Alg 2

3 Linear Systems and Matrices

Before

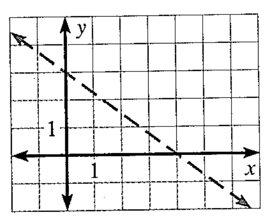
In previous chapters, you learned the following skills, which you'll use in Chapter 3: graphing equations, solving equations, and graphing inequalities.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. The **linear inequality** that represents the graph shown at the right is __?__.



2. The **graph of a linear inequality** in two variables is the set of all points in a coordinate plane that are __?__ of the inequality.

SKILLS CHECK

Graph the equation. ***(Review p. 89 for 3.1.)***

3. $x + y = 4$
4. $y = 3x - 3$
5. $-2x + 3y = -12$

Solve the equation. ***(Review p. 18 for 3.2, 3.4.)***

6. $2x - 12 = 16$
7. $-3x - 7 = 12$
8. $-2x + 5 = 2x - 5$

Graph the inequality in a coordinate plane. ***(Review p. 132 for 3.3.)***

9. $y \geq -x + 2$
10. $x + 4y < -16$
11. $3x + 5y > -5$

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Now

In Chapter 3, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 221. You will also use the key vocabulary listed below.

Big Ideas

1. **Solving systems of equations using a variety of methods**
2. **Graphing systems of equations and inequalities**
3. **Using matrices**

KEY VOCABULARY

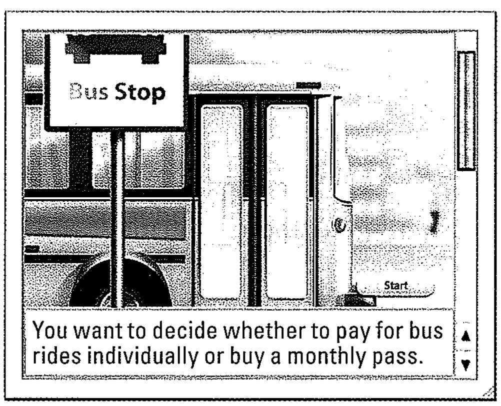
- system of two linear equations, *p. 153*
- consistent, *p. 154*
- inconsistent, *p. 154*
- independent, *p. 154*
- dependent, *p. 154*
- substitution method, *p. 160*
- elimination method, *p. 161*
- system of linear inequalities, *p. 168*
- system of three linear equations, *p. 178*
- ordered triple, *p. 178*
- matrix, *p. 187*
- determinant, *p. 203*
- Cramer's rule, *p. 205*
- identity matrix, *p. 210*
- inverse matrices, *p. 210*

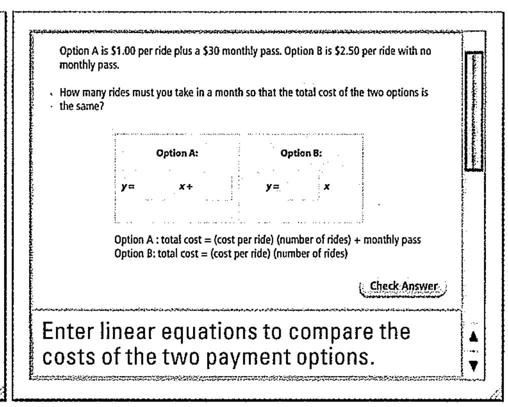
Why?

You can use systems of linear equations to solve real-world problems. For example, you can determine which of two payment options for riding a bus is more cost-effective.

Animated Algebra

The animation illustrated below for Example 4 on page 155 helps you answer this question: After how many bus rides will the cost of two payment options be the same?





Animated Algebra at classzone.com

Other animations for Chapter 3: pages 153, 161, 168, 196, 211, and 212

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Keystrokes

3.1 Solving Linear Systems Using Tables

MATERIALS • graphing calculator

QUESTION **How can you solve a system of linear equations using a table?**

An example of a *system of linear equations* in two variables x and y is the following:

$$y = 2x + 4 \qquad \textbf{Equation 1}$$
$$y = -3x + 44 \qquad \textbf{Equation 2}$$

A *solution* of a system of equations in two variables is an ordered pair (x, y) that is a solution of both equations. One way to solve a system is to use the *table* feature of a graphing calculator.

EXPLORE **Solve a system**

Use a table to solve the system of equations above.

STEP 1 ***Enter equations***

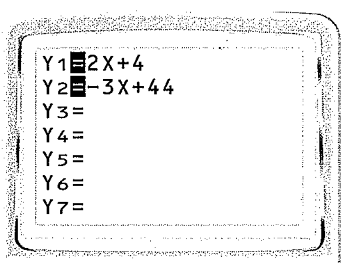
Press [Y=] to enter the equations. Enter Equation 1 as y_1 and Equation 2 as y_2.

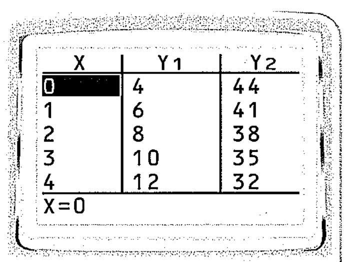
STEP 2 ***Make a table***

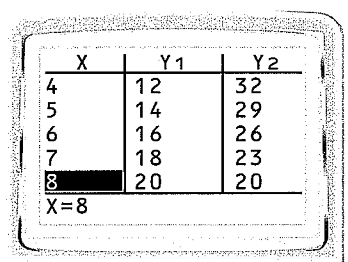
Set the starting x-value of the table to 0 and the step value to 1. Then use the *table* feature to make a table.

STEP 3 ***Find the solution***

Scroll through the table until you find an x-value for which y_1 and y_2 are equal. The table shows $y_1 = y_2 = 20$ when $x = 8$.







▶ The solution of the system is (8, 20).

DRAW CONCLUSIONS **Use your observations to complete these exercises**

Use a table to solve the system. If you are using a graphing calculator, you may need to first solve the equations in the system for y before entering them.

1. $y = 2x + 5$
   $y = -x + 2$

2. $y = 4x + 1$
   $y = 4x - 8$

3. $y = 4x - 3$
   $y = \frac{3}{2}x + 2$

4. $8x - 4y = 16$
   $-6x + 3y = 3$

5. $6x - 2y = -2$
   $-3x - 7y = 17$

6. $x + y = 11$
   $-x - y = -11$

7. Based on your results in Exercises 1–6, make a conjecture about the number of solutions a system of linear equations can have.

3.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS4 for Exs. 9, 21, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 29, 30, 37, and 39

◆ = **MULTIPLE REPRESENTATIONS** Ex. 38

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A consistent system that has exactly one solution is called ___?___.

2. ★ **WRITING** *Explain* how to identify the solution(s) of a system from the graphs of the equations in the system.

EXAMPLE 1 on p. 153 for Exs. 3–16

GRAPH AND CHECK Graph the linear system and estimate the solution. Then check the solution algebraically.

3. $y = -3x + 2$; $y = 2x - 3$
4. $y = 5x + 2$; $y = 3x$
5. $y = -x + 3$; $-x - 3y = -1$
6. $x + 2y = 2$; $x - 4y = 14$
7. $y = 2x - 10$; $x - 4y = 5$
8. $-x + 6y = -12$; $x + 6y = 12$
9. $y = -3x - 2$; $5x + 2y = -2$
10. $y = -3x - 13$; $-x - 2y = -4$
11. $x - 7y = 6$; $-3x + 21y = -18$
12. $y = 4x + 3$; $20x - 5y = -15$
13. $5x - 4y = 3$; $3x + 2y = 15$
14. $7x + y = -17$; $3x - 10y = 24$

15. ★ **MULTIPLE CHOICE** What is the solution of the system?

$$-4x - y = 2$$
$$7x + 2y = -5$$

Ⓐ (2, −6) Ⓑ (−1, 6) Ⓒ (1, −6) Ⓓ (−3, 8)

16. **ERROR ANALYSIS** A student used the check shown to conclude that (0, −1) is a solution of this system:

$$3x - 2y = 2$$
$$x + 2y = 6$$

$$3x - 2y = 2$$
$$3(0) - 2(-1) \stackrel{?}{=} 2$$
$$2 = 2$$

Describe and correct the student's error.

EXAMPLES 2 and 3 on p. 154 for Exs. 17–29

SOLVE AND CLASSIFY Solve the system. Then classify the system as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

17. $y = -1$; $3x + y = 5$
18. $2x - y = 4$; $x - 2y = -1$
19. $y = 3x + 2$; $y = 3x - 2$
20. $y = 2x - 1$; $-6x + 3y = -3$
21. $-20x + 12y = -24$; $5x - 3y = 6$
22. $4x - 5y = 0$; $3x - 5y = -5$
23. $3x + 7y = 6$; $2x + 9y = 4$
24. $4x + 5y = 3$; $6x + 9y = 9$
25. $8x + 9y = 15$; $5x - 2y = 17$
26. $\frac{1}{2}x - 3y = 10$; $\frac{1}{4}x + 2y = -2$
27. $3x - 2y = -15$; $x - \frac{2}{3}y = -5$
28. $\frac{5}{2}x - y = -4$; $5x - 2y = \frac{1}{4}$

29. ★ **MULTIPLE CHOICE** How would you classify the system?

$$-12x + 16y = 10$$
$$3x + 4y = -6$$

Ⓐ Consistent and independent Ⓑ Consistent and dependent

Ⓒ Inconsistent Ⓓ None of these

30. ★ **OPEN-ENDED MATH** Write a system of two linear equations that has the given number of solutions.

a. One solution **b.** No solution **c.** Infinitely many solutions

GRAPH AND CHECK Graph the system and estimate the solution(s). Then check the solution(s) algebraically.

31. $y = |x + 2|$
$y = x$

32. $y = |x - 1|$
$y = -x + 4$

33. $y = |x| - 2$
$y = 2$

34. **CHALLENGE** State the conditions on the constants a, b, c, and d for which the system below is **(a)** consistent and independent, **(b)** consistent and dependent, and **(c)** inconsistent.

$$y = ax + b$$
$$y = cx + d$$

PROBLEM SOLVING

EXAMPLE 4
on p. 155
for Exs. 35–39

35. **WORK SCHEDULE** You worked 14 hours last week and earned a total of \$96 before taxes. Your job as a lifeguard pays \$8 per hour, and your job as a cashier pays \$6 per hour. How many hours did you work at each job?

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36. **LAW ENFORCEMENT** During one calendar year, a state trooper issued a total of 375 citations for warnings and speeding tickets. Of these, there were 37 more warnings than speeding tickets. How many warnings and how many speeding tickets were issued?

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37. ★ **SHORT RESPONSE** A gym offers two options for membership plans. Option A includes an initiation fee of \$121 and costs \$1 per day. Option B has no initiation fee but costs \$12 per day. After how many days will the total costs of the gym membership plans be equal? How does your answer change if the daily cost of Option B increases? *Explain.*

38. **MULTIPLE REPRESENTATIONS** The price of refrigerator A is \$600, and the price of refrigerator B is \$1200. The cost of electricity needed to operate the refrigerators is \$50 per year for refrigerator A and \$40 per year for refrigerator B.

a. Writing Equations Write an equation for the cost of owning refrigerator A and an equation for the cost of owning refrigerator B.

b. Graphing Equations Graph the equations from part (a). After how many years are the total costs of owning the refrigerators equal?

c. Checking Reasonableness Is your solution from part (b) reasonable in this situation? *Explain.*

39. ★ **EXTENDED RESPONSE** The table below gives the winning times (in seconds) in the Olympic 100 meter freestyle swimming event for the period 1972–2000.

Years since 1972, x	0	4	8	12	16	20	24	28
Men's time, m	51.2	50.0	50.4	49.8	48.6	49.0	48.7	48.3
Women's time, w	58.6	55.7	54.8	55.9	54.9	54.6	54.4	53.8

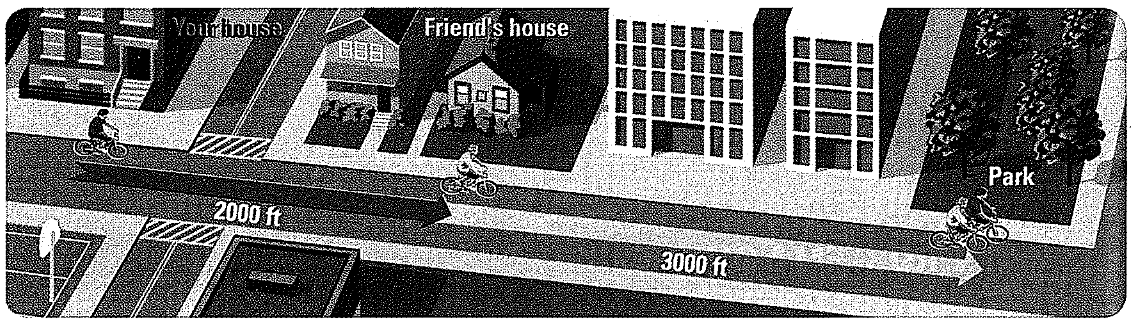
a. Use a graphing calculator to fit a line to the data pairs (x, m).

b. Use a graphing calculator to fit a line to the data pairs (x, w).

c. Graph the lines and predict when the women's performance will catch up to the men's performance.

d. Do you think your prediction from part (c) is reasonable? *Explain.*

40. **CHALLENGE** Your house and your friend's house are both on a street that passes by a park, as shown below.



At 1:00 P.M., you and your friend leave your houses on bicycles and head toward the park. You travel at a speed of 25 feet per second, and your friend also travels at a constant speed. You both reach the park at the same time.

a. Write and graph an equation giving your distance d (in feet) from the park after t seconds.

b. At what speed does your friend travel to the park? *Explain* how you found your answer.

c. Write an equation giving your friend's distance d (in feet) from the park after t seconds. Graph the equation in the same coordinate plane you used for part (a).

MIXED REVIEW

Solve the equation.

41. $8x + 1 = 3x - 14$ *(p. 18)*
42. $-4(x + 3) = 5x + 9$ *(p. 18)*
43. $x + 2 = \frac{3}{2}x - \frac{5}{4}$ *(p. 18)*
44. $|x - 18| = 9$ *(p. 51)*
45. $|2x + 5| = 12$ *(p. 51)*
46. $|5x - 18| = 17$ *(p. 51)*

PREVIEW
Prepare for Lesson 3.2 in Exs. 47–52.

Solve the equation for y. Then find the value of y for the given value of x. *(p. 26)*

47. $3x - 2y = 8; x = -2$
48. $-5x + y = -12; x = 9$
49. $8x - 3y = 10; x = 8$
50. $8x - 2y = 7; x = -1$
51. $16x + 9y = -24; x = -6$
52. $-12x + 9y = -60; x = -7$

53. **VETERINARY MEDICINE** The normal body temperature of a dog is 38°C. Your dog's temperature is 101°F. Does your dog have a fever? *Explain.* *(p. 26)*

Graphing Calculator **ACTIVITY** *Use after Lesson 3.1*

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Keystrokes

3.1 Graph Systems of Equations

QUESTION **How can you solve a system of linear equations using a graphing calculator?**

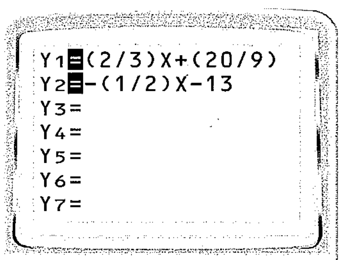
In Lesson 3.1, you learned to *estimate* the solution of a linear system by graphing. You can use the *intersect* feature of a graphing calculator to get an answer that is very close to, and sometimes *exactly* equal to, the actual solution.

EXAMPLE **Solve a system**

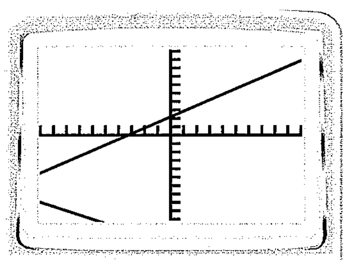
Use a graphing calculator to solve the system.

$6x - 9y = -20$ **Equation 1**
$2x + 4y = -52$ **Equation 2**

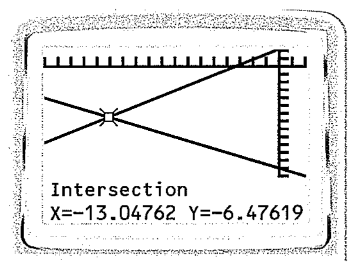
STEP 1 ***Enter equations***
Solve each equation for y. Then enter the revised equations into a graphing calculator.



STEP 2 ***Graph equations***
Graph the equations in the standard viewing window.



STEP 3 ***Find the solution***
Adjust the viewing window, and use the *intersect* feature to find the intersection point.



▶ The solution is about $(-13.05, -6.48)$.

PRACTICE

Solve the linear system using a graphing calculator.

1. $y = -x + 2$
$y = 2x - 5$

2. $y = -2x + 15$
$y = 5x - 4$

3. $-9x + 7y = 14$
$-3x + y = -17$

4. $-11x - 6y = -6$
$4x + 2y = 10$

5. $5x + 8y = -48$
$x + 3y = 27$

6. $-2x + 16y = 56$
$4x + 7y = -35$

7. **VACATION** Your family is planning a 7 day trip to Texas. You estimate that it will cost \$275 per day in San Antonio and \$400 per day in Dallas. Your budget for the 7 days is \$2300. How many days should you spend in each city?

8. **MOVIE TICKETS** In one day, a movie theater collected \$4600 from 800 people. The price of admission is \$7 for an adult and \$5 for a child. How many adults and how many children were admitted to the movie theater that day?

3.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS5 for Exs. 5, 29, and 59

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 40, 50, 57, 58, and 60

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: To solve a linear system where one of the coefficients is 1 or −1, it is usually easiest to use the _?_ method.

2. ★ **WRITING** *Explain* how to use the elimination method to solve a linear system.

EXAMPLES 1 and 4 on pp. 160–163 for Exs. 3–14

SUBSTITUTION METHOD **Solve the system using the substitution method.**

3. $2x + 5y = 7$; $x + 4y = 2$
4. $3x + y = 16$; $2x - 3y = -4$
5. $6x - 2y = 5$; $-3x + y = 7$
6. $x + 4y = 1$; $3x + 2y = -12$
7. $3x - y = 2$; $6x + 3y = 14$
8. $3x - 4y = -5$; $-x + 3y = -5$
9. $3x + 2y = 6$; $x - 4y = -12$
10. $6x - 3y = 15$; $-2x + y = -5$
11. $3x + y = -1$; $2x + 3y = 18$
12. $2x - y = 1$; $8x + 4y = 6$
13. $3x + 7y = 13$; $x + 3y = -7$
14. $2x + 5y = 10$; $-3x + y = 36$

EXAMPLES 2 and 4 on pp. 161–163 for Exs. 15–27

ELIMINATION METHOD **Solve the system using the elimination method.**

15. $2x + 6y = 17$; $2x - 10y = 9$
16. $4x - 2y = -16$; $-3x + 4y = 12$
17. $3x - 4y = -10$; $6x + 3y = -42$
18. $4x - 3y = 10$; $8x - 6y = 20$
19. $5x - 3y = -3$; $2x + 6y = 0$
20. $10x - 2y = 16$; $5x + 3y = -12$
21. $2x + 5y = 14$; $3x - 2y = -36$
22. $7x + 2y = 11$; $-2x + 3y = 29$
23. $3x + 4y = 18$; $6x + 8y = 18$
24. $2x + 5y = 13$; $6x + 2y = -13$
25. $4x - 5y = 13$; $6x + 2y = 48$
26. $6x - 4y = 14$; $2x + 8y = 21$

27. **ERROR ANALYSIS** *Describe* and correct the error in the first step of solving the system.

$$3x + 2y = 7$$
$$5x + 4y = 15$$

$$\begin{array}{rcl} -6x - 4y &=& 7 \\ 5x + 4y &=& 15 \\ \hline -x &=& 22 \\ x &=& -22 \end{array}$$

CHOOSING A METHOD **Solve the system using any algebraic method.**

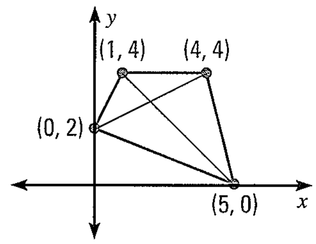
28. $3x + 2y = 11$; $4x + y = -2$
29. $2x - 3y = 8$; $-4x + 5y = -10$
30. $3x + 7y = -1$; $2x + 3y = 6$
31. $4x - 10y = 18$; $-2x + 5y = -9$
32. $3x - y = -2$; $5x + 2y = 15$
33. $x + 2y = -8$; $3x - 4y = -24$
34. $2x + 3y = -6$; $3x - 4y = 25$
35. $3x + y = 15$; $-x + 2y = -19$
36. $4x - 3y = 8$; $-8x + 6y = 16$
37. $4x - y = -10$; $6x + 2y = -1$
38. $7x + 5y = -12$; $3x - 4y = 1$
39. $2x + y = -1$; $-4x + 6y = 6$

40. ★ **MULTIPLE CHOICE** What is the solution of the linear system?

$$3x + 2y = 4$$
$$6x - 3y = -27$$

Ⓐ $(-2, -5)$ Ⓑ $(-2, 5)$ Ⓒ $(2, -5)$ Ⓓ $(2, 5)$

GEOMETRY Find the coordinates of the point where the diagonals of the quadrilateral intersect.

41.


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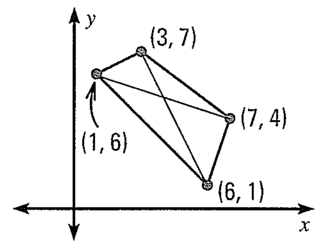

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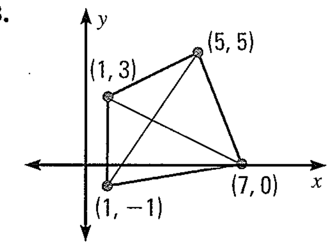

SOLVING LINEAR SYSTEMS Solve the system using any algebraic method.

44. $0.02x - 0.05y = -0.38$
$0.03x + 0.04y = 1.04$

45. $0.05x - 0.03y = 0.21$
$0.07x + 0.02y = 0.16$

46. $\frac{2}{3}x + 3y = -34$
$x - \frac{1}{2}y = -1$

47. $\frac{1}{2}x + \frac{2}{3}y = \frac{5}{6}$
$\frac{5}{12}x + \frac{7}{12}y = \frac{3}{4}$

48. $\frac{x+3}{4} + \frac{y-1}{3} = 1$
$2x - y = 12$

49. $\frac{x-1}{2} + \frac{y+2}{3} = 4$
$x - 2y = 5$

50. ★ **OPEN-ENDED MATH** Write a system of linear equations that has $(-1, 4)$ as its only solution. Verify that $(-1, 4)$ is a solution using either the substitution method or the elimination method.

SOLVING NONLINEAR SYSTEMS Use the elimination method to solve the system.

51. $7y + 18xy = 30$
$13y - 18xy = 90$

52. $xy - x = 14$
$5 - xy = 2x$

53. $2xy + y = 44$
$32 - xy = 3y$

54. **CHALLENGE** Find values of r, s, and t that produce the indicated solution(s).

$$-3x - 5y = 9$$
$$rx + sy = t$$

a. No solution **b.** Infinitely many solutions **c.** A solution of $(2, -3)$

PROBLEM SOLVING

EXAMPLE 3 on p. 162 for Exs. 55–59

55. **GUITAR SALES** In one week, a music store sold 9 guitars for a total of \$3611. Electric guitars sold for \$479 each and acoustic guitars sold for \$339 each. How many of each type of guitar were sold?

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56. **COUNTY FAIR** An adult pass for a county fair costs \$2 more than a children's pass. When 378 adult and 214 children's passes were sold, the total revenue was \$2384. Find the cost of an adult pass.

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57. ★ **SHORT RESPONSE** A company produces gas mowers and electric mowers at two factories. The company has orders for 2200 gas mowers and 1400 electric mowers. The production capacity of each factory (in mowers per week) is shown in the table.

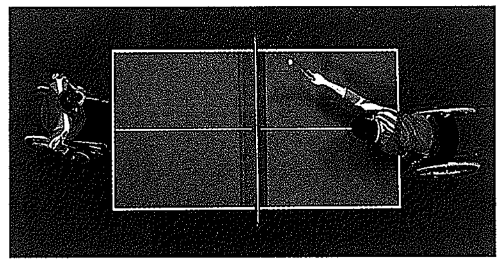
	Factory A	Factory B
Gas mowers	200	400
Electric mowers	100	300

Describe how the company can fill its orders by operating the factories simultaneously at full capacity. Write and solve a linear system to support your answer.

58. ★ **MULTIPLE CHOICE** The cost of 11 gallons of regular gasoline and 16 gallons of premium gasoline is \$58.55. Premium costs \$.20 more per gallon than regular. What is the cost of a gallon of premium gasoline?

(A) \$2.05 (B) \$2.25 (C) \$2.29 (D) \$2.55

59. **TABLE TENNIS** One evening, 76 people gathered to play doubles and singles table tennis. There were 26 games in progress at one time. A doubles game requires 4 players and a singles game requires 2 players. How many games of each kind were in progress at one time if all 76 people were playing?



60. ★ **EXTENDED RESPONSE** A local hospital is holding a two day marathon walk to raise funds for a new research facility. The total distance of the marathon is 26.2 miles. On the first day, Martha starts walking at 10:00 A.M. She walks 4 miles per hour. Carol starts two hours later than Martha but decides to run to catch up to Martha. Carol runs at a speed of 6 miles per hour.

a. Write an equation to represent the distance Martha travels.

b. Write an equation to represent the distance Carol travels.

c. Solve the system of equations to find when Carol will catch up to Martha.

d. Carol wants to reduce the time she takes to catch up to Martha by 1 hour. How can she do this by changing her starting time? How can she do this by changing her speed? *Explain* whether your answers are reasonable.

61. **BUSINESS** A nut wholesaler sells a mix of peanuts and cashews. The wholesaler charges \$2.80 per pound for peanuts and \$5.30 per pound for cashews. The mix is to sell for \$3.30 per pound. How many pounds of peanuts and how many pounds of cashews should be used to make 100 pounds of the mix?

62. **AVIATION** Flying with the wind, a plane flew 1000 miles in 5 hours. Flying against the wind, the plane could fly only 500 miles in the same amount of time. Find the speed of the plane in calm air and the speed of the wind.

63. **CHALLENGE** For a recent job, an electrician earned \$50 per hour, and the electrician's apprentice earned \$20 per hour. The electrician worked 4 hours more than the apprentice, and together they earned a total of \$550. How much money did each person earn?

○ = **WORKED-OUT SOLUTIONS on p. WS1**

★ = **STANDARDIZED TEST PRACTICE**

MIXED REVIEW

Solve the equation.

64. $-5x + 4 = 29$ *(p. 18)*

65. $6(2a - 3) = -30$ *(p. 18)*

66. $1.2m = 2.3m - 2.2$ *(p. 18)*

67. $|x + 3| = 4$ *(p. 51)*

68. $|2x + 11| = 3$ *(p. 51)*

69. $|-x + 7| = 13$ *(p. 51)*

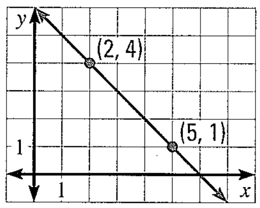
Tell whether the lines are *parallel, perpendicular,* or *neither.* *(p. 82)*

70. Line 1: through (2, 10) and (1, 5)
Line 2: through (3, −7) and (8, −8)

71. Line 1: through (4, 5) and (9, −2)
Line 2: through (6, −6) and (−2, −1)

Write an equation of the line. *(p. 98)*

72.



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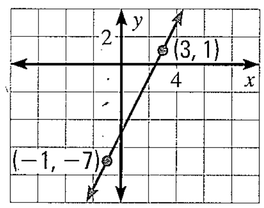

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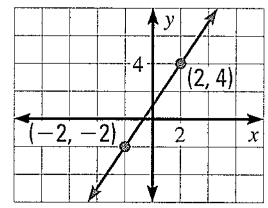


PREVIEW Prepare for Lesson 3.3 in Exs. 75–80.

Graph the inequality in a coordinate plane. *(p. 132)*

75. $x < -3$

76. $y \geq 2$

77. $2x + y > 1$

78. $y \leq -x + 4$

79. $4x - y \geq 5$

80. $y < -3x + 2$

QUIZ *for Lessons 3.1–3.2*

Graph the linear system and estimate the solution. Then check the solution algebraically. *(p. 153)*

1. $3x + y = 11$
$x - 2y = -8$

2. $2x + y = -5$
$-x + 3y = 6$

3. $x - 2y = -2$
$3x + y = -20$

Solve the system. Then classify the system as *consistent and independent, consistent and dependent,* or *inconsistent.* *(p. 153)*

4. $4x + 8y = 8$
$x + 2y = 6$

5. $-5x + 3y = -5$
$y = \frac{5}{3}x + 1$

6. $x - 2y = 2$
$2x - y = -5$

Solve the system using the substitution method. *(p. 160)*

7. $3x - y = -4$
$x + 3y = -28$

8. $x + 5y = 1$
$-3x + 4y = 16$

9. $6x + y = -6$
$4x + 3y = 17$

Solve the system using the elimination method. *(p. 160)*

10. $2x - 3y = -1$
$2x + 3y = -19$

11. $3x - 2y = 10$
$-6x + 4y = -20$

12. $2x + 3y = 17$
$5x + 8y = 20$

13. **HOME ELECTRONICS** To connect a VCR to a television set, you need a cable with special connectors at both ends. Suppose you buy a 6 foot cable for \$15.50 and a 3 foot cable for \$10.25. Assuming that the cost of a cable is the sum of the cost of the two connectors and the cost of the cable itself, what would you expect to pay for a 4 foot cable? *Explain* how you got your answer.

3.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS5 for Exs. 9, 19, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 3, 26, 27, 36, and 39

◆ = **MULTIPLE REPRESENTATIONS** Ex. 37

SKILL PRACTICE

1. **VOCABULARY** What must be true in order for an ordered pair to be a solution of a system of linear inequalities?

2. ★ **WRITING** *Describe* how to graph a system of linear inequalities.

EXAMPLES 1, 2, and 3 on pp. 168–169 for Exs. 3–16

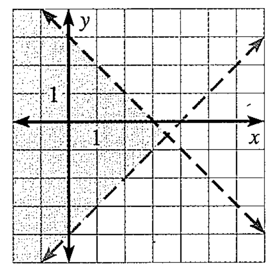
3. ★ **MULTIPLE CHOICE** Which system of inequalities is represented by the graph?

(A) $x + y > 3$
$-x + y < -4$

(B) $-x + y \geq -4$
$x + y \leq 3$

(C) $-2x + y > -4$
$2x + y < 3$

(D) $-x + y > -4$
$x + y < 3$



SYSTEMS OF TWO INEQUALITIES **Graph the system of inequalities.**

4. $x > -1$
$x < 3$

5. $x \leq 2$
$y \leq 5$

6. $y \geq 5$
$y \leq 1$

7. $-x + y < -3$
$-x + y > 4$

8. $y < 10$
$y > |x|$

9. $4x - 4y \geq -16$
$-x + 2y \geq -4$

10. $-x \geq y$
$-x + y \geq -5$

11. $y > |x| - 4$
$3y < -2x + 9$

12. $x + y \geq -3$
$-6x + 4y < 14$

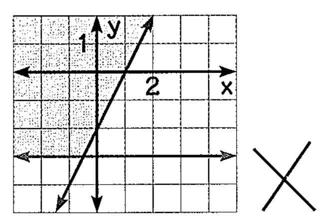
13. $2y < -5x - 10$
$5x + 2y > -2$

14. $3x - y > 12$
$-x + 8y > -4$

15. $x - 4y \leq -10$
$y \leq 3|x - 1|$

16. **ERROR ANALYSIS** *Describe* and correct the error in graphing the system of inequalities.

$y \geq -3$
$y \leq 2x - 2$



EXAMPLE 4 on p. 170 for Exs. 17–25

SYSTEMS OF THREE OR MORE INEQUALITIES **Graph the system of inequalities.**

17. $x < 6$
$y > -1$
$y < x$

18. $x \geq -8$
$y \leq -1$
$y < -2x - 4$

19. $3x + 2y > -6$
$-5x + 2y > -2$
$y < 5$

20. $x + y < 5$
$2x - y > 0$
$-x + 5y > -20$

21. $x \geq 2$
$-3x + y < -1$
$4x + 3y < 12$

22. $y \geq x$
$x + 3y < 5$
$2x + y \geq -3$

23. $y \geq 0$
$x > 3$
$x + y \geq -2$
$y < 4x$

24. $x + y < 5$
$x + y > -5$
$x - y < 4$
$x - y > -2$

25. $x \leq 10$
$x \geq -2$
$3x + 2y < 6$
$6x + 4y > -12$

26. ★ **MULTIPLE CHOICE** Which quadrant of the coordinate plane contains no solutions of the system of inequalities?

$$y \le -|x-3| + 2$$
$$4x - 5y \le 20$$

Ⓐ Quadrant I Ⓑ Quadrant II Ⓒ Quadrant III Ⓓ Quadrant IV

27. ★ **OPEN-ENDED MATH** Write a system of two linear inequalities that has $(2, -1)$ as a solution.

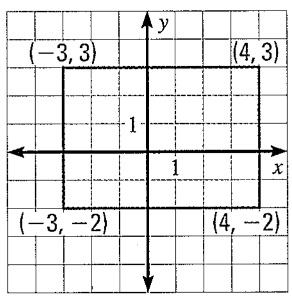
ABSOLUTE VALUE SYSTEMS Graph the system of inequalities.

28. $y < |x|$
$y > -|x|$

29. $y \le |x-2|$
$y \ge |x| - 2$

30. $y \le -|x-3| + 2$
$y > |x-3| - 1$

CHALLENGE Write a system of linear inequalities for the shaded region.

31.


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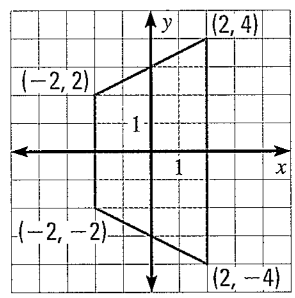

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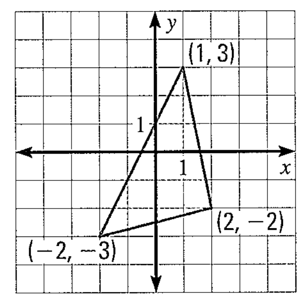

PROBLEM SOLVING

EXAMPLE 4 on p. 170 for Exs. 34–39

34. **SUMMER JOBS** You can work at most 20 hours next week. You need to earn at least \$92 to cover your weekly expenses. Your dog-walking job pays \$7.50 per hour and your job as a car wash attendant pays \$6 per hour. Write a system of linear inequalities to model the situation.

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35. **VIDEO GAME SALE** An online media store is having a sale, as described in the ad shown. Use the information in the ad to write and graph a system of inequalities for the regular video game prices and possible sale prices. Then use the graph to estimate the range of possible sale prices for games that are regularly priced at \$20.

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36. ★ **SHORT RESPONSE** A book on the care of tropical fish states that the pH level of the water should be between 8.0 and 8.3 pH units and the temperature of the water should be between 76°F and 80°F. Let x be the pH level and y be the temperature. Write and graph a system of inequalities that describes the proper pH level and temperature of the water. *Compare* this graph to the graph you would obtain if the temperatures were given in degrees Celsius.

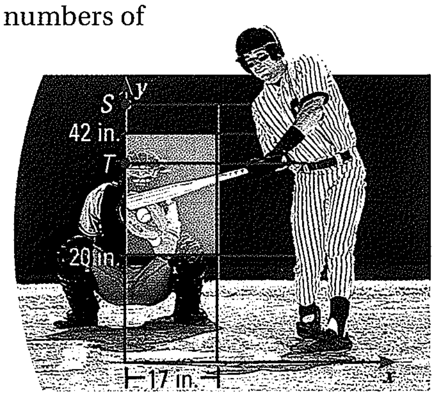
37. ◈ **MULTIPLE REPRESENTATIONS** The Junior-Senior Prom Committee must consist of 5 to 8 representatives from the junior and senior classes. The committee must include at least 2 juniors and at least 2 seniors. Let x be the number of juniors and y be the number of seniors.

a. Writing a System Write a system of inequalities to describe the situation.

b. Graphing a System Graph the system you wrote in part (a).

c. Finding Solutions Give two possible solutions for the numbers of juniors and seniors on the prom committee.

38. **BASEBALL** In baseball, the strike zone is a rectangle the width of home plate that extends from the batter's knees to a point halfway between the shoulders S and the top T of the uniform pants. The width of home plate is 17 inches. Suppose a batter's knees are 20 inches above the ground and the point halfway between his shoulders and the top of his pants is 42 inches above the ground. Write and graph a system of inequalities that represents the strike zone.



39. ★ **EXTENDED RESPONSE** A person's theoretical maximum heart rate (in heartbeats per minute) is $220 - x$ where x is the person's age in years ($20 \le x \le 65$). When a person exercises, it is recommended that the person strive for a heart rate that is at least 50% of the maximum and at most 75% of the maximum.

a. Write a system of linear inequalities that describes the given information.

b. Graph the system you wrote in part (a).

c. A 40-year-old person has a heart rate of 158 heartbeats per minute when exercising. Is the person's heart rate in the target zone? *Explain.*

40. **CHALLENGE** You and a friend are trying to guess the number of pennies in a jar. You both agree that the jar contains at least 500 pennies. You guess that there are x pennies, and your friend guesses that there are y pennies. The actual number of pennies in the jar is 1000. Write and graph a system of inequalities describing the values of x and y for which your guess is closer than your friend's guess to the actual number of pennies.

MIXED REVIEW

Evaluate the expression for the given values of x and y. *(p. 10)*

41. $6x - 8y$ when $x = 4$ and $y = -1$

42. $12x + 3y$ when $x = 4$ and $y = -5$

43. $x^2 - 2xy + 3y$ when $x = -2$ and $y = 3$

44. $4x^2y^2 - xy$ when $x = 5$ and $y = -6$

Solve the inequality. Then graph the solution. *(p. 41)*

45. $x - 8 \le -5$

46. $5x - 11 > -x + 7$

47. $9x + 2 \ge -3x - 13$

PREVIEW
Prepare for Lesson 3.4 in Exs. 48–53.

Solve the system of linear equations. *(p. 160)*

48. $-5x + y = -11$
$4x - y = 7$

49. $9x + 4y = -7$
$3x - 5y = -34$

50. $4x + 9y = -10$
$8x + 18y = 20$

51. $x - 5y = 18$
$2x + 3y = 10$

52. $16x - 12y = -8$
$8x - 6y = -4$

53. $16x + 5y = -4$
$8x - 2y = 7$

Extension

Use after Lesson 3.3

Use Linear Programming

GOAL Solve linear programming problems.

Key Vocabulary
- **constraints**
- **objective function**
- **linear programming**
- **feasible region**

BUSINESS A potter wants to make and sell serving bowls and plates. A bowl uses 5 pounds of clay. A plate uses 4 pounds of clay. The potter has 40 pounds of clay and wants to make at least 4 bowls.

Let x be the number of bowls made and let y be the number of plates made. You can represent the information above using linear inequalities called **constraints**.

$x \geq 4$	**Make at least 4 bowls.**
$y \geq 0$	**Number of plates cannot be negative.**
$5x + 4y \leq 40$	**Can use up to 40 pounds of clay.**

The profit on a bowl is \$35 and the profit on a plate is \$30. The potter's total profit P is given by the equation below, called the **objective function**.

$$P = 35x + 30y$$

It is reasonable for the potter to want to maximize profit subject to the given constraints. The process of maximizing or minimizing a linear objective function subject to constraints that are linear inequalities is called **linear programming**.

If the constraints are graphed, all of the points in the intersection are the combinations of bowls and plates that the potter can make. The intersection of the graphs is called the **feasible region**.

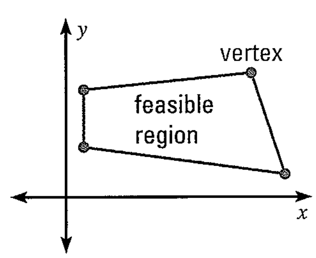
The following result tells you how to determine the optimal solution of a linear programming problem.

READING
A feasible region is *bounded* if it is completely enclosed by line segments.

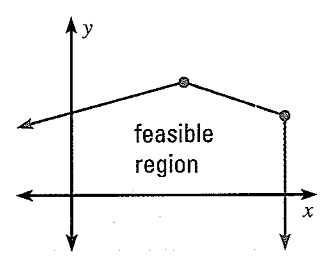
KEY CONCEPT *For Your Notebook*

Optimal Solution of a Linear Programming Problem

If the feasible region for a linear programming problem is bounded, then the objective function has both a maximum value and a minimum value on the region. Moreover, the maximum and minimum values each occur at a vertex of the feasible region.



Bounded region



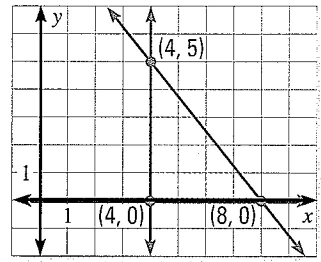
Unbounded region

EXAMPLE 1 Use linear programming to maximize profit

BUSINESS How many bowls and how many plates should the potter described on page 174 make in order to maximize profit?

Solution

STEP 1 **Graph** the system of constraints:



$x \geq 4$	**Make at least 4 bowls.**
$y \geq 0$	**Number of plates cannot be negative.**
$5x + 4y \leq 40$	**Can use up to 40 pounds of clay.**

STEP 2 **Evaluate** the profit function $P = 35x + 30y$ at each vertex of the feasible region.

At (4, 0): $P = 35(\mathbf{4}) + 30(\mathbf{0}) = 140$
At (8, 0): $P = 35(\mathbf{8}) + 30(\mathbf{0}) = 280$
At (4, 5): $P = 35(\mathbf{4}) + 30(\mathbf{5}) = 290$ ⟵ **Maximum**

▶ The potter can maximize profit by making 4 bowls and 5 plates.

EXAMPLE 2 Solve a linear programming problem

Find the minimum value and the maximum value of the objective function $C = 4x + 5y$ subject to the following constraints.

$x \geq 0$

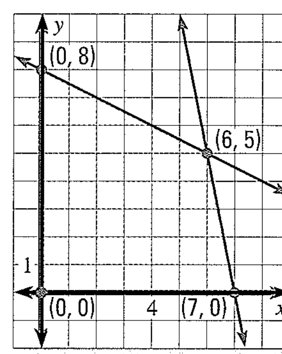
$y \geq 0$

$x + 2y \leq 16$

$5x + y \leq 35$

Solution

STEP 1 **Graph** the system of constraints. Find the coordinates of the vertices of the feasible region by solving systems of two linear equations. For example, the solution of the system



$$x + 2y = 16$$
$$5x + y = 35$$

gives the vertex (6, 5). The other three vertices are (0, 0), (7, 0), and (0, 8).

STEP 2 **Evaluate** the function $C = 4x + 5y$ at each of the vertices.

At (0, 0): $C = 4(\mathbf{0}) + 5(\mathbf{0}) = 0$ ⟵ **Minimum**
At (7, 0): $C = 4(\mathbf{7}) + 5(\mathbf{0}) = 28$
At (6, 5): $C = 4(\mathbf{6}) + 5(\mathbf{5}) = 49$ ⟵ **Maximum**
At (0, 8): $C = 4(\mathbf{0}) + 5(\mathbf{8}) = 40$

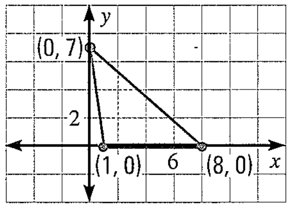
▶ The minimum value of C is 0. It occurs when $x = 0$ and $y = 0$.
The maximum value of C is 49. It occurs when $x = 6$ and $y = 5$.

PRACTICE

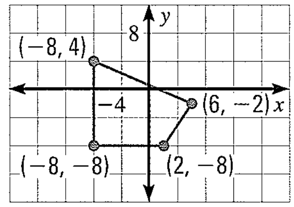
EXAMPLES 1 and 2 on p. 175 for Exs. 1–9

CHECKING VERTICES **Find the minimum and maximum values of the objective function for the given feasible region.**

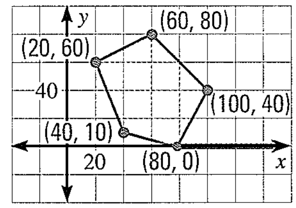
1. $C = x + 2y$



2. $C = 4x - 2y$



3. $C = 3x + 5y$



FINDING VALUES **Find the minimum and maximum values of the objective function subject to the given constraints.**

4. Objective function:
$C = 3x + 4y$

Constraints:
$x \geq 0$
$y \geq 0$
$x + y \leq 5$

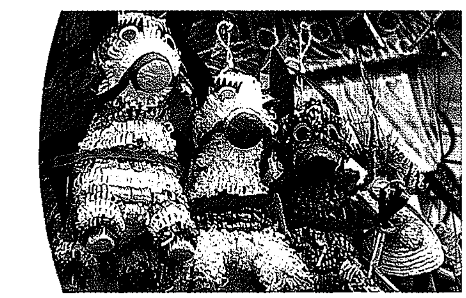
5. Objective function:
$C = 2x + 5y$

Constraints:
$x \leq 5$
$y \geq 3$
$-3x + 5y \leq 30$

6. Objective function:
$C = 3x + y$

Constraints:
$x \geq 0$
$y \geq -2$
$y \geq -x$
$x - 4y \geq -16$

7. CRAFT FAIR Piñatas are made to sell at a craft fair. It takes 2 hours to make a mini piñata and 3 hours to make a regular-sized piñata. The owner of the craft booth will make a profit of \$12 for each mini piñata sold and \$24 for each regular-sized piñata sold. If the craft booth owner has no more than 30 hours available to make piñatas and wants to have at least 12 piñatas to sell, how many of each size piñata should be made to maximize profit?



8. MANUFACTURING A company manufactures two types of printers, an inkjet printer and a laser printer. The company can make a total of 60 printers per day, and it has 120 labor-hours per day available. It takes 1 labor-hour to make an inkjet printer and 3 labor-hours to make a laser printer. The profit is \$40 per inkjet printer and \$60 per laser printer. How many of each type of printer should the company make to maximize its daily profit?

9. FARM STAND SALES You have 140 tomatoes and 13 onions left over from your garden. You want to use these to make jars of tomato sauce and jars of salsa to sell at a farm stand. A jar of tomato sauce requires 10 tomatoes and 1 onion, and a jar of salsa requires 5 tomatoes and $\frac{1}{4}$ onion. You will make a profit of \$2 on every jar of tomato sauce sold and a profit of \$1.50 on every jar of salsa sold. The owner of the farm stand wants at least three times as many jars of tomato sauce as jars of salsa. How many jars of each should you make to maximize profit?

10. CHALLENGE Consider the objective function $C = 2x + 3y$. Draw a feasible region that satisfies the given condition.

a. C has a maximum value but no minimum value on the region.

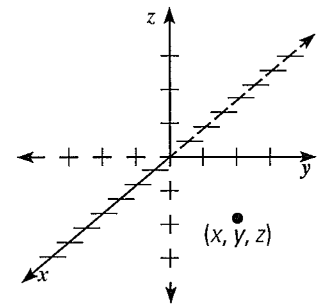
b. C has a minimum value but no maximum value on the region.

3.4 Graphing Linear Equations in Three Variables

MATERIALS • graph paper • ruler

QUESTION **What is the graph of a linear equation in three variables?**

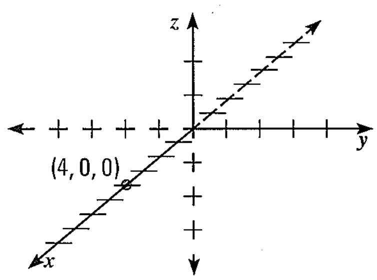
A *linear equation in three variables* has the form $ax + by + cz = d$. You can graph this type of equation in a three-dimensional coordinate system formed by three axes that divide space into eight *octants*. Each point in space is represented by an *ordered triple* (x, y, z).



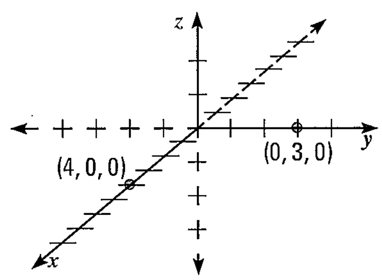
The graph of any equation in three variables is the set of all points (x, y, z) whose coordinates make the equation true. For a linear equation in three variables, the graph is a plane.

EXPLORE **Graph $3x + 4y + 6z = 12$**

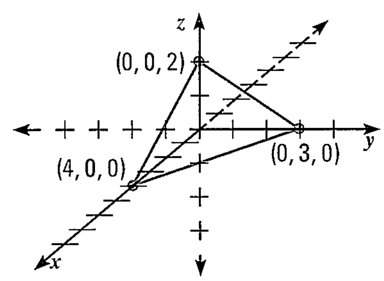
STEP 1 ***Find x-intercept***
Find the x-intercept by setting y and z equal to 0 and solving the resulting equation, $3x = 12$. The x-intercept is 4, so plot $(4, 0, 0)$.



STEP 2 ***Find y-intercept***
Find the y-intercept by setting x and z equal to 0 and solving the resulting equation, $4y = 12$. The y-intercept is 3, so plot $(0, 3, 0)$.



STEP 3 ***Find z-intercept***
Find the z-intercept by setting x and y equal to 0 and solving the resulting equation, $6z = 12$. The z-intercept is 2, so plot $(0, 0, 2)$. Then connect the points.



The triangular region shown in Step 3 is the portion of the graph of $3x + 4y + 6z = 12$ that lies in the first octant.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

Sketch the graph of the equation.

1. $4x + 3y + 2z = 12$
2. $2x + 2y + 3z = 6$
3. $x + 5y + 3z = 15$
4. $5x - y - 2z = 10$
5. $-7x + 7y + 2z = 14$
6. $2x + 9y - 3z = -18$

7. Suppose three linear equations in three variables are graphed in the same coordinate system. In how many different ways can the planes intersect? *Explain* your reasoning.

3.4 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS5 for Exs. 11, 25, and 45

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 24, 34, 45, and 47

SKILL PRACTICE

1. **VOCABULARY** Write a linear equation in three variables. What is the graph of such an equation?

2. ★ **WRITING** *Explain* how to use the substitution method to solve a system of three linear equations in three variables.

EXAMPLES 1, 2, and 3 on pp. 179–180 for Exs. 3–14

CHECKING SOLUTIONS Tell whether the given ordered triple is a solution of the system.

3. $(1, 4, -3)$
$2x - y + z = -5$
$5x + 2y - 2z = 19$
$x - 3y + z = -5$

4. $(-1, -2, 5)$
$4x - y + 3z = 13$
$x + y + z = 2$
$x + 3y - 2z = -17$

5. $(6, 0, -3)$
$x + 4y - 2z = 12$
$3x - y + 4z = 6$
$-x + 3y + z = -9$

6. $(-5, 1, 0)$
$3x + 4y - 2z = -11$
$2x + y - z = 11$
$x + 4y + 3z = -1$

7. $(2, 8, 4)$
$3x - y + 5z = 34$
$x + 3y - 6z = 2$
$-3x + y - 2z = -6$

8. $(0, -4, 7)$
$2x + 4y - z = -23$
$x - 5y - 3z = -1$
$-x + y + 4z = 24$

ELIMINATION METHOD Solve the system using the elimination method.

9. $3x + y + z = 14$
$-x + 2y - 3z = -9$
$5x - y + 5z = 30$

10. $2x - y + 2z = -7$
$-x + 2y - 4z = 5$
$x + 4y - 6z = -1$

11. $3x - y + 2z = 4$
$6x - 2y + 4z = -8$
$2x - y + 3z = 10$

12. $4x - y + 2z = -18$
$-x + 2y + z = 11$
$3x + 3y - 4z = 44$

13. $5x + y - z = 6$
$x + y + z = 2$
$3x + y = 4$

14. $2x + y - z = 9$
$-x + 6y + 2z = -17$
$5x + 7y + z = 4$

EXAMPLE 4 on p. 181 for Exs. 15–20

SUBSTITUTION METHOD Solve the system using the substitution method.

15. $x + y - z = 4$
$3x + 2y + 4z = 17$
$-x + 5y + z = 8$

16. $2x - y - z = 15$
$4x + 5y + 2z = 10$
$-x - 4y + 3z = -20$

17. $4x + y + 5z = -40$
$-3x + 2y + 4z = 10$
$x - y - 2z = -2$

18. $x + 3y - z = 12$
$2x + 4y - 2z = 6$
$-x - 2y + z = -6$

19. $2x - y + z = -2$
$6x + 3y - 4z = 8$
$-3x + 2y + 3z = -6$

20. $3x + 5y - z = 12$
$x + y + z = 0$
$-x + 2y + 2z = -27$

ERROR ANALYSIS ***Describe*** **and correct the error in the first step of solving the system.**

$$\begin{aligned} 2x + y - 2z &= 23 \\ 3x + 2y + z &= 11 \\ x - y + z &= -2 \end{aligned}$$

21.
$$\begin{array}{r} 2x + y - 2z = 23 \\ 6x + 2y + 2z = 22 \\ \hline 8x + 3y = 45 \end{array}$$

22.
$z = 11 + 3x + 2y$
$2x + y - 2(11 + 3x + 2y) = 23$
$-4x - 3y = 45$

23. ★ **MULTIPLE CHOICE** Which ordered triple is a solution of the system?

$$\begin{aligned} 2x + 5y + 3z &= 10 \\ 3x - y + 4z &= 8 \\ 5x - 2y + 7z &= 12 \end{aligned}$$

Ⓐ (7, 1, −3) Ⓑ (7, −1, −3) Ⓒ (7, 1, 3) Ⓓ (−7, 1, −3)

24. ★ **MULTIPLE CHOICE** Which ordered triple describes all of the solutions of the system?

$$\begin{aligned} 2x - 2y - z &= 6 \\ -x + y + 3z &= -3 \\ 3x - 3y + 2z &= 9 \end{aligned}$$

Ⓐ $(-x, x + 2, 0)$ Ⓑ $(x, x - 3, 0)$ Ⓒ $(x + 2, x, 0)$ Ⓓ $(0, y, y + 4)$

CHOOSING A METHOD **Solve the system using any algebraic method.**

25. $x + 5y - 2z = -1$
$-x - 2y + z = 6$
$-2x - 7y + 3z = 7$

26. $4x + 5y + 3z = 15$
$x - 3y + 2z = -6$
$-x + 2y - z = 3$

27. $6x + y - z = -2$
$x + 6y + 3z = 23$
$-x + y + 2z = 5$

28. $x + 2y = -1$
$3x - y + 4z = 17$
$-4x + 2y - 3z = -30$

29. $2x - y + 2z = -21$
$x + 5y - z = 25$
$-3x + 2y + 4z = 6$

30. $4x - 8y + 2z = 10$
$-3x + y - 2z = 6$
$2x - 4y + z = 8$

31. $-x + 5y - z = -16$
$2x + 3y + 4z = 18$
$x + y - z = -8$

32. $2x - y + 4z = 19$
$-x + 3y - 2z = -7$
$4x + 2y + 3z = 37$

33. $x + y + z = 3$
$3x - 4y + 2z = -28$
$-x + 5y + z = 23$

34. ★ **OPEN-ENDED MATH** Write a system of three linear equations in three variables that has the given number of solutions.

a. One solution **b.** No solution **c.** Infinitely many solutions

SYSTEMS WITH FRACTIONS **Solve the system using any algebraic method.**

35. $x + \frac{1}{2}y + \frac{1}{2}z = \frac{5}{2}$
$\frac{3}{4}x + \frac{1}{4}y + \frac{3}{2}z = \frac{7}{4}$
$\frac{1}{3}x + \frac{3}{2}y + \frac{2}{3}z = \frac{13}{6}$

36. $\frac{1}{3}x + \frac{5}{6}y + \frac{2}{3}z = \frac{4}{3}$
$\frac{1}{6}x + \frac{2}{3}y + \frac{1}{4}z = \frac{5}{6}$
$\frac{2}{3}x + \frac{1}{6}y + \frac{3}{2}z = \frac{4}{3}$

37. **REASONING** For what values of a, b, and c does the linear system shown have (−1, 2, −3) as its only solution? *Explain* your reasoning.

$$\begin{aligned} x + 2y - 3z &= a \\ -x - y + z &= b \\ 2x + 3y - 2z &= c \end{aligned}$$

CHALLENGE **Solve the system of equations. *Describe* each step of your solution.**

38. $w + x + y + z = 2$
$2w - x + 2y - z = 1$
$-w + 2x - y + 2z = -2$
$3w + x + y - z = -5$

39. $2w + x - 3y + z = 4$
$w - 3x + y + z = 32$
$-w + 2x + 2y - z = -10$
$w + x - y + 3z = 14$

40. $w + 2x + 5y = 11$
$-2w + x + 4y + 2z = -7$
$w + 2x - 2y + 5z = 3$
$-3w + x = -1$

41. $2w + 7x - 3y = 41$
$-w - 2x + y = -13$
$-2w + 4x + z = 12$
$-w - x + y = -8$

PROBLEM SOLVING

EXAMPLE 4
on p. 181
for Exs. 42–47

42. PIZZA SPECIALS At a pizza shop, two small pizzas, a liter of soda, and a salad cost \$14; one small pizza, a liter of soda, and three salads cost \$15; and three small pizzas and a liter of soda cost \$16. What is the cost of one small pizza? of one liter of soda? of one salad?

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43. HEALTH CLUB The juice bar at a health club receives a delivery of juice at the beginning of each month. Over a three month period, the health club received 1200 gallons of orange juice, 900 gallons of pineapple juice, and 1000 gallons of grapefruit juice. The table shows the composition of each juice delivery. How many gallons of juice did the health club receive in each delivery?

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Juice	1st delivery	2nd delivery	3rd delivery
Orange	70%	50%	30%
Pineapple	20%	30%	30%
Grapefruit	10%	20%	40%

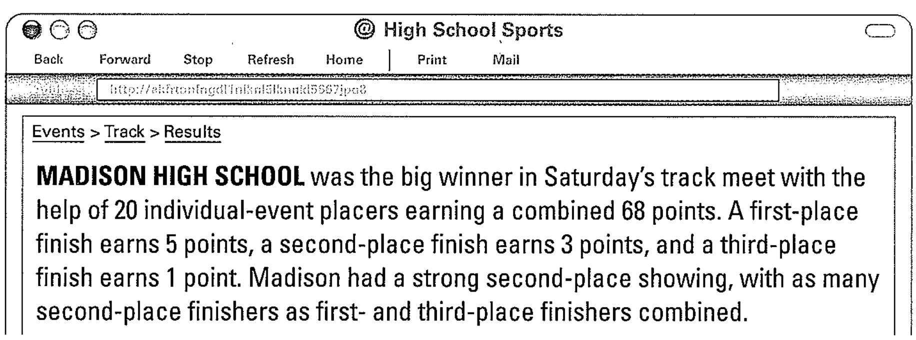
44. MULTI-STEP PROBLEM You make a tape of your friend's three favorite TV shows: a comedy, a drama, and a reality show. An episode of the comedy lasts 30 minutes, while an episode of the drama or the reality show lasts 60 minutes. The tape can hold 360 minutes of programming. You completely fill the tape with 7 episodes and include twice as many episodes of the drama as the comedy.

a. Write a system of equations to represent this situation.

b. Solve the system from part (a). How many episodes of each show are on the tape?

c. How would your answer to part (b) change if you completely filled the tape with only 5 episodes but still included twice as many episodes of the drama as the comedy?

45. ★ **SHORT RESPONSE** The following Internet announcement describes the results of a high school track meet.



a. Write and solve a system of equations to find the number of athletes who finished in first place, in second place, and in third place.

b. Suppose the announcement had claimed that the Madison athletes scored a total of 70 points instead of 68 points. Show that this claim must be false because the solution of the resulting linear system is not reasonable.

46. **FIELD TRIP** You and two friends buy snacks for a field trip. You spend a total of \$8, Jeff spends \$9, and Curtis spends \$9. The table shows the amounts of mixed nuts, granola, and dried fruit that each person purchased. What is the price per pound of each type of snack?

	Mixed nuts	Granola	Dried fruit
You	1 lb	0.5 lb	1 lb
Jeff	2 lb	0.5 lb	0.5 lb
Curtis	1 lb	2 lb	0.5 lb

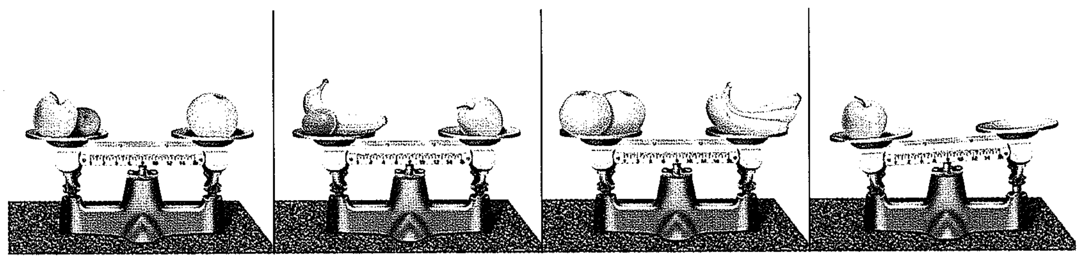
47. ★ **EXTENDED RESPONSE** A florist must make 5 identical bridesmaid bouquets for a wedding. She has a budget of \$160 and wants 12 flowers for each bouquet. Roses cost \$2.50 each, lilies cost \$4 each, and irises cost \$2 each. She wants twice as many roses as the other two types of flowers combined.

a. **Write** Write a system of equations to represent this situation.

b. **Solve** Solve the system of equations. How many of each type of flower should be in each bouquet?

c. **Analyze** Suppose there is no limitation on the total cost of the bouquets. Does the problem still have a unique solution? If so, state the unique solution. If not, give three possible solutions.

48. **CHALLENGE** Write a system of equations to represent the first three pictures below. Use the system to determine how many tangerines will balance the apple in the final picture. *Note:* The first picture shows that one tangerine and one apple balance one grapefruit.



MIXED REVIEW

PREVIEW
Prepare for Lesson 3.5 in Exs. 49–52.

Perform the indicated operation. *(p. 975)*

49. $15 + (-8)$ 50. $-4 - (-13)$ 51. $15 \cdot 7$ 52. $-4(-8)$

Find the slope of the line passing through the given points. Then tell whether the line *rises, falls, is horizontal,* or *is vertical.* *(p. 82)*

53. $(1, -4), (2, 6)$ 54. $(4, 2), (-18, 1)$ 55. $(6, -6), (-6, 6)$

56. $(-5, 2), (-5, 10)$ 57. $(-2, 4), (-6, 8)$ 58. $(-7, 3), (5, 3)$

Solve the system using any algebraic method. *(p. 160)*

59. $3x - y = -7$
$2x + 3y = 21$

60. $3x + 2y = -3$
$4x - 3y = -38$

61. $5x + y = 11$
$2x + 3y = -19$

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 3.1–3.4

1. **MULTI-STEP PROBLEM** You are making jewelry to sell at a craft fair. The cost of materials is \$3.50 to make one necklace and \$2.50 to make one bracelet. You sell the necklaces for \$9.00 each and the bracelets for \$7.50 each. You spend a total of \$121 on materials and sell all of the jewelry for a total of \$324.
   a. Write a system of equations that represents this situation.
   b. Solve the system to find how many necklaces and how many bracelets were sold.

2. **MULTI-STEP PROBLEM** You are making gift baskets. Each basket will contain three different kinds of candles: tapers, pillars, and jar candles. Tapers cost \$1 each, pillars cost \$4 each, and jar candles cost \$6 each. You put 8 candles costing a total of \$24 in each basket, and you include as many tapers as pillars and jar candles combined.
   a. Write a system of equations that represents this situation.
   b. Solve the system of equations. How many of each type of candle is in a basket?
   c. Suppose there are no restrictions on the value of the candles included in each basket. Does the problem still have a unique solution? If so, state the unique solution. If not, give three possible solutions.

3. **OPEN-ENDED** Write a system of linear inequalities whose graph is the interior of a right triangle.

4. **SHORT RESPONSE** A restaurant has 20 tables. Each table can seat either 4 people or 6 people. The restaurant can seat a total of 90 people.
   a. How many 4 seat tables and how many 6 seat tables does the restaurant have?
   b. *Describe* three ways in which the restaurant can increase its capacity to 140 people by buying additional 4 seat and 6 seat tables.

5. **GRIDDED ANSWER** At a snack booth, one soda, one pretzel, and two hot dogs cost \$7; two sodas, one pretzel, and two hot dogs cost \$8; and one soda and four hot dogs cost \$10. What is the price (in dollars) of one hot dog?

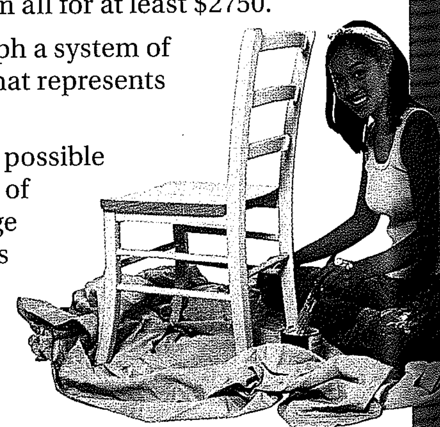
6. **GRIDDED ANSWER** Your school is planning a 5 hour outing at a community park. The park rents bicycles for \$8 per hour and inline skates for \$6 per hour. The total budget per student is \$34. A student bikes and skates the entire time and uses all the money budgeted. How many hours does the student spend inline skating?

7. **EXTENDED RESPONSE** The table shows the expected life spans at birth for men and women born in the years 1996–2000.

Years since 1996, x	Men's life span (years), m	Women's life span (years), w
0	73.0	79.0
1	73.6	79.4
2	73.8	79.5
3	73.9	79.4
4	74.3	79.7

   a. Write an equation of the best-fitting line for the data pairs (x, m).
   b. Write an equation of the best-fitting line for the data pairs (x, w).
   c. Graph the equations from parts (a) and (b) and estimate the coordinates of the intersection point. *Explain* what this point represents.

8. **MULTI-STEP PROBLEM** A store orders wooden chairs, hand paints them, and sells the chairs for a profit. A small chair costs the store \$45 and sells for \$80. A large chair costs the store \$70 and sells for \$110. The store wants to pay no more than \$2000 for its next order of chairs and wants to sell them all for at least \$2750.
   a. Write and graph a system of inequalities that represents this situation.
   b. Identify three possible combinations of small and large wooden chairs that the store can buy and sell.



SOLVING MATRIX EQUATIONS You can use what you know about matrix operations and matrix equality to solve an equation involving matrices.

EXAMPLE 4 Solve a matrix equation

Solve the matrix equation for x and y.

$$3\left(\begin{bmatrix} 5x & -2 \\ 6 & -4 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ -5 & -y \end{bmatrix}\right) = \begin{bmatrix} -21 & 15 \\ 3 & -24 \end{bmatrix}$$

Solution

Simplify the left side of the equation.

$$3\left(\begin{bmatrix} 5x & -2 \\ 6 & -4 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ -5 & -y \end{bmatrix}\right) = \begin{bmatrix} -21 & 15 \\ 3 & -24 \end{bmatrix}$$ **Write original equation.**

$$3\begin{bmatrix} 5x+3 & 5 \\ 1 & -4-y \end{bmatrix} = \begin{bmatrix} -21 & 15 \\ 3 & -24 \end{bmatrix}$$ **Add matrices inside parentheses.**

$$\begin{bmatrix} \mathbf{15x+9} & \mathbf{15} \\ \mathbf{3} & \mathbf{-12-3y} \end{bmatrix} = \begin{bmatrix} \mathbf{-21} & \mathbf{15} \\ \mathbf{3} & \mathbf{-24} \end{bmatrix}$$ **Perform scalar multiplication.**

Equate corresponding elements and solve the two resulting equations.

$$\mathbf{15x + 9 = -21} \qquad \mathbf{-12 - 3y = -24}$$
$$x = -2 \qquad y = 4$$

▶ The solution is $x = -2$ and $y = 4$.

✓ **GUIDED PRACTICE** for Examples 3 and 4

5. In Example 3, find $B - A$ and explain what information this matrix gives.

6. Solve $-2\left(\begin{bmatrix} -3x & -1 \\ 4 & y \end{bmatrix} + \begin{bmatrix} 9 & -4 \\ -5 & 3 \end{bmatrix}\right) = \begin{bmatrix} 12 & 10 \\ 2 & -18 \end{bmatrix}$ for x and y.

3.5 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS6 for Exs. 5, 21, and 33
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 28, 29, 33, and 34

SKILL PRACTICE

1. VOCABULARY Copy and complete: The _?_ of a matrix with 3 rows and 4 columns are 3×4.

2. ★ WRITING *Describe* how to determine whether two matrices are equal.

EXAMPLE 1
on p. 187
for Exs. 3–9

3. ERROR ANALYSIS *Describe* and correct the error in adding the matrices.

$$\begin{bmatrix} 9 \\ -5 \end{bmatrix} + \begin{bmatrix} 4.1 \\ 3.8 \end{bmatrix} = \begin{bmatrix} 9 & 4.1 \\ -5 & 3.8 \end{bmatrix} \quad \times$$

ADDING AND SUBTRACTING MATRICES Perform the indicated operation, if possible. If not possible, state the reason.

4. $\begin{bmatrix} 5 & 2 \\ -1 & 8 \end{bmatrix} + \begin{bmatrix} -8 & 10 \\ -6 & 3 \end{bmatrix}$

5. $\begin{bmatrix} 10 & -8 \\ 5 & -3 \end{bmatrix} - \begin{bmatrix} 12 & -3 \\ 3 & -4 \end{bmatrix}$

6. $\begin{bmatrix} 4 & -5 \\ 8 & 1 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

7. $\begin{bmatrix} 1.2 & 5.3 \\ 0.1 & 4.4 \\ 6.2 & 0.7 \end{bmatrix} + \begin{bmatrix} 2.4 & -0.6 \\ 6.1 & 3.1 \\ 8.1 & -1.9 \end{bmatrix}$

8. $\begin{bmatrix} 8 & 3 \\ 9 & -1 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 5 & -1 & 0 \\ 6 & 2 & -3 \\ 8 & -1 & 2 \end{bmatrix}$

9. $\begin{bmatrix} 7 & -3 \\ 12 & 5 \\ -4 & 11 \end{bmatrix} - \begin{bmatrix} 9 & 2 \\ -2 & 6 \\ 6 & 5 \end{bmatrix}$

EXAMPLE 2
on p. 188
for Exs. 10–15

MULTIPLYING BY A SCALAR Perform the indicated operation.

10. $2\begin{bmatrix} -1 & 4 \\ 3 & -6 \end{bmatrix}$

11. $-3\begin{bmatrix} 2 & 0 & -5 \\ 4 & 7 & -3 \end{bmatrix}$

12. $-4\begin{bmatrix} 2 & -3 & -2 \\ -\frac{5}{8} & \frac{11}{2} & \frac{7}{4} \end{bmatrix}$

13. $1.5\begin{bmatrix} -2 & 3.4 & 1.6 \\ 5.4 & 0 & -3 \end{bmatrix}$

14. $\frac{1}{2}\begin{bmatrix} -2 & 8 & 12 \\ 20 & -1 & 0 \\ -8 & 10 & 2 \end{bmatrix}$

15. $-2.2\begin{bmatrix} 6 & 3.1 & 4.5 \\ -1 & 0 & 2.5 \\ 5.5 & -1.8 & 6.4 \end{bmatrix}$

MATRIX OPERATIONS Use matrices A, B, C, and D to evaluate the matrix expression.

$A = \begin{bmatrix} 5 & -4 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 18 & -12 \\ -6 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 1.8 & -1.5 & 10.6 \\ -8.8 & 3.4 & 0 \end{bmatrix}$ $D = \begin{bmatrix} 7.2 & 0 & -5.4 \\ 2.1 & -1.9 & 3.3 \end{bmatrix}$

16. $A + B$

17. $B - A$

18. $4A - B$

19. $\frac{2}{3}B$

20. $C + D$

21. $C + 3D$

22. $D - 2C$

23. $0.5C - D$

EXAMPLE 4
on p. 190
for Exs. 24–27

SOLVING MATRIX EQUATIONS Solve the matrix equation for x and y.

24. $\begin{bmatrix} -1 & 3x \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} -1 & -18 \\ 2y & 5 \end{bmatrix}$

25. $\begin{bmatrix} -2x & 6 \\ 1 & -8 \end{bmatrix} + 2\begin{bmatrix} 5 & -1 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -9 & 4 \\ -13 & y \end{bmatrix}$

26. $2\begin{bmatrix} 8 & -x \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 3 & -9 \\ 10 & -4y \end{bmatrix} = \begin{bmatrix} 13 & 4 \\ 0 & 16 \end{bmatrix}$

27. $4x\begin{bmatrix} -1 & 2 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 8 & -16 \\ -24 & 3y \end{bmatrix}$

28. ★ **MULTIPLE CHOICE** Based on the equation below, what is the value of the expression $3x - 2y$?

$$\begin{bmatrix} 2x & 0 \\ 0.5 & -0.75 \end{bmatrix} = \begin{bmatrix} 6.4 & 0 \\ 0.5 & 3y \end{bmatrix}$$

Ⓐ 7.15 Ⓑ 9.1 Ⓒ 10.1 Ⓓ 20.7

29. ★ **OPEN-ENDED MATH** Find two matrices A and B such that $2A - 3B = \begin{bmatrix} 5 & 0 \\ -1 & 2 \end{bmatrix}$.

30. **CHALLENGE** Find the matrix X that makes the equation true.

a. $X + \begin{bmatrix} -5 & 0 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 7 & -8 \\ -3 & 5 \end{bmatrix}$

b. $X - \begin{bmatrix} 2 & 3 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ -1 & 3 \end{bmatrix}$

c. $-X + \begin{bmatrix} -3 & 1 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 8 & -9 \\ 0 & 10 \end{bmatrix}$

d. $3X - \begin{bmatrix} 11 & -6 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -13 & 15 \\ -19 & 2 \end{bmatrix}$

PROBLEM SOLVING

EXAMPLE 3 on p. 189 for Exs. 31–34

31. SNOWBOARD SALES A sporting goods store sells snowboards in several different styles and lengths. The matrices below show the number of each type of snowboard sold in 2003 and 2004. Write a matrix giving the change in sales for each type of snowboard from 2003 to 2004.

$$\begin{array}{l} \textbf{Sales for 2003} \\ \begin{array}{l} \\ \text{Freeride} \\ \text{Alpine} \\ \text{Freestyle} \end{array} \begin{array}{c} \begin{array}{cccc} \text{150 cm} & \text{155 cm} & \text{160 cm} & \text{165 cm} \end{array} \\ \begin{bmatrix} 32 & 42 & 29 & 20 \\ 12 & 17 & 25 & 16 \\ 28 & 40 & 32 & 21 \end{bmatrix} \end{array} \end{array} \qquad \begin{array}{l} \textbf{Sales for 2004} \\ \begin{array}{c} \begin{array}{cccc} \text{150 cm} & \text{155 cm} & \text{160 cm} & \text{165 cm} \end{array} \\ \begin{bmatrix} 32 & 47 & 30 & 19 \\ 5 & 16 & 20 & 14 \\ 29 & 39 & 36 & 31 \end{bmatrix} \end{array} \end{array}$$

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32. FUEL ECONOMY A car dealership sells four different models of cars. The fuel economy (in miles per gallon) is shown below for each model. Organize the data using a matrix. Then write a new matrix giving the fuel economy figures for next year's models if each measure of fuel economy increases by 8%.

Economy car: 32 mpg in city driving, 40 mpg in highway driving

Mid-size car: 24 mpg in city driving, 34 mpg in highway driving

Mini-van: 18 mpg in city driving, 25 mpg in highway driving

SUV: 19 mpg in city driving, 22 mpg in highway driving

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33. ★ **EXTENDED RESPONSE** In a certain city, an electronics chain has a downtown store and a store in the mall. Each store carries three models of digital camera. Sales of the cameras for May and June are shown.

May Downtown sales: 31 of model A, 42 of model B, 18 of model C
Mall sales: 22 of model A, 25 of model B, 11 of model C

June Downtown sales: 25 of model A, 36 of model B, 12 of model C
Mall sales: 38 of model A, 32 of model B, 15 of model C

a. Organize the information using two matrices M and J that represent the sales for May and June, respectively.

b. Find $M + J$ and describe what this matrix sum represents.

c. Write a matrix giving the average monthly sales for the two month period.

34. ★ **SHORT RESPONSE** The matrices below show the numbers of female athletes who participated in selected NCAA sports and the average team size for each sport during the 2000–2001 and 2001–2002 seasons. Does the matrix $A + B$ give meaningful information? *Explain.*

$$\begin{array}{l} \textbf{2000–2001 } (A) \\ \begin{array}{l} \\ \text{Basketball} \\ \text{Gymnastics} \\ \text{Skiing} \\ \text{Soccer} \end{array} \begin{array}{c} \begin{array}{cc} \text{Athletes} & \text{Team size} \end{array} \\ \begin{bmatrix} 14{,}439 & 14.5 \\ 1{,}397 & 15.7 \\ 526 & 11.9 \\ 18{,}548 & 22.5 \end{bmatrix} \end{array} \end{array} \qquad \begin{array}{l} \textbf{2001–2002 } (B) \\ \begin{array}{l} \\ \text{Basketball} \\ \text{Gymnastics} \\ \text{Skiing} \\ \text{Soccer} \end{array} \begin{array}{c} \begin{array}{cc} \text{Athletes} & \text{Team size} \end{array} \\ \begin{bmatrix} 14{,}524 & 14.3 \\ 1{,}440 & 16.2 \\ 496 & 11.0 \\ 19{,}467 & 22.4 \end{bmatrix} \end{array} \end{array}$$

◯ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

35. CHALLENGE A rectangle has vertices (1, 1), (1, 4), (5, 1), and (5, 4). Write a 2×4 matrix A whose columns are the vertices of the rectangle. Multiply matrix A by 3. In the same coordinate plane, draw the rectangles represented by the matrices A and $3A$. How are the rectangles related?

MIXED REVIEW

PREVIEW Prepare for Lesson 3.6 in Exs. 36–41.

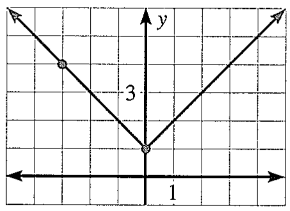
Perform the indicated operation. *(p. 975)*

36. $5 + (-8)$ **37.** $-7 + 6$ **38.** $8(-7)$

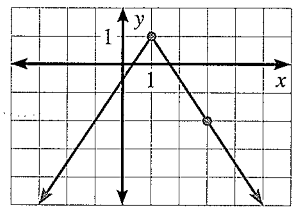
39. $-6 - (-15)$ **40.** $-5(-9)$ **41.** $6 - (-18)$

Write an equation of the graph shown. *(p. 123)*

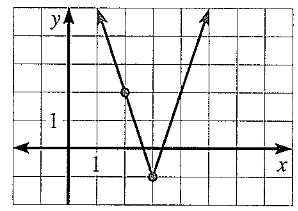
42.



43.



44.



Check whether the ordered pairs are solutions of the inequality. *(p. 132)*

45. $x + 2y \leq -3$; (0, 3), (−5, 1) **46.** $5x - y > 2$; (−5, 0), (5, 23)

47. $-8x - 3y < 5$; (−1, 1), (3, −9) **48.** $21x - 10y > 4$; (2, 3), (−1, 0)

QUIZ for Lessons 3.3–3.5

Graph the system of inequalities. *(p. 168)*

1. $y < 6$
$x + y > -2$

2. $x \geq -1$
$-2x + y \leq 5$

3. $x + 3y > 3$
$x + 3y < -9$

4. $x - y \geq 4$
$2x + 4y \geq -10$

5. $x + 2y \leq 10$
$y \geq |x + 2|$

6. $-y < x$
$2y < 5x + 9$

Solve the system using any algebraic method. *(p. 178)*

7. $2x - y - 3z = 5$
$x + 2y - 5z = -11$
$-x - 3y = 10$

8. $x + y + z = -3$
$2x - 3y + z = 9$
$4x - 5y + 2z = 16$

9. $2x - 4y + 3z = 1$
$6x + 2y + 10z = 19$
$-2x + 5y - 2z = 2$

Use matrices A, B, and C to evaluate the matrix expression, if possible. If not possible, state the reason. *(p. 187)*

$$A = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} -4 & 3 \\ 8 & 10 \end{bmatrix} \qquad C = \begin{bmatrix} -6 & -2 & 9 \\ 1 & -4 & -1 \end{bmatrix}$$

10. $A + B$ **11.** $B - 2A$ **12.** $3A + C$ **13.** $\frac{2}{3}C$

14. APPLES You have \$25 to spend on 21 pounds of three kinds of apples. Empire apples cost \$1.40 per pound, Red Delicious apples cost \$1.10 per pound, and Golden Delicious apples cost \$1.30 per pound. You want the weight of the Red Delicious apples to equal twice the combined weight of the other two kinds. How many pounds of each kind of apple should you buy? *(p. 178)*

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3.5 Use Matrix Operations

QUESTION **How can you use a graphing calculator to perform matrix operations?**

EXAMPLE **Perform operations with matrices**

Using matrices A and B below, find $A + B$ and $3A - 2B$.

$$A = \begin{bmatrix} 8 & -1 & 2 \\ 3 & -7 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & -5 \\ -4 & 6 & 10 \end{bmatrix}$$

STEP 1 ***Enter matrix A***

Enter the dimensions and elements of matrix A.

```
MATRIX[A] 2×3
[8   -1   2]
[3   -7   9]

2,3=9
```

STEP 2 ***Enter matrix B***

Enter the dimensions and elements of matrix B.

```
MATRIX[B] 2×3
[1   0   -5]
[-4  6   10]

2,3=10
```

STEP 3 ***Perform calculations***

From the home screen, calculate $A + B$ and $3A - 2B$.

```
[A]+[B]
  [[9    -1    -3]
   [-1   -1    19]]
3[A]-2[B]
  [[22   -3    16]
   [17   -33   7 ]]
```

PRACTICE

Use a graphing calculator to perform the indicated operation(s).

1. $\begin{bmatrix} 7 & 3 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 12 & -8 \\ 3 & -6 \end{bmatrix}$

2. $2.6\begin{bmatrix} 12.4 & 6.8 & -1.2 \\ -0.8 & 5.6 & -3.2 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 1 & -2 \\ -1 & 5 & 6 \\ 4 & 13 & 0 \end{bmatrix} + \begin{bmatrix} -9 & 10 & -3 \\ 0 & 6 & 1 \\ 14 & 7 & -8 \end{bmatrix}$

4. $3\begin{bmatrix} 4 & -3 \\ 8 & -7 \\ -1 & 2 \end{bmatrix} - 2\begin{bmatrix} -5 & 8 \\ -7 & 9 \\ 4 & -3 \end{bmatrix}$

5. BOOK SALES The matrices below show book sales (in thousands of dollars) at a chain of bookstores for July and August. The book formats are hardcover and paperback. The categories of books are romance (R), mystery (M), science fiction (S), and children's (C). Find the total sales of each format and category for July and August.

$$\begin{array}{l} \\ \\ \textbf{Hardcover} \\ \textbf{Paperback} \end{array} \begin{array}{c} \textbf{July} \\ \begin{array}{cccc} \textbf{R} & \textbf{M} & \textbf{S} & \textbf{C} \end{array} \\ \begin{bmatrix} 18 & 16 & 21 & 13 \\ 36 & 20 & 14 & 30 \end{bmatrix} \end{array} \qquad \begin{array}{c} \textbf{August} \\ \begin{array}{cccc} \textbf{R} & \textbf{M} & \textbf{S} & \textbf{C} \end{array} \\ \begin{bmatrix} 26 & 20 & 17 & 8 \\ 40 & 24 & 8 & 20 \end{bmatrix} \end{array}$$

3.6 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS6 for Exs. 13, 23, and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 21, 35, 41, and 44

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The product of matrices A and B is defined provided the number of _?_ in A is equal to the number of _?_ in B.

2. ★ **WRITING** Suppose A and B are two matrices and AB is defined. *Explain* how to find the element in the first row and first column of AB.

EXAMPLE 1 on p. 195 for Exs. 3–9

MATRIX PRODUCTS State whether the product AB is defined. If so, give the dimensions of AB.

3. $A: 2 \times 2, B: 2 \times 2$

4. $A: 3 \times 4, B: 4 \times 2$

5. $A: 2 \times 1, B: 2 \times 2$

6. $A: 1 \times 2, B: 2 \times 3$

7. $A: 4 \times 3, B: 2 \times 3$

8. $A: 2 \times 1, B: 1 \times 5$

9. ★ **MULTIPLE CHOICE** If A is a 2×3 matrix and B is a 3×2 matrix, what are the dimensions of AB?

Ⓐ 2×2 Ⓑ 3×3 Ⓒ 3×2 Ⓓ 2×3

EXAMPLE 2 on p. 196 for Exs. 10–21

MULTIPLYING MATRICES Find the product. If the product is not defined, state the reason.

10. $\begin{bmatrix} 3 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

11. $\begin{bmatrix} 1 \\ 4 \end{bmatrix} \begin{bmatrix} -2 & 1 \end{bmatrix}$

12. $\begin{bmatrix} -1 & 0 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 4 & -6 \end{bmatrix}$

13. $\begin{bmatrix} 9 & -3 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}$

14. $\begin{bmatrix} 5 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 6 & 2 \end{bmatrix}$

15. $\begin{bmatrix} 5 & 2 \\ 0 & -4 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ -2 & 0 \end{bmatrix}$

16. $\begin{bmatrix} 0 & -4 \\ 2 & 5 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 2 & 8 \\ 3 & 0 \\ -5 & -2 \end{bmatrix}$

17. $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 12 & -4 \end{bmatrix} \begin{bmatrix} 9 & 1 \\ 4 & -3 \\ -2 & 4 \end{bmatrix}$

18. $\begin{bmatrix} 2 & 5 \\ -1 & 4 \\ 3 & -7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ -3 & 10 & -4 \end{bmatrix}$

ERROR ANALYSIS *Describe* and correct the error in finding the element in the first row and first column of the matrix product.

19.

$$\begin{bmatrix} 3 & -1 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 1 & -6 \end{bmatrix} =$$

$$\begin{bmatrix} 3(7) + (-1)(0) & \\ & \end{bmatrix} = \begin{bmatrix} 21 & \\ & \end{bmatrix}$$

20.

$$\begin{bmatrix} 2 & 5 \\ 1 & 7 \end{bmatrix} \begin{bmatrix} 4 & -8 \\ 3 & -1 \end{bmatrix} =$$

$$\begin{bmatrix} 2(4) + 1(-8) & \\ & \end{bmatrix} = \begin{bmatrix} 0 & \\ & \end{bmatrix}$$

21. ★ **MULTIPLE CHOICE** What is the product of $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$ and $\begin{bmatrix} 4 & -1 \\ 0 & -3 \end{bmatrix}$?

Ⓐ $\begin{bmatrix} -4 & 12 \\ 3 & -3 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 4 & 11 \\ 12 & 3 \end{bmatrix}$ Ⓒ $\begin{bmatrix} -4 & 11 \\ 12 & -3 \end{bmatrix}$ Ⓓ $\begin{bmatrix} 4 & -11 \\ 0 & 3 \end{bmatrix}$

EXAMPLE 3
on p. 197
for Exs. 22–29

EVALUATING EXPRESSIONS **Using the given matrices, evaluate the expression.**

$$A = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}, C = \begin{bmatrix} -6 & 3 \\ 4 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 3 & 2 \\ -3 & 1 & 4 \\ 2 & 1 & -2 \end{bmatrix}, E = \begin{bmatrix} -3 & 1 & 4 \\ 7 & 0 & -2 \\ 3 & 4 & -1 \end{bmatrix}$$

22. $3AB$ **23.** $-\frac{1}{2}AC$ **24.** $AB + AC$ **25.** $AB - BA$

26. $E(D + E)$ **27.** $(D + E)D$ **28.** $-2(BC)$ **29.** $4AC + 3AB$

SOLVING MATRIX EQUATIONS **Solve for x and y.**

30. $\begin{bmatrix} -2 & 1 & 2 \\ 3 & 2 & 4 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 19 \\ y \end{bmatrix}$

31. $\begin{bmatrix} 4 & 1 & 3 \\ -2 & x & 1 \end{bmatrix} \begin{bmatrix} 9 & -2 \\ 2 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} y & -4 \\ -13 & 8 \end{bmatrix}$

FINDING POWERS **Using the given matrix, find $A^2 = AA$ and $A^3 = AAA$.**

32. $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ **33.** $A = \begin{bmatrix} -4 & 1 \\ 2 & -1 \end{bmatrix}$ **34.** $A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 3 & 2 \\ -2 & -1 & 0 \end{bmatrix}$

35. ★ **OPEN-ENDED MATH** Find two matrices A and B such that $A \neq B$ and $AB = BA$.

36. **CHALLENGE** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, and let k be a scalar. Prove the associative property of scalar multiplication for 2×2 matrices by showing that $k(AB) = (kA)B = A(kB)$.

PROBLEM SOLVING

EXAMPLE 4
on p. 198
for Exs. 37–42

In Exercises 37 and 38, write an inventory matrix and a cost per item matrix. Then use matrix multiplication to write a total cost matrix.

37. **SOFTBALL** A softball team needs to buy 12 bats, 45 balls, and 15 uniforms. Each bat costs $21, each ball costs $4, and each uniform costs $30.

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38. **ART SUPPLIES** A teacher is buying supplies for two art classes. For class 1, the teacher buys 24 tubes of paint, 12 brushes, and 17 canvases. For class 2, the teacher buys 20 tubes of paint, 14 brushes, and 15 canvases. Each tube of paint costs $3.35, each brush costs $1.75, and each canvas costs $4.50.

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39. **MULTI-STEP PROBLEM** Tickets to the senior class play cost $2 for students, $5 for adults, and $4 for senior citizens. At Friday night's performance, there were 120 students, 150 adults, and 40 senior citizens in attendance. At Saturday night's performance, there were 192 students, 215 adults, and 54 senior citizens in attendance. Organize the information using matrices. Then use matrix multiplication to find the income from ticket sales for Friday and Saturday nights' performances.

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

40. **SUMMER OLYMPICS** The top three countries in the final medal standings for the 2004 Summer Olympics were the United States, China, and Russia. Each gold medal is worth 3 points, each silver medal is worth 2 points, and each bronze medal is worth 1 point. Organize the information using matrices. How many points did each country score?

Medals Won

	Gold	Silver	Bronze
USA	35	39	29
China	32	17	14
Russia	27	27	38

41. ★ **SHORT RESPONSE** Matrix S gives the numbers of three types of cars sold in February by two car dealers, dealer A and dealer B. Matrix P gives the profit for each type of car sold. Which matrix is defined, SP or PS? Find this matrix and explain what its elements represent.

Matrix S

$$\begin{array}{l c} & \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{array}{l} \text{Compact} \\ \text{Mid-size} \\ \text{Full-size} \end{array} & \begin{bmatrix} 21 & 16 \\ 40 & 33 \\ 15 & 19 \end{bmatrix} \end{array}$$

Matrix P

$$\begin{array}{l c} & \begin{array}{ccc} \text{Compact} & \text{Mid-size} & \text{Full-size} \end{array} \\ \text{Profit} & \begin{bmatrix} \$650 & \$825 & \$1050 \end{bmatrix} \end{array}$$

42. **GRADING** Your overall grade in math class is a weighted average of three components: homework, quizzes, and tests. Homework counts for 20% of your grade, quizzes count for 30%, and tests count for 50%. The spreadsheet below shows the grades on homework, quizzes, and tests for five students. Organize the information using a matrix, then multiply the matrix by a matrix of weights to find each student's overall grade.

	A	B	C	D
1	Name	Homework	Quizzes	Test
2	Jean	82	88	86
3	Ted	92	88	90
4	Pat	82	73	81
5	Al	74	75	78
6	Matt	88	92	90

43. **MULTI-STEP PROBLEM** Residents of a certain suburb commute to a nearby city either by driving or by using public transportation. Each year, 20% of those who drive switch to public transportation, and 5% of those who use public transportation switch to driving.

a. The information above can be represented by the *transition matrix*

$$T = \begin{bmatrix} 1-p & q \\ p & 1-q \end{bmatrix}$$

where p is the percent of commuters who switch from driving to public transportation and q is the percent of commuters who switch from public transportation to driving. (Both p and q are expressed as decimals.) Write a transition matrix for the given situation.

b. Suppose 5000 commuters drive and 8000 commuters take public transportation. Let M_0 be the following matrix:

$$M_0 = \begin{bmatrix} 5000 \\ 8000 \end{bmatrix}$$

Find $M_1 = TM_0$. What does this matrix represent?

c. Find $M_2 = TM_1$, $M_3 = TM_2$, and $M_4 = TM_3$. What do these matrices represent?

44. ★ **EXTENDED RESPONSE** Two students have a business selling handmade scarves. The scarves come in four different styles: plain, with the class year, with the school name, and with the school mascot. The costs of making each style of scarf are \$10, \$15, \$20, and \$20, respectively. The prices of each style of scarf are \$15, \$20, \$25, and \$30, respectively.

a. Write a 4×1 matrix C that gives the cost of making each style of scarf and a 4×1 matrix P that gives the price of each style of scarf.

b. The sales for the first three years of the business are shown below.

Year 1: *0 plain, 20 class year, 100 school name, 0 school mascot*

Year 2: *10 plain, 100 class year, 50 school name, 30 school mascot*

Year 3: *20 plain, 300 class year, 100 school name, 50 school mascot*

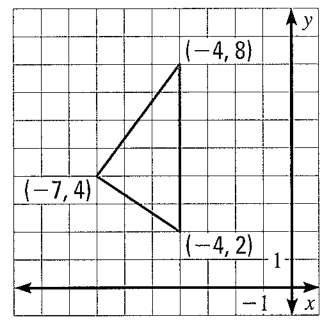
Write a 3×4 matrix S that gives the sales for the first three years.

c. Find SC and SP. What do these matrices represent?

d. Find $SP - SC$. What does this matrix represent?

45. **CHALLENGE** Matrix A is a 90° rotational matrix. Matrix B contains the coordinates of the vertices of the triangle shown in the graph.

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -7 & -4 & -4 \\ 4 & 8 & 2 \end{bmatrix}$$



a. Find AB. Draw the triangle whose vertices are given by AB.

b. Find the 180° and 270° rotations of the original triangle by using repeated multiplication of the 90° rotational matrix. What are the coordinates of the vertices of the rotated triangles?

MIXED REVIEW

Graph the equation.

46. $3x + y = 6$ ***(p. 89)***

47. $2x - 3y = 7$ ***(p. 89)***

48. $x + 4y = 10$ ***(p. 89)***

49. $y = |x| - 6$ ***(p. 123)***

50. $y = -|x + 4| - 5$ ***(p. 123)***

51. $y = 2|x - 2| + 7$ ***(p. 123)***

Write an equation of the line that satisfies the given conditions. ***(p. 98)***

52. slope: 2, passes through $(0, -4)$

53. slope: -3, passes through $(5, 2)$

54. slope: $-\frac{2}{3}$, passes through $(0, -1)$

55. slope: $\frac{3}{4}$, passes through $(0, 3)$

56. passes through $(4, 8)$ and $(1, 2)$

57. passes through $(-8, 8)$ and $(0, 1)$

PREVIEW
Prepare for Lesson 3.7 in Exs. 58–63.

Solve the system of linear equations using any algebraic method. ***(p. 160)***

58. $3x + 2y = 5$
$-x + 3y = 13$

59. $3x - 5y = 11$
$2x + 5y = 24$

60. $3x - y = 4$
$-2x + 3y = -26$

61. $4x - 3y = 17$
$2x + 5y = 15$

62. $4x - 2y = 14$
$-2x + y = -7$

63. $x + 4y = 4$
$3x - 2y = 19$

EXTRA PRACTICE for Lesson 3.6, p. 1012

ONLINE QUIZ at classzone.com

✓ **GUIDED PRACTICE** for Examples 3 and 4

Use Cramer's rule to solve the linear system.

5. $3x - 4y = -15$
$2x + 5y = 13$

6. $4x + 7y = 2$
$-3x - 2y = -8$

7. $3x - 4y + 2z = 18$
$4x + y - 5z = -13$
$2x - 3y + z = 11$

3.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS6 for Exs. 11, 23, and 43

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 28, 38, 42, and 45

SKILL PRACTICE

1. VOCABULARY Copy and complete: The _?_ of a 2×2 matrix is the difference of the products of the elements on the diagonals.

2. ★ WRITING *Explain* Cramer's rule and how it is used.

EXAMPLE 1 on p. 203 for Exs. 3–21

2 × 2 DETERMINANTS Evaluate the determinant of the matrix.

3. $\begin{bmatrix} 2 & -1 \\ 4 & -5 \end{bmatrix}$ **4.** $\begin{bmatrix} 7 & 1 \\ 0 & 3 \end{bmatrix}$ **5.** $\begin{bmatrix} -4 & 3 \\ 1 & -7 \end{bmatrix}$ **6.** $\begin{bmatrix} 1 & -3 \\ 2 & 6 \end{bmatrix}$

7. $\begin{bmatrix} 10 & -6 \\ -7 & 5 \end{bmatrix}$ **8.** $\begin{bmatrix} 0 & 3 \\ 5 & -3 \end{bmatrix}$ **9.** $\begin{bmatrix} 9 & -3 \\ 7 & 2 \end{bmatrix}$ **10.** $\begin{bmatrix} -5 & 12 \\ 4 & 6 \end{bmatrix}$

3 × 3 DETERMINANTS Evaluate the determinant of the matrix.

11. $\begin{bmatrix} -1 & 12 & 4 \\ 0 & 2 & -5 \\ 3 & 0 & 1 \end{bmatrix}$ **12.** $\begin{bmatrix} 1 & 2 & 3 \\ 5 & -8 & 1 \\ 2 & 4 & 3 \end{bmatrix}$ **13.** $\begin{bmatrix} 5 & 0 & 2 \\ -3 & 9 & -2 \\ 1 & -4 & 0 \end{bmatrix}$ **14.** $\begin{bmatrix} -7 & 4 & 5 \\ 1 & 2 & -4 \\ -10 & 1 & 6 \end{bmatrix}$

15. $\begin{bmatrix} 12 & 5 & 8 \\ 0 & 6 & -8 \\ 1 & 10 & 4 \end{bmatrix}$ **16.** $\begin{bmatrix} -4 & 3 & -9 \\ 12 & 6 & 0 \\ 8 & -12 & 0 \end{bmatrix}$ **17.** $\begin{bmatrix} -2 & 6 & 0 \\ 8 & 15 & 3 \\ 4 & -1 & 7 \end{bmatrix}$ **18.** $\begin{bmatrix} 5 & 7 & 6 \\ -4 & 0 & 8 \\ 1 & 8 & 7 \end{bmatrix}$

ERROR ANALYSIS *Describe* **and correct the error in evaluating the determinant.**

19.

$$\left|\begin{array}{ccc|cc} 2 & 0 & -1 & 2 & 0 \\ 4 & 1 & 6 & 4 & 1 \\ -3 & 2 & 5 & -3 & 2 \end{array}\right.$$

$= 10 + 0 + (-8) + (3 + 24 + 0)$
$= 2 + 27 = 29$

20.

$$\left.\begin{array}{cc|ccc} 3 & 0 & 3 & 0 & 1 \\ 2 & 2 & 2 & 2 & -3 \\ -3 & 5 & -3 & 5 & 0 \end{array}\right|$$

$= -18 + 0 + 0 - (-18 + 0 - 6)$
$= -18 - (-24) = 6$

21. ★ MULTIPLE CHOICE Which matrix has the greatest determinant?

Ⓐ $\begin{bmatrix} -4 & 1 \\ 6 & 3 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 1 & 6 \\ 3 & 8 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 5 & -3 \\ 7 & -1 \end{bmatrix}$ Ⓓ $\begin{bmatrix} 5 & -2 \\ 1 & 5 \end{bmatrix}$

EXAMPLE 2
on p. 204
for Exs. 22–28

AREA OF A TRIANGLE Find the area of the triangle with the given vertices.

22. $A(1, 5), B(4, 6), C(7, 3)$

23. $A(4, 2), B(4, 8), C(8, 5)$

24. $A(-4, 6), B(0, 3), C(6, 6)$

25. $A(-4, -4), B(-1, 2), C(2, -6)$

26. $A(5, -4), B(6, 3), C(8, -1)$

27. $A(-6, 1), B(-2, -6), C(0, 3)$

28. ★ **MULTIPLE CHOICE** What is the area of the triangle with vertices $(-3, 4)$, $(6, 3)$, and $(2, -1)$?

(A) 20 (B) 26 (C) 30 (D) 40

EXAMPLES 3 and 4
on pp. 205–206
for Exs. 29–37

USING CRAMER'S RULE Use Cramer's rule to solve the linear system.

29. $3x + 5y = 3$
$-x + 2y = 10$

30. $2x - y = -2$
$x + 2y = 14$

31. $5x + y = -40$
$2x - 5y = 11$

32. $-x + y + z = -3$
$4x - y + 4z = -14$
$x + 2y - z = 9$

33. $-x - 2y + 4z = -28$
$x + y + 2z = -11$
$2x + y - 3z = 30$

34. $4x + y + 3z = 7$
$2x - 5y + 4z = -19$
$x - y + 2z = -2$

35. $5x - y - 2z = -6$
$x + 3y + 4z = 16$
$2x - 4y + z = -15$

36. $x + y + z = -8$
$3x - 3y + 2z = -21$
$-x + 2y - 2z = 11$

37. $3x - y + z = 25$
$-x + 2y - 3z = -17$
$x + y + z = 21$

38. ★ **OPEN-ENDED MATH** Write a 2×2 matrix that has a determinant of 5.

39. **CHALLENGE** Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ -2 & -4 \end{bmatrix}$.

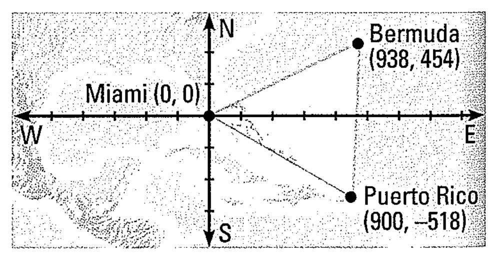
a. How is det AB related to det A and det B?

b. How is det kA related to det A if k is a scalar? Give an algebraic justification for your answer.

PROBLEM SOLVING

EXAMPLE 2
on p. 204
for Exs. 40–41

40. **BERMUDA TRIANGLE** The Bermuda Triangle is a large triangular region in the Atlantic Ocean. The triangle is formed by imaginary lines connecting Bermuda, Puerto Rico, and Miami, Florida. (In the map, the coordinates are measured in miles.) Use a determinant to estimate the area of the Bermuda Triangle.



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41. **GARDENING** You are planning to turn a triangular region of your yard into a garden. The vertices of the triangle are $(0, 0)$, $(5, 2)$, and $(3, 6)$ where the coordinates are measured in feet. Find the area of the triangular region.

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EXAMPLES 3 and 4
on pp. 205–206
for Exs. 42–44

42. ★ **SHORT RESPONSE** The attendance at a rock concert was 6700 people. The tickets for the concert cost \$40 for floor seats and \$25 for all other seats. The total income of ticket sales was \$185,500. Write a linear system that models this situation. Solve the system in three ways: using Cramer's rule, using the substitution method, and using the elimination method. *Compare* the methods, and explain which one you prefer in this situation.

○ = **WORKED-OUT SOLUTIONS on p. WS1**

★ = **STANDARDIZED TEST PRACTICE**

43. **MULTI-STEP PROBLEM** An ice cream shop sells the following sizes of ice cream cones: single scoop for \$.90, double scoop for \$1.20, and triple scoop for \$1.60. One day, a total of 120 cones are sold for \$134, as many single-scoop cones are sold as double-scoop and triple-scoop cones combined.

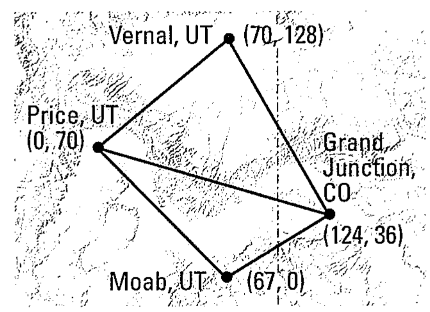
a. Use a linear system and Cramer's rule to find how many of each size of cone are sold.

b. The next day, the shop raises prices by 10%. As a result, the number of each size of cone sold falls by 5%. What is the revenue from cone sales?

44. **SCIENCE** The atomic weights of three compounds are shown in the table. Use a linear system and Cramer's rule to find the atomic weights of fluorine (F), sodium (Na), and chlorine (Cl).

Compound	Formula	Atomic weight
Sodium fluoride	FNa	42
Sodium chloride	NaCl	58.5
Chlorine pentafluoride	ClF_5	130.5

45. ★ **EXTENDED RESPONSE** In Utah and Colorado, an area called the Dinosaur Diamond is known for containing many dinosaur fossils. The map at the right shows the towns at the four vertices of the diamond. The coordinates given are measured in miles.



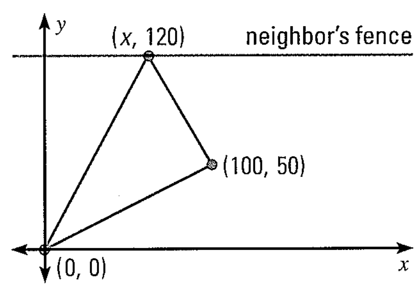
a. Find the area of the top triangular region.

b. Find the area of the bottom triangular region.

c. What is the total area of the Dinosaur Diamond?

d. *Describe* another way in which you can divide the Dinosaur Diamond into two triangles in order to find its area.

46. **CHALLENGE** A farmer is fencing off a triangular region of a pasture, as shown. The area of the region should be 5000 square feet. The farmer has planted the first two fence posts at (0, 0) and (100, 50). He wants to plant the final post along his neighbor's fence, which lies on the horizontal line $y = 120$. At which *two* points could the farmer plant the final post so that the triangular region has the desired area?



MIXED REVIEW

Evaluate the function for the given value of *x*. *(p. 72)*

47. $f(x) = x - 12; f(8)$

48. $f(x) = 4x + 8; f(7)$

49. $f(x) = x^2 - 10; f(-5)$

50. $f(x) = -x^2 + 2x; f(3)$

51. $f(x) = -x^2 - x + 5; f(4)$

52. $f(x) = x^2 - 2x + 4; f(-2)$

Graph the system of linear inequalities. *(p. 168)*

53. $x + y \geq 3$
$4x + y < 4$

54. $2x - y \geq 2$
$5x - y \geq 2$

55. $4x - 3y > 1$
$-x + y \geq 4$

56. $y < -x - 5$
$y < 3x + 1$

PREVIEW
Prepare for Lesson 3.8 in Exs. 57–59.

Find the product. *(p. 195)*

57. $\begin{bmatrix} 2 & -4 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 1 & 7 \end{bmatrix}$

58. $\begin{bmatrix} -6 & -8 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} 0 & 5 \\ 7 & 1 \end{bmatrix}$

59. $\begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -5 & 10 \\ 2 & 0 \end{bmatrix}$

3.8 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS7 for Exs. 3, 25, and 47

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 34, 41, and 46

◆ = **MULTIPLE REPRESENTATIONS** Ex. 45

SKILL PRACTICE

1. **VOCABULARY** Identify the matrix of variables and the matrix of constants in the matrix equation. $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

2. ★ **WRITING** *Explain* how to find the inverse of a 2×2 matrix A where $\det A \neq 0$.

EXAMPLE 1 on p. 210 for Exs. 3–12

FINDING INVERSES **Find the inverse of the matrix.**

3. $\begin{bmatrix} 1 & -5 \\ -1 & 4 \end{bmatrix}$
4. $\begin{bmatrix} -2 & 3 \\ -3 & 4 \end{bmatrix}$
5. $\begin{bmatrix} 6 & 2 \\ 5 & 2 \end{bmatrix}$
6. $\begin{bmatrix} -7 & -9 \\ 2 & 3 \end{bmatrix}$
7. $\begin{bmatrix} -4 & -6 \\ 4 & 7 \end{bmatrix}$
8. $\begin{bmatrix} 6 & -22 \\ -12 & 20 \end{bmatrix}$
9. $\begin{bmatrix} -24 & 60 \\ -6 & 30 \end{bmatrix}$
10. $\begin{bmatrix} \frac{4}{3} & \frac{5}{6} \\ -4 & -1 \end{bmatrix}$

11. **ERROR ANALYSIS** *Describe* and correct the error in finding the inverse of the matrix $\begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$.

$$\begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}^{-1} = 6\begin{bmatrix} 5 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 30 & -24 \\ -6 & 12 \end{bmatrix} \times$$

12. ★ **MULTIPLE CHOICE** What is the inverse of the matrix $\begin{bmatrix} 10 & -3 \\ 3 & -1 \end{bmatrix}$?

Ⓐ $\begin{bmatrix} -10 & 3 \\ -3 & 1 \end{bmatrix}$ Ⓑ $\begin{bmatrix} -1 & 3 \\ -3 & 10 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 1 & -3 \\ 3 & -10 \end{bmatrix}$ Ⓓ $\begin{bmatrix} 10 & -3 \\ 3 & -1 \end{bmatrix}$

EXAMPLE 2 on p. 211 for Exs. 13–18

SOLVING EQUATIONS **Solve the matrix equation.**

13. $\begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}X = \begin{bmatrix} 2 & 3 \\ -1 & 6 \end{bmatrix}$
14. $\begin{bmatrix} 6 & 8 \\ 2 & 3 \end{bmatrix}X = \begin{bmatrix} 4 & 3 \\ 0 & -2 \end{bmatrix}$
15. $\begin{bmatrix} -1 & 0 \\ 6 & 4 \end{bmatrix}X = \begin{bmatrix} 3 & -1 \\ 4 & 5 \end{bmatrix}$
16. $\begin{bmatrix} -3 & 6 \\ 1 & 2 \end{bmatrix}X = \begin{bmatrix} 5 & -1 \\ 8 & 2 \end{bmatrix}$
17. $\begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix}X = \begin{bmatrix} 3 & -1 & 0 \\ 6 & 8 & 4 \end{bmatrix}$
18. $\begin{bmatrix} -5 & 2 \\ -9 & 3 \end{bmatrix}X = \begin{bmatrix} 4 & 5 & 0 \\ 3 & 1 & 6 \end{bmatrix}$

EXAMPLE 3 on p. 211 for Exs. 19–24

FINDING INVERSES **Use a graphing calculator to find the inverse of matrix A. Check the result by showing that $AA^{-1} = I$ and $A^{-1}A = I$.**

19. $A = \begin{bmatrix} 1 & 1 & -2 \\ -2 & 0 & 3 \\ 3 & 1 & 0 \end{bmatrix}$
20. $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 1 & 4 & 4 \end{bmatrix}$
21. $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 10 \\ 3 & -1 & 2 \end{bmatrix}$
22. $A = \begin{bmatrix} -2 & 5 & -1 \\ 0 & 8 & 1 \\ 12 & -5 & 0 \end{bmatrix}$
23. $A = \begin{bmatrix} 3 & -8 & 0 \\ 2 & 4 & 1 \\ -1 & 0 & -6 \end{bmatrix}$
24. $A = \begin{bmatrix} 4 & 1 & 5 \\ -2 & 2 & 1 \\ 3 & -1 & 6 \end{bmatrix}$

EXAMPLE 4
on p. 212
for Exs. 25–34

SYSTEMS OF TWO EQUATIONS **Use an inverse matrix to solve the linear system.**

25. $4x - y = 10$
$-7x - 2y = -25$

26. $4x + 7y = -16$
$2x + 3y = -4$

27. $3x - 2y = 5$
$6x - 5y = 14$

28. $x - y = 4$
$9x - 10y = 45$

29. $-2x - 9y = -2$
$4x + 16y = 8$

30. $2x - 7y = -6$
$-x + 5y = 3$

31. $6x + y = -2$
$-x + 3y = -25$

32. $2x + y = -2$
$2x + 5y = 38$

33. $5x + 7y = 20$
$3x + 5y = 16$

34. ★ **MULTIPLE CHOICE** What is the solution of the system shown?

$3x - 5y = -26$
$-x + 2y = 10$

Ⓐ (3, 7) Ⓑ (7, −1) Ⓒ (−2, 4) Ⓓ (68, 110)

EXAMPLE 5
on p. 213
for Exs. 35–40

SYSTEMS OF THREE EQUATIONS **Use an inverse matrix and a graphing calculator to solve the linear system.**

35. $x - y - 3z = 2$
$5x + 2y + z = -17$
$-3x - y = 8$

36. $-3x + y - 8z = 18$
$x - 2y + z = -11$
$2x - 2y + 5z = -17$

37. $2x + 4y + 5z = 5$
$x + 2y + 3z = 4$
$5x - 4y - 2z = -3$

38. $4x - y - z = -20$
$6x - z = -27$
$-x + 4y + 5z = 23$

39. $3x + 2y - z = 14$
$-x - 5y + 4z = -48$
$4x + y + z = 2$

40. $6x + y + 2z = 11$
$x - y + z = -5$
$-x + 4y - z = 14$

41. ★ **OPEN-ENDED MATH** Write a 2×2 matrix that has no inverse.

42. **CHALLENGE** Solve the linear system using the given inverse of the coefficient matrix.

$2w + 5x - 4y + 6z = 0$
$2x + y - 7z = 52$
$4w + 8x - 7y + 14z = -25$
$3w + 6x - 5y + 10z = -16$

$$A^{-1} = \begin{bmatrix} -10 & 4 & 27 & -29 \\ 5 & -2 & -16 & 18 \\ 4 & -2 & -17 & 20 \\ 2 & -1 & -7 & 8 \end{bmatrix}$$

PROBLEM SOLVING

EXAMPLES 4 and 5
on pp. 212–213
for Exs. 43–48

43. **AVIATION** A pilot has 200 hours of flight time in single-engine airplanes and twin-engine airplanes. Renting a single-engine airplane costs \$60 per hour, and renting a twin-engine airplane costs \$240 per hour. The pilot has spent \$21,000 on airplane rentals. Use an inverse matrix to find how many hours the pilot has flown each type of airplane.

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44. **BASKETBALL** During the 2003–2004 NBA season, Dirk Nowitzki of the Dallas Mavericks made a total of 976 shots and scored 1680 points. His shots consisted of 3-point field goals, 2-point field goals, and 1-point free throws. He made 135 more 2-point field goals than free throws. Use an inverse matrix to find how many of each type of shot he made.

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45. **MULTIPLE REPRESENTATIONS** A cooking class wants to use up 8 cups of buttermilk and 11 eggs by baking rolls and muffins to freeze. A batch of rolls uses 2 cups of buttermilk and 3 eggs. A batch of muffins uses 1 cup of buttermilk and 1 egg.

a. **Writing a System** Write a system of equations for this situation.

b. **Writing a Matrix Equation** Write the system of equations from part (a) as a matrix equation $AX = B$.

c. **Solving a System** Use an inverse matrix to solve the system of equations. How many batches of each recipe should the class make?

46. ★ **EXTENDED RESPONSE** A company sells party platters with varying assortments of meats and cheeses. A basic platter with 2 cheeses and 3 meats costs \$18, a medium platter with 3 cheeses and 5 meats costs \$28, and a super platter with 7 cheeses and 10 meats costs \$60.

a. Write and solve a system of equations using the information about the basic platter and the medium platter.

b. Write and solve a system of equations using the information about the medium platter and the super platter.

c. *Compare* the results from parts (a) and (b) and make a conjecture about why there is a discrepancy.

47. **NUTRITION** The table shows the calories, fat, and carbohydrates per ounce for three brands of cereal. How many ounces of each brand should be combined to get 500 calories, 3 grams of fat, and 100 grams of carbohydrates? Round your answers to the nearest tenth of an ounce.

Cereal	Calories	Fat	Carbohydrates
Bran Crunchies	78	1 g	22 g
Toasted Oats	104	0 g	25.5 g
Whole Wheat Flakes	198	0.6 g	23.8 g

48. **MULTI-STEP PROBLEM** You need 9 square feet of glass mosaic tiles to decorate a wall of your kitchen. You want the area of the red tiles to equal the combined area of the yellow and blue tiles. The cost of a sheet of glass tiles having an area of 0.75 square foot is \$6.50 for red, \$4.50 for yellow, and \$8.50 for blue. You have \$80 to spend.

a. Write a system of equations to represent this situation.

b. Rewrite the system as a matrix equation.

c. Use an inverse matrix to find how many sheets of each color tile you should buy.

Mosaic tiles

49. **GEOMETRY** The columns of matrix T below give the coordinates of the vertices of a triangle. Matrix A is a transformation matrix.

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 4 & 2 \end{bmatrix}$$

a. Find AT and AAT. Then draw the original triangle and the two transformed triangles. What transformation does A represent?

b. *Describe* how to use matrices to obtain the original triangle represented by T from the transformed triangle represented by AAT.

50. **CHALLENGE** Verify the formula on page 210 for the inverse of a 2×2 matrix by showing that $AB = I$ and $BA = I$ for the matrices A and B given below.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad B = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

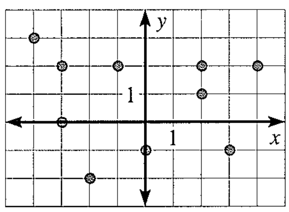
MIXED REVIEW

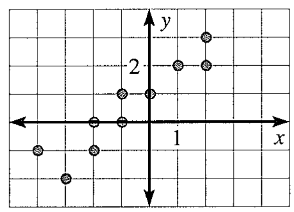
PREVIEW Prepare for Lesson 4.1 in Exs. 51–56.

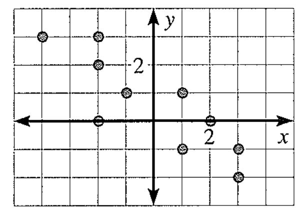
Graph the equation or inequality.

51. $y = -5x + 3$ *(p. 89)*
52. $y = \frac{2}{3}x + 2$ *(p. 89)*
53. $y = -\frac{1}{8}x - 2$ *(p. 89)*
54. $y \geq 2|x|$ *(p. 132)*
55. $y < |x - 2|$ *(p. 132)*
56. $y \leq -2|x + 1|$ *(p. 132)*

Tell whether x and y have a *positive correlation*, a *negative correlation*, or *approximately no correlation*. *(p. 113)*

57. 

58. 

59. 

Solve the matrix equation for x and y. *(p. 187)*

60. $\begin{bmatrix} 5 & 4x \\ 18 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -20 \\ 3y & 6 \end{bmatrix}$

61. $\begin{bmatrix} -3x & -9 \\ 13 & -5 \end{bmatrix} + \begin{bmatrix} 4 & 12 \\ -5y & 16 \end{bmatrix} = \begin{bmatrix} -20 & 3 \\ 18 & 11 \end{bmatrix}$

QUIZ for Lessons 3.6–3.8

Using the given matrices, evaluate the expression. *(p. 195)*

$$A = \begin{bmatrix} 1 & -4 \\ 5 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} -6 & -1 \\ 2 & 4 \end{bmatrix}$$

1. $2AB$
2. $AB + AC$
3. $A(B + C)$
4. $(B - A)C$

Evaluate the determinant of the matrix. *(p. 203)*

5. $\begin{bmatrix} 5 & 4 \\ -2 & -3 \end{bmatrix}$
6. $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & 3 & -1 \end{bmatrix}$
7. $\begin{bmatrix} 2 & -1 & 5 \\ -3 & 6 & 9 \\ -2 & 3 & 1 \end{bmatrix}$

Use an inverse matrix to solve the linear system. *(p. 210)*

8. $x + 3y = -2$
   $2x + 7y = -6$
9. $3x - 4y = 5$
   $2x - 3y = 3$
10. $-3x + 2y = -13$
    $6x - 5y = 24$
11. $3x - y = -4$
    $2x - 2y = -8$
12. $7x + 4y = 6$
    $5x + 3y = -25$
13. $4x + y = -2$
    $-6x + y = 18$

14. **BOATING** You are making a triangular sail for a sailboat. The vertices of the sail are (0, 2), (12, 2), and (12, 26) where the coordinates are measured in feet. Find the area of the sail. *(p. 203)*

c tiles

TATIONS

PROBLEM SOLVING WORKSHOP

LESSON 3.8

Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 213

MULTIPLE REPRESENTATIONS In Example 5 on page 213, you solved a linear system using an inverse matrix. You can also solve systems using *augmented matrices*. An **augmented matrix** for a system contains the system's coefficient matrix and matrix of constants.

Linear System

$$x - 4y = 9$$
$$-6x + 7y = -2$$

Augmented Matrix

$$\left[\begin{array}{cc|c} 1 & -4 & 9 \\ -6 & 7 & -2 \end{array}\right]$$

Recall from Lesson 3.2 that an equation in a system can be multiplied by a constant, or a multiple of one equation can be added to another equation. Similar operations can be performed on the rows of an augmented matrix to solve the corresponding system.

KEY CONCEPT — *For Your Notebook*

Elementary Row Operations for Augmented Matrices

Two augmented matrices are *row-equivalent* if their corresponding systems have the same solution(s). Any of these row operations performed on an augmented matrix will produce a matrix that is row-equivalent to the original:

- Interchange two rows.
- Multiply a row by a nonzero constant.
- Add a multiple of one row to another row.

PROBLEM

GIFTS A company sells three types of movie gift baskets. A basic basket with 2 movie passes and 1 package of microwave popcorn costs $15.50. A medium basket with 2 movie passes, 2 packages of popcorn, and 1 DVD costs $37. A super basket with 4 movie passes, 3 packages of popcorn, and 2 DVDs costs $72.50. Find the cost of each item in the gift baskets.

METHOD

Using an Augmented Matrix You need to write a linear system, write the corresponding augmented matrix, and use row operations to transform the augmented matrix into a matrix with 1's along the main diagonal and 0's below the main diagonal. Such a matrix is in *triangular form* and can be used to solve for the variables in the system.

Let m be the cost of a movie pass, p be the cost of a package of popcorn, and d be the cost of a DVD.

STEP 1 **Write** a linear system and then write an augmented matrix.

$$\begin{aligned} 2m + p &= 15.5 \\ 2m + 2p + d &= 37 \\ 4m + 3p + 2d &= 72.5 \end{aligned} \qquad \left[\begin{array}{ccc|c} 2 & 1 & 0 & 15.5 \\ 2 & 2 & 1 & 37 \\ 4 & 3 & 2 & 72.5 \end{array}\right]$$

STEP 2 **Add** −2 times the first row to the third row.

$$(-2)R_1 + R_3 \longrightarrow \left[\begin{array}{ccc|c} 2 & 1 & 0 & 15.5 \\ 2 & 2 & 1 & 37 \\ 0 & 1 & 2 & 41.5 \end{array}\right]$$

STEP 3 **Add** −1 times the first row to the second row.

$$(-1)R_1 + R_2 \longrightarrow \left[\begin{array}{ccc|c} 2 & 1 & 0 & 15.5 \\ 0 & 1 & 1 & 21.5 \\ 0 & 1 & 2 & 41.5 \end{array}\right]$$

STEP 4 **Add** −1 times the second row to the third row.

$$(-1)R_2 + R_3 \longrightarrow \left[\begin{array}{ccc|c} 2 & 1 & 0 & 15.5 \\ 0 & 1 & 1 & 21.5 \\ 0 & 0 & 1 & 20 \end{array}\right]$$

STEP 5 **Multiply** the first row by 0.5.

$$0.5R_1 \longrightarrow \left[\begin{array}{ccc|c} 1 & 0.5 & 0 & 7.75 \\ 0 & 1 & 1 & 21.5 \\ 0 & 0 & 1 & 20 \end{array}\right]$$

The third row of the matrix tells you that $d = 20$. Substitute 20 for d in the equation for the second row, $p + d = 21.5$, to obtain $p + 20 = 21.5$, or $p = 1.5$. Then substitute 1.5 for p in the equation for the first row, $m + 0.5p = 7.75$, to obtain $m + 0.5(1.5) = 7.75$, or $m = 7$.

▶ A movie pass costs \$7, a package of popcorn costs \$1.50, and a DVD costs \$20.

PRACTICE

1. **WHAT IF?** In the problem on page 218, suppose a basic basket costs \$17.75, a medium basket costs \$34.50, and a super basket costs \$67.25. Use an augmented matrix to find the cost of each item.

2. **FINANCE** You have \$18,000 to invest. You want an overall annual return of 8%. The expected annual returns are 10% for a stock fund, 7% for a bond fund, and 5% for a money market fund. You want to invest as much in stocks as in bonds and the money market combined. Use an augmented matrix to find how much to invest in each fund.

3. **BIRDSEED** A pet store sells 20 pounds of birdseed for \$10.85. The birdseed is made from two kinds of seeds, sunflower seeds and thistle seeds. Sunflower seeds cost \$.34 per pound and thistle seeds cost \$.79 per pound. Use an augmented matrix to find how many pounds of each variety are in the mixture.

4. **REASONING** Solve the given system using an augmented matrix. What can you say about the system's solution(s)?

$$\begin{aligned} x - 2y + 4z &= -10 \\ 5x + y - z &= 24 \\ 3x - 6y + 12z &= -30 \end{aligned}$$

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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Lessons 3.5–3.8

1. **MULTI-STEP PROBLEM** The cost (in thousands of dollars) of a 30 second commercial on two cable TV networks is shown below for two cities. The cost varies based on when the commercial airs: daytime (D), prime time (P), and late night (L).

Costs in City A

	D	P	L
Network 1	4.5	6	2.5
Network 2	5.5	8	2.5

Costs in City B

	D	P	L
Network 1	4	6.5	3.25
Network 2	5	8.5	3.25

a. Organize this information using two matrices A and B that give the costs for city A and city B, respectively.

b. Find $B - A$ and describe what each element in $B - A$ represents.

c. Next year, all of the costs will increase by 10%. Write two matrices that give next year's costs for the two cities.

2. **MULTI-STEP PROBLEM** A person has 85 coins in nickels, dimes, and quarters with a combined value of \$13.25. There are twice as many quarters as dimes.

a. Write a system of equations that models this situation.

b. Write the system from part (a) as a matrix equation $AX = B$.

c. Use an inverse matrix to find the number of each type of coin.

3. **OPEN-ENDED** Consider the following matrices.

$$A = \begin{bmatrix} 3 & 2 \\ 8 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 0 & 5 \\ 1 & 4 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

a. Write a 2×2 matrix C such that $C \neq A$ but $\det C = \det A$.

b. Write a 3×3 matrix D such that $D \neq B$ but $\det D = \det B$.

4. **SHORT RESPONSE** A store has three departments: clothing (C), housewares (H), and electronics (E). Matrix A shows the total sales (in dollars) for two salespeople, Mary and Mark, in each department. Matrix B shows the commission on sales in each department. Which matrix is defined, AB or BA? Find this matrix and explain what its elements represent.

Matrix *A* **Matrix *B***

$$\begin{array}{c} \\ \text{C} \\ \text{H} \\ \text{E} \end{array} \begin{array}{c} \begin{array}{cc} \text{Mary} & \text{Mark} \end{array} \\ \begin{bmatrix} 175 & 270 \\ 370 & 225 \\ 200 & 255 \end{bmatrix} \end{array} \qquad \begin{array}{c} \begin{array}{ccc} \text{C} & \text{H} & \text{E} \end{array} \\ \begin{bmatrix} 3\% & 5\% & 8\% \end{bmatrix} \end{array}$$

5. **EXTENDED RESPONSE** The atomic weights of three compounds are shown in the table.

Compound	Formula	Atomic weight
Nitric acid	HNO_3	63
Nitrous oxide	N_2O	44
Water	H_2O	18

a. Write a linear system using the formula for each compound. Let H, N, and O represent the atomic weights of hydrogen, nitrogen, and oxygen, respectively.

b. Write the coefficient matrix for the system and evaluate its determinant.

c. Can you use Cramer's rule to solve the system? If so, find the atomic weights of hydrogen, nitrogen, and oxygen. If not, explain why not.

6. **GRIDDED ANSWER** A farmer harvests his crops and receives \$2.35 per bushel of corn, \$5.40 per bushel of soybeans, and \$3.60 per bushel of wheat. The farmer harvests a total of 1700 bushels of crops and receives a total of \$4837. The amount of corn harvested is 3.25 times the combined amount of soybeans and wheat harvested. How many bushels of wheat were harvested?

3 CHAPTER SUMMARY

BIG IDEAS — *For Your Notebook*

Big Idea 1

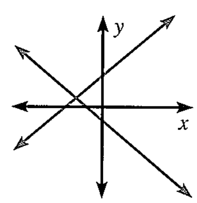
Solving Systems of Equations Using a Variety of Methods

Method	When to use
Graphing: Graph each equation in the system. A point where the graphs intersect is a solution.	The equations have only two variables and are given in a form that is easy to graph.
Substitution: Solve one equation for one of the variables and substitute into the other equation(s).	One of the variables in the system has a coefficient of 1 or −1.
Elimination: Multiply equations by constants, then add the revised equations to eliminate a variable.	None of the variables in the system have a coefficient of 1 or −1.
Cramer's rule: Use determinants to find the solution.	The determinant of the coefficient matrix is not zero.
Inverse matrices: Write the system as a matrix equation $AX = B$. Multiply each side by A^{-1} on the left to obtain the solution $X = A^{-1}B$.	The determinant of the coefficient matrix is not zero.

Big Idea 2

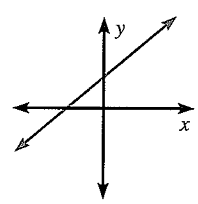
Graphing Systems of Equations and Inequalities

System of equations with 1 solution



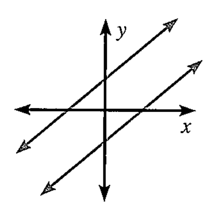
Intersecting lines

System of equations with many solutions



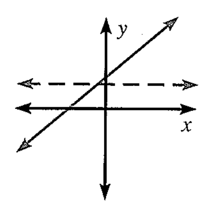
Coinciding lines

System of equations with no solution



Parallel lines

System of inequalities



Shaded region

Big Idea 3

Using Matrices

Addition, subtraction, and scalar multiplication	Matrix multiplication	Inverse matrices
$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$ $k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} =$ $\begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$	If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{\lvert A \rvert}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ or $A^{-1} = \frac{1}{ad-cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

3 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- system of two linear equations in two variables, *p. 153*
- solution of a system of linear equations, *p. 153*
- consistent, inconsistent, independent, dependent, *p. 154*
- substitution method, *p. 160*
- elimination method, *p. 161*
- system of linear inequalities in two variables, *p. 168*
- solution, graph of a system of inequalities, *p. 168*
- linear equation in three variables, *p. 178*
- system of three linear equations in three variables, *p. 178*
- solution of a system of three linear equations, *p. 178*
- ordered triple, *p. 178*
- matrix, *p. 187*
- dimensions, elements of a matrix, *p. 187*
- equal matrices, *p. 187*
- scalar, *p. 188*
- scalar multiplication, *p. 188*
- determinant, *p. 203*
- Cramer's rule, *p. 205*
- coefficient matrix, *p. 205*
- identity matrix, inverse matrices, *p. 210*
- matrix of variables, *p. 212*
- matrix of constants, *p. 212*

VOCABULARY EXERCISES

1. Copy and complete: A system of linear equations with at least one solution is __?__, while a system with no solution is __?__.
2. Copy and complete: A solution (x, y, z) of a system of linear equations in three variables is called a(n) __?__.
3. **WRITING** *Explain* when the product of two matrices is defined.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 3.

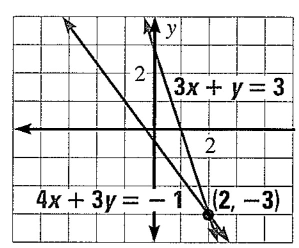
3.1 Solve Linear Systems by Graphing

pp. 153–158

EXAMPLE

Graph the system and estimate the solution. Check the solution algebraically.

$3x + y = 3$ **Equation 1**
$4x + 3y = -1$ **Equation 2**



Graph both equations. From the graph, the lines appear to intersect at $(2, -3)$. You can check this algebraically.

$3(2) + (-3) = 3$ ✓ **Equation 1 checks.**

$4(2) + 3(-3) = -1$ ✓ **Equation 2 checks.**

EXERCISES

EXAMPLE 1 on p. 153 for Exs. 4–6

Graph the system and estimate the solution. Check the solution algebraically.

4. $2x - y = 9$
   $x + 3y = 8$
5. $2x - 3y = -2$
   $x + y = -6$
6. $3x + y = 6$
   $-x + 2y = 12$

3.2 Solve Linear Systems Algebraically

pp. 160–167

EXAMPLE

Solve the system using the elimination method.

$2x + 5y = 8$ **Equation 1**
$4x + 3y = -12$ **Equation 2**

Multiply Equation 1 by -2 so that the coefficients of x differ only in sign.

$$\begin{array}{lcr} 2x + 5y = 8 & \xrightarrow{\times -2} & -4x - 10y = -16 \\ 4x + 3y = -12 & \longrightarrow & \underline{4x + 3y = -12} \end{array}$$

Add the revised equations and solve for y.

$$-7y = -28$$
$$y = 4$$

Substitute the value of y into one of the original equations and solve for x.

$2x + 5(4) = 8$ **Substitute 4 for y in Equation 1.**

$2x = -12$ **Subtract $5(4) = 20$ from each side.**

$x = -6$ **Divide each side by 2.**

▶ The solution is $(-6, 4)$.

EXERCISES

EXAMPLES 2 and 3 on pp. 161–162 for Exs. 7–10

Solve the system using the elimination method.

7. $3x + 2y = 5$
$-2x + 3y = 27$

8. $3x + 5y = 5$
$2x - 3y = 16$

9. $2x + 3y = 9$
$-3x + y = 25$

10. FUEL COSTS The cost of 14 gallons of regular gasoline and 10 gallons of premium gasoline is \$46.68. Premium costs \$.30 more per gallon than regular. What is the cost per gallon of each type of gasoline?

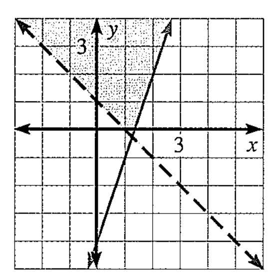
3.3 Graph Systems of Linear Inequalities

pp. 168–173

EXAMPLE

Graph the system of linear inequalities.

$3x - y \leq 4$ **Inequality 1**
$x + y > 1$ **Inequality 2**



Graph each inequality in the system. Use a different color for each half-plane. Then identify the region that is common to both graphs. It is the region that is shaded purple.

EXERCISES

EXAMPLE 1 on p. 168 for Exs. 11–13

Graph the system of linear inequalities.

11. $4x + y < 1$
$-x + 2y \leq 5$

12. $2x + 3y > 6$
$2x - y \leq 8$

13. $x + 3y \geq 5$
$-x + 2y < 4$

3 CHAPTER REVIEW

3.4 Solve Systems of Linear Equations in Three Variables *pp. 178–185*

EXAMPLE

Solve the system.

$$\begin{aligned} 2x + y + 3z &= 5 && \textbf{Equation 1} \\ -x + 3y + z &= -14 && \textbf{Equation 2} \\ 3x - y - 2z &= 11 && \textbf{Equation 3} \end{aligned}$$

Rewrite the system as a linear system in two variables. Add -3 times Equation 1 to Equation 2. Then add Equation 1 and Equation 3.

$$\begin{array}{rrrl} -6x & -\ 3y & -\ 9z & = -15 \\ -x & +\ 3y & +\ z & = -14 \\ \hline -7x & & -\ 8z & = -29 \end{array} \qquad \begin{array}{rrrl} 2x & +\ y & +\ 3z & = 5 \\ 3x & -\ y & -\ 2z & = 11 \\ \hline 5x & & +\ z & = 16 \end{array}$$

Solve the new linear system for both of its variables.

$$\begin{array}{rrl} -7x & -\ 8z & = -29 \\ 40x & +\ 8z & = 128 \\ \hline 33x & & = 99 \\ & x & = 3 \\ & z & = 1 \end{array} \qquad \begin{array}{l} \textbf{Add new Equation 1 to} \\ \textbf{8 times new Equation 2.} \\ \\ \textbf{Solve for } \boldsymbol{x}\textbf{.} \\ \textbf{Substitute into new Equation 1 or 2 to find } \boldsymbol{z}\textbf{.} \end{array}$$

Substituting $x = 3$ and $z = 1$ into one of the original equations and solving for y gives $y = -4$. The solution is $(3, -4, 1)$.

EXERCISES

EXAMPLES 1 and 4 on pp. 179–181 for Exs. 14–17

Solve the system.

14. $x - y + z = 10$
$4x + y - 2z = 15$
$-3x + 5y - z = -18$

15. $6x - y + 4z = 6$
$-x - 3y + z = 31$
$2x + 2y - 5z = -42$

16. $5x + y - z = 40$
$x + 7y + 4z = 44$
$-x + 3y + z = 16$

17. MUSIC Fifteen band members from a school were selected to play in the state orchestra. Twice as many students who play a wind instrument were selected as students who play a string or percussion instrument combined. Of the students selected, one fifth play a string instrument. How many of the students selected play each type of instrument?

3.5 Perform Basic Matrix Operations *pp. 187–193*

EXAMPLE

Perform the indicated operation.

a. $\begin{bmatrix} 4 & -1 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} -5 & 2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 4 + (-5) & -1 + 2 \\ 2 + (-3) & 5 + 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 6 \end{bmatrix}$

b. $4\begin{bmatrix} -2 & 0 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 4(-2) & 4(0) \\ 4(3) & 4(5) \end{bmatrix} = \begin{bmatrix} -8 & 0 \\ 12 & 20 \end{bmatrix}$

EXAMPLES 2 and 3 on pp. 188–189 for Exs. 18–23

EXERCISES

Perform the indicated operation.

18. $\begin{bmatrix} 4 & -5 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ -7 & 4 \end{bmatrix}$

19. $\begin{bmatrix} -1 & 8 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 7 & -4 \\ 6 & -1 \end{bmatrix}$

20. $\begin{bmatrix} 10 & -4 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 2 & 7 \end{bmatrix}$

21. $\begin{bmatrix} -2 & 3 & 5 \\ -1 & 6 & -2 \end{bmatrix} - \begin{bmatrix} -4 & 7 & 5 \\ -8 & 0 & -9 \end{bmatrix}$

22. $-3\begin{bmatrix} 5 & -2 \\ 3 & 6 \end{bmatrix}$

23. $8\begin{bmatrix} 8 & 4 & 5 \\ -1 & 6 & -2 \end{bmatrix}$

3.6 Multiply Matrices

pp. 195–202

EXAMPLE

Find AB if $A = \begin{bmatrix} 2 & -3 \\ -1 & 0 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 3 & 1 \end{bmatrix}$.

$$AB = \begin{bmatrix} 2 & -3 \\ -1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2(-2) + (-3)(3) & 2(3) + (-3)(1) \\ -1(-2) + 0(3) & -1(3) + 0(1) \\ 4(-2) + 5(3) & 4(3) + 5(1) \end{bmatrix}$$

$$= \begin{bmatrix} -13 & 3 \\ 2 & -3 \\ 7 & 17 \end{bmatrix}$$

EXAMPLES 2 and 4 on pp. 196–198 for Exs. 24–28

EXERCISES

Find the product.

24. $\begin{bmatrix} -1 & -1 \end{bmatrix}\begin{bmatrix} 8 & 2 \\ -6 & -9 \end{bmatrix}$

25. $\begin{bmatrix} 11 & 7 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 0 & -5 \\ 4 & -3 \end{bmatrix}$

26. $\begin{bmatrix} 4 & -1 \\ 1 & 7 \end{bmatrix}\begin{bmatrix} 5 & -2 & 4 \\ 3 & 12 & 6 \end{bmatrix}$

27. $\begin{bmatrix} -2 & 5 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 6 & -3 & 5 \\ 2 & 0 & -1 \end{bmatrix}$

28. MANUFACTURING A company manufactures three models of flat-screen color TVs: a 19 inch model, a 27 inch model, and a 32 inch model. The TVs are shipped to two warehouses. The numbers of units shipped to each warehouse are given in matrix A, and the prices of the models are given in matrix B. Write a matrix that gives the total value of the TVs in each warehouse.

Matrix A

$$\begin{array}{rc} & \begin{array}{ccc} \textbf{19 in.} & \textbf{27 in.} & \textbf{32 in.} \end{array} \\ \begin{array}{r} \textbf{Warehouse 1} \\ \textbf{Warehouse 2} \end{array} & \begin{bmatrix} 5{,}000 & 6{,}000 & 8{,}000 \\ 4{,}000 & 10{,}000 & 5{,}000 \end{bmatrix} \end{array}$$

Matrix B

$$\begin{array}{rc} & \textbf{Price} \\ \begin{array}{r} \textbf{19 inch} \\ \textbf{27 inch} \\ \textbf{32 inch} \end{array} & \begin{bmatrix} \$109.99 \\ \$319.99 \\ \$549.99 \end{bmatrix} \end{array}$$

3.7 Evaluate Determinants and Apply Cramer's Rule

pp. 203–209

EXAMPLE

Evaluate the determinant of $\begin{bmatrix} 2 & 1 \\ 5 & 7 \end{bmatrix}$.

$$\begin{vmatrix} 2 & 1 \\ 5 & 7 \end{vmatrix} = 2(7) - 5(1) = 14 - 5 = 9$$

EXERCISES

EXAMPLES 1 and 2 on pp. 203–204 for Exs. 29–32

Evaluate the determinant of the matrix.

29. $\begin{bmatrix} -4 & 2 \\ 5 & 8 \end{bmatrix}$ **30.** $\begin{bmatrix} 3 & -5 \\ 2 & 6 \end{bmatrix}$ **31.** $\begin{bmatrix} 3 & 0 \\ 1 & 6 \end{bmatrix}$

32. SCHOOL SPIRIT You are making a large triangular pennant for your school football team. The vertices of the triangle are (0, 0), (0, 50), and (70, 20) where the coordinates are measured in inches. How many square *feet* of material will you need to make the pennant?

3.8 Use Inverse Matrices to Solve Linear Systems

pp. 210–217

EXAMPLE

Use an inverse matrix to solve the linear system at the right.

$$x - 2y = 14$$
$$2x + y = 8$$

Write the linear system as a matrix equation $AX = B$.

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 8 \end{bmatrix}$$

Find the inverse of the coefficient matrix A.

$$A^{-1} = \frac{1}{1-(-4)}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.4 \\ -0.4 & 0.2 \end{bmatrix}$$

Then multiply the matrix of constants by A^{-1} on the left.

$$X = A^{-1}B = \begin{bmatrix} 0.2 & 0.4 \\ -0.4 & 0.2 \end{bmatrix}\begin{bmatrix} 14 \\ 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

▸ The solution of the system is (6, −4).

EXERCISES

EXAMPLE 4 on p. 212 for Exs. 33–35

Use an inverse matrix to solve the linear system.

33. $x + 4y = 11$
$2x - 5y = 9$

34. $3x + y = -1$
$-x + 2y = 12$

35. $3x + 2y = -11$
$4x - 3y = 8$

3 CHAPTER TEST

Graph the linear system and estimate the solution. Then check the solution algebraically.

1. $4x + y = 5$
$3x - y = 2$

2. $x + 2y = -6$
$-6x - 2y = -14$

3. $2x - 3y = 15$
$x - \frac{3}{2}y = -3$

4. $3x - y = 12$
$-x + 8y = -4$

Graph the system of linear inequalities.

5. $2x + y < 6$
$y > -2$

6. $x - 3y \geq 9$
$\frac{1}{3}x - y \leq 3$

7. $x - 2y \leq -14$
$y \geq |x|$

8. $-3x + 4y > -12$
$y < -2|x| + 5$

Solve the system using any algebraic method.

9. $3x + y = -9$
$x - 2y = -10$

10. $2x + 3y = -2$
$4x + 7y = -6$

11. $x + 4y = -26$
$-5x - 2y = -14$

12. $x - y + z = -3$
$2x - y + 5z = 4$
$4x + 2y - z = 2$

13. $x + y + z = 3$
$-x + 3y + 2z = -8$
$5y + z = 2$

14. $2x - 5y - z = 17$
$x + y + 3z = 19$
$-4x + 6y + z = -20$

Use the given matrices to evaluate the expression, if possible. If not possible, state the reason.

$$A = \begin{bmatrix} 1 & -2 \\ 4 & -3 \end{bmatrix}, B = \begin{bmatrix} 3 & 5 \\ -1 & 0 \end{bmatrix}, C = \begin{bmatrix} -6 & 8 \\ 10 & 15 \end{bmatrix}, D = \begin{bmatrix} -1 & 3 & -2 \\ 2 & 0 & -1 \end{bmatrix}, E = \begin{bmatrix} 4 & -1 & 3 \\ 6 & -2 & 1 \end{bmatrix}$$

15. $2A + B$

16. $C - 3B$

17. $A - 2D$

18. $4D + E$

19. AC

20. DE

21. $(A + B)D$

22. $A(C - B)$

Evaluate the determinant of the matrix.

23. $\begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$

24. $\begin{bmatrix} -4 & 5 \\ 2 & -1 \end{bmatrix}$

25. $\begin{bmatrix} -1 & 3 & 1 \\ 0 & 2 & -3 \\ 5 & 1 & -2 \end{bmatrix}$

26. $\begin{bmatrix} 2 & 0 & -1 \\ 5 & -3 & 2 \\ 1 & 4 & 6 \end{bmatrix}$

Use an inverse matrix to solve the linear system.

27. $3x + 4y = 6$
$4x + 5y = 7$

28. $2x - 7y = -36$
$x - 3y = -16$

29. $5x + 3y = -5$
$-9x - 6y = 12$

30. $3x + 2y = 15$
$-x + 4y = -33$

31. FINANCE A total of \$15,000 is invested in two corporate bonds that pay 5% and 7% simple annual interest. The investor wants to earn \$880 in interest per year from the bonds. How much should be invested in each bond?

32. TICKET SALES For the opening day of a carnival, 800 admission tickets were sold. The receipts totaled \$3775. Tickets for children cost \$3 each, tickets for adults cost \$8 each, and tickets for senior citizens cost \$5 each. There were twice as many children's tickets sold as adult tickets. How many of each type of ticket were sold?

33. BOATING On a certain river, a motorboat can travel 34 miles per hour with the current and 28 miles per hour against the current. Find the speed of the motorboat in still water and the speed of the current.