplane to the first town?

c. What is the distance y between the two towns? *Explain* the process you used to find your answer.

37. **CHALLENGE** You measure the angle of elevation from the ground to the top of a building as 32°. When you move 50 meters closer to the building, the angle of elevation is 53°. How high is the building?

MIXED REVIEW

PREVIEW
Prepare for Lesson 13.2 in Exs. 38–43.

Perform the indicated conversion. *(p. 2)*

38. 3 years to seconds

39. 10 pints to gallons

40. 500 feet to yards

41. 9.4 kilograms to grams

42. 2 tons to ounces

43. 5.6 meters to millimeters

Solve the system using any algebraic method. *(p. 160)*

44. $2x - y = 8$
$3x + 4y = 23$

45. $4x + 3y = -1$
$3x + y = 3$

46. $9x - 12y = 20$
$-6x + 8y = 11$

47. $12x + 5y = 4$
$3x - 10y = -8$

48. $-2x + 7y = 16$
$3x - 5y = -2$

49. $4x - 6y = 18$
$-2x + y = -3$

Classify the conic section and write its equation in standard form. *(p. 650)*

50. $x^2 + y^2 + 4x + 6y - 17 = 0$

51. $x^2 - 4y^2 + 6x + 16y + 137 = 0$

52. $y^2 - 4y + 16x + 116 = 0$

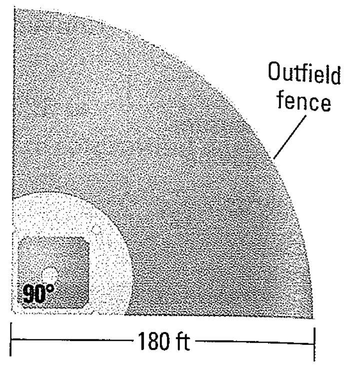
53. $9x^2 + 25y^2 + 162x + 250y + 454 = 0$

54. $x^2 + 8x + 4y + 28 = 0$

55. $x^2 - y^2 + 14x + 16y - 5 = 0$

EXAMPLE 4 Solve a multi-step problem

SOFTBALL A softball field forms a sector with the dimensions shown. Find the length of the outfield fence and the area of the field.



Solution

STEP 1 **Convert** the measure of the central angle to radians.

$$90° = 90°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{2} \text{ radians}$$

AVOID ERRORS
You must write the measure of an angle in radians when using the formulas for the arc length and area of a sector.

STEP 2 **Find** the arc length and the area of the sector.

Arc length: $s = r\theta = 180\left(\frac{\pi}{2}\right) = 90\pi \approx 283$ feet

Area: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(180)^2\left(\frac{\pi}{2}\right) = 8100\pi \approx 25{,}400 \text{ ft}^2$

▶ The length of the outfield fence is about 283 feet. The area of the field is about 25,400 square feet.

✓ GUIDED PRACTICE for Example 4

9. **WHAT IF?** In Example 4, estimate the length of the outfield fence and the area of the field if the outfield fence is 220 feet from home plate.

13.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS22 for Exs. 11, 23, and 51
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, 31, 50, and 53

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: An angle is in standard position if its vertex is at the _?_ and its _?_ lies on the positive x-axis.

2. ★ **WRITING** How does the sign of an angle's measure determine its direction of rotation?

EXAMPLES 1 and 3 on pp. 859–861 for Exs. 3–14

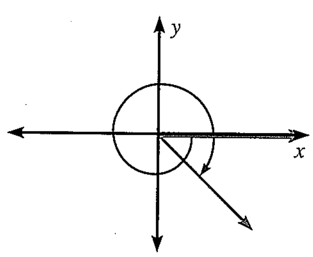
VISUAL THINKING **Match the angle measure with the angle.**

3. $-240°$

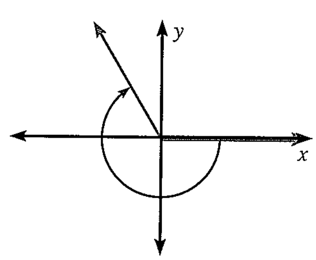
4. $600°$

5. $-\frac{9\pi}{4}$

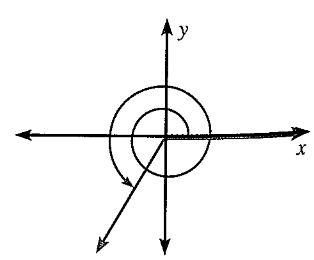
A.



B.



C.

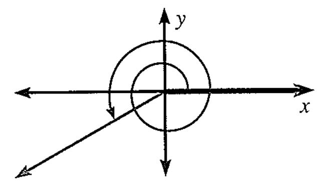


DRAWING ANGLES **Draw an angle with the given measure in standard position.**

6. 110° **7.** −10° **8.** 450° **9.** −900°

10. 6π **11.** $\frac{5\pi}{18}$ **12.** $-\frac{5\pi}{3}$ **13.** $\frac{26\pi}{9}$

14. ★ **MULTIPLE CHOICE** Which angle measure is shown in the diagram?



Ⓐ −150° Ⓑ 210°

Ⓒ 570° Ⓓ 930°

EXAMPLES 2 and 3
on pp. 860–861 for Exs. 15–22

FINDING COTERMINAL ANGLES **Find one positive angle and one negative angle that are coterminal with the given angle.**

15. 70° **16.** 255° **17.** −125° **18.** 820°

19. $\frac{9\pi}{2}$ **20.** $-\frac{7\pi}{6}$ **21.** $\frac{28\pi}{9}$ **22.** $\frac{20\pi}{3}$

EXAMPLE 3
on p. 861 for Exs. 23–31

CONVERTING MEASURES **Convert the degree measure to radians or the radian measure to degrees.**

23. 40° **24.** 315° **25.** −260° **26.** 500°

27. $\frac{\pi}{9}$ **28.** $-\frac{\pi}{4}$ **29.** 5π **30.** $\frac{14\pi}{15}$

31. ★ **MULTIPLE CHOICE** Which angle measure is equivalent to $\frac{13\pi}{6}$ radians?

Ⓐ 30° Ⓑ 390° Ⓒ 750° Ⓓ 1110°

EXAMPLE 4
on p. 862 for Exs. 32–38

FINDING ARC LENGTH AND AREA **Find the arc length and area of a sector with the given radius *r* and central angle *θ*.**

32. $r = 4$ in., $\theta = \frac{\pi}{6}$ **33.** $r = 3$ m, $\theta = \frac{5\pi}{12}$ **34.** $r = 15$ cm, $\theta = 45°$

35. $r = 12$ ft, $\theta = 150°$ **36.** $r = 18$ m, $\theta = 25°$ **37.** $r = 25$ in., $\theta = 270°$

38. **ERROR ANALYSIS** *Describe* and correct the error in finding the area of a sector with a radius of 6 centimeters and a central angle of 40°.

$$A = \frac{1}{2}(6)^2(40) = 720 \text{ cm}^2 \quad \times$$

HINT
For Exs. 39–46, set your calculator in radian mode.

EVALUATING FUNCTIONS **Evaluate the trigonometric function using a calculator if necessary. If possible, give an exact answer.**

39. $\cos\frac{\pi}{3}$ **40.** $\sin\frac{\pi}{4}$ **41.** $\tan\frac{\pi}{6}$ **42.** $\sec\frac{\pi}{9}$

43. $\cot\frac{\pi}{8}$ **44.** $\cos\frac{\pi}{6}$ **45.** $\sin\frac{3\pi}{7}$ **46.** $\csc\frac{4\pi}{15}$

47. **CHALLENGE** A rotating object that passes through an angle θ during time t has an angular velocity v given by the formula $v = \frac{\theta}{t}$. Find the angular velocity of the hour hand, the minute hand, and the second hand on a 12 hour clock. Give all answers in degrees per hour.

PROBLEM SOLVING

EXAMPLES 1 and 3 on pp. 859–861 for Exs. 48–50

48. ASTRONOMY In astronomy, the *terminator* is the day-night line on a planet that divides the planet into daytime and nighttime regions. The terminator moves across the planet's surface as the planet rotates. It takes about 4 hours for Earth's terminator to move across the continental United States. Through what angle has Earth rotated during this time? Give the answer in both degrees and radians.

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49. CD PLAYER When a CD player reads information from the outer edge of a CD, the CD spins about 200 revolutions per minute. At that speed, through what angle does a point on the CD spin in one minute? Give the answer in both degrees and radians.

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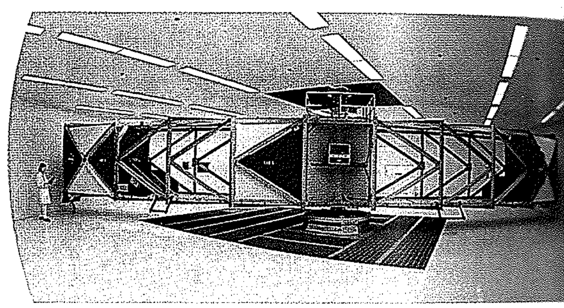
50. ★ SHORT RESPONSE You work every Saturday from 9:00 A.M. to 5:00 P.M. Draw a diagram that shows the rotation completed by the hour hand of a clock during this time. Find the measure of the angle generated by the hour hand in both degrees and radians. *Compare* this angle with the angle generated by the minute hand from 9:00 A.M. to 5:00 P.M.

EXAMPLE 4 on p. 862 for Exs. 51–53

(51.) MULTI-STEP PROBLEM A scientist performed an experiment to study the effects of gravitational force on humans. In order for humans to experience twice Earth's gravity, they were placed in a centrifuge 58 feet long and spun at a rate of about 15 revolutions per minute.



a. Through how many radians did the people rotate each second?

b. Find the length of the arc through which the people rotated each second.

52. MULTI-STEP PROBLEM In the shot put event at the 2004 Summer Olympic Games, the winning shot was 21.16 meters. For a shot put to be fair, it must land within a sector having a central angle of 34.92°.

a. If the officials drew an arc across the fair landing area marking the farthest throw, how long would the arc be?

b. All fair shot puts in the 2004 Olympics landed within a sector bounded by the arc from part (a). What is the area of this sector?

53. ★ EXTENDED RESPONSE A spiral staircase has 15 steps. Each step is a sector with a radius of 42 inches and a central angle of $\frac{\pi}{8}$.

a. What is the length of the arc formed by the outer edge of a step?

b. Through what angle would you rotate by climbing the stairs? Include a sixteenth turn for stepping up on the landing. *Explain* your reasoning.

c. How many square inches of carpeting would you need to cover the 15 steps?

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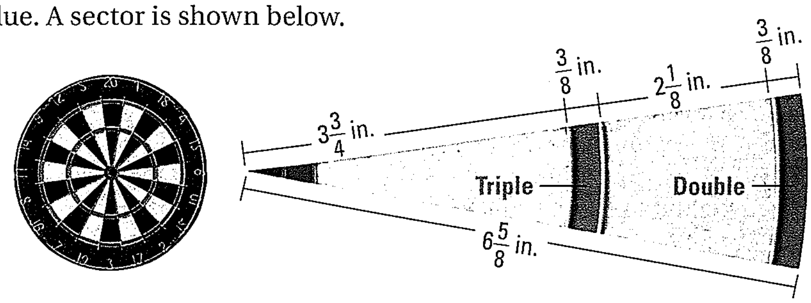
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

54. **CHALLENGE** A dartboard is divided into 20 sectors. Each sector is worth a point value from 1 to 20 and has shaded regions that double or triple this value. A sector is shown below.



a. Find the areas of the entire sector, the double region, and the triple region.

b. A dart you throw randomly lands somewhere inside the sector. What is the probability that it lands in the double region? in the triple region?

MIXED REVIEW

PREVIEW
Prepare for Lesson 13.3 in Exs. 55–62.

Simplify the expression. *(p. 266)*

55. $\sqrt{54}$ 56. $\sqrt{320}$ 57. $\sqrt{36} \cdot \sqrt{18}$ 58. $\sqrt{3} \cdot \sqrt{60}$

59. $\sqrt{\frac{5}{49}}$ 60. $\sqrt{\frac{27}{64}}$ 61. $\frac{\sqrt{12}}{\sqrt{7}}$ 62. $\frac{\sqrt{28}}{\sqrt{8}}$

Write an equation of the line tangent to the given circle at the given point. *(p. 626)*

63. $x^2 + y^2 = 53$; (7, 2) 64. $x^2 + y^2 = 40$; (2, 6) 65. $x^2 + y^2 = 146$; (5, 11)

Find the number of permutations or combinations.

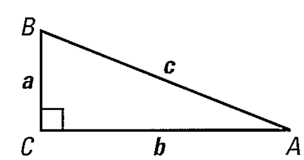
66. ${}_8P_3$ *(p. 682)* 67. ${}_7P_4$ *(p. 682)* 68. ${}_{12}P_2$ *(p. 682)* 69. ${}_{10}P_8$ *(p. 682)*

70. ${}_9C_5$ *(p. 690)* 71. ${}_{15}C_7$ *(p. 690)* 72. ${}_6C_5$ *(p. 690)* 73. ${}_8C_4$ *(p. 690)*

QUIZ for Lessons 13.1–13.2

Solve $\triangle ABC$ using the diagram and the given measurements. *(p. 852)*

1. $A = 50°, a = 14$ 2. $A = 25°, b = 10$
3. $B = 70°, a = 5$ 4. $B = 42°, c = 18$
5. $A = 15°, a = 9$ 6. $B = 37°, c = 12$



Find one positive angle and one negative angle that are coterminal with the given angle. *(p. 859)*

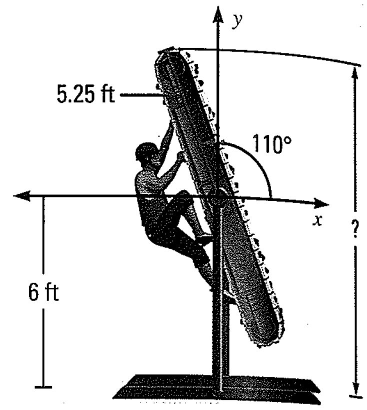
7. 115° 8. 290° 9. $\frac{4\pi}{9}$ 10. $\frac{7\pi}{5}$

11. Find the arc length and area of a sector with a radius of 8 inches and a central angle of $\theta = 115°$. *(p. 859)*

12. **ESCALATOR** The escalator at the Wilshire/Vermont Metro Rail Station in Los Angeles has an angle of elevation of 30°. The length of the escalator is 152 feet. What is the height of the escalator? *(p. 852)*

EXAMPLE 6 Model with a trigonometric function

ROCK CLIMBING A rock climber is using a rock climbing treadmill that is 10.5 feet long. The climber begins by lying horizontally on the treadmill, which is then rotated about its midpoint by 110° so that the rock climber is climbing towards the top. If the midpoint of the treadmill is 6 feet above the ground, how high above the ground is the top of the treadmill?



Solution

$\sin \theta = \frac{y}{r}$ **Use definition of sine.**

$\sin 110° = \frac{y}{5.25}$ **Substitute 110° for θ and $\frac{10.5}{2} = 5.25$ for r.**

$4.9 \approx y$ **Solve for y.**

▶ The top of the treadmill is about $6 + 4.9 = 10.9$ feet above the ground.

✓ **GUIDED PRACTICE for Examples 5 and 6**

10. **TRACK AND FIELD** Estimate the horizontal distance traveled by a track and field long jumper who jumps at an angle of 20° and with an initial speed of 27 feet per second.

11. **WHAT IF?** In Example 6, how high is the top of the rock climbing treadmill if it is rotated 100° about its midpoint?

13.3 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS22 for Exs. 5, 17, and 37
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 33, 37, and 39

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A(n) __?__ is an angle in standard position whose terminal side lies on an axis.

2. ★ **WRITING** Given an angle θ in Quadrant III, explain how you can use a reference angle to find $\cos \theta$.

EXAMPLE 1 on p. 866 for Exs. 3–11

USING A POINT Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ.

3. (8, 15)
4. (−9, 12)
5. (−7, −24)
6. (5, −12)
7. (2, −2)
8. (−6, 9)
9. (−3, −5)
10. $(5, -\sqrt{11})$

11. ★ **MULTIPLE CHOICE** Let (−7, −4) be a point on the terminal side of an angle θ in standard position. What is the value of $\tan \theta$?

Ⓐ $-\frac{7}{4}$ Ⓑ $-\frac{4}{7}$ Ⓒ $\frac{4}{7}$ Ⓓ $\frac{7}{4}$

EXAMPLE 2
on p. 867
for Exs. 12–15

QUADRANTAL ANGLES **Evaluate the six trigonometric functions of θ.**

12. $\theta = 0°$ **13.** $\theta = \frac{\pi}{2}$ **14.** $\theta = 540°$ **15.** $\theta = \frac{7\pi}{2}$

EXAMPLE 3
on p. 868
for Exs. 16–23

FINDING REFERENCE ANGLES **Sketch the angle. Then find its reference angle.**

16. $-100°$ **17.** $150°$ **18.** $320°$ **19.** $-370°$

20. $-\frac{5\pi}{6}$ **21.** $\frac{8\pi}{3}$ **22.** $\frac{15\pi}{4}$ **23.** $-\frac{13\pi}{6}$

EXAMPLE 4
on p. 869
for Exs. 24–31

EVALUATING FUNCTIONS **Evaluate the function without using a calculator.**

24. $\sec 135°$ **25.** $\tan 240°$ **26.** $\sin(-150°)$ **27.** $\csc(-420°)$

28. $\cos \frac{7\pi}{4}$ **29.** $\cot\left(-\frac{8\pi}{3}\right)$ **30.** $\tan\left(-\frac{3\pi}{4}\right)$ **31.** $\sec \frac{11\pi}{6}$

32. **ERROR ANALYSIS** Let (4, 3) be a point on the terminal side of an angle θ in standard position. *Describe* and correct the error in finding $\tan \theta$.

$$\tan \theta = \frac{x}{y} = \frac{4}{3} \quad \times$$

33. ★ **SHORT RESPONSE** Write $\tan \theta$ as the ratio of two other trigonometric functions. Use this ratio to explain why $\tan 90°$ is undefined but $\cot 90° = 0$.

34. **CHALLENGE** Five of the most famous numbers in mathematics — 0, 1, π, e, and i — are related by the simple equation $e^{\pi i} + 1 = 0$. Derive this equation using Euler's formula: $e^{a+bi} = e^{a}(\cos b + i \sin b)$.

PROBLEM SOLVING

EXAMPLE 5
on p. 869
for Exs. 35–36

In Exercises 35 and 36, use the formula in Example 5 on page 869.

35. **FOOTBALL** You and a friend each kick a football with an initial speed of 49 feet per second. Your kick is projected at an angle of 45° and your friend's kick is projected at an angle of 60°. About how much farther will your football travel than your friend's football?

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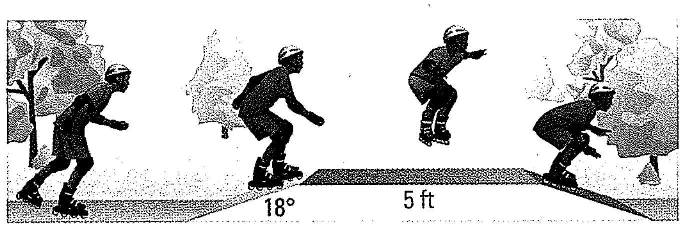
36. **IN-LINE SKATING** At what speed must the in-line skater launch himself off the ramp in order to land on the other side of the ramp?



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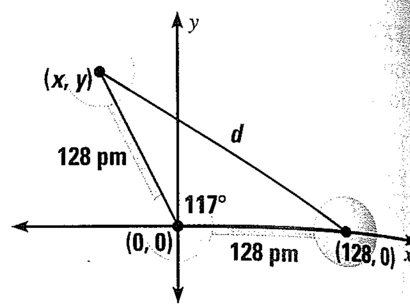
EXAMPLE 6
on p. 870
for Exs. 37–38

37. ★ **SHORT RESPONSE** A Ferris wheel has a radius of 75 feet. You board a car at the bottom of the Ferris wheel, which is 10 feet above the ground, and rotate 255° counterclockwise before the ride temporarily stops. How high above the ground are you when the ride stops? If the radius of the Ferris wheel is doubled, is your height above the ground doubled? *Explain.*

38. MULTI-STEP PROBLEM When two atoms in a molecule are bonded to a common atom, chemists are interested in both the bond angle and the lengths of the bonds. An ozone molecule (O_3) is made up of two oxygen atoms bonded to a third oxygen atom, as shown.



a. In the diagram, coordinates are given in picometers (pm). (*Note:* 1 pm $= 10^{-12}$ m.) Find the coordinates (x, y) of the center of the oxygen atom in Quadrant II.

b. Find the distance d (in picometers) between the centers of the two unbonded oxygen atoms.

39. ★ EXTENDED RESPONSE A sprinkler at ground level is used to water a garden. The water leaving the sprinkler has an initial speed of 25 feet per second.

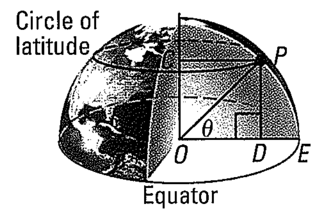
a. Calculate Copy the table below. Use the formula in Example 5 on page 869 to complete the table.

Angle of sprinkler, θ	25°	30°	35°	40°	45°	50°	55°	60°	65°
Horizontal distance water travels, d	?	?	?	?	?	?	?	?	?

b. Interpret What value of θ appears to maximize the horizontal distance traveled by the water? Use the formula for horizontal distance traveled and the unit circle to explain why your answer makes sense.

c. Compare *Compare* the horizontal distance traveled by the water when $\theta = (45 - k)°$ with the distance when $\theta = (45 + k)°$.

40. CHALLENGE The latitude of a point on Earth is the degree measure of the shortest arc from that point to the equator. For example, the latitude of point P in the diagram equals the degree measure of arc PE. At what latitude θ is the circumference of the circle of latitude at P half the distance around the equator?



MIXED REVIEW

PREVIEW
Prepare for Lesson 13.4 in Exs. 41–46.

Graph the function f. Then use the graph to determine whether the inverse of f is a function. *(p. 438)*

41. $f(x) = 5x + 2$ **42.** $f(x) = -x + 7$ **43.** $f(x) = x^2 + 5$

44. $f(x) = 4x^2, x \geq 0$ **45.** $f(x) = 0.25x^2$ **46.** $f(x) = |x - 7|$

Find the range and standard deviation of the data set. *(p. 744)*

47. 3, 5, 2, 3, 7, 11, 8, 4 **48.** 18, 12, 15, 9, 13, 7, 4, 17

49. 5.9, 8.2, 3.7, 6.1, 2.9, 1.8, 5.7 **50.** 54, 60, 57, 53, 59, 51, 56, 62

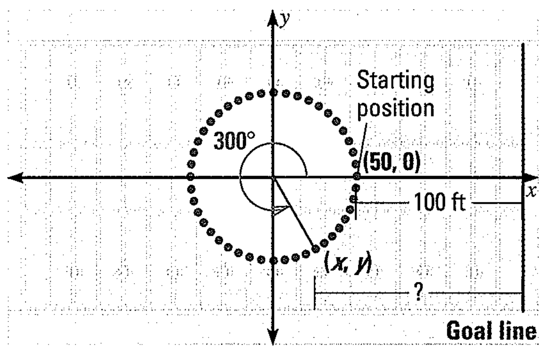
Find the sum of the series.

51. $\sum_{i=1}^{15} (3i + 2)$ *(p. 802)* **52.** $\sum_{i=1}^{18} (4i + 1)$ *(p. 802)* **53.** $\sum_{i=1}^{24} (17 - 2i)$ *(p. 802)*

54. $\sum_{i=1}^{5} 2(3)^{i-1}$ *(p. 810)* **55.** $\sum_{i=1}^{7} \frac{1}{4}\left(\frac{3}{2}\right)^{i-1}$ *(p. 810)* **56.** $\sum_{i=1}^{\infty} 8\left(\frac{1}{2}\right)^{i-1}$ *(p. 820)*

Lessons 13.1–13.3

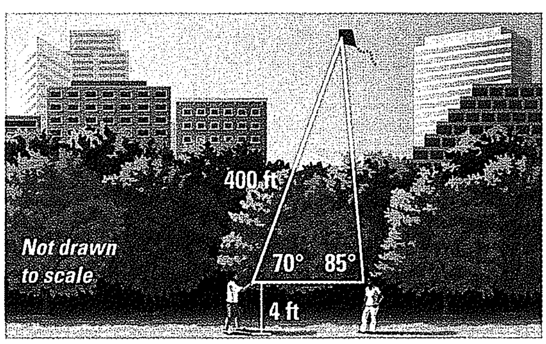
1. **MULTI-STEP PROBLEM** Your school's marching band is performing at halftime during a football game. In the last formation, the band members form a circle 100 feet wide in the center of the field. You start at a point on the circle 100 feet from the goal line, march 300° around the circle, and then walk toward the goal line to exit the field.



   a. How far from the goal line are you at the point where you leave the circle?

   b. How far do you march around the circle?

2. **MULTI-STEP PROBLEM** You are flying a kite at an angle of 70°. You have let out a total of 400 feet of string and are holding the reel steady 4 feet above the ground.



   a. How high above the ground is the kite?

   b. A friend watching the kite estimates that the angle of elevation to the kite is 85°. How far from your friend are you standing?

3. **GRIDDED ANSWER** What is the reference angle, in degrees, for the angle $\theta = 560°$?

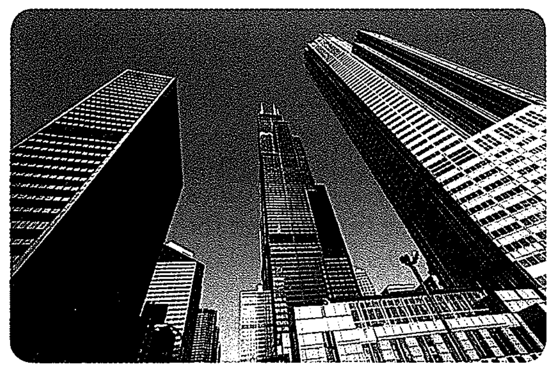
4. **OPEN-ENDED** What is the measure, in degrees, of an angle for which the secant is positive and the cotangent is negative?

5. **SHORT RESPONSE** The top of the Space Needle in Seattle, Washington, is a revolving, circular restaurant. The restaurant has a radius of 47.25 feet and makes one complete revolution in about an hour. You have dinner at a window table from 7:00 P.M. to 8:55 P.M.

   a. How many feet do you revolve?

   b. Do diners seated 5 feet away from the windows revolve the same distance? *Explain.*

6. **MULTI-STEP PROBLEM** You are standing 100 meters from the main entrance of the Sears Tower in Chicago, Illinois. You estimate that the angle of elevation to the top of the skyscraper is 77°.



   a. What is the approximate height h of the Sears Tower?

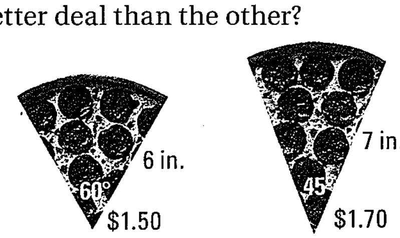
   b. Suppose one of your friends is at the top of the Sears Tower. What is the straight-line distance d between you and your friend?

7. **EXTENDED RESPONSE** A pizza shop offers two choices for individual pizza slices, as shown.

   a. Find the area of each slice of pizza.

   b. Which slice is the better deal? *Explain* your reasoning.

   c. How could you change the price of the 7 inch slice so that neither slice offers a better deal than the other?



13.4 Investigating Inverse Trigonometric Functions

MATERIALS • paper and pencil

QUESTION **Do the sine and cosine functions have inverse functions?**

EXPLORE **Determine if a trigonometric function has an inverse function**

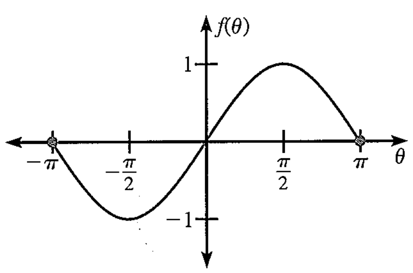
STEP 1 ***Make a table*** Copy and complete the table to find the values of $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$ for each of the given values of θ.

θ	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$f(\theta) = \sin\theta$	?	?	?	?	?	?	?	?	?
$g(\theta) = \cos\theta$	?	?	?	?	?	?	?	?	?

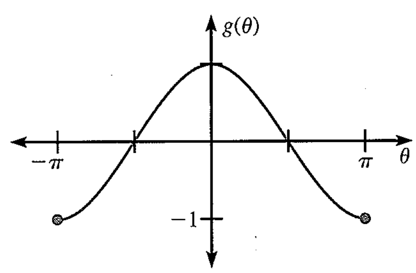
STEP 2 ***Analyze sine*** Use the table to explain why $f(\theta) = \sin\theta$ does not have an inverse function on the domain $-\pi \le \theta \le \pi$.

STEP 3 ***Analyze cosine*** Does $g(\theta) = \cos\theta$ have an inverse function on the domain $-\pi \le \theta \le \pi$? Explain why or why not.

STEP 4 ***Use graphs*** The graphs of $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$ are shown for the domain $-\pi \le \theta \le \pi$. Explain how the graphs justify your answers in Steps 2 and 3.



$f(\theta) = \sin\theta$



$g(\theta) = \cos\theta$

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Use the graph of $f(\theta) = \sin\theta$ in Step 4 to choose a restricted domain for which the sine function does have an inverse function. *Explain* how you made your choice.
2. Give a restricted domain for which $g(\theta) = \cos\theta$ has an inverse function. *Explain* how you chose the domain.
3. Are the domains that you wrote in Exercises 1 and 2 the *only* domains for which the trigonometric functions have inverse functions? *Explain.*

13.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS22 for Exs. 7, 23, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 30, 31, 37, and 38

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The _?_ sine of $\frac{1}{2}$ is $\frac{\pi}{6}$, or 30°.

2. ★ **WRITING** *Explain* why $\tan^{-1} 3$ is defined, but $\cos^{-1} 3$ is undefined.

EXAMPLE 1 on p. 876 for Exs. 3–11

EVALUATING EXPRESSIONS Evaluate the expression without using a calculator. Give your answer in both radians and degrees.

3. $\sin^{-1} 1$
4. $\tan^{-1} (-1)$
5. $\cos^{-1} 0$
6. $\cos^{-1} (-2)$
7. $\sin^{-1} \frac{\sqrt{3}}{2}$
8. $\sin^{-1} \frac{1}{2}$
9. $\tan^{-1} \left(-\frac{\sqrt{3}}{3}\right)$
10. $\cos^{-1} \left(-\frac{1}{2}\right)$

11. ★ **MULTIPLE CHOICE** What is the value of the expression $\cos^{-1} \frac{\sqrt{2}}{2}$?

Ⓐ 0° Ⓑ 30° Ⓒ 45° Ⓓ 60°

USING A CALCULATOR Use a calculator to evaluate the expression in both radians and degrees.

12. $\sin^{-1} 0.18$
13. $\tan^{-1} 2.6$
14. $\cos^{-1} 0.36$
15. $\cos^{-1} (-0.4)$
16. $\tan^{-1} (-0.75)$
17. $\sin^{-1} (-0.2)$
18. $\sin^{-1} 0.8$
19. $\cos^{-1} 0.99$

EXAMPLE 2 on p. 876 for Exs. 20–26

SOLVING EQUATIONS Solve the equation for θ.

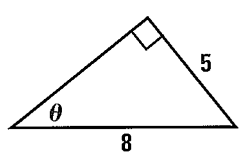
20. $\cos \theta = -0.82$; $180° < \theta < 270°$
21. $\sin \theta = -0.45$; $180° < \theta < 270°$
22. $\sin \theta = 0.15$; $90° < \theta < 180°$
23. $\tan \theta = 3.2$; $180° < \theta < 270°$
24. $\tan \theta = -5.3$; $90° < \theta < 180°$
25. $\cos \theta = 0.25$; $270° < \theta < 360°$

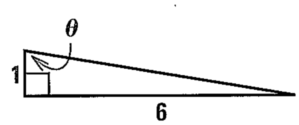
26. **ERROR ANALYSIS** *Describe* and correct the error in solving the equation $\sin \theta = 0.7$ where $90° < \theta < 180°$.

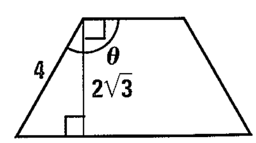
> The angle whose sine is 0.7 is $\sin^{-1} 0.7 \approx 44.4°$, so $\theta \approx 44.4°$. ✗

EXAMPLE 3 on p. 877 for Exs. 27–29

FINDING ANGLES Find the measure of the angle θ.

27.


28.


29.


30. ★ **OPEN-ENDED MATH** Suppose $\cos \theta > 0$ and $\sin \theta < 0$. Give a possible value of θ such that $-360° \le \theta \le 0°$.

31. ★ **OPEN-ENDED MATH** Suppose $\sin \theta < 0$ and $\tan \theta > 0$. Give a possible value of θ such that $360° \le \theta \le 720°$.

CHALLENGE Rewrite the expression so that it does not involve trigonometric functions or inverse trigonometric functions.

32. $\csc (\sin^{-1} x)$
33. $\cot (\tan^{-1} x)$
34. $\sec (\cos^{-1} x)$

PROBLEM SOLVING

EXAMPLE 4
on p. 877
for Exs. 35–37

35. LADDER ANGLE A fire truck has a 100 foot ladder whose base is 10 feet above the ground. A firefighter extends a ladder toward a burning building to reach a window 90 feet above the ground. Draw a diagram to represent this situation. At what angle should the firefighter set the ladder?

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36. ANGLE OF DESCENT An airplane is flying at an altitude of 31,000 feet when it begins its descent for landing. If the runway is 104 miles away, at what angle does the airplane descend?

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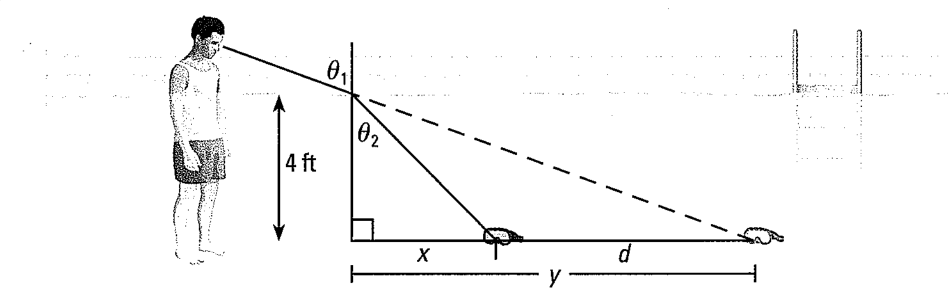
37. ★ **SHORT RESPONSE** Different types of granular substances naturally settle at different angles when stored in cone-shaped piles. The angle θ is called the *angle of repose*. When rock salt is stored in a cone-shaped pile 11 feet high, the diameter of the pile's base is about 34 feet. Find the angle of repose for rock salt. If another pile of rock salt is 15 feet high, what is the diameter of its base? *Explain.*

38. ★ **EXTENDED RESPONSE** If you are in shallow water and look at an object below the surface of the water, the object will look farther away from you than it really is. This is because when light rays pass between air and water, the water *refracts*, or bends, the light rays. The *index of refraction* for water is 1.333. This is the ratio of the sine of θ_1 to the sine of θ_2 for the angles θ_1 and θ_2 shown below.



a. You are in 4 feet of water in the shallow end of a pool. You look down at some goggles at angle $\theta_1 = 70°$ (measured from a line perpendicular to the surface of the water). Find θ_2.

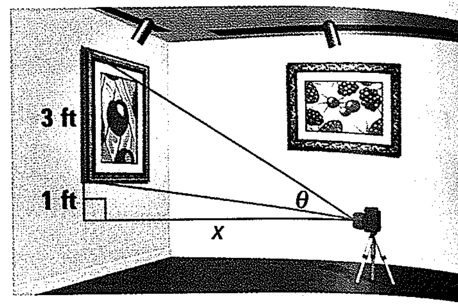
b. Find the distances x and y.

c. Find the distance d between where the goggles are and where they appear to be.

d. *Explain* what happens to d as you move closer to the goggles.

39. CYCLING As a spectator at a cycling road race, you are sitting 100 feet from the center of a straightaway. A cyclist traveling 30 miles per hour passes in front of you. At what angle do you have to turn your head to see the cyclist t seconds later? Assume the cyclist is still on the straightaway and is traveling at a constant speed. (*Hint:* First convert 30 miles per hour to a speed v in feet per second. The expression vt represents the distance, in feet, traveled by the cyclist.)

40. CHALLENGE You want to photograph a painting with a camera mounted on a tripod. The painting is 3 feet tall, and the bottom of the painting is 1 foot above the camera lens, as shown. How far should the camera be positioned from the wall in order to have the largest possible viewing angle θ when you take the photograph? (*Hint:* Write an equation for θ in terms of x only, and then use a graphing calculator to find the value of x that maximizes θ.)



MIXED REVIEW

Solve the equation.

41. $x + 4 = -\frac{1}{4}x - \frac{3}{8}$ *(p. 18)*
42. $18x^2 + x - 5 = 0$ *(p. 259)*
43. $12x^2 = 8x + 15$ *(p. 259)*
44. $3x^2 - 30x - 9 = 0$ *(p. 284)*
45. $27x^3 - 64 = 0$ *(p. 353)*
46. $\sqrt[3]{x + 12} = 5$ *(p. 452)*
47. $(6x - 11)^{5/2} = 243$ *(p. 452)*
48. $8^{x-4} = 32^{3x-8}$ *(p. 515)*
49. $10^{2x} - 6 = 12$ *(p. 515)*

PREVIEW
Prepare for Lesson 13.5 in Exs. 50–55.

Solve the rational equation. Check for extraneous solutions. *(p. 589)*

50. $\frac{4}{x} = \frac{9}{x + 5}$
51. $\frac{2}{x - 6} = \frac{10}{x}$
52. $\frac{3}{2 + x} = \frac{-9}{4x}$
53. $\frac{5}{x + 1} + 3 = \frac{-7}{x + 1}$
54. $\frac{1}{x + 3} = \frac{x}{3x + 16}$
55. $\frac{3x}{x - 4} = 2 + \frac{12}{x - 4}$

QUIZ *for Lessons 13.3–13.4*

Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ. *(p. 866)*

1. $(6, -2)$
2. $(-7, 5)$
3. $(4, 8)$
4. $(-12, -3)$

Evaluate the expression without using a calculator. *(p. 866)*

5. $\cos 150°$
6. $\tan \frac{8\pi}{3}$
7. $\sin (-840°)$
8. $\sec\left(-\frac{15\pi}{4}\right)$

Evaluate the expression without using a calculator. Give your answer in both radians and degrees. *(p. 875)*

9. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
10. $\sin^{-1}(-1)$
11. $\tan^{-1}\frac{\sqrt{3}}{3}$
12. $\cos^{-1}\frac{1}{2}$

Solve the equation for θ. *(p. 875)*

13. $\sin \theta = 0.3;\ 90° < \theta < 180°$
14. $\tan \theta = 6;\ 180° < \theta < 270°$
15. $\cos \theta = -0.72;\ 90° < \theta < 180°$
16. $\sin \theta = -0.55;\ 270° < \theta < 360°$

17. **ACROBATICS** A stuntman uses a 30 foot rope to swing 136° between two platforms of equal height, grazing the ground in the middle of the swing. If the rope stays taut throughout the swing, how far above the ground was the stuntman at the beginning and the end of the swing? How far apart are the two platforms? *(p. 875)*

EXTRA PRACTICE for Lesson 13.4, p. 1022 **ONLINE QUIZ** at classzone.com

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Keystrokes

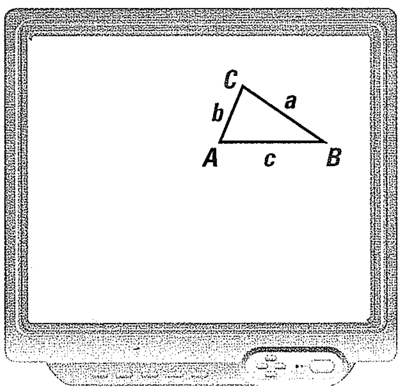
13.5 Explore the Law of Sines

QUESTION **How can you use geometry software to explore the law of sines?**

EXPLORE **Investigate a relationship between the angles and sides of a triangle**

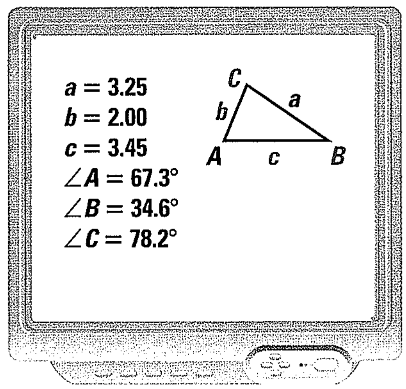
STEP 1 ***Draw a triangle***

Draw $\triangle ABC$. Label the vertices and sides as shown.



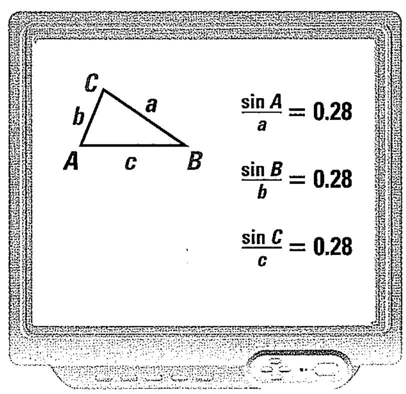
STEP 2 ***Measure parts of triangle***

Find the side lengths a, b, and c. Also find the measures of angles A, B, and C.



STEP 3 ***Calculate ratios***

Find the ratios $\frac{\sin A}{a}$, $\frac{\sin B}{b}$, and $\frac{\sin C}{c}$.



DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. What are the values of the ratios $\frac{\sin A}{a}$, $\frac{\sin B}{b}$, and $\frac{\sin C}{c}$ for your triangle? What do you notice about these values?

2. Change the shape of your triangle by dragging its vertices, and observe how the ratios you found in Step 3 change. Make a conjecture about how these ratios are related for *any* triangle.

13.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS23 for Exs. 13, 31, and 45

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 28, 41, 47, and 48

◆ = **MULTIPLE REPRESENTATIONS** Ex. 45

SKILL PRACTICE

1. **VOCABULARY** What information do you need to use the law of sines?

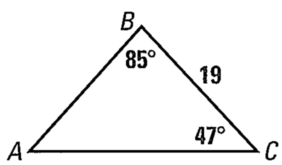
2. ★ **WRITING** Suppose a, b, and A are given for $\triangle ABC$ where $A < 90°$. Under what conditions would you have no triangle? one triangle? two triangles?

EXAMPLES 1, 2, 3, and 4 on pp. 882–884 for Exs. 3–28

IDENTIFYING CASES State the case (AAS, ASA, or SSA) applicable to the given measurements. Then decide whether the measurements determine *one triangle*, *two triangles*, or *no triangle*.

3. $A = 112°, a = 9, b = 4$
4. $A = 40°, C = 75°, c = 20$
5. $A = 52°, a = 32, b = 42$
6. $A = 37°, a = 8, b = 14$
7. $A = 28°, B = 64°, c = 55$
8. $A = 149°, a = 7, b = 10$
9. $B = 34°, b = 5, a = 16$
10. $B = 70°, b = 85, c = 88$
11. $C = 48°, c = 28, b = 20$

SOLVING TRIANGLES Solve $\triangle ABC$.

12.


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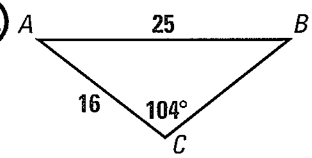

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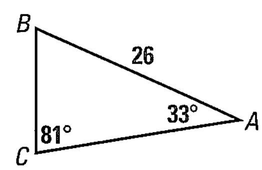

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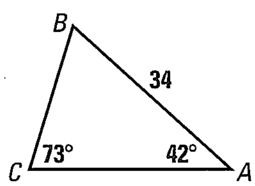

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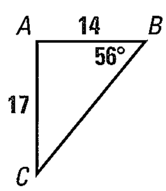
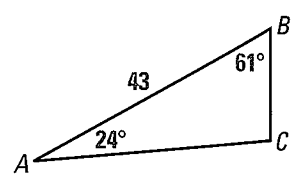
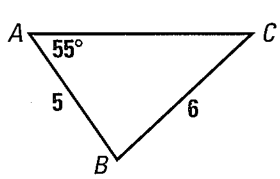


17.


SOLVING TRIANGLES Solve $\triangle ABC$. (*Hint:* Some of the "triangles" have no solution and some have two solutions.)

18. $A = 73°, a = 18, b = 11$
19. $A = 26°, C = 35°, b = 13$
20. $B = 102°, C = 43°, b = 21$
21. $A = 38°, a = 19, b = 25$
22. $A = 55°, B = 64°, c = 34$
23. $A = 114°, a = 15, b = 10$
24. $C = 98°, c = 29, a = 33$
25. $A = 49°, B = 32°, b = 44$
26. $B = 21°, b = 17, c = 32$

27. **ERROR ANALYSIS** *Describe* and correct the error in finding the measure of angle C in the triangle below.



$$\frac{\sin C}{6} = \frac{\sin 55°}{5}$$

$$\sin C = \frac{6 \sin 55°}{5} \approx 0.9830$$

$$C \approx 79.4°$$

28. ★ **MULTIPLE CHOICE** What is the side length c in $\triangle ABC$ if $A = 32°$, $C = 67°$, and $b = 31$ ft?

Ⓐ 16.6 ft Ⓑ 28.9 ft Ⓒ 33.3 ft Ⓓ 57.8 ft

EXAMPLE 5
on p. 885
for Exs. 29–41

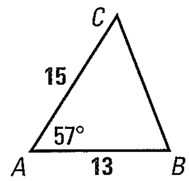
FINDING AREA **Find the area of $\triangle ABC$ with the given side lengths and included angle.**

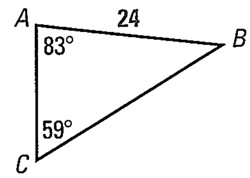
29. $B = 124°, a = 9, c = 11$ 30. $A = 68°, b = 13, c = 7$ 31. $A = 34°, b = 29, c = 36$

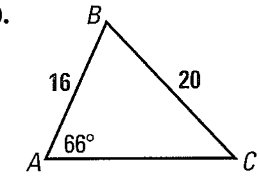
32. $C = 79°, a = 25, b = 17$ 33. $B = 57°, a = 9, c = 5$ 34. $C = 96°, a = 7, b = 15$

35. $A = 130°, b = 23, c = 20$ 36. $B = 60°, a = 19, c = 14$ 37. $C = 29°, a = 38, b = 31$

FINDING AREA **Find the area of $\triangle ABC$.**

38.


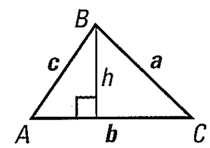
39.


40.


41. ★ **MULTIPLE CHOICE** What is the area of $\triangle ABC$ if $B = 52°$, $a = 29$, and $c = 24$?

Ⓐ 274 units^2 Ⓑ 348 units^2 Ⓒ 548 units^2 Ⓓ 696 units^2

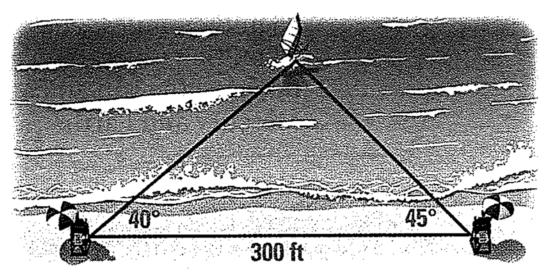
42. **CHALLENGE** Using the triangle shown at the right as a reference, derive the formulas for the area of a triangle given on page 885. Then use the area formulas to derive the law of sines.



PROBLEM SOLVING

EXAMPLE 1
on p. 882
for Ex. 43

43. **LIFEGUARDS** Two lifeguards are watching a windsurfer. Use the information in the diagram to find the distance from each lifeguard to the windsurfer.



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EXAMPLE 2
on p. 883
for Ex. 44

44. **NEW YORK CITY** You are on the observation deck of the Empire State Building looking at the Chrysler Building. When you turn 145° clockwise, you see the Statue of Liberty. You know that the Chrysler Building and the Empire State Building are about 0.6 mile apart and that the Chrysler Building and the Statue of Liberty are about 5.7 miles apart. Estimate the distance between the Empire State Building and the Statue of Liberty.

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EXAMPLE 5
on p. 885
for Exs. 45–46

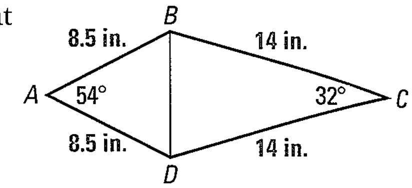
45. ◆ **MULTIPLE REPRESENTATIONS** You are fertilizing a triangular garden. One side of the garden is 62 feet long and another side is 54 feet long. The angle opposite the 62 foot side is 58°.

a. **Drawing a Diagram** Draw a diagram to represent this situation.

b. **Solving a Triangle** Use the law of sines to solve the triangle you drew in part (a).

c. **Applying a Formula** One bag of fertilizer covers an area of 200 square feet. How many bags of fertilizer will you need to cover the entire garden?

46. **MULTI-STEP PROBLEM** Quadrilateral *ABCD* shown at the right is a kite.



a. Find the area of $\triangle ABD$.

b. Find the area of $\triangle BCD$.

c. What is the area of the kite?

47. ★ **SHORT RESPONSE** A building is constructed on top of a cliff that is 300 meters high. A person standing on level ground below the cliff observes that the angle of elevation to the top of the building is 72°, and the angle of elevation to the top of the cliff is 63°.

a. How far away is the person from the base of the cliff?

b. *Describe* two different methods you can use to find the height of the building. Use one of these methods to find the building's height.

48. ★ **EXTENDED RESPONSE** Use a graphing calculator to explore how the included angle in the formulas on page 885 affects a triangle's area.

a. **Model** Choose lengths for two sides of the triangle. Let x represent the measure (in degrees) of the included angle. Write an equation that gives the triangle's area y as a function of x.

b. **Graphing Calculator** Enter the equation from part (a) into a graphing calculator. Use the *table* feature to examine values of the area for $0° < x° < 180°$. Does the area always increase as x increases? *Explain.*

c. **Interpret** What value of x maximizes the triangle's area? What is the maximum area, and how is it related to the side lengths you chose in part (a)?

49. **CHALLENGE** The distance between Mercury and the sun is approximately 36 million miles. The distance between Earth and the sun is approximately 93 million miles. If on a certain day the angle (measured from Earth) between the sun and Mercury is 22°, what are the possible distances between Mercury and Earth?

MIXED REVIEW

Perform the indicated operation. *(p. 420)*

50. $6\sqrt{13} - \sqrt{13}$

51. $5(250)^{1/3} - 10(54)^{1/3}$

52. $-2\sqrt[4]{160} + 4\sqrt[4]{810}$

53. $5(20)^{1/2} - 3(45)^{1/2}$

54. $9\sqrt[3]{56} - \sqrt[3]{189}$

55. $-6(88)^{1/3} + 9(297)^{1/3}$

Perform the indicated operation and simplify. *(p. 573)*

56. $\frac{5x^5}{x^3y} \cdot \frac{xy}{20xy^2}$

57. $\frac{40x^3y^3}{2xyz} \div \frac{10xy}{x^2yz^3}$

58. $\frac{3x^2 - 9}{x - 1} \cdot \frac{x + 7}{6x^2 - 18}$

59. $\frac{4x^2 - 16}{x^2 - 25} \cdot \frac{x + 5}{4x - 8}$

60. $\frac{x^2 + 5x + 6}{3x^2 + 13x + 14} \div (x + 3)$

61. $\frac{6x^2 + 11x + 4}{2x^2 + 3x - 1} \div \frac{6x - 8}{x + 1}$

PREVIEW
Prepare for Lesson 13.6 in Exs. 62–67.

Solve the equation for θ. *(p. 875)*

62. $\cos\theta = 0.75$; $270° < \theta < 360°$

63. $\cos\theta = -0.6$; $180° < \theta < 270°$

64. $\cos\theta = -0.35$; $90° < \theta < 180°$

65. $\cos\theta = 0.92$; $270° < \theta < 360°$

66. $\cos\theta = -0.28$; $180° < \theta < 270°$

67. $\cos\theta = 0.47$; $270° < \theta < 360°$

13.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS23 for Exs. 17, 25, and 45
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 20, 33, 34, 45, and 47

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In a triangle with sides of length a, b, and c, $\frac{1}{2}(a + b + c)$ is called the _?_.

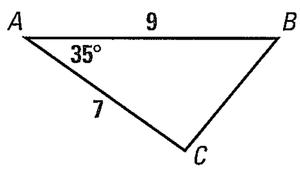
2. ★ **WRITING** Express Heron's formula in words.

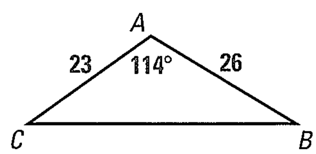
EXAMPLES 1 and 2 on pp. 889–890 for Exs. 3–20

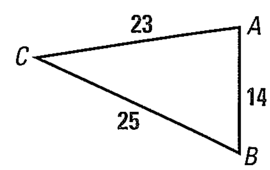
CHOOSING A METHOD For the given case, tell whether you would use the *law of sines* or the *law of cosines* to solve the triangle.

3. SSS
4. ASA
5. SSA
6. SAS
7. AAS

SOLVING TRIANGLES Solve $\triangle ABC$.

8. 

9. 

10. 

SOLVING TRIANGLES Solve $\triangle ABC$.

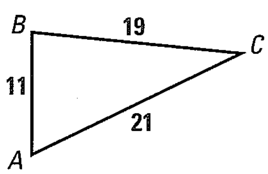
11. $B = 25°, a = 8, c = 6$
12. $A = 103°, b = 15, c = 24$
13. $a = 18, b = 28, c = 13$
14. $a = 38, b = 31, c = 35$
15. $C = 48°, a = 17, b = 20$
16. $B = 63°, a = 29, c = 38$
17. $a = 10, b = 3, c = 12$
18. $a = 23, b = 24, c = 20$
19. $C = 96°, a = 35, b = 43$

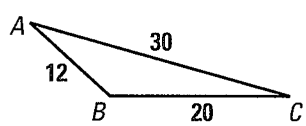
20. ★ **MULTIPLE CHOICE** What is the measure of angle B in $\triangle ABC$ if $a = 17$, $b = 29$, and $c = 14$?

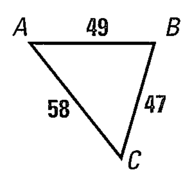
Ⓐ 18.7° Ⓑ 22.9° Ⓒ 111.2° Ⓓ 138.4°

EXAMPLE 4 on p. 891 for Exs. 21–33

FINDING AREA Find the area of $\triangle ABC$.

21. 

22. 

23. 

FINDING AREA Find the area of $\triangle ABC$ with the given side lengths.

24. $a = 12, b = 7, c = 8$
25. $a = 5, b = 11, c = 10$
26. $a = 25, b = 24, c = 19$
27. $a = 14, b = 20, c = 28$
28. $a = 31, b = 23, c = 17$
29. $a = 81, b = 67, c = 71$
30. $a = 43, b = 59, c = 48$
31. $a = 51, b = 51, c = 43$
32. $a = 38, b = 25, c = 61$

33. ★ **MULTIPLE CHOICE** What is the area of $\triangle ABC$ if $a = 21$, $b = 16$, and $c = 13$?

Ⓐ 66 units^2 Ⓑ 104 units^2 Ⓒ 1350 units^2 Ⓓ 4368 units^2

34. ★ **SHORT RESPONSE** Use the law of cosines to show that the measure of each angle of an equilateral triangle is 60°. *Explain* your reasoning.

35. ERROR ANALYSIS *Describe* and correct the error in finding the measure of angle A in $\triangle ABC$ if $a = 18$, $b = 15$, and $c = 10$.

$$\cos A = \frac{15^2 + 10^2 - 18^2}{2(18)(15)} \approx 0.0019$$

$$A \approx \cos^{-1} 0.0019 \approx 89.9°$$

CHOOSING A METHOD **Use the law of sines, the law of cosines, or the Pythagorean theorem to solve $\triangle ABC$.**

36. $A = 72°, B = 44°, b = 14$

37. $B = 98°, C = 37°, a = 18$

38. $C = 65°, a = 12, b = 21$

39. $B = 90°, a = 15, c = 6$

40. $C = 40°, b = 36, c = 27$

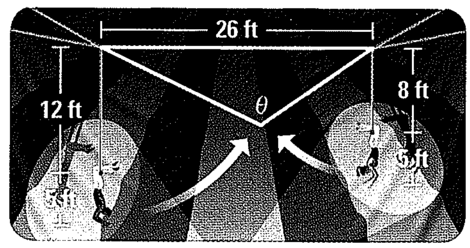
41. $a = 34, b = 19, c = 27$

42. CHALLENGE Given $\triangle ABC$ with height h, derive the law of cosines. *Explain* how the Pythagorean theorem is related to the law of cosines.

PROBLEM SOLVING

EXAMPLE 3
on p. 890
for Ex. 43

43. TRAPEZE ARTISTS The diagram shows the paths of two trapeze artists who are both 5 feet long when hanging by their knees. The "flyer" on the left bar is preparing to make hand-to-hand contact with the "catcher" on the right bar. At what angle θ will the two meet?



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EXAMPLE 4
on p. 891
for Exs. 44–45

44. RESEARCH TRIANGLE Raleigh, Durham, and Chapel Hill are three cities in North Carolina that form what is known as the Research Triangle. It is about 18 miles from Raleigh to Durham, 23 miles from Raleigh to Chapel Hill, and 8 miles from Chapel Hill to Durham. Find the area of the Research Triangle.

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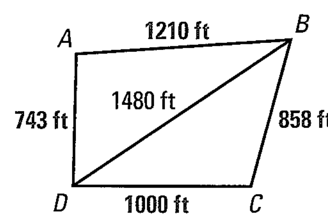
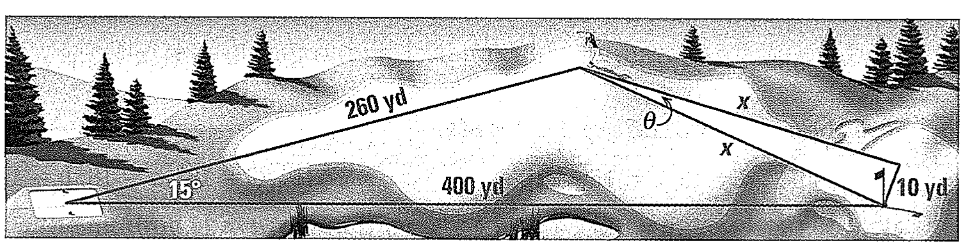
45. ★ **SHORT RESPONSE** The diagram shows the dimensions of a plot of land. What is the area of the land in acres? (Use the fact that 1 acre = 43,560 square feet.) *Explain* how you could also determine the area by first finding the length of $\overline{AC}$.



46. MULTI-STEP PROBLEM A golfer hits a drive 260 yards on a hole that is 400 yards long. The shot is 15° off target.



a. What is the distance x from the golfer's ball to the hole?

b. Assume the golfer is able to hit the ball precisely the distance found in part (a). What is the maximum angle θ by which the ball can be off target in order to land no more than 10 yards from the hole?

47. ★ **EXTENDED RESPONSE** Starting at the same point in a forest, two hikers take different paths. The first hiker walks due north at a speed of 2 miles per hour. The second hiker walks 60° east of north at a speed of 3 miles per hour.

a. How far apart are the hikers after 1 hour?

b. The two hikers carry walkie-talkies with a range of 10 miles. After how much time are the hikers out of range of each other?

c. Suppose after two hours the first hiker stops and tells the second hiker to meet her. How long will it take the second hiker to meet the first hiker? In what direction should the second hiker walk? *Explain* your reasoning.

48. CHALLENGE An airplane flies 55° east of north from city A to city B, a distance of 470 miles. Another airplane flies 7° north of east from city A to city C, a distance of 890 miles. What is the distance between cities B and C?

MIXED REVIEW

PREVIEW Prepare for Lesson 14.1 in Exs. 49–54.

Graph the function.

49. $y = 9 - 4x$ *(p. 89)*

50. $y = |4x - 5|$ *(p. 123)*

51. $f(x) = x^2 - 10x + 12$ *(p. 236)*

52. $y = 2\sqrt{x + 7} - 3$ *(p. 446)*

53. $y = 1.5e^{-x}$ *(p. 492)*

54. $g(x) = \ln(x + 4)$ *(p. 499)*

Divide using synthetic division. *(p. 362)*

55. $(x^3 + 8x^2 + 17x + 1) \div (x + 4)$

56. $(x^3 - 9x^2 + 26x - 17) \div (x - 3)$

57. $(x^4 + x^3 + 3x^2 + 7x + 4) \div (x + 2)$

58. $(x^4 + x^3 - 31x^2 + 8x - 22) \div (x - 5)$

Perform the indicated operation and state the domain. *(p. 428)*

59. $f + g; f(x) = x + 7, g(x) = 10x$

60. $f - g; f(x) = 5x^{1/2}, g(x) = -4x^{1/2}$

61. $f \cdot g; f(x) = 3x - 2, g(x) = 2x^3$

62. $f(g(x)); f(x) = x^2 - 6, g(x) = 8x + 11$

QUIZ *for Lessons 13.5–13.6*

Solve $\triangle ABC$. *(pp. 882 and 889)*

1. $A = 50°, B = 74°, c = 12$

2. $C = 66°, a = 18, c = 17$

3. $a = 20, b = 14, c = 23$

4. $C = 118°, a = 26, b = 34$

5. $A = 102°, C = 25°, a = 31$

6. $a = 49, b = 52, c = 38$

7. $B = 53°, a = 41, c = 29$

8. $A = 112°, B = 48°, c = 5$

Find the area of $\triangle ABC$. *(pp. 882 and 889)*

9. $B = 94°, a = 13, c = 15$

10. $C = 18°, a = 16, b = 11$

11. $a = 18, b = 25, c = 19$

12. $a = 27, b = 21, c = 37$

13. $a = 62, b = 47, c = 53$

14. $A = 70°, b = 44, c = 36$

15. GEOMETRY The base of a right triangular prism has sides of length 8 centimeters, 10 centimeters, and 13 centimeters. The height of the prism is 5 centimeters. What is the volume of the prism? *(p. 889)*

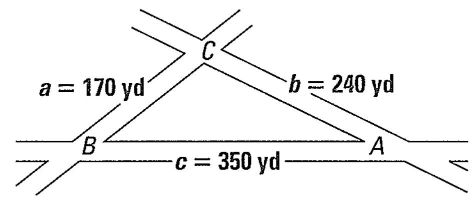
Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 891

MULTIPLE REPRESENTATIONS In Example 4 on page 891, you found the area of a triangle given the lengths of its sides by using Heron's formula. You can also find the area of the triangle by writing and solving a system of equations.

PROBLEM

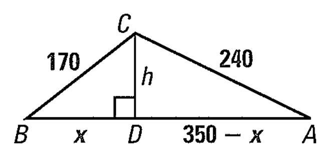
URBAN PLANNING The intersection of three streets forms a piece of land called a traffic triangle. Find the area of the traffic triangle shown.



METHOD

Using a System of Equations Use a system of quadratic equations to find the triangle's height h. Then find the area of the triangle using the formula $A = \frac{1}{2}bh$.

STEP 1 **Draw** a new diagram of the triangle as shown. Let h be the height of the triangle. The altitude labeled by h divides $\overline{AB}$ into two segments of length x and $350 - x$.



STEP 2 **Use** the Pythagorean theorem to write a system of quadratic equations.

$$h^2 + x^2 = 170^2$$
$$h^2 + (350 - x)^2 = 240^2$$

STEP 3 **Solve** the first equation for h^2 to get $h^2 = 170^2 - x^2$. Substitute this expression for h^2 in the second equation, and solve for x.

$$170^2 - x^2 + (350 - x)^2 = 240^2$$
$$28{,}900 - x^2 + 122{,}500 - 700x + x^2 = 57{,}600$$
$$-700x = -93{,}800$$
$$x = 134$$

STEP 4 **Use** the Pythagorean theorem to find that $h = \sqrt{170^2 - 134^2} \approx 104.6$.
So the area of the triangle is $A = \frac{1}{2}bh \approx \frac{1}{2}(350)(104.6) \approx 18{,}300$.

▶ The area of the triangle is about 18,300 square yards.

PRACTICE

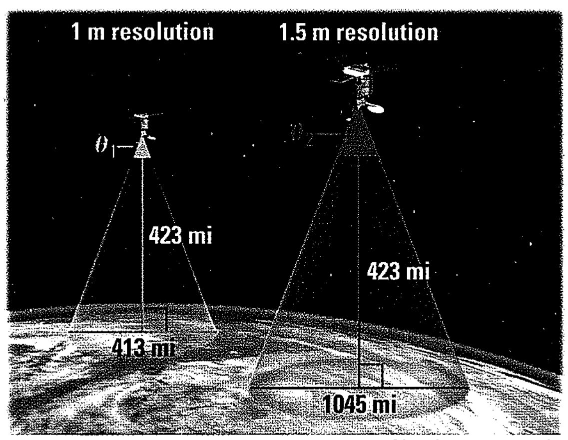
FINDING AREAS Use the method above to find the area of $\triangle ABC$ with the given side lengths.

1. $a = 12, b = 17, c = 26$
2. $a = 63, b = 92, c = 87$
3. $a = 101, b = 94, c = 153$
4. **WHAT IF?** Suppose $a = 200$ yd in the problem above. Find the area of the triangle.
5. **GARDEN AREA** A triangular garden has sides with lengths 50 feet, 38 feet, and 43 feet. Use the method above to find the area of the garden.

Lessons 13.4–13.6

1. **MULTI-STEP PROBLEM** You are buying a triangular piece of property. Two sides of the triangle are 540 yards and 330 yards long and have an included angle of 100°.
   a. How long is the third side of the property?
   b. The price of the land is $2500 per acre (1 acre = 4840 square yards). How much does the land cost?

2. **MULTI-STEP PROBLEM** The IKONOS satellite takes images of Earth's surface from a height of about 423 miles.

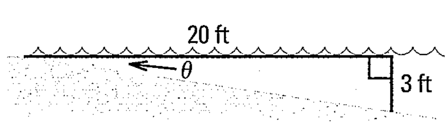


Not drawn to scale

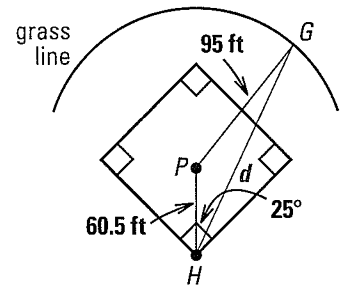
   a. IKONOS can take photographs that show objects 1 meter across provided the objects lie within a region about 413 miles across, as shown above. Find the value of θ_1.
   b. The largest region IKONOS can view is about 1045 miles across, as shown above. What is the maximum angle θ_2 through which IKONOS can rotate?

3. **OPEN-ENDED** A triangle has a 30° angle. Give the lengths of two sides that include the angle and produce a triangle with an area of 40 square inches.

4. **GRIDDED ANSWER** After walking 20 feet into the water at a beach, you notice that the depth of the water is 3 feet. Find the angle θ at which the beach slopes. Round your answer to the nearest tenth of a degree.

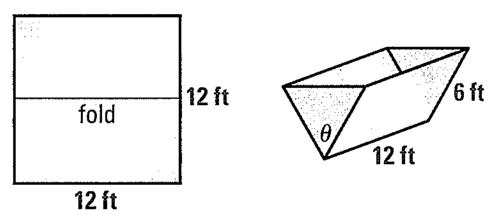


5. **SHORT RESPONSE** On a baseball field, the pitcher's mound at *P* is 60.5 feet from home plate at *H* and 95 feet from an arc where the outfield grass begins.



A ball is hit 25° to the right of the pitcher's mound and travels to the edge of the grass. What distance *d* must an outfielder at *G* throw the ball to make an out at home plate? *Explain.*

6. **EXTENDED RESPONSE** A trough can be made by folding a rectangular piece of metal in half and then enclosing the ends. The volume of water the trough can hold depends on how far you bend the metal.



   a. Predict the value of θ that will maximize the volume of the trough shown.
   b. Find the volume of the trough as a function of θ. (*Hint:* You will need to find the area of one of the triangular faces.)
   c. *Describe* how the volume changes as θ increases from 0° to 180°.
   d. What value of θ maximizes the volume? *Compare* this value with your prediction.

7. **SHORT RESPONSE** You are building a triangular concrete patio that has sides of length 8 feet, 11 feet, and 15 feet, and a thickness of 0.5 foot. If one bag of cement makes 0.33 cubic foot of concrete, how many bags of cement do you need to build the patio? *Explain* your reasoning.

13 CHAPTER SUMMARY

Animated Algebra
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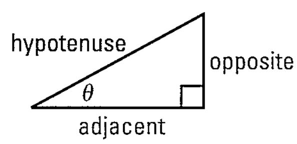
BIG IDEAS — *For Your Notebook*

Big Idea 1

Using Trigonometric Functions



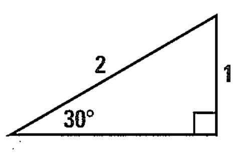
sine	cosine	tangent
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$	$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$	$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$
cosecant	**secant**	**cotangent**
$\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$	$\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$	$\cot\theta = \dfrac{\text{adj}}{\text{opp}}$

Big Idea 2

Using Inverse Trigonometric Functions

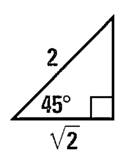
Inverse trigonometric functions can be used to solve trigonometric equations.

If $-1 \le a \le 1$, then the inverse sine of a is an angle θ, written $\sin^{-1} a = \theta$, where $\sin\theta = a$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.



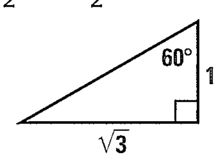
$\sin^{-1}\frac{1}{2} = 30°$

If $-1 \le a \le 1$, then the inverse cosine of a is an angle θ, written $\cos^{-1} a = \theta$, where $\cos\theta = a$ and $0 \le \theta \le \pi$.



$\cos^{-1}\frac{\sqrt{2}}{2} = 45°$

If a is any real number, then the inverse tangent of a is an angle θ, written $\tan^{-1} a = \theta$, where $\tan\theta = a$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.



$\tan^{-1}\sqrt{3} = 60°$

Big Idea 3

Applying the Law of Sines and Law of Cosines

Use the table below to help you remember when to apply each law.

If you know this information ...	use this law ...	to find this information.
angle-angle-side	Law of sines	remaining sides*
angle-side-angle	Law of sines	remaining sides*
side-side-angle	Law of sines	remaining side and one angle*
side-angle-side	Law of cosines	remaining side and one angle*
side-side-side	Law of cosines	two angles*

* Find the remaining angle by using the triangle sum theorem.

13 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- sine, *p. 852*
- cosine, *p. 852*
- tangent, *p. 852*
- cosecant, *p. 852*
- secant, *p. 852*
- cotangent, *p. 852*
- angle of elevation, *p. 855*
- angle of depression, *p. 855*
- initial side of an angle, *p. 859*
- terminal side of an angle, *p. 859*
- standard position of an angle, *p. 859*
- coterminal angles, *p. 860*
- radian, *p. 860*
- sector, *p. 861*
- central angle, *p. 861*
- unit circle, *p. 867*
- quadrantal angle, *p. 867*
- reference angle, *p. 868*
- inverse sine, *p. 875*
- inverse cosine, *p. 875*
- inverse tangent, *p. 875*
- law of sines, *p. 882*
- law of cosines, *p. 889*

VOCABULARY EXERCISES

1. **WRITING** *Describe* an angle in standard position.
2. Identify the relationship between the angles −225° and 135°.
3. What is the name of a circle with center at the origin and radius 1 unit?
4. Copy and complete: If $\cos\theta = a$ and $0 \le \theta \le \pi$, then the _?_ of a equals θ.
5. **WRITING** State the law of sines in words.

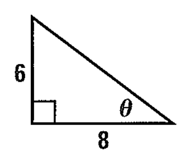
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 13.

13.1 Use Trigonometry with Right Triangles — *pp. 852–858*

EXAMPLE

Evaluate the six trigonometric functions of the angle θ.



From the Pythagorean theorem, the length of the hypotenuse is $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} = \frac{4}{5} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} = \frac{3}{4}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{10}{6} = \frac{5}{3} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{10}{8} = \frac{5}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{8}{6} = \frac{4}{3}$$

EXERCISES

EXAMPLES 1 and 3 on pp. 852–854 for Exs. 6–7

6. In $\triangle ABC$, $a = 4$, $b = 5$, and $C = 90°$. Evaluate the six trigonometric functions of angle B.
7. **HOT AIR BALLOON** You are standing 50 meters from a hot air balloon that is preparing to take off. The angle of elevation to the top of the balloon is 28°. Find the height of the balloon.

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Chapter Review Practice

13.2 Define General Angles and Use Radian Measure *pp. 859–865*

EXAMPLE

Convert 110° to radians and $\frac{7\pi}{12}$ radians to degrees.

$$110° = 110°\left(\frac{\pi \text{ radians}}{180°}\right) \qquad \frac{7\pi}{12}\text{ radians} = \left(\frac{7\pi}{12}\text{ radians}\right)\left(\frac{180°}{\pi \text{ radians}}\right)$$

$$= \frac{11\pi}{18}\text{ radians} \qquad = 105°$$

EXERCISES

EXAMPLE 3 on p. 861 for Exs. 8–11

Convert the degree measure to radians or the radian measure to degrees.

8. 145° **9.** −80° **10.** $\frac{4\pi}{3}$ **11.** $\frac{11\pi}{6}$

13.3 Evaluate Trigonometric Functions of Any Angle *pp. 866–872*

EXAMPLE

Evaluate sec 120°.

The reference angle is $\theta' = 180° - 120° = 60°$. The secant function is negative in Quadrant II, so you can write:

$$\sec 120° = -\sec 60° = -2$$

EXERCISES

EXAMPLE 4 on p. 869 for Exs. 12–15

Evaluate the function without using a calculator.

12. tan 330° **13.** csc (−405°) **14.** $\sin \frac{13\pi}{6}$ **15.** $\sec \frac{11\pi}{3}$

13.4 Evaluate Inverse Trigonometric Functions *pp. 875–880*

EXAMPLE

Evaluate $\tan^{-1} 1$ in both radians and degrees.

When $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, or $-90° < \theta < 90°$, the angle θ whose tangent is 1 is:

$$\theta = \tan^{-1} 1 = \frac{\pi}{4} \quad \text{or} \quad \theta = \tan^{-1} 1 = 45°$$

EXERCISES

EXAMPLES 1 and 4 on pp. 876–877 for Exs. 16–17

16. Evaluate $\sin^{-1}(-0.5)$ in both radians and degrees.

17. RAMP You use a 12 foot ramp to load items into a van. If the floor of the van is 4 feet off the ground, what is the angle of elevation of the ramp?

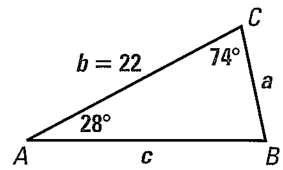
13 CHAPTER REVIEW

13.5 Apply the Law of Sines

pp. 882–888

EXAMPLE

Solve $\triangle ABC$ with $A = 28°$, $C = 74°$, and $b = 22$.



Find angle B: $B = 180° - 28° - 74° = 78°$.

Use the law of sines to solve for a and c.

$$\frac{a}{\sin 28°} = \frac{22}{\sin 78°} \qquad \frac{c}{\sin 74°} = \frac{22}{\sin 78°}$$

$$a = \frac{22 \sin 28°}{\sin 78°} \approx 10.6 \qquad c = \frac{22 \sin 74°}{\sin 78°} \approx 21.6$$

▶ For $\triangle ABC$, $B = 78°$, $a \approx 10.6$, and $c \approx 21.6$.

EXERCISES

EXAMPLES 1, 2, 3 and 4 on pp. 882–884 for Exs. 18–21

Solve $\triangle ABC$. (*Hint:* Some of the "triangles" may have no solution and some may have two solutions.)

18. $A = 43°$, $C = 83°$, $b = 12$

19. $B = 104°$, $b = 25$, $c = 18$

20. $C = 55°$, $a = 17$, $c = 15$

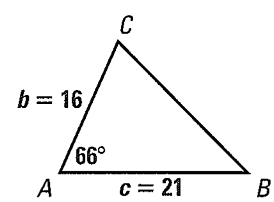
21. $B = 60°$, $C = 73°$, $b = 20$

13.6 Apply the Law of Cosines

pp. 889–894

EXAMPLE

Solve $\triangle ABC$ with $A = 66°$, $b = 16$, and $c = 21$.



Use the law of cosines to find the length a.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 16^2 + 21^2 - 2(16)(21) \cos 66°$$

$$a^2 \approx 423.7$$

$$a \approx 20.6$$

Now find angle B and angle C.

$$\frac{\sin B}{16} = \frac{\sin 66°}{20.6}$$

$$\sin B = \frac{16 \sin 66°}{20.6} \approx 0.7095$$

$$B = \sin^{-1} 0.7095 \approx 45.2°$$

$$C \approx 180° - 66° - 45.2° \approx 68.8°$$

▶ For $\triangle ABC$, $B \approx 45.2°$, $C \approx 68.8°$, and $a \approx 20.6$.

EXERCISES

EXAMPLES 1 and 2 on pp. 889–890 for Exs. 22–24

Solve $\triangle ABC$.

22. $a = 19$, $b = 11$, $c = 14$

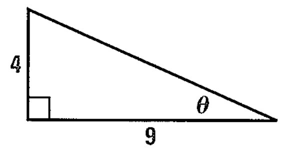
23. $B = 75°$, $a = 20$, $c = 17$

24. $a = 30$, $b = 35$, $c = 39$

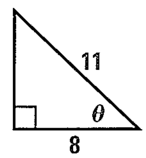
CHAPTER TEST

Evaluate the six trigonometric functions of the angle θ.

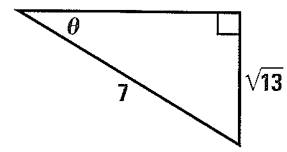
1.



2.



3.



Convert the degree measure to radians or the radian measure to degrees.

4. $260°$ **5.** $-50°$ **6.** $\frac{4\pi}{5}$ **7.** $\frac{8\pi}{3}$

Evaluate the function without using a calculator.

8. $\tan 150°$ **9.** $\sec(-480°)$ **10.** $\sin\left(-\frac{5\pi}{3}\right)$ **11.** $\cos\frac{11\pi}{6}$

Evaluate the expression in both radians and degrees without using a calculator.

12. $\cos^{-1} 1$ **13.** $\tan^{-1}\sqrt{3}$ **14.** $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ **15.** $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Solve $\triangle ABC$. (*Hint:* Some of the "triangles" may have no solution and some may have two solutions.)

16. $A = 47°, C = 32°, c = 12$

17. $a = 24, b = 12, c = 17$

18. $B = 63°, a = 11, b = 8$

19. $C = 101°, a = 23, b = 19$

20. $a = 24, b = 30, c = 21$

21. $A = 26°, B = 77°, c = 50$

Find the area of $\triangle ABC$.

22. $A = 81°, b = 16, c = 18$

23. $a = 8, b = 6, c = 7$

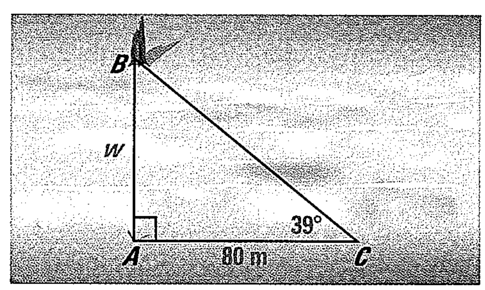
24. $a = 25, b = 24, c = 38$

25. $C = 111°, a = 7, b = 13$

26. $a = 16, b = 33, c = 24$

27. $B = 61°, a = 12, c = 18$

28. SURVEYING To measure the width of a river, you plant a stake at point A on one side of the riverbank, directly across from a tree stump at point B on the other side of the riverbank. From point A, you walk 80 meters along the riverbank to point C. You find the measure of angle C to be 39°. What is the width w of the river?



29. CONSTRUCTION A crane has a 200 foot arm with a lower end that is 5 feet off the ground. The arm has to reach to the top of a building that is 160 feet high. At what angle θ should the arm be set?

30. NAVIGATION A boat travels 40 miles due west before turning 20° and traveling an additional 25 miles. How far is the boat from its point of departure?