Alg 2

5 Polynomials and Polynomial Functions

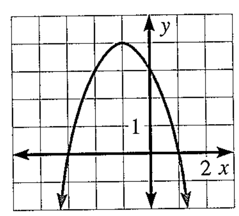
Before

In previous chapters, you learned the following skills, which you'll use in Chapter 5: graphing functions, factoring, and solving equations.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.



1. The **zeros** of the function graphed are _?_.
2. The **maximum value** of the function graphed is _?_.
3. The **standard form** of a quadratic equation in one variable is _?_ where $a \neq 0$.

SKILLS CHECK

Graph the function. Label the vertex and the axis of symmetry. ***(Review pp. 236, 245 for 5.2.)***

4. $y = -2(x - 1)^2 + 4$
5. $y = 3(x - 2)(x + 3)$
6. $y = -x^2 - 4x + 4$

Factor the expression. ***(Review pp. 252, 259 for 5.4.)***

7. $x^2 + 9x + 20$
8. $2x^2 + 5x - 3$
9. $9x^2 - 64$

Solve the equation. ***(Review pp. 252, 259 for 5.4–5.7.)***

10. $2x^2 + x + 6 = 0$
11. $10x^2 + 13x = 3$
12. $x^2 + 6x + 2 = 20$

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Now

In Chapter 5, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 401. You will also use the key vocabulary listed below.

Big Ideas

1. **Graphing polynomial functions**
2. **Performing operations with polynomials**
3. **Solving polynomial equations and finding zeros**

KEY VOCABULARY

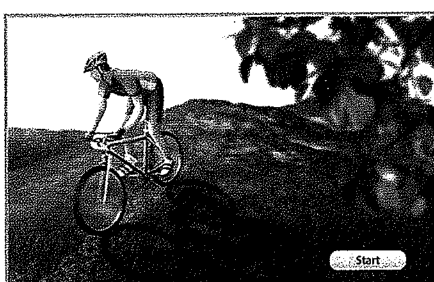
- polynomial, *p. 337*
- polynomial function, *p. 337*
- synthetic substitution, *p. 338*
- end behavior, *p. 339*
- factored completely, *p. 353*
- factor by grouping, *p. 354*
- quadratic form, *p. 355*
- polynomial long division, *p. 362*
- synthetic division, *p. 363*
- repeated solution, *p. 379*
- local maximum, *p. 388*
- local minimum, *p. 388*
- finite differences, *p. 393*

Why?

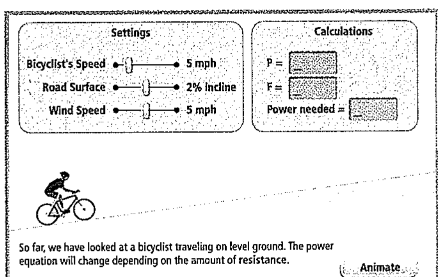
You can use polynomial functions to model real-life situations. For example, you can use a polynomial function to model the relationship between the speed of an object and the power needed to maintain that speed.

Animated Algebra

The animation illustrated below for Exercise 61 on page 351 helps you answer this question: How does the power needed to keep a bicycle moving at a constant speed change as the conditions change?



The power exerted by a bicyclist depends on speed and resistance.



Use the sliders to see how the road slope and wind speed affect the resistance.

Animated Algebra at classzone.com

Other animations for Chapter 5: pages 331, 340, 371, 388, 396, and 401

✓ **GUIDED PRACTICE** for Examples 3, 4, and 5

Simplify the expression. Tell which properties of exponents you used.

5. $x^{-6}x^5x^3$
6. $(7y^2z^5)(y^{-4}z^{-1})$
7. $\left(\frac{s^3}{t^{-4}}\right)^2$
8. $\left(\frac{x^4y^{-2}}{x^3y^6}\right)^3$

5.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS9 for Exs. 17, 31, and 51

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 36, 46, 51, and 53

SKILL PRACTICE

1. **VOCABULARY** State the name of the property illustrated.
   a. $a^m \cdot a^n = a^{m+n}$
   b. $a^{-m} = \frac{1}{a^m}, a \neq 0$
   c. $(ab)^m = a^m b^m$

2. ★ **WRITING** Is the number 25.2×10^{-3} in scientific notation? *Explain.*

EXAMPLE 1 on p. 330 for Exs. 3–14

EVALUATING NUMERICAL EXPRESSIONS **Evaluate the expression. Tell which properties of exponents you used.**

3. $3^3 \cdot 3^2$
4. $(4^{-2})^3$
5. $(-5)(-5)^4$
6. $(2^4)^2$
7. $\frac{5^2}{5^5}$
8. $\left(\frac{3}{5}\right)^4$
9. $\left(\frac{2}{7}\right)^{-3}$
10. $9^3 \cdot 9^{-1}$
11. $\frac{3^4}{3^{-2}}$
12. $\left(\frac{2}{3}\right)^{-5}\left(\frac{2}{3}\right)^4$
13. $6^3 \cdot 6^0 \cdot 6^{-5}$
14. $\left(\left(\frac{1}{2}\right)^{-5}\right)^2$

EXAMPLE 2 on p. 331 for Exs. 15–23

SCIENTIFIC NOTATION **Write the answer in scientific notation.**

15. $(4.2 \times 10^3)(1.5 \times 10^6)$
16. $(1.2 \times 10^{-3})(6.7 \times 10^{-7})$
17. $(6.3 \times 10^5)(8.9 \times 10^{-12})$
18. $(7.2 \times 10^9)(9.4 \times 10^8)$
19. $(2.1 \times 10^{-4})^3$
20. $(4.0 \times 10^3)^4$
21. $\frac{8.1 \times 10^{12}}{5.4 \times 10^9}$
22. $\frac{1.1 \times 10^{-3}}{5.5 \times 10^{-8}}$
23. $\frac{(7.5 \times 10^8)(4.5 \times 10^{-4})}{1.5 \times 10^7}$

EXAMPLES 3 and 4 on pp. 331–332 for Exs. 24–39

SIMPLIFYING ALGEBRAIC EXPRESSIONS **Simplify the expression. Tell which properties of exponents you used.**

24. $\frac{w^{-2}}{w^6}$
25. $(2^2y^3)^5$
26. $(p^3q^2)^{-1}$
27. $(w^3x^{-2})(w^6x^{-1})$
28. $(5s^{-2}t^4)^{-3}$
29. $(3a^3b^5)^{-3}$
30. $\frac{x^{-1}y^2}{x^2y^{-1}}$
31. $\frac{3c^3d}{9cd^{-1}}$
32. $\frac{4r^4s^5}{24r^4s^{-5}}$
33. $\frac{2a^3b^{-4}}{3a^5b^{-2}}$
34. $\frac{y^{11}}{4z^3} \cdot \frac{8z^7}{y^7}$
35. $\frac{x^2y^{-3}}{3y^2} \cdot \frac{y^2}{x^{-4}}$

36. ★ **MULTIPLE CHOICE** What is the simplified form of $\frac{2x^2y}{6xy^{-1}}$?

(A) $\frac{y^2}{3}$ (B) $\frac{xy^2}{3}$ (C) $\frac{x}{3}$ (D) $\frac{1}{3}$

ERROR ANALYSIS *Describe* **and correct the error in simplifying the expression.**

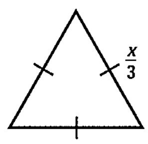
37. $\frac{x^{10}}{x^2} = x^5$ ✗

38. $x^5 \cdot x^3 = x^{15}$ ✗

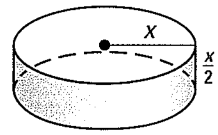
39. $(-3)^2(-3)^4 = 9^6$ ✗

GEOMETRY **Write an expression for the figure's area or volume in terms of x.**

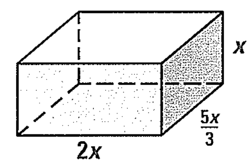
40. $A = \frac{\sqrt{3}}{4}s^2$



41. $V = \pi r^2 h$



42. $V = \ell wh$



REASONING **Write an expression that makes the statement true.**

43. $x^{15}y^{12}z^8 = x^4y^7z^{11} \cdot ?$

44. $3x^3y^2 = \frac{12x^2y^5}{?}$

45. $(a^5b^4)^2 = a^{14}b^{-1} \cdot ?$

46. ★ **OPEN-ENDED MATH** Find three different ways to complete the following statement so that it is true: $x^{12}y^{16} = (x^?y^?)(x^?y^?)$.

CHALLENGE **Refer to the properties of exponents on page 330.**

47. Show how the negative exponent property can be derived from the quotient of powers property and the zero exponent property.

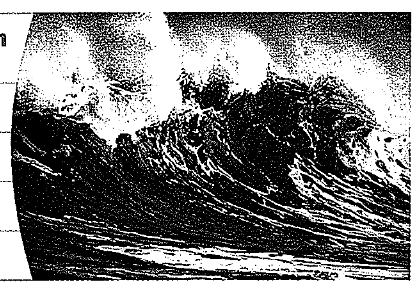
48. Show how the quotient of powers property can be derived from the product of powers property and the negative exponent property.

PROBLEM SOLVING

EXAMPLE 2
on p. 331
for Exs. 49–50

49. **OCEAN VOLUME** The table shows the surface areas and average depths of four oceans. Calculate the volume of each ocean by multiplying the surface area of each ocean by its average depth. Write your answers in scientific notation.

Ocean	Surface area (square meters)	Average depth (meters)
Pacific	1.56×10^{14}	4.03×10^3
Atlantic	7.68×10^{13}	3.93×10^3
Indian	6.86×10^{13}	3.96×10^3
Arctic	1.41×10^{13}	1.21×10^3



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50. **EARTH SCIENCE** The continents of Earth move at a very slow rate. The South American continent has been moving about 0.000022 mile per year for the past 125,000,000 years. How far has the continent moved in that time? Write your answer in scientific notation.

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EXAMPLE 5
on p. 332
for Exs. 51–52

51. ★ **SHORT RESPONSE** A typical cultured black pearl is made by placing a bead with a diameter of 6 millimeters inside an oyster. The resulting pearl has a diameter of about 9 millimeters. *Compare* the volume of the resulting pearl with the volume of the bead.

52. **MULTI-STEP PROBLEM** A can of tennis balls consists of three spheres of radius r stacked vertically inside a cylinder of radius r and height h.

a. Write an expression for the total volume of the three tennis balls in terms of r.

b. Write an expression for the volume of the cylinder in terms of r and h.

c. Write an expression for h in terms of r using the fact that the height of the cylinder is the sum of the diameters of the three tennis balls.

d. What fraction of the can's volume is taken up by the tennis balls?

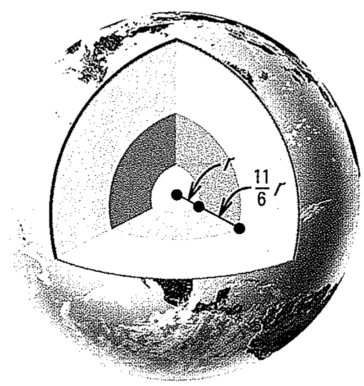
53. ★ **EXTENDED RESPONSE** You can think of a penny as a cylinder with a radius of about 9.53 millimeters and a height of about 1.55 millimeters.

a. **Calculate** Approximate the volume of a penny. Give your answer in cubic meters.

b. **Estimate** Approximate the volume of your classroom in cubic meters. *Explain* how you obtained your answer.

c. **Interpret** Use your results from parts (a) and (b) to estimate how many pennies it would take to fill your classroom. Do you think your answer is an overestimate or an underestimate? *Explain.*

54. **CHALLENGE** Earth's core is approximately spherical in shape and is divided into a solid inner core (the yellow region in the diagram shown) and a liquid outer core (the dark orange region in the diagram).



a. Earth's radius is about 5 times as great as the radius of Earth's inner core. Find the ratio of Earth's total volume to the volume of Earth's inner core.

b. Find the ratio of the volume of Earth's outer core to the volume of Earth's inner core.

MIXED REVIEW

PREVIEW
Prepare for Lesson 5.2 in Exs. 55–60.

Graph the function.

55. $y = -x + 4$ *(p. 89)*
56. $y = 2x - 5$ *(p. 89)*
57. $y = x^2 + 4$ *(p. 236)*
58. $y = -2x^2 - 1$ *(p. 236)*
59. $y = (x - 5)^2 - 3$ *(p. 245)*
60. $y = 3x(x + 4)$ *(p. 245)*

Use an inverse matrix to solve the linear system. *(p. 210)*

61. $x + y = 2$
$7x + 8y = 21$

62. $-x - 2y = 3$
$2x + 8y = 1$

63. $4x + 3y = 6$
$6x - 2y = 10$

Write the expression as a complex number in standard form. *(p. 275)*

64. $(8 + 3i) - (7 + 4i)$
65. $(5 - 2i) - (-9 + 6i)$
66. $i(3 + i)$
67. $(12 + 5i) - (7 - 8i)$
68. $(5 + 4i)(2 + 3i)$
69. $(8 - 4i)(1 + 6i)$

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Keystrokes

5.2 End Behavior of Polynomial Functions

MATERIALS • graphing calculator

QUESTION **How is the end behavior of a polynomial function related to the function's equation?**

Functions of the form $f(x) = \pm x^n$, where n is a positive integer, are examples of *polynomial functions*. The *end behavior* of a polynomial function's graph is its behavior as x approaches positive infinity ($+\infty$) or as x approaches negative infinity ($-\infty$).

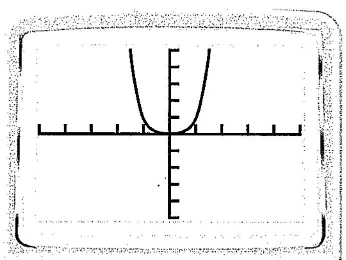
EXPLORE **Investigate the end behavior of $f(x) = \pm x^n$ where n is even**

Graph the function. Describe the end behavior of the graph.

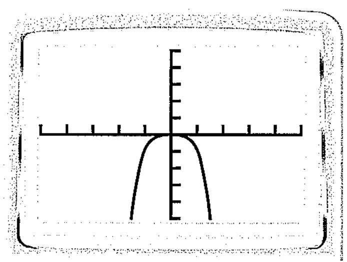
a. $f(x) = x^4$ **b.** $f(x) = -x^4$

STEP 1 ***Graph functions*** Graph each function on a graphing calculator.

a.



b.



STEP 2 ***Describe end behavior*** Summarize the end behavior of each function.

Function	As x approaches $-\infty$	As x approaches $+\infty$
a. $f(x) = x^4$	$f(x)$ approaches $+\infty$	$f(x)$ approaches $+\infty$
b. $f(x) = -x^4$	$f(x)$ approaches $-\infty$	$f(x)$ approaches $-\infty$

DRAW CONCLUSIONS **Use your observations to complete these exercises**

Graph the function. Then describe its end behavior as shown above.

1. $f(x) = x^5$ **2.** $f(x) = -x^5$ **3.** $f(x) = x^6$ **4.** $f(x) = -x^6$

5. Make a conjecture about the end behavior of each family of functions.

a. $f(x) = x^n$ where n is odd **b.** $f(x) = -x^n$ where n is odd

c. $f(x) = x^n$ where n is even **d.** $f(x) = -x^n$ where n is even

6. Make a conjecture about the end behavior of the function $f(x) = x^6 - x$. *Explain* your reasoning.

✓ **GUIDED PRACTICE** for Examples 5 and 6

Graph the polynomial function.

9. $f(x) = x^4 + 6x^2 - 3$ **10.** $f(x) = -x^3 + x^2 + x - 1$ **11.** $f(x) = 4 - 2x^3$

12. **WHAT IF?** If wind speed is measured in miles per hour, the model in Example 6 becomes $E = 0.0051s^4$. Graph this model. What wind speed is needed to generate a wave with 2000 foot-pounds of energy per square foot?

5.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS10 for Exs. 21, 27, and 57

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 24, 37, 50, 52, and 59

◆ = **MULTIPLE REPRESENTATIONS** Ex. 56

SKILL PRACTICE

1. **VOCABULARY** Identify the degree, type, leading coefficient, and constant term of the polynomial function $f(x) = 6 + 2x^2 - 5x^4$.

2. ★ **WRITING** *Explain* what is meant by the end behavior of a polynomial function.

EXAMPLE 1 on p. 337 for Exs. 3–8

POLYNOMIAL FUNCTIONS **Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.**

3. $f(x) = 8 - x^2$ **4.** $f(x) = 6x + 8x^4 - 3$ **5.** $g(x) = \pi x^4 + \sqrt{6}$

6. $h(x) = x^3\sqrt{10} + 5x^{-2} + 1$ **7.** $h(x) = -\frac{5}{2}x^3 + 3x - 10$ **8.** $g(x) = 8x^3 - 4x^2 + \frac{2}{x}$

EXAMPLE 2 on p. 338 for Exs. 9–14

DIRECT SUBSTITUTION **Use direct substitution to evaluate the polynomial function for the given value of x.**

9. $f(x) = 5x^3 - 2x^2 + 10x - 15;\ x = -1$ **10.** $f(x) = 8x + 5x^4 - 3x^2 - x^3;\ x = 2$

11. $g(x) = 4x^3 - 2x^5;\ x = -3$ **12.** $h(x) = 6x^3 - 25x + 20;\ x = 5$

13. $h(x) = x + \frac{1}{2}x^4 - \frac{3}{4}x^3 + 10;\ x = -4$ **14.** $g(x) = 4x^5 + 6x^3 + x^2 - 10x + 5;\ x = -2$

EXAMPLE 3 on p. 338 for Exs. 15–23

SYNTHETIC SUBSTITUTION **Use synthetic substitution to evaluate the polynomial function for the given value of x.**

15. $f(x) = 5x^3 - 2x^2 - 8x + 16;\ x = 3$ **16.** $f(x) = 8x^4 + 12x^3 + 6x^2 - 5x + 9;\ x = -2$

17. $g(x) = x^3 + 8x^2 - 7x + 35;\ x = -6$ **18.** $h(x) = -8x^3 + 14x - 35;\ x = 4$

19. $f(x) = -2x^4 + 3x^3 - 8x + 13;\ x = 2$ **20.** $g(x) = 6x^5 + 10x^3 - 27;\ x = -3$

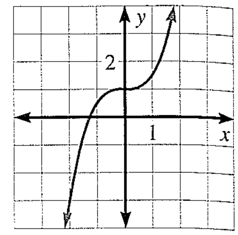
(21.) $h(x) = -7x^3 + 11x^2 + 4x;\ x = 3$ **22.** $f(x) = x^4 + 3x - 20;\ x = 4$

23. **ERROR ANALYSIS** *Describe* and correct the error in evaluating the polynomial function $f(x) = -4x^4 + 9x^2 - 21x + 7$ when $x = -2$.

−2	−4	9	−21	7
		8	−34	110
	−4	17	−55	117 ✗

EXAMPLE 4
on p. 339
for Exs. 24–27

24. ★ **MULTIPLE CHOICE** The graph of a polynomial function is shown. What is true about the function's degree and leading coefficient?



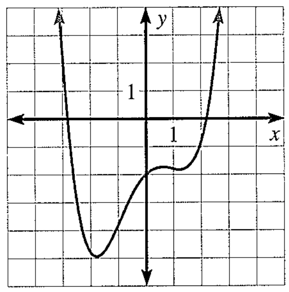
Ⓐ The degree is odd and the leading coefficient is positive.

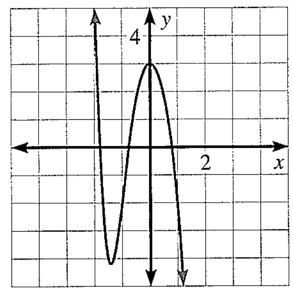
Ⓑ The degree is odd and the leading coefficient is negative.

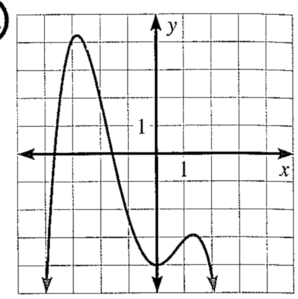
Ⓒ The degree is even and the leading coefficient is positive.

Ⓓ The degree is even and the leading coefficient is negative.

USING END BEHAVIOR ***Describe*** **the degree and leading coefficient of the polynomial function whose graph is shown.**

25. 

26. 

(27.) 

DESCRIBING END BEHAVIOR ***Describe*** **the end behavior of the graph of the polynomial function by completing these statements:** $f(x) \to$ **_?_ as** $x \to -\infty$ **and** $f(x) \to$ **_?_ as** $x \to +\infty$**.**

28. $f(x) = 10x^4$
29. $f(x) = -x^6 + 4x^3 - 3x$
30. $f(x) = -2x^3 + 7x - 4$
31. $f(x) = x^7 + 3x^4 - x^2$
32. $f(x) = 3x^{10} - 16x$
33. $f(x) = -6x^5 + 14x^2 + 20$
34. $f(x) = 0.2x^3 - x + 45$
35. $f(x) = 5x^8 + 8x^7$
36. $f(x) = -x^{273} + 500x^{271}$

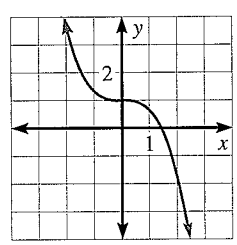
37. ★ **OPEN-ENDED MATH** Write a polynomial function f of degree 5 such that the end behavior of the graph of f is given by $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$. Then graph the function to verify your answer.

EXAMPLE 5
on p. 340
for Exs. 38–50

GRAPHING POLYNOMIALS **Graph the polynomial function.**

38. $f(x) = x^3$
39. $f(x) = -x^4$
40. $f(x) = x^5 + 3$
41. $f(x) = x^4 - 2$
42. $f(x) = -x^3 + 5$
43. $f(x) = x^3 - 5x$
44. $f(x) = -x^4 + 8x$
45. $f(x) = x^5 + x$
46. $f(x) = -x^3 + 3x^2 - 2x + 5$
47. $f(x) = x^5 + x^2 - 4$
48. $f(x) = x^4 - 5x^2 + 6$
49. $f(x) = -x^4 + 3x^3 - x + 1$

50. ★ **MULTIPLE CHOICE** Which function is represented by the graph shown?



Ⓐ $f(x) = \frac{1}{3}x^3 + 1$ Ⓑ $f(x) = -\frac{1}{3}x^3 + 1$

Ⓒ $f(x) = \frac{1}{3}x^3 - 1$ Ⓓ $f(x) = -\frac{1}{3}x^3 - 1$

51. **VISUAL THINKING** Suppose $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$. *Describe* the end behavior of $g(x) = -f(x)$.

52. ★ **SHORT RESPONSE** A cubic polynomial function f has leading coefficient 2 and constant term -5. If $f(1) = 0$ and $f(2) = 3$, what is $f(-5)$? *Explain* how you found your answer.

53. **CHALLENGE** Let $f(x) = x^3$ and $g(x) = x^3 - 2x^2 + 4x$.

a. Copy and complete the table.

b. Use the numbers in the table to complete this statement: As $x \to +\infty$, $\frac{f(x)}{g(x)} \to \underline{\ ?\ }$.

c. *Explain* how the result from part (b) shows that the functions f and g have the same end behavior as $x \to +\infty$.

x	$f(x)$	$g(x)$	$\frac{f(x)}{g(x)}$
10	?	?	?
20	?	?	?
50	?	?	?
100	?	?	?
200	?	?	?

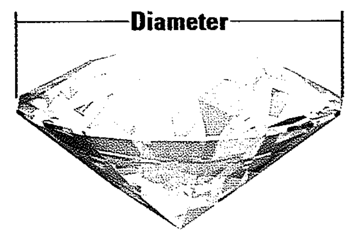
PROBLEM SOLVING

EXAMPLE 6 on p. 340 for Exs. 54–59

54. **DIAMONDS** The weight of an ideal round-cut diamond can be modeled by

$$w = 0.0071d^3 - 0.090d^2 + 0.48d$$

where w is the diamond's weight (in carats) and d is its diameter (in millimeters). According to the model, what is the weight of a diamond with a diameter of 15 millimeters?



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55. **SKATEBOARDING** From 1992 to 2003, the number of people in the United States who participated in skateboarding can be modeled by

$$S = -0.0076t^4 + 0.14t^3 - 0.62t^2 + 0.52t + 5.5$$

where S is the number of participants (in millions) and t is the number of years since 1992. Graph the model. Then use the graph to estimate the first year that the number of skateboarding participants was greater than 8 million.

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56. **MULTIPLE REPRESENTATIONS** From 1987 to 2003, the number of indoor movie screens M in the United States can be modeled by

$$M = -11.0t^3 + 267t^2 - 592t + 21{,}600$$

where t is the number of years since 1987.

a. Classifying a Function State the degree and type of the function.

b. Making a Table Make a table of values for the function.

c. Sketching a Graph Use your table to graph the function.

57. **SNOWBOARDING** From 1992 to 2003, the number of people in the United States who participated in snowboarding can be modeled by

$$S = 0.0013t^4 - 0.021t^3 + 0.084t^2 + 0.037t + 1.2$$

where S is the number of participants (in millions) and t is the number of years since 1992. Graph the model. Use the graph to estimate the first year that the number of snowboarding participants was greater than 2 million.

58. **MULTI-STEP PROBLEM** From 1980 to 2002, the number of quarterly periodicals P published in the United States can be modeled by

$$P = 0.138t^4 - 6.24t^3 + 86.8t^2 - 239t + 1450$$

where t is the number of years since 1980.

a. *Describe* the end behavior of the graph of the model.

b. Graph the model on the domain $0 \le t \le 22$.

c. Use the model to predict the number of quarterly periodicals in the year 2010. Is it appropriate to use the model to make this prediction? *Explain.*

59. ★ **EXTENDED RESPONSE** The weights of Sarus crane chicks S and hooded crane chicks H (both in grams) during the 10 days following hatching can be modeled by the functions

$$S = -0.122t^3 + 3.49t^2 - 14.6t + 136$$

$$H = -0.115t^3 + 3.71t^2 - 20.6t + 124$$

where t is the number of days after hatching.

a. **Calculate** According to the models, what is the difference in weight between 5-day-old Sarus crane chicks and hooded crane chicks?

b. **Graph** Sketch the graphs of the two models.

c. **Apply** A biologist finds that the weight of a crane chick 3 days after hatching is 130 grams. What species of crane is the chick more likely to be? *Explain* how you found your answer.

60. **CHALLENGE** The weight y (in pounds) of a rainbow trout can be modeled by $y = 0.000304x^3$ where x is the length of the trout (in inches).

a. Write a function that relates the weight y and length x of a rainbow trout if y is measured in kilograms and x is measured in centimeters. Use the fact that 1 kilogram $\approx$ 2.20 pounds and 1 centimeter $\approx$ 0.394 inch.

b. Graph the original function and the function from part (a) in the same coordinate plane. What type of transformation can you apply to the graph of $y = 0.000304x^3$ to produce the graph from part (a)?

MIXED REVIEW

Solve the equation or inequality.

61. $2b + 11 = 15 - 6b$ *(p. 18)*

62. $2.7n + 4.3 = 12.94$ *(p. 18)*

63. $-7 < 6y - 1 < 5$ *(p. 41)*

64. $x^2 - 14x + 48 = 0$ *(p. 252)*

65. $-24q^2 - 90q = 21$ *(p. 259)*

66. $z^2 + 5z < 36$ *(p. 300)*

The variables x and y vary directly. Write an equation that relates x and y. Then find the value of x when $y = -3$. *(p. 107)*

67. $x = 4, y = 12$

68. $x = 3, y = -21$

69. $x = 10, y = -4$

70. $x = 0.8, y = 0.2$

71. $x = -0.45, y = -0.35$

72. $x = -6.5, y = 3.9$

PREVIEW
Prepare for Lesson 5.3 in Exs. 73–78.

Write the quadratic function in standard form. *(p. 245)*

73. $y = (x + 3)(x - 7)$

74. $y = 8(x - 4)(x + 2)$

75. $y = -3(x - 5)^2 - 25$

76. $y = 2.5(x - 6)^2 + 9.3$

77. $y = \frac{1}{2}(x - 4)^2$

78. $y = -\frac{5}{3}(x + 4)(x + 9)$

EXTRA PRACTICE for Lesson 5.2, p. 1014 **ONLINE QUIZ** at classzone.com

Graphing Calculator **ACTIVITY** *Use after Lesson 5.2*

@HomeTutor
classzone.com
Keystrokes

5.2 Set a Good Viewing Window

QUESTION **What is a good viewing window for a polynomial function?**

When you graph a function with a graphing calculator, you should choose a viewing window that displays the important characteristics of the graph.

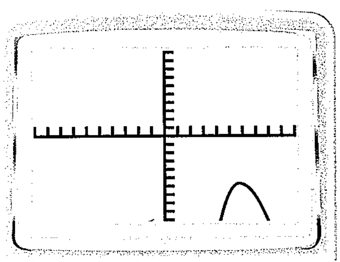
EXAMPLE **Graph a polynomial function**

Graph $f(x) = 0.2x^3 - 5x^2 + 38x - 97$.

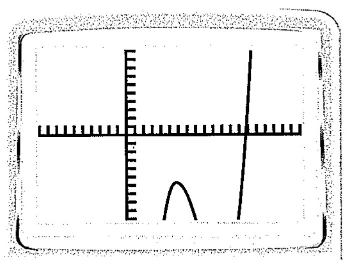
STEP 1 ***Graph the function***
Graph the function in the standard viewing window.

STEP 2 ***Adjust horizontally***
Adjust the horizontal scale so that the end behavior of the graph as $x \to +\infty$ is visible.

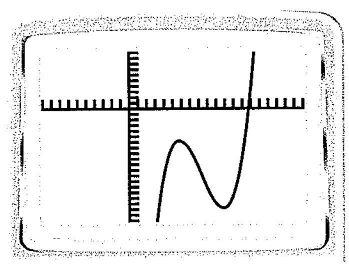
STEP 3 ***Adjust vertically***
Adjust the vertical scale so that the turning points and end behavior of the graph as $x \to -\infty$ are visible.



$-10 \le x \le 10, -10 \le y \le 10$



$-10 \le x \le 20, -10 \le y \le 10$



$-10 \le x \le 20, -20 \le y \le 10$

PRACTICE

Find intervals for x and y that describe a good viewing window for the graph of the polynomial function.

1. $f(x) = x^3 + 4x^2 - 8x + 11$
2. $f(x) = -x^3 + 36x^2 - 10$
3. $f(x) = x^4 - 4x^2 + 2$
4. $f(x) = -x^4 - 2x^3 + 3x^2 - 4x + 5$
5. $f(x) = -x^4 + 3x^3 + 15x$
6. $f(x) = 2x^4 - 7x^3 + x - 8$
7. $f(x) = -x^5 + 9x^3 - 12x + 18$
8. $f(x) = x^5 - 7x^4 + 25x^3 - 40x^2 + 13x$

9. **REASONING** Let $g(x) = f(x) + c$ where $f(x)$ and $g(x)$ are polynomial functions and c is a positive constant. How is a good viewing window for the graph of $f(x)$ related to a good viewing window for the graph of $g(x)$?

10. **BASEBALL** From 1994 to 2003, the average salary S (in thousands of dollars) for major league baseball players can be modeled by

$$S(x) = -4.10x^3 + 67.4x^2 - 121x + 1170$$

where x is the number of years since 1994. Find intervals for the horizontal and vertical axes that describe a good viewing window for the graph of S.

5.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS10 for Exs. 11, 21, and 61

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 47, 56, and 63

SKILL PRACTICE

1. **VOCABULARY** When you add or subtract polynomials, you add or subtract the coefficients of _?_.

2. ★ **WRITING** *Explain* how a polynomial subtraction problem is equivalent to a polynomial addition problem.

EXAMPLES 1 and 2 on p. 346 for Exs. 3–15

ADDING AND SUBTRACTING POLYNOMIALS **Find the sum or difference.**

3. $(3x^2 - 5) + (7x^2 - 3)$
4. $(x^2 - 3x + 5) - (-4x^2 + 8x + 9)$
5. $(4y^2 + 9y - 5) - (4y^2 - 5y + 3)$
6. $(z^2 + 5z - 7) + (5z^2 - 11z - 6)$
7. $(3s^3 + s) + (4s^3 - 2s^2 + 7s + 10)$
8. $(2a^2 - 8) - (a^3 + 4a^2 - 12a + 4)$
9. $(5c^2 + 7c + 1) + (2c^3 - 6c + 8)$
10. $(4t^3 - 11t^2 + 4t) - (-7t^2 - 5t + 8)$
11. $(5b - 6b^3 + 2b^4) - (9b^3 + 4b^4 - 7)$
12. $(3y^2 - 6y^4 + 5 - 6y) + (5y^4 - 6y^3 + 4y)$
13. $(x^4 - x^3 + x^2 - x + 1) + (x + x^4 - 1 - x^2)$
14. $(8v^4 - 2v^2 + v - 4) - (3v^3 - 12v^2 + 8v)$

15. ★ **MULTIPLE CHOICE** What is the result when $2x^4 - 8x^2 - x + 10$ is subtracted from $8x^4 - 4x^3 - x + 2$?

(A) $-6x^4 + 4x^3 - 8x^2 + 8$
(B) $6x^4 - 4x^3 + 8x^2 - 8$
(C) $10x^4 - 8x^3 - 4x^2 + 12$
(D) $6x^4 + 4x^3 - 2x - 8$

EXAMPLE 3 on p. 347 for Exs. 16–25

MULTIPLYING POLYNOMIALS **Find the product of the polynomials.**

16. $x(2x^2 - 5x + 7)$
17. $5x^2(6x + 2)$
18. $(y - 7)(y + 6)$
19. $(3z + 1)(z - 3)$
20. $(w + 4)(w^2 + 6w - 11)$
21. $(2a - 3)(a^2 - 10a - 2)$
22. $(5c^2 - 4)(2c^2 + c - 3)$
23. $(-x^2 + 4x + 1)(x^2 - 8x + 3)$
24. $(-d^2 + 4d + 3)(3d^2 - 7d + 6)$
25. $(3y^2 + 6y - 1)(4y^2 - 11y - 5)$

ERROR ANALYSIS ***Describe*** **and correct the error in simplifying the expression.**

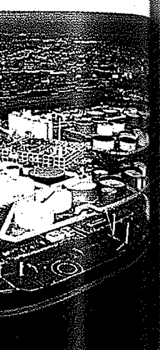
26.
$$(x^2 - 3x + 4) - (x^3 + 7x - 2)$$
$$= x^2 - 3x + 4 - x^3 + 7x - 2$$
$$= -x^3 + x^2 + 4x + 2$$

27.
$$(2x - 7)^3 = (2x)^3 - 7^3$$
$$= 8x^3 - 343$$

EXAMPLE 4 on p. 347 for Exs. 28–37

MULTIPLYING THREE BINOMIALS **Find the product of the binomials.**

28. $(x + 4)(x - 6)(x - 5)$
29. $(x + 1)(x - 7)(x + 3)$
30. $(z - 4)(-z + 2)(z + 8)$
31. $(a - 6)(2a + 5)(a + 1)$
32. $(3p + 1)(p + 3)(p + 1)$
33. $(b - 2)(2b - 1)(-b + 1)$
34. $(2s + 1)(3s - 2)(4s - 3)$
35. $(w - 6)(4w - 1)(-3w + 5)$
36. $(4x - 1)(-2x - 7)(-5x - 4)$
37. $(3q - 8)(-9q + 2)(q - 2)$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ch, California

EXAMPLE 5
on p. 348
for Exs. 38–47

SPECIAL PRODUCTS **Find the product.**

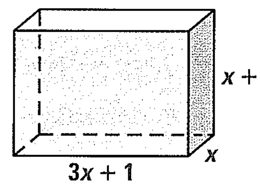
38. $(x+5)(x-5)$
39. $(w-9)^2$
40. $(y+4)^3$
41. $(2c+5)^2$
42. $(3t-4)^3$
43. $(5p-3)(5p+3)$
44. $(7x-y)^3$
45. $(2a+9b)(2a-9b)$
46. $(3z+7y)^3$

47. ★ **MULTIPLE CHOICE** Which expression is equivalent to $(3x-2y)^2$?

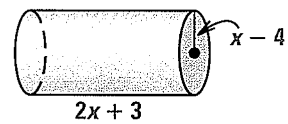
(A) $9x^2-4y^2$
(B) $9x^2+4y^2$
(C) $9x^2+12xy+4y^2$
(D) $9x^2-12xy+4y^2$

GEOMETRY **Write the figure's volume as a polynomial in standard form.**

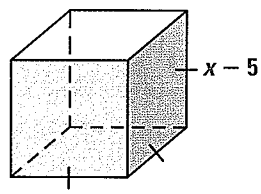
48. $V=\ell wh$



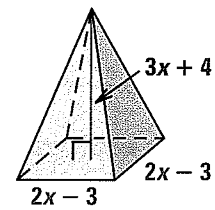
49. $V=\pi r^2h$



50. $V=s^3$



51. $V=\frac{1}{3}Bh$



SPECIAL PRODUCTS **Verify the special product pattern by multiplying.**

52. $(a+b)(a-b)=a^2-b^2$
53. $(a+b)^2=a^2+2ab+b^2$
54. $(a+b)^3=a^3+3a^2b+3ab^2+b^3$
55. $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

56. ★ **EXTENDED RESPONSE** Let $p(x)=x^4-7x+14$ and $q(x)=x^2-5$.

a. What is the degree of the polynomial $p(x)+q(x)$?

b. What is the degree of the polynomial $p(x)-q(x)$?

c. What is the degree of the polynomial $p(x)\cdot q(x)$?

d. In general, if $p(x)$ and $q(x)$ are polynomials such that $p(x)$ has degree m, $q(x)$ has degree n, and $m>n$, what are the degrees of $p(x)+q(x)$, $p(x)-q(x)$, and $p(x)\cdot q(x)$?

57. **FINDING A PATTERN** Look at the following polynomial factorizations.

$x^2-1=(x-1)(x+1)$

$x^3-1=(x-1)(x^2+x+1)$

$x^4-1=(x-1)(x^3+x^2+x+1)$

a. Factor x^5-1 and x^6-1 into the product of $x-1$ and another polynomial. Check your answers by multiplying.

b. In general, how can x^n-1 be factored? Show that this factorization works by multiplying the factors.

58. **CHALLENGE** Suppose $f(x)=(x+a)(x+b)(x+c)(x+d)$. If $f(x)$ is written in standard form, show that the coefficient of x^3 is the sum of a, b, c, and d, and the constant term is the product of a, b, c, and d.

○ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLE 6 on p. 348 for Exs. 59–61

59. HIGHER EDUCATION Since 1970, the number (in thousands) of males M and females F attending institutes of higher education can be modeled by

$$M = 0.091t^3 - 4.8t^2 + 110t + 5000 \quad \text{and} \quad F = 0.19t^3 - 12t^2 + 350t + 3600$$

where t is the number of years since 1970. Write a model for the total number of people attending institutes of higher education.

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60. ELECTRONICS From 1999 to 2004, the number of DVD players D (in millions) sold in the United States and the average price per DVD player P (in dollars) can be modeled by

$$D = 4.11t + 4.44 \quad \text{and} \quad P = 6.82t^2 - 61.7t + 265$$

where t is the number of years since 1999. Write a model for the total revenue R from DVD sales. According to the model, what was the total revenue in 2002?

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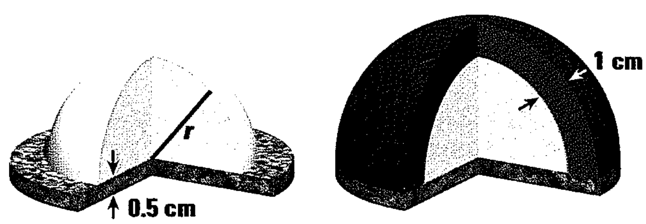
61. BICYCLING The equation $P = 0.00267sF$ gives the power P (in horsepower) needed to keep a certain bicycle moving at speed s (in miles per hour), where F is the force (in pounds) of road and air resistance. On level ground, the equation

$$F = 0.0116s^2 + 0.789$$

models the force F. Write a model (in terms of s only) for the power needed to keep the bicycle moving at speed s on level ground. How much power is needed to keep the bicycle moving at 10 miles per hour?

***Animated* Algebra** at classzone.com

62. MULTI-STEP PROBLEM A dessert is made by taking a hemispherical mound of marshmallow on a 0.5 centimeter thick cookie and covering it with a chocolate shell 1 centimeter thick. Use the diagrams to write two polynomial functions in standard form: $M(r)$ for the combined volume of the marshmallow plus cookie, and $D(r)$ for the volume of the entire dessert. Then use $M(r)$ and $D(r)$ to write a function $C(r)$ for the volume of the chocolate.



Marshmallow on cookie **Chocolate layer added**

63. ★ SHORT RESPONSE From 1997 to 2002, the number of NCAA lacrosse teams for men L_m and women L_w, as well as the average size of a men's team S_m and a women's team S_w, can be modeled by

$$L_m = 5.57t + 182 \quad \text{and} \quad S_m = -0.127t^3 + 0.822t^2 - 1.02t + 31.5$$

$$L_w = 12.2t + 185 \quad \text{and} \quad S_w = -0.0662t^3 + 0.437t^2 - 0.725t + 22.3$$

where t is the number of years since 1997. Write a model for the *total* number of people N on NCAA lacrosse teams. *Explain* how you obtained your model.

64. **CHALLENGE** From 1970 to 2002, the circulation C (in millions) of Sunday newspapers in the United States can be modeled by

$$C = -0.00105t^3 + 0.0281t^2 + 0.465t + 48.8$$

where t is the number of years since 1970. Rewrite C as a function of s, where s is the number of years since 1975.

MIXED REVIEW

PREVIEW Prepare for Lesson 5.4 in Exs. 65–72.

Solve the equation.

65. $2x - 7 = 11$ ***(p. 18)***

66. $10 - 3x = 25$ ***(p. 18)***

67. $4t - 7 = 2t$ ***(p. 18)***

68. $y^2 - 2y - 48 = 0$ ***(p. 252)***

69. $w^2 - 15w + 54 = 0$ ***(p. 252)***

70. $x^2 + 9x + 14 = 0$ ***(p. 252)***

71. $4z^2 + 21z - 18 = 0$ ***(p. 259)***

72. $9a^2 - 30a + 25 = 0$ ***(p. 259)***

Solve the system of equations. ***(p. 178)***

73. $x + y - 2z = -4$
$3x - y + z = 22$
$-x + 2y + 3z = -9$

74. $x - 2y + z = -13$
$-x + 4y + z = 35$
$3x + 2y + 4z = 28$

75. $3x - y - 2z = 20$
$-x + 3y - z = -16$
$-2x - y + 3z = -5$

Evaluate the determinant of the matrix. ***(p. 203)***

76. $\begin{bmatrix} 3 & -4 \\ 3 & 1 \end{bmatrix}$

77. $\begin{bmatrix} 5 & 7 \\ -4 & 9 \end{bmatrix}$

78. $\begin{bmatrix} -1 & 8 & 0 \\ 3 & 4 & -3 \\ -5 & 2 & 1 \end{bmatrix}$

79. $\begin{bmatrix} 2 & 3 & -4 \\ -6 & 1 & 5 \\ -3 & -1 & -2 \end{bmatrix}$

QUIZ *for Lessons 5.1–5.3*

Evaluate the expression. ***(p. 330)***

1. $3^5 \cdot 3^{-1}$

2. $(2^4)^2$

3. $\left(\frac{2}{3^{-2}}\right)^2$

4. $\left(\frac{3}{5}\right)^{-2}$

Simplify the expression. ***(p. 330)***

5. $(x^4y^{-2})(x^{-3}y^8)$

6. $(a^2b^{-5})^{-3}$

7. $\frac{x^3y^7}{x^{-4}y^0}$

8. $\frac{c^3d^{-2}}{c^5d^{-1}}$

Graph the polynomial function. ***(p. 337)***

9. $g(x) = 2x^3 - 3x + 1$

10. $h(x) = x^4 - 4x + 2$

11. $f(x) = -2x^3 + x^2 - 5$

Perform the indicated operation. ***(p. 346)***

12. $(x^3 + x^2 - 6) - (2x^2 + 4x - 8)$

13. $(-3x^2 + 4x - 10) + (x^2 - 9x + 15)$

14. $(x - 5)(x^2 - 5x + 7)$

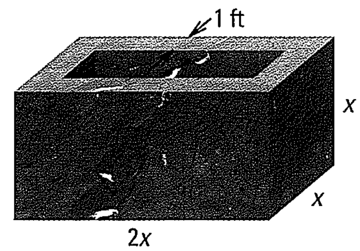
15. $(x + 3)(x - 6)(3x - 1)$

16. **NATIONAL DEBT** On July 21, 2004, the national debt of the United States was about $7,282,000,000,000. The population of the United States at that time was about 294,000,000. Suppose the national debt was divided evenly among everyone in the United States. How much would each person owe? ***(p. 330)***

EXAMPLE 6 Solve a polynomial equation

CITY PARK You are designing a marble basin that will hold a fountain for a city park. The basin's sides and bottom should be 1 foot thick. Its outer length should be twice its outer width and outer height.

What should the outer dimensions of the basin be if it is to hold 36 cubic feet of water?



ANOTHER WAY
For alternative methods to solving the problem in Example 6, turn to page 360 for the **Problem Solving Workshop**.

Solution

Volume (cubic feet)	=	Interior length (feet)	·	Interior width (feet)	·	Interior height (feet)
36	**=**	$\mathbf{(2x - 2)}$	·	$\mathbf{(x - 2)}$	·	$\mathbf{(x - 1)}$

$36 = (2x - 2)(x - 2)(x - 1)$ **Write equation.**

$0 = 2x^3 - 8x^2 + 10x - 40$ **Write in standard form.**

$0 = 2x^2(x - 4) + 10(x - 4)$ **Factor by grouping.**

$0 = (2x^2 + 10)(x - 4)$ **Distributive property**

▶ The only real solution is $x = 4$. The basin is 8 ft long, 4 ft wide, and 4 ft high.

✓ **GUIDED PRACTICE** for Example 6

11. **WHAT IF?** In Example 6, what should the basin's dimensions be if it is to hold 40 cubic feet of water and have outer length $6x$, width $3x$, and height x?

5.4 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS on p. WS10 for Exs. 7, 23, and 61**
★ = **STANDARDIZED TEST PRACTICE Exs. 2, 9, 41, 63, and 64**

SKILL PRACTICE

1. **VOCABULARY** The expression $8x^6 + 10x^3 - 3$ is in _?_ form because it can be written as $2u^2 + 5u - 3$ where $u = 2x^3$.

2. ★ **WRITING** What condition must the factorization of a polynomial satisfy in order for the polynomial to be factored completely?

EXAMPLE 1
on p. 353 for Exs. 3–9

MONOMIAL FACTORS **Factor the polynomial completely.**

3. $14x^2 - 21x$
4. $30b^3 - 54b^2$
5. $c^3 + 9c^2 + 18c$
6. $z^3 - 6z^2 - 72z$
7. $3y^5 - 48y^3$
8. $54m^5 + 18m^4 + 9m^3$

9. ★ **MULTIPLE CHOICE** What is the complete factorization of $2x^7 - 32x^3$?

(A) $2x^3(x + 2)(x - 2)(x^2 + 4)$
(B) $2x^3(x^2 + 2)(x^2 - 2)$
(C) $2x^3(x^2 + 4)^2$
(D) $2x^3(x + 2)^2(x - 2)^2$

EXAMPLE 2 on p. 354 for Exs. 10–17

SUM OR DIFFERENCE OF CUBES **Factor the polynomial completely.**

10. $x^3 + 8$ **11.** $y^3 - 64$ **12.** $27m^3 + 1$ **13.** $125n^3 + 216$

14. $27a^3 - 1000$ **15.** $8c^3 + 343$ **16.** $192w^3 - 3$ **17.** $-5z^3 + 320$

EXAMPLE 3 on p. 354 for Exs. 18–23

FACTORING BY GROUPING **Factor the polynomial completely.**

18. $x^3 + x^2 + x + 1$ **19.** $y^3 - 7y^2 + 4y - 28$ **20.** $n^3 + 5n^2 - 9n - 45$

21. $3m^3 - m^2 + 9m - 3$ **22.** $25s^3 - 100s^2 - s + 4$ **23.** $4c^3 + 8c^2 - 9c - 18$

EXAMPLE 4 on p. 355 for Exs. 24–29

QUADRATIC FORM **Factor the polynomial completely.**

24. $x^4 - 25$ **25.** $a^4 + 7a^2 + 6$ **26.** $3s^4 - s^2 - 24$

27. $32z^5 - 2z$ **28.** $36m^6 + 12m^4 + m^2$ **29.** $15x^5 - 72x^3 - 108x$

EXAMPLE 5 on p. 355 for Exs. 30–41

ERROR ANALYSIS ***Describe*** **and correct the error in finding all real-number solutions.**

30.

$$8x^3 - 27 = 0$$
$$(2x + 3)(4x^2 + 6x + 9) = 0$$
$$x = -\frac{3}{2}$$

31.

$$3x^3 - 48x = 0$$
$$3x(x^2 - 16) = 0$$
$$x^2 - 16 = 0$$
$$x = -4 \text{ or } x = 4$$

SOLVING EQUATIONS **Find the real-number solutions of the equation.**

32. $y^3 - 5y^2 = 0$ **33.** $18s^3 = 50s$ **34.** $g^3 + 3g^2 - g - 3 = 0$

35. $m^3 + 6m^2 - 4m - 24 = 0$ **36.** $4w^4 + 40w^2 - 44 = 0$ **37.** $4z^5 = 84z^3$

38. $5b^3 + 15b^2 + 12b = -36$ **39.** $x^6 - 4x^4 - 9x^2 + 36 = 0$ **40.** $48p^5 = 27p^3$

41. ★ **MULTIPLE CHOICE** What are the real-number solutions of the equation $3x^4 - 27x^2 + 9x = x^3$?

(A) $-1, 0, 3$ (B) $-3, 0, 3$ (C) $-3, 0, \frac{1}{3}, 3$ (D) $-3, -\frac{1}{3}, 0, 3$

CHOOSING A METHOD **Factor the polynomial completely using any method.**

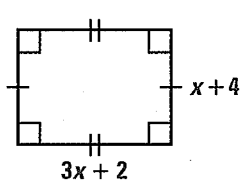
42. $16x^3 - 44x^2 - 42x$ **43.** $n^4 - 4n^2 - 60$ **44.** $-4b^4 - 500b$

45. $36a^3 - 15a^2 + 84a - 35$ **46.** $18c^4 + 57c^3 - 10c^2$ **47.** $2d^4 - 13d^2 - 45$

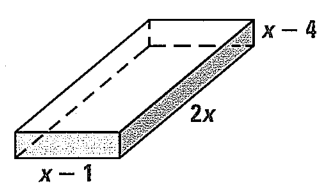
48. $32x^5 - 108x^2$ **49.** $8y^6 - 38y^4 - 10y^2$ **50.** $z^5 - 3z^4 - 16z + 48$

GEOMETRY **Find the possible value(s) of x.**

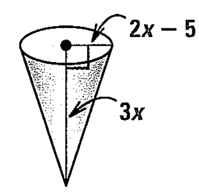
51. Area = 48



52. Volume = 40



53. Volume = 125π



CHOOSING A METHOD **Factor the polynomial completely using any method.**

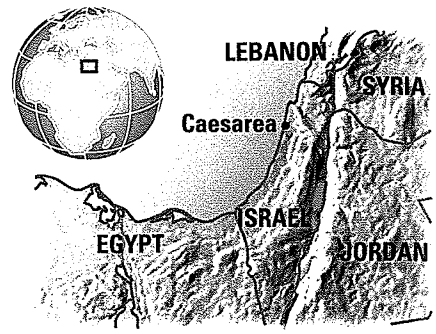
54. $x^3y^6 - 27$ **55.** $7ac^2 + bc^2 - 7ad^2 - bd^2$ **56.** $x^{2n} - 2x^n + 1$

57. **CHALLENGE** Factor $a^5b^2 - a^2b^4 + 2a^4b - 2ab^3 + a^3 - b^2$ completely.

PROBLEM SOLVING

EXAMPLE 6 on p. 356 for Exs. 58–63

58. **ARCHAEOLOGY** At the ruins of Caesarea, archaeologists discovered a huge hydraulic concrete block with a volume of 945 cubic meters. The block's dimensions are x meters high by $12x - 15$ meters long by $12x - 21$ meters wide. What is the height of the block?



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59. **CHOCOLATE MOLD** You are designing a chocolate mold shaped like a hollow rectangular prism for a candy manufacturer. The mold must have a thickness of 1 centimeter in all dimensions. The mold's outer dimensions should also be in the ratio 1 : 3 : 6. What should the outer dimensions of the mold be if it is to hold 112 cubic centimeters of chocolate?

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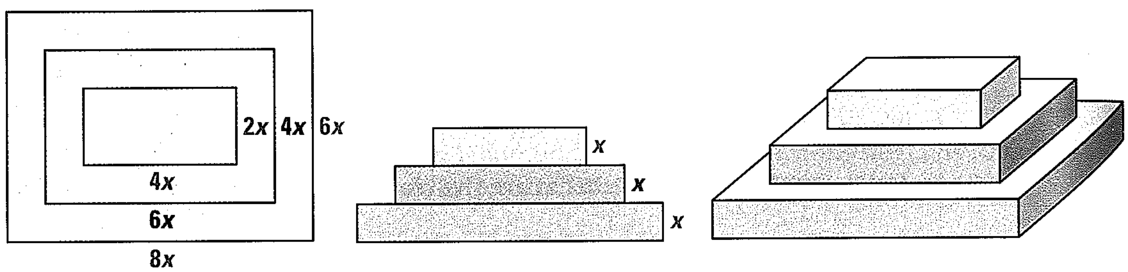
60. **MULTI-STEP PROBLEM** A production crew is assembling a three-level platform inside a stadium for a performance. The platform has the dimensions shown in the diagrams, and has a total volume of 1250 cubic feet.



a. **Write Expressions** What is the volume, in terms of x, of each of the three levels of the platform?

b. **Write an Equation** Use what you know about the total volume to write an equation involving x.

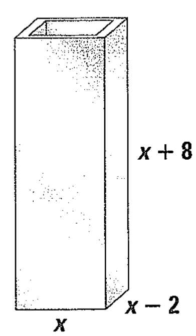
c. **Solve** Solve the equation from part (b). Use your solution to calculate the dimensions of each of the three levels of the platform.

(61.) **SCULPTURE** Suppose you have 250 cubic inches of clay with which to make a sculpture shaped as a rectangular prism. You want the height and width each to be 5 inches less than the length. What should the dimensions of the prism be?

62. **MANUFACTURING** A manufacturer wants to build a rectangular stainless steel tank with a holding capacity of 670 gallons, or about 89.58 cubic feet. The tank's walls will be one half inch thick, and about 6.42 cubic feet of steel will be used for the tank. The manufacturer wants the outer dimensions of the tank to be related as follows:

- The width should be 2 feet less than the length.
- The height should be 8 feet more than the length.

What should the outer dimensions of the tank be?



○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

63. ★ **SHORT RESPONSE** A platform shaped like a rectangular prism has dimensions $x - 2$ feet by $3 - 2x$ feet by $3x + 4$ feet. *Explain* why the volume of the platform cannot be $\frac{7}{3}$ cubic feet.

64. ★ **EXTENDED RESPONSE** In 2000 B.C., the Babylonians solved polynomial equations using tables of values. One such table gave values of $y^3 + y^2$. To be able to use this table, the Babylonians sometimes had to manipulate the equation, as shown below.

$ax^3 + bx^2 = c$	**Original equation**
$\frac{a^3x^3}{b^3} + \frac{a^2x^2}{b^2} = \frac{a^2c}{b^3}$	**Multiply each side by** $\frac{a^2}{b^3}$.
$\left(\frac{ax}{b}\right)^3 + \left(\frac{ax}{b}\right)^2 = \frac{a^2c}{b^3}$	**Rewrite cubes and squares.**

They then found $\frac{a^2c}{b^3}$ in the $y^3 + y^2$ column of the table. Because the corresponding y-value was $y = \frac{ax}{b}$, they could conclude that $x = \frac{by}{a}$.

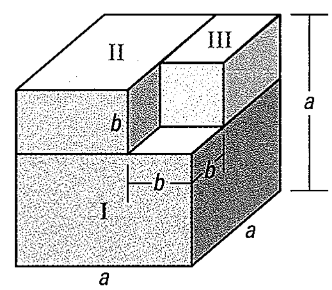
a. Calculate $y^3 + y^2$ for $y = 1, 2, 3, \ldots, 10$. Record the values in a table.

b. Use your table and the method described above to solve $x^3 + 2x^2 = 96$.

c. Use your table and the method described above to solve $3x^3 + 2x^2 = 512$.

d. How can you modify the method described above for equations of the form $ax^4 + bx^3 = c$?

65. **CHALLENGE** Use the diagram to complete parts (a)–(c).



a. *Explain* why $a^3 - b^3$ is equal to the sum of the volumes of solid I, solid II, and solid III.

b. Write an algebraic expression for the volume of each of the three solids. Leave your expressions in factored form.

c. Use the results from parts (a) and (b) to derive the factoring pattern for $a^3 - b^3$ given on page 354.

MIXED REVIEW

Graph the function.

66. $f(x) = -2|x - 3| + 5$ ***(p. 123)***

67. $y = \frac{1}{2}x^2 + 4x + 5$ ***(p. 236)***

68. $y = 3(x + 4)^2 + 7$ ***(p. 245)***

69. $f(x) = x^3 - 2x - 5$ ***(p. 337)***

Graph the inequality in a coordinate plane. *(p. 132)*

70. $y \le 2x - 3$

71. $y > -5 - x$

72. $y < 0.5x + 5$

73. $4x + 12y \le 4$

74. $9x - 9y \ge 27$

75. $\frac{2}{5}x + \frac{5}{2}y > 5$

PREVIEW
Prepare for Lesson 5.5 in Exs. 76–79.

Use synthetic substitution to evaluate the polynomial function for the given value of x. *(p. 337)*

76. $f(x) = 5x^4 - 3x^3 + 4x^2 - x + 10;\ x = 2$

77. $f(x) = -3x^5 + x^3 - 6x^2 + 2x + 4;\ x = -3$

78. $f(x) = 5x^5 - 4x^3 + 12x^2 + 20;\ x = -2$

79. $f(x) = -6x^4 + 9x - 15;\ x = 4$

PROBLEM SOLVING WORKSHOP
LESSON 5.4

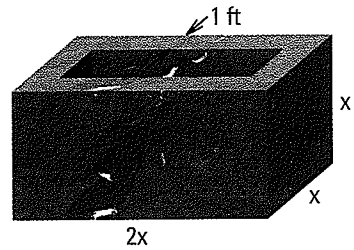
Using ALTERNATIVE METHODS

Another Way to Solve Example 6, page 356

MULTIPLE REPRESENTATIONS In Example 6 on page 356, you solved a polynomial equation by factoring. You can also solve a polynomial equation using a table or a graph.

PROBLEM

CITY PARK You are designing a marble basin that will hold a fountain for a city park. The basin's sides and bottom should be 1 foot thick. Its outer length should be twice its outer width and outer height.



What should the outer dimensions of the basin be if it is to hold 36 cubic feet of water?

METHOD 1

Using a Table One alternative approach is to write a function for the volume of the basin and make a table of values for the function. Using the table, you can find the value of x that makes the volume of the basin 36 cubic feet.

STEP 1 **Write** the function. From the diagram, you can see that the volume y of water the basin can hold is given by this function:

$$y = (2x - 2)(x - 2)(x - 1)$$

STEP 2 **Make** a table of values for the function. Use only positive values of x because the basin's dimensions must be positive.

X	Y1	
1	0	
2	0	
3	8	
4	36	
5	96	

Y1=96

STEP 3 **Identify** the value of x for which $y = 36$. The table shows that $\boldsymbol{y = 36}$ when $\boldsymbol{x = 4}$.

X	Y1	
1	0	
2	0	
3	8	
4	36	
5	96	

Y1=96

▶ The volume of the basin is 36 cubic feet when x is 4 feet. So, the outer dimensions of the basin should be as follows:

Length $= 2x = 8$ feet

Width $= x = 4$ feet

Height $= x = 4$ feet

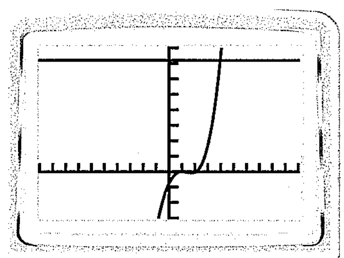
METHOD 2

Using a Graph Another approach is to make a graph. You can use the graph to find the value of x that makes the volume of the basin 36 cubic feet.

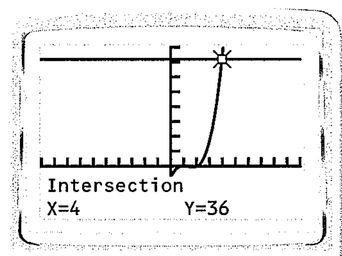
STEP 1 **Write** the function. From the diagram, you can see that the volume y of water the basin can hold is given by this function:

$$y = (2x - 2)(x - 2)(x - 1)$$

STEP 2 **Graph** the equations $y = 36$ and $y = (x - 1)(2x - 2)(x - 2)$. Choose a viewing window that shows the intersection of the graphs.



STEP 3 **Identify** the coordinates of the intersection point. On a graphing calculator, you can use the *intersect* feature. The intersection point is (4, 36).



▶ The volume of the basin is 36 cubic feet when x is 4 feet. So, the outer dimensions of the basin should be as follows:

Length = $2x$ = 8 feet

Width = x = 4 feet

Height = x = 4 feet

PRACTICE

SOLVING EQUATIONS Solve the polynomial equation using a table or using a graph.

1. $x^3 + 4x^2 - 8x = 96$
2. $x^3 - 9x^2 - 14x + 7 = -33$
3. $2x^3 - 11x^2 + 3x + 5 = 59$
4. $x^4 + x^3 - 15x^2 - 8x + 6 = -45$
5. $-x^4 + 2x^3 + 6x^2 + 17x - 4 = 32$
6. $-3x^4 + 4x^3 + 8x^2 + 4x - 11 = 13$
7. $4x^4 - 16x^3 + 29x^2 - 95x = -150$

8. **WHAT IF?** In the problem on page 360, suppose the basin is to hold 200 cubic feet of water. Find the outer dimensions of the basin using a table and using a graph.

9. **PACKAGING** A factory needs a box that has a volume of 1728 cubic inches. The width should be 4 inches less than the height, and the length should be 6 inches greater than the height. Find the dimensions of the box using a table and using a graph.

10. **AGRICULTURE** From 1970 to 2002, the average yearly pineapple consumption P (in pounds) per person in the United States can be modeled by the function

$$P(x) = 0.0000984x^4 - 0.00712x^3 + 0.162x^2 - 1.11x + 12.3$$

where x is the number of years since 1970. In what year was the pineapple consumption about 9.97 pounds per person? Solve the problem using a table and a graph.

5.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS10 for Exs. 17, 25, and 43

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 35, 39, 44, and 45

◆ = **MULTIPLE REPRESENTATIONS** Ex. 38

SKILL PRACTICE

1. **VOCABULARY** State the remainder theorem.

2. ★ **WRITING** Synthetic division has been used to divide $f(x) = x^4 - 5x^2 + 8x - 2$ by $x + 3$. *Explain* what the colored numbers represent in the division problem.

−3	1	0	−5	8	−2
		−3	9	−12	12
	1	−3	4	−4	10

EXAMPLES 1 and 2 on pp. 362–363 for Exs. 3–10

USING LONG DIVISION **Divide using polynomial long division.**

3. $(x^2 + x - 17) \div (x - 4)$
4. $(3x^2 - 11x - 26) \div (x - 5)$
5. $(x^3 + 3x^2 + 3x + 2) \div (x - 1)$
6. $(8x^2 + 34x - 1) \div (4x - 1)$
7. $(3x^3 + 11x^2 + 4x + 1) \div (x^2 + x)$
8. $(7x^3 + 11x^2 + 7x + 5) \div (x^2 + 1)$
9. $(5x^4 - 2x^3 - 7x^2 - 39) \div (x^2 + 2x - 4)$
10. $(4x^4 + 5x - 4) \div (x^2 - 3x - 2)$

EXAMPLE 3 on p. 363 for Exs. 11–20

USING SYNTHETIC DIVISION **Divide using synthetic division.**

11. $(2x^2 - 7x + 10) \div (x - 5)$
12. $(4x^2 - 13x - 5) \div (x - 2)$
13. $(x^2 + 8x + 1) \div (x + 4)$
14. $(x^2 + 9) \div (x - 3)$
15. $(x^3 - 5x^2 - 2) \div (x - 4)$
16. $(x^3 - 4x + 6) \div (x + 3)$
17. $(x^4 - 5x^3 - 8x^2 + 13x - 12) \div (x - 6)$
18. $(x^4 + 4x^3 + 16x - 35) \div (x + 5)$

ERROR ANALYSIS ***Describe*** **and correct the error in using synthetic division to divide** $x^3 - 5x + 3$ **by** $x - 2$.

19.

2	1	0	−5	3
		2	4	−2
	1	2	−1	1

$$\frac{x^3 - 5x + 3}{x - 2} = x^3 + 2x^2 - x + 1$$

20.

2	1	−5	3
		2	−6
	1	−3	−3

$$\frac{x^3 - 5x + 3}{x - 2} = x^2 - 3x - \frac{3}{x - 2}$$

EXAMPLE 4 on p. 364 for Exs. 21–28

FACTOR **Given polynomial $f(x)$ and a factor of $f(x)$, factor $f(x)$ completely.**

21. $f(x) = x^3 - 10x^2 + 19x + 30;\ x - 6$
22. $f(x) = x^3 + 6x^2 + 5x - 12;\ x + 4$
23. $f(x) = x^3 - 2x^2 - 40x - 64;\ x - 8$
24. $f(x) = x^3 + 18x^2 + 95x + 150;\ x + 10$
25. $f(x) = x^3 + 2x^2 - 51x + 108;\ x + 9$
26. $f(x) = x^3 - 9x^2 + 8x + 60;\ x + 2$
27. $f(x) = 2x^3 - 15x^2 + 34x - 21;\ x - 1$
28. $f(x) = 3x^3 - 2x^2 - 61x - 20;\ x - 5$

EXAMPLE 5 on p. 365 for Exs. 29–35

FIND ZEROS **Given polynomial function f and a zero of f, find the other zeros.**

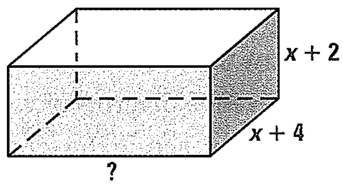
29. $f(x) = x^3 - 2x^2 - 21x - 18;\ -3$
30. $f(x) = 4x^3 - 25x^2 - 154x + 40;\ 10$
31. $f(x) = 10x^3 - 81x^2 + 71x + 42;\ 7$
32. $f(x) = 3x^3 + 34x^2 + 72x - 64;\ -4$
33. $f(x) = 2x^3 - 10x^2 - 71x - 9;\ 9$
34. $f(x) = 5x^3 - x^2 - 18x + 8;\ -2$

35. ★ **MULTIPLE CHOICE** One zero of $f(x) = 4x^3 + 15x^2 - 63x - 54$ is $x = -6$. What is another zero of f?

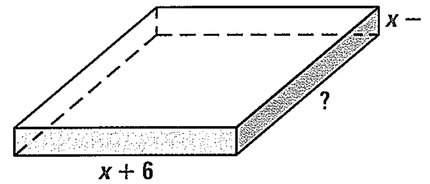
Ⓐ -9 Ⓑ -3 Ⓒ -1 Ⓓ 3

GEOMETRY You are given an expression for the volume of the rectangular prism. Find an expression for the missing dimension.

36. $V = 2x^3 + 17x^2 + 46x + 40$



37. $V = x^3 + 13x^2 + 34x - 48$



38. **MULTIPLE REPRESENTATIONS** Consider the polynomial function $f(x) = x^3 - 5x^2 - 12x + 36$.

a. **Zeros of a Function** Given that $f(2) = 0$, find the other zeros of f.

b. **Factors of an Expression** Based on your results from part (a), what are the factors of the polynomial $x^3 - 5x^2 - 12x + 36$?

c. **Solutions of an Equation** What are the solutions of the polynomial equation $x^3 - 5x^2 - 12x + 36 = 0$?

39. ★ **MULTIPLE CHOICE** What is the value of k such that $x - 5$ is a factor of $x^3 - x^2 + kx - 30$?

Ⓐ -14 Ⓑ -2 Ⓒ 26 Ⓓ 32

40. **CHALLENGE** It can be shown that $2x - 1$ is a factor of the polynomial function $f(x) = 30x^3 + 7x^2 - 39x + 14$.

a. What can you conclude is a zero of f?

b. Use synthetic division to write $f(x)$ in the form $(x - k) \cdot q(x)$.

c. Write $f(x)$ as the product of linear factors with integer coefficients.

PROBLEM SOLVING

EXAMPLE 6 on p. 365 for Exs. 41–43

41. **CLOTHING** The profit P (in millions of dollars) for a T-shirt manufacturer can be modeled by $P = -x^3 + 4x^2 + x$ where x is the number of T-shirts produced (in millions). Currently, the company produces 4 million T-shirts and makes a profit of \$4,000,000. What lesser number of T-shirts could the company produce and still make the same profit?

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42. **MP3 PLAYERS** The profit P (in millions of dollars) for a manufacturer of MP3 players can be modeled by $P = -4x^3 + 12x^2 + 16x$ where x is the number of MP3 players produced (in millions). Currently, the company produces 3 million MP3 players and makes a profit of \$48,000,000. What lesser number of MP3 players could the company produce and still make the same profit?

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43. **WOMEN'S BASKETBALL** From 1985 to 2003, the total attendance A (in thousands) at NCAA women's basketball games and the number T of NCAA women's basketball teams can be modeled by

$$A = -1.95x^3 + 70.1x^2 - 188x + 2150 \quad \text{and} \quad T = 14.8x + 725$$

where x is the number of years since 1985. Write a function for the average attendance per team from 1985 to 2003.

44. ★ **EXTENDED RESPONSE** The price p (in dollars) that a radio manufacturer is able to charge for a radio is given by $p = 40 - 4x^2$ where x is the number (in millions) of radios produced. It costs the company \$15 to make a radio.

   a. Write an expression for the company's total revenue in terms of x.

   b. Write a function for the company's profit P by subtracting the total cost to make x radios from the expression in part (a).

   c. Currently, the company produces 1.5 million radios and makes a profit of \$24,000,000. Write and solve an equation to find a lesser number of radios that the company could produce and still make the same profit.

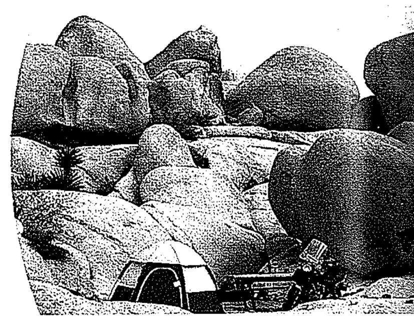
   d. Do all the solutions in part (c) make sense in this situation? *Explain.*

45. ★ **SHORT RESPONSE** Since 1990, overnight stays S and total visits V (both in millions) to national parks can be modeled by

$$S = -0.00722x^4 + 0.176x^3 - 1.40x^2 + 3.39x + 17.6$$

$$V = 3.10x + 256$$

where x is the number of years since 1990. Write a function for the percent of visits to national parks that were overnight stays. *Explain* how you constructed your function.



Joshua Tree National Park, California

46. **CHALLENGE** The profit P (in millions of dollars) for a DVD manufacturer can be modeled by $P = -6x^3 + 72x$ where x is the number of DVDs produced (in millions). Show that 2 million DVDs is the only production level for the company that yields a profit of \$96,000,000.

MIXED REVIEW

Tell whether the given ordered pairs are solutions of the inequality. *(p. 132)*

47. $x - 4y < 5$; (1, 4), (4, −1)

48. $3x + 2y \geq 1$; (−2, 4), (1, −3)

49. $5x - 2y > 10$; (4, 6), (8, 10)

50. $6x + 5y \leq 15$; (−5, 10), (−1, 4)

PREVIEW
Prepare for Lesson 5.6 in Exs. 51–56.

Solve the equation.

51. $x^2 + 3x - 40 = 0$ *(p. 252)*

52. $5x^2 + 13x + 6 = 0$ *(p. 259)*

53. $x^2 + 7x + 2 = 0$ *(p. 292)*

54. $4x^2 + 15x + 10 = 0$ *(p. 292)*

55. $2x^2 + 15x + 31 = 0$ *(p. 292)*

56. $x^2 + 2x + 10 = 0$ *(p. 292)*

Perform the indicated operation. *(p. 346)*

57. $(x^2 - 4x + 15) + (-3x^2 + 6x - 12)$

58. $(2x^2 - 5x + 8) - (5x^2 - 7x - 7)$

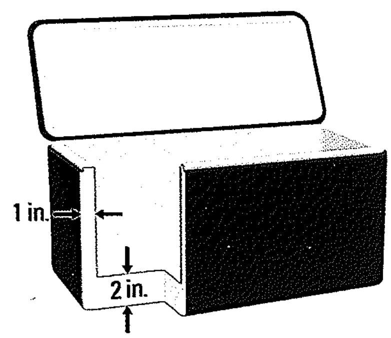
59. $(3x - 4)(3x^3 + 2x^2 - 8)$

60. $(3x - 5)^3$

Lessons 5.1–5.5

1. **MULTI-STEP PROBLEM** The average distance between Earth and the sun is 164,000,000,000 yards.
   a. Write the distance in scientific notation.
   b. The length of a football field, including the end zones, is 1.20×10^2 yards. How many football fields stretched end-to-end would it take to reach from Earth to the sun?

2. **MULTI-STEP PROBLEM** You are designing a rectangular picnic cooler with length 4 times its width and height 2 times its width. The cooler has insulation that is 1 inch thick on each of the four sides and 2 inches thick on the top and bottom.



   a. Let x represent the width of the cooler. Write a polynomial function $T(x)$ in standard form for the volume of the rectangular prism formed by the cooler's outer surfaces.
   b. Write a polynomial function $C(x)$ in standard form for the volume of the inside of the cooler.
   c. Let $I(x)$ be a polynomial function for the volume of insulation. How is $I(x)$ related to $T(x)$ and $C(x)$?
   d. Write $I(x)$ in standard form. What is the volume of the insulation when $x = 8$ inches?

3. **SHORT RESPONSE** In biology, a cell with a higher surface area-to-volume ratio can exchange materials with its environment faster than a cell with a lower ratio. *Explain* whether a cubic cell with side length x or a spherical cell with diameter x can exchange materials with its environment faster.

4. **OPEN-ENDED** Write a polynomial function that has degree 4 and end behavior given by $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$. Then graph the function to check your answer.

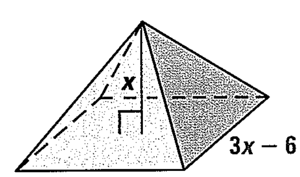
5. **EXTENDED RESPONSE** From 1995 to 2003, the average monthly cell phone bill C (in dollars) for subscribers in the United States can be modeled by

$$C = -0.027t^4 + 0.32t^3 - 0.25t^2 - 4.9t + 51$$

   where t is the number of years since 1995.
   a. Classify the function by degree and type.
   b. Make a table of values for the function.
   c. Sketch a graph of the function. Do you think the model will accurately predict cell phone bills for years beyond 2003? *Explain.*

6. **EXTENDED RESPONSE** The price p (in dollars) that a camera manufacturer is able to charge for a camera is given by $p = 100 - 10x^2$ where x is the number (in millions) of cameras produced. It costs the company \$30 to make a camera. Currently, the company produces 2 million cameras and makes a profit of \$60,000,000.
   a. Write a function that gives the total revenue R in terms of x.
   b. Write a function that gives the company's profit P in terms of x.
   c. Write and solve an equation to find other values of x that yield a profit of \$60,000,000.
   d. Do all the solutions in part (c) make sense in this situation? *Explain.*

7. **GRIDDED ANSWER** For the city park commission, you are designing a marble sculpture in the shape of a pyramid with a square base, as shown below. The volume of the sculpture is 48 cubic feet. What is the height x in feet of the sculpture?



5.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS11 for Exs. 7, 21, and 47

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 38, 39, 40, and 50

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If a polynomial function has integer coefficients, then every rational zero of the function has the form $\frac{p}{q}$, where p is a factor of the _?_ and q is a factor of the _?_.

2. ★ **WRITING** *Describe* a method you can use to shorten the list of possible rational zeros when using the rational zero theorem.

EXAMPLE 1 on p. 370 for Exs. 3–10

LISTING RATIONAL ZEROS List the possible rational zeros of the function using the rational zero theorem.

3. $f(x) = x^3 - 3x + 28$
4. $g(x) = x^3 - 4x^2 + x - 10$
5. $f(x) = 2x^4 + 6x^3 - 7x + 9$
6. $h(x) = 2x^3 + x^2 - x - 18$
7. $g(x) = 4x^5 + 3x^3 - 2x - 14$
8. $f(x) = 3x^4 + 5x^3 - 3x + 42$
9. $h(x) = 8x^4 + 4x^3 - 10x + 15$
10. $h(x) = 6x^3 - 3x^2 + 12$

EXAMPLE 2 on p. 371 for Exs. 11–18

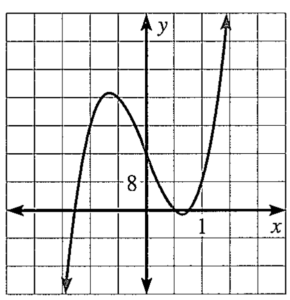
FINDING REAL ZEROS Find all real zeros of the function.

11. $f(x) = x^3 - 12x^2 + 35x - 24$
12. $f(x) = x^3 - 5x^2 - 22x + 56$
13. $g(x) = x^3 - 31x - 30$
14. $h(x) = x^3 + 8x^2 - 9x - 72$
15. $h(x) = x^4 + 7x^3 + 26x^2 + 44x + 24$
16. $f(x) = x^4 - 2x^3 - 9x^2 + 10x - 24$
17. $f(x) = x^4 + 2x^3 - 9x^2 - 2x + 8$
18. $g(x) = x^4 - 16x^2 - 40x - 25$

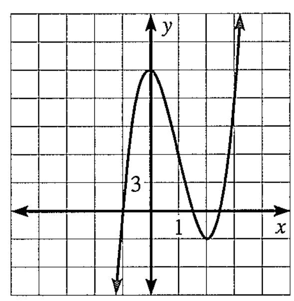
EXAMPLE 3 on p. 372 for Exs. 19–35

ELIMINATING POSSIBLE ZEROS Use the graph to shorten the list of possible rational zeros of the function. Then find all real zeros of the function.

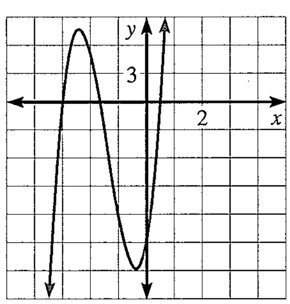
19. $f(x) = 4x^3 - 20x + 16$



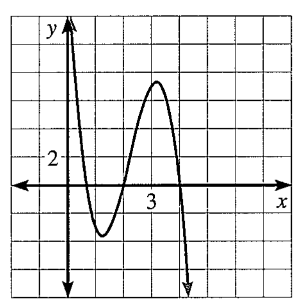
20. $f(x) = 4x^3 - 12x^2 - x + 15$



21. $f(x) = 6x^3 + 25x^2 + 16x - 15$



22. $f(x) = -3x^3 + 20x^2 - 36x + 16$



23. ★ **MULTIPLE CHOICE** According to the rational zero theorem, which is *not* a possible zero of the function $f(x) = 2x^4 - 5x^3 + 10x^2 - 9$?

Ⓐ -9 Ⓑ $-\frac{1}{2}$ Ⓒ $\frac{5}{2}$ Ⓓ 3

FINDING REAL ZEROS **Find all real zeros of the function.**

24. $f(x) = 2x^3 + 2x^2 - 8x - 8$
25. $g(x) = 2x^3 - 7x^2 + 9$
26. $h(x) = 2x^3 - 3x^2 - 14x + 15$
27. $f(x) = 3x^3 + 4x^2 - 35x - 12$
28. $f(x) = 3x^3 + 19x^2 + 4x - 12$
29. $g(x) = 2x^3 + 5x^2 - 11x - 14$
30. $g(x) = 2x^4 + 9x^3 + 5x^2 + 3x - 4$
31. $h(x) = 2x^4 - x^3 - 7x^2 + 4x - 4$
32. $h(x) = 3x^4 - 6x^3 - 32x^2 + 35x - 12$
33. $f(x) = 2x^4 - 9x^3 + 37x - 30$
34. $f(x) = x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12$
35. $h(x) = 2x^5 + 5x^4 - 3x^3 - 2x^2 - 5x + 3$

ERROR ANALYSIS ***Describe*** **and correct the error in listing the possible rational zeros of the function.**

36.

$f(x) = x^3 + 7x^2 + 2x + 14$

Possible zeros: ✗

1, 2, 7, 14

37.

$f(x) = 6x^3 - 3x^2 + 12x + 5$

Possible zeros:

$\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{6}{5}$ ✗

38. ★ **OPEN-ENDED MATH** Write a polynomial function f that has a leading coefficient of 4 and has 12 possible rational zeros according to the rational zero theorem.

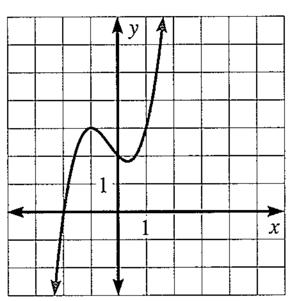
39. ★ **MULTIPLE CHOICE** Which of the following is *not* a zero of the function $f(x) = 40x^5 - 42x^4 - 107x^3 + 107x^2 + 33x - 36$?

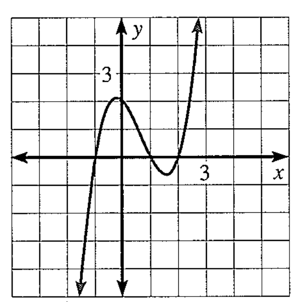
Ⓐ $-\frac{3}{2}$ Ⓑ $-\frac{3}{8}$ Ⓒ $\frac{3}{4}$ Ⓓ $\frac{4}{5}$

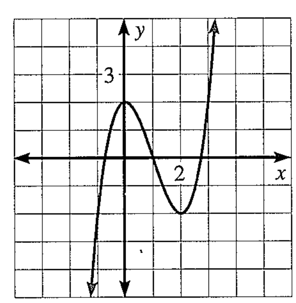
40. ★ **SHORT RESPONSE** Let a_n be the leading coefficient of a polynomial function f and a_0 be the constant term. If a_n has r factors and a_0 has s factors, what is the largest number of possible rational zeros of f that can be generated by the rational zero theorem? *Explain* your reasoning.

MATCHING **Find all real zeros of the function. Then match each function with its graph.**

41. $f(x) = x^3 - 2x^2 - x + 2$
42. $g(x) = x^3 - 3x^2 + 2$
43. $h(x) = x^3 + x^2 - x + 2$

A.


B.


C.


44. **CHALLENGE** Is it possible for a cubic function to have more than three real zeros? Is it possible for a cubic function to have no real zeros? *Explain.*

PROBLEM SOLVING

EXAMPLE 4
on p. 373
for Exs. 45–48

45. MANUFACTURING At a factory, molten glass is poured into molds to make paperweights. Each mold is a rectangular prism with a height 4 inches greater than the length of each side of its square base. Each mold holds 63 cubic inches of molten glass. What are the dimensions of the mold?

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46. SWIMMING POOL You are designing a rectangular swimming pool that is to be set into the ground. The width of the pool is 5 feet more than the depth, and the length is 35 feet more than the depth. The pool holds 2000 cubic feet of water. What are the dimensions of the pool?

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GEOMETRY In Exercises 47 and 48, write a polynomial equation to model the situation. Then list the possible rational solutions of the equation.

47. A rectangular prism has edges of lengths x, $x - 1$, and $x - 2$ and a volume of 24.

48. A pyramid has a square base with sides of length x, a height of $2x - 5$, and a volume of 3.

49. MULTI-STEP PROBLEM From 1994 to 2003, the amount of athletic equipment E (in millions of dollars) sold domestically can be modeled by

$$E(t) = -10t^3 + 140t^2 - 20t + 18{,}150$$

where t is the number of years since 1994. Use the following steps to find the year when about $20,300,000,000 of athletic equipment was sold.

a. Write a polynomial equation that can be used to find the answer.

b. List the possible whole-number solutions of the equation in part (a) that are less than 10.

c. Use synthetic division to determine which of the possible solutions in part (b) is an actual solution. Then calculate the year which corresponds to the solution.

50. ★ EXTENDED RESPONSE Since 1990, the number of U.S. travelers to foreign countries F (in thousands) can be modeled by

$$F(t) = 12t^4 - 264t^3 + 2028t^2 - 3924t + 43{,}916$$

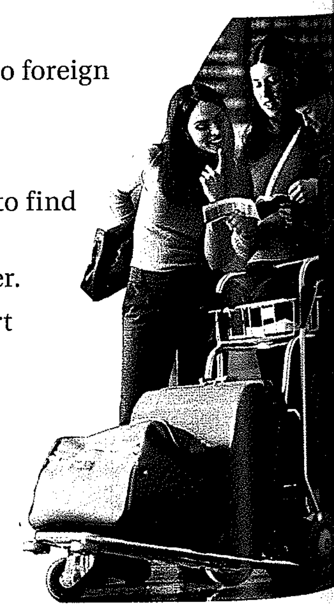
where t is the number of years since 1990. Use the following steps to find the year when there were about 56,300,000 travelers.

a. Write a polynomial equation that can be used to find the answer.

b. List the possible whole-number solutions of the equation in part (a) that are less than or equal to 10.

c. Use synthetic division to determine which of the possible solutions in part (b) is an actual solution.

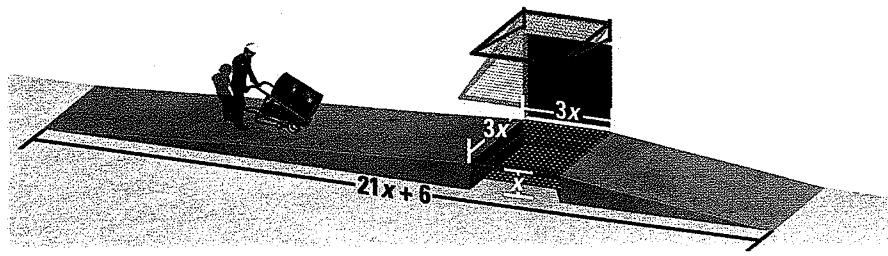
d. Graph the function $F(t)$ and explain why there are no other reasonable solutions. Then calculate the year which corresponds to the solution.



○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

51. **CHALLENGE** You are building a pair of ramps for a loading platform. The left ramp is twice as long as the right ramp. If 150 cubic feet of concrete are used to build the two ramps, what are the dimensions of each ramp?



MIXED REVIEW

Solve the equation.

52. $4x - 6 = 18$ ***(p. 18)***
53. $3y + 7 = -14$ ***(p. 18)***
54. $|2p + 5| = 15$ ***(p. 51)***
55. $49z^2 - 14z + 1 = 0$ ***(p. 259)***
56. $8x^2 - 30x + 7 = 0$ ***(p. 259)***
57. $-3(q + 2)^2 = -18$ ***(p. 266)***

Solve the matrix equation. ***(p. 210)***

58. $\begin{bmatrix} 1 & 5 \\ -2 & -1 \end{bmatrix} X = \begin{bmatrix} -3 & 5 \\ 6 & -1 \end{bmatrix}$
59. $\begin{bmatrix} -2 & 1 \\ 4 & 0 \end{bmatrix} X = \begin{bmatrix} 6 & 0 \\ -1 & 10 \end{bmatrix}$
60. $\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix} X = \begin{bmatrix} -3 & 1 & 2 \\ 0 & -4 & -1 \end{bmatrix}$
61. $\begin{bmatrix} 2 & -8 \\ 3 & -7 \end{bmatrix} X = \begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & -3 \end{bmatrix}$

PREVIEW
Prepare for Lesson 5.7 in Exs. 62–67.

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation. ***(p. 292)***

62. $x^2 - 4x + 11 = 0$
63. $s^2 - 14s + 49 = 0$
64. $3t^2 - 8t - 5 = 0$
65. $-2y^2 - 5y - 3 = 0$
66. $81p^2 + 18p + 1 = 0$
67. $7r^2 + 5 = 0$

QUIZ *for Lessons 5.4–5.6*

Factor the polynomial completely. ***(p. 353)***

1. $2x^3 - 54$
2. $x^3 - 3x^2 + 2x - 6$
3. $x^3 + x^2 + x + 1$
4. $6x^5 - 150x$
5. $3x^4 - 24x^2 + 48$
6. $2x^3 - 3x^2 - 12x + 18$

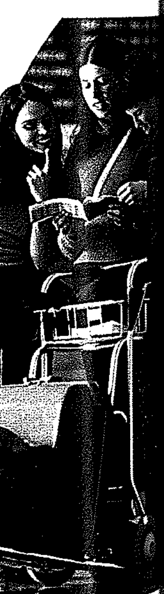
Divide using polynomial long division or synthetic division. ***(p. 362)***

7. $(x^4 + x^3 - 8x^2 + 5x + 5) \div (x^2 + 5x - 2)$
8. $(4x^3 + 27x^2 + 3x + 64) \div (x + 7)$

Find all real zeros of the function. ***(p. 370)***

9. $f(x) = 2x^3 - 19x^2 + 50x + 30$
10. $f(x) = x^3 - 4x^2 - 25x - 56$
11. $f(x) = x^4 + 4x^3 - 13x^2 - 4x + 12$
12. $f(x) = 4x^4 - 5x^2 + 42x - 20$

13. **LANDSCAPING** You are a landscape artist designing a square patio that is to be made from 128 cubic feet of concrete. The thickness of the patio is 15.5 feet less than each side length. What are the dimensions of the patio? ***(p. 370)***



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5.6 Use the Location Principle

QUESTION **How can you use the Location Principle to identify zeros of a polynomial function?**

You can use the following result, called the *Location Principle,* to help you find zeros of polynomial functions:

If f is a polynomial function and a and b are two numbers such that $f(a) < 0$ and $f(b) > 0$, then f has at least one real zero between a and b.

EXAMPLE **Find zeros of a polynomial function**

Find all real zeros of $f(x) = 6x^3 + 5x^2 - 17x - 6$.

STEP 1 ***Enter values for x***

Enter "x" into cell A1. Enter "0" into cell A2. Type "=A2+1" into cell A3. Select cells A3 through A7, and use the *fill down* command to fill in values of x.

	A	B
1	x	
2	0	
3	1	
4	2	
5	3	
6	4	
7	5	

STEP 2 ***Enter values for f(x)***

Enter "$f(x)$" into cell B1. Enter "=6*A2^3+5*A2^2−17*A2−6" into cell B2. Select cells B2 through B7, and use the *fill down* command to fill in the values of $f(x)$.

	A	B
1	x	f(x)
2	0	−6
3	1	−12
4	2	28
5	3	150
6	4	390
7	5	784

STEP 3 ***Use Location Principle***

The spreadsheet in Step 2 shows that $f(1) < 0$ and $f(2) > 0$. So, by the Location Principle, f has a zero between 1 and 2. The rational zero theorem shows that the only possible *rational* zero between 1 and 2 is $\frac{3}{2}$. Synthetic division confirms that $\frac{3}{2}$ is a zero and that f can be factored as:

$$f(x) = \left(x - \frac{3}{2}\right)(6x^2 + 14x + 4) = (2x - 3)(3x^2 + 7x + 2) = (2x - 3)(3x + 1)(x + 2)$$

▶ The zeros of f are $\frac{3}{2}$, $-\frac{1}{3}$, and -2.

PRACTICE

Find all real zeros of the function.

1. $f(x) = 6x^3 - 10x^2 - 6x + 10$

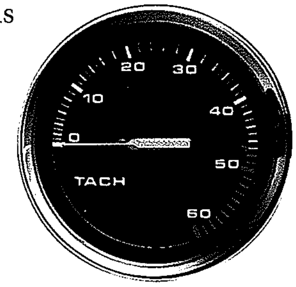
2. $f(x) = 24x^4 - 38x^3 - 191x^2 - 157x - 28$

3. $f(x) = 36x^3 + 109x^2 - 341x + 70$

4. $f(x) = 12x^4 + 25x^3 - 160x^2 - 305x - 132$

EXAMPLE 6 Approximate real zeros of a polynomial model

TACHOMETER A tachometer measures the speed (in revolutions per minute, or RPMs) at which an engine shaft rotates. For a certain boat, the speed x of the engine shaft (in 100s of RPMs) and the speed s of the boat (in miles per hour) are modeled by



$$s(x) = 0.00547x^3 - 0.225x^2 + 3.62x - 11.0$$

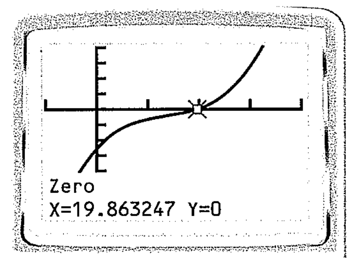
What is the tachometer reading when the boat travels 15 miles per hour?

Solution

Substitute 15 for $s(x)$ in the given function. You can rewrite the resulting equation as:

$$0 = 0.00547x^3 - 0.225x^2 + 3.62x - 26.0$$

Then, use a graphing calculator to approximate the real zeros of $f(x) = 0.00547x^3 - 0.225x^2 + 3.62x - 26.0$.



From the graph, there is one real zero: $x \approx 19.9$.

▶ The tachometer reading is about 1990 RPMs.

✓ GUIDED PRACTICE for Examples 5 and 6

11. Approximate the real zeros of $f(x) = 3x^5 + 2x^4 - 8x^3 + 4x^2 - x - 1$.

12. WHAT IF? In Example 6, what is the tachometer reading when the boat travels 20 miles per hour?

5.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS11 for Exs. 15, 37, and 61

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 33, 51, 52, 63, and 64

SKILL PRACTICE

1. VOCABULARY Copy and complete: For the equation $(x - 1)^2(x + 2) = 0$, a(n) _?_ solution is 1 because the factor $x - 1$ appears twice.

2. ★ WRITING *Explain* the difference between complex conjugates and irrational conjugates.

EXAMPLE 1 on p. 379 for Exs. 3–9

NUMBER OF SOLUTIONS OR ZEROS Identify the number of solutions or zeros.

3. $x^4 + 2x^3 - 4x^2 + x - 10 = 0$

4. $5y^3 - 3y^2 + 8y = 0$

5. $9t^6 - 14t^3 + 4t - 1 = 0$

6. $f(z) = -7z^4 + z^2 - 25$

7. $g(s) = 12s^7 - 9s^6 + 4s^5 - s^3 - 20s + 50$

8. $h(x) = -x^{12} + 7x^8 + 5x^4 - 8x + 6$

9. ★ MULTIPLE CHOICE How many zeros does the function $f(x) = 16x - 22x^3 + 6x^6 + 19x^5 - 3$ have?

Ⓐ 1 Ⓑ 3 Ⓒ 5 Ⓓ 6

EXAMPLE 2
on p. 380
for Exs. 10–19

FINDING ZEROS **Find all zeros of the polynomial function.**

10. $f(x) = x^4 - 6x^3 + 7x^2 + 6x - 8$

11. $f(x) = x^4 + 5x^3 - 7x^2 - 29x + 30$

12. $g(x) = x^4 - 9x^2 - 4x + 12$

13. $h(x) = x^3 + 5x^2 - 4x - 20$

14. $f(x) = x^4 + 15x^2 - 16$

(15.) $f(x) = x^4 + x^3 + 2x^2 + 4x - 8$

16. $h(x) = x^4 + 4x^3 + 7x^2 + 16x + 12$

17. $g(x) = x^4 - 2x^3 - 3x^2 + 2x + 2$

18. $g(x) = 4x^4 + 4x^3 - 11x^2 - 12x - 3$

19. $h(x) = 2x^4 + 13x^3 + 19x^2 - 10x - 24$

EXAMPLE 3
on p. 381
for Exs. 20–32

WRITING POLYNOMIAL FUNCTIONS **Write a polynomial function *f* of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.**

20. 1, 2, 3

21. −2, 1, 3

22. −5, −1, 2

23. −3, 1, 6

24. 2, $-i$, i

25. $3i$, $2 - i$

26. −1, 2, $-3i$

27. 5, 5, $4 + i$

28. 4, $-\sqrt{5}$, $\sqrt{5}$

29. −4, 1, $2 - \sqrt{6}$

30. −2, −1, 2, 3, $\sqrt{11}$

31. 3, $4 + 2i$, $1 + \sqrt{7}$

32. ERROR ANALYSIS *Describe* and correct the error in writing a polynomial function with rational coefficients and zeros 2 and $1 + i$.

$$\begin{aligned} f(x) &= (x - 2)[x - (1 + i)] \\ &= x(x - 1 - i) - 2(x - 1 - i) \\ &= x^2 - x - ix - 2x + 2 + 2i \\ &= x^2 - (3 + i)x + (2 + 2i) \end{aligned}$$

33. ★ OPEN-ENDED MATH Write a polynomial function of degree 5 with zeros 1, 2, and $-i$.

EXAMPLE 4
on p. 382
for Exs. 34–41

CLASSIFYING ZEROS **Determine the possible numbers of positive real zeros, negative real zeros, and imaginary zeros for the function.**

34. $f(x) = x^4 - x^2 - 6$

35. $g(x) = -x^3 + 5x^2 + 12$

36. $g(x) = x^3 - 4x^2 + 8x + 7$

(37.) $h(x) = x^5 - 2x^3 - x^2 + 6x + 5$

38. $h(x) = x^5 - 3x^3 + 8x - 10$

39. $f(x) = x^5 + 7x^4 - 4x^3 - 3x^2 + 9x - 15$

40. $g(x) = x^6 + x^5 - 3x^4 + x^3 + 5x^2 + 9x - 18$

41. $f(x) = x^7 + 4x^4 - 10x + 25$

EXAMPLE 5
on p. 382
for Exs. 42–49

APPROXIMATING ZEROS **Use a graphing calculator to graph the function. Then use the *zero* (or *root*) feature to approximate the real zeros of the function.**

42. $f(x) = x^3 - x^2 - 8x + 5$

43. $f(x) = -x^4 - 4x^2 + x + 8$

44. $g(x) = x^3 - 3x^2 + x + 6$

45. $h(x) = x^4 - 5x - 3$

46. $h(x) = 3x^3 - x^2 - 5x + 3$

47. $g(x) = x^4 - x^3 + 2x^2 - 6x - 3$

48. $f(x) = 2x^6 + x^4 + 31x^2 - 35$

49. $g(x) = x^5 - 16x^3 - 3x^2 + 42x + 30$

50. REASONING Two zeros of $f(x) = x^3 - 6x^2 - 16x + 96$ are 4 and −4. *Explain* why the third zero must also be a real number.

51. ★ SHORT RESPONSE *Describe* the possible numbers of positive real, negative real, and imaginary zeros for a cubic function with rational coefficients.

52. ★ MULTIPLE CHOICE Which is *not* a possible classification of the zeros of $f(x) = x^5 - 4x^3 + 6x^2 + 12x - 6$ according to Descartes' rule of signs?

Ⓐ 3 positive real zeros, 2 negative real zeros, and 0 imaginary zeros

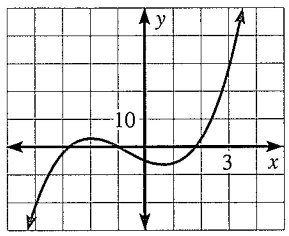
Ⓑ 3 positive real zeros, 0 negative real zeros, and 2 imaginary zeros

Ⓒ 1 positive real zero, 4 negative real zeros, and 0 imaginary zeros

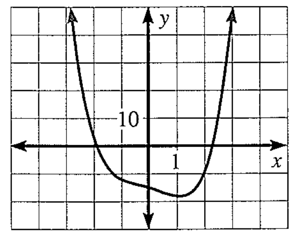
Ⓓ 1 positive real zero, 2 negative real zeros, and 2 imaginary zeros

CLASSIFYING ZEROS **Determine the numbers of positive real zeros, negative real zeros, and imaginary zeros for the function with the given degree and graph. *Explain* your reasoning.**

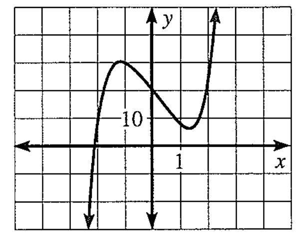
53. Degree: 3



54. Degree: 4



55. Degree: 5



CHALLENGE **Show that the given number is a zero of the given function but that the conjugate of the number is *not* a zero.**

56. $f(x) = x^3 - 2x^2 + 2x + 5i;\ 2 - i$

57. $g(x) = x^3 + 2x^2 + 2i - 2;\ -1 + i$

58. *Explain* why the results of Exercises 56 and 57 do not contradict the complex conjugate theorem on page 380.

PROBLEM SOLVING

EXAMPLE 6 on p. 383 for Exs. 59–62

59. BUSINESS For the 12 years that a grocery store has been open, its annual revenue R (in millions of dollars) can be modeled by the function

$$R = 0.0001(-t^4 + 12t^3 - 77t^2 + 600t + 13{,}650)$$

where t is the number of years since the store opened. In which year(s) was the revenue \$1.5 million?

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60. ENVIRONMENT From 1990 to 2003, the number N of inland lakes in Michigan infested with zebra mussels can be modeled by the function

$$N = -0.028t^4 + 0.59t^3 - 2.5t^2 + 8.3t - 2.5$$

where t is the number of years since 1990. In which year did the number of infested inland lakes first reach 120?

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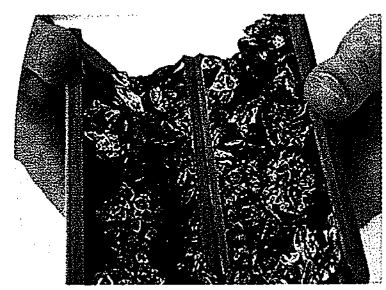


Pipe clogged with zebra mussels

61. PHYSIOLOGY A study group found that a person's score S on a step-climbing exercise test was related to his or her amount of hemoglobin x (in grams per 100 milliliters of blood) by this function:

$$S = -0.015x^3 + 0.6x^2 - 2.4x + 19$$

Given that the normal range of hemoglobin is 12–18 grams per 100 milliliters of blood, what is the most likely amount of hemoglobin for a person who scores 75?

62. POPULATION From 1890 to 2000, the American Indian, Eskimo, and Aleut population P (in thousands) can be modeled by the function

$$P = 0.0035t^3 - 0.235t^2 + 4.87t + 243$$

where t is the number of years since 1890. In which year did the population first reach 722,000?

63. ★ **SHORT RESPONSE** A 60-inch-long bookshelf is warped under 180 pounds of books. The deflection d of the bookshelf (in inches) is given by

$$d = (2.724 \times 10^{-7})x^4 - (3.269 \times 10^{-5})x^3 + (9.806 \times 10^{-4})x^2$$

where x is the distance (in inches) from the bookshelf's left end. Approximate the real zeros of the function on the domain $0 \le x \le 60$. *Explain* why all your answers make sense in this situation.

64. ★ **EXTENDED RESPONSE** You plan to save $1000 each year towards buying a used car in four years. At the end of each summer, you deposit $1000 earned from summer jobs into your bank account. The table shows the value of your deposits over the four year period. In the table, g is the growth factor $1 + r$ where r is the annual interest rate expressed as a decimal.

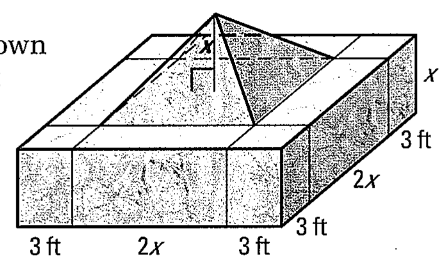
	Year 1	Year 2	Year 3	Year 4
Value of 1st deposit	1000	$1000g$	$1000g^2$	$1000g^3$
Value of 2nd deposit	–	1000	?	?
Value of 3rd deposit	–	–	1000	?
Value of 4th deposit	–	–	–	1000

a. **Apply** Copy and complete the table.

b. **Model** Write a polynomial function that gives the value v of your account at the end of the fourth summer in terms of g.

c. **Reasoning** You want to buy a car that costs about $4300. What growth factor do you need to obtain this amount? What annual interest rate do you need? *Explain* how you found your answers.

65. **CHALLENGE** A monument with the dimensions shown is to be built using 1000 cubic feet of marble. What is the value of x?



MIXED REVIEW

Evaluate the determinant of the matrix. ***(p. 203)***

66. $\begin{bmatrix} 5 & -9 & 2 \\ 4 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}$ 67. $\begin{bmatrix} 3 & 12 & -1 \\ 5 & 9 & 0 \\ -6 & 4 & -2 \end{bmatrix}$ 68. $\begin{bmatrix} 15 & 4 & -9 \\ 10 & 0 & 2 \\ -8 & 2 & -7 \end{bmatrix}$ 69. $\begin{bmatrix} -2 & 1 & -3 \\ 7 & 0 & 2 \\ -6 & 2 & -4 \end{bmatrix}$

PREVIEW
Prepare for Lesson 5.8 in Exs. 70–75.

Graph the function.

70. $y = 2(x + 5)(x - 3)$ ***(p. 245)*** 71. $y = (x - 2)(x - 9)$ ***(p. 245)*** 72. $y = 5(x + 1)(x + 9)$ ***(p. 245)***

73. $y = x^3 - 5x + 1$ ***(p. 337)*** 74. $y = x^4 - 16$ ***(p. 337)*** 75. $y = x^5 - 3$ ***(p. 337)***

Write a quadratic function in intercept form whose graph has the given x-intercepts and passes through the given point. ***(p. 309)***

76. x-intercepts: −2, 4
point: (2, −4)

77. x-intercepts: −5, −1
point: (−2, 6)

78. x-intercepts: 2, 7
point: (4, −2)

5.8 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS11 for Exs. 3, 19, and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 30, 32, 33, and 43

◆ = **MULTIPLE REPRESENTATIONS** Ex. 42

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A local maximum or local minimum of a polynomial function occurs at a __?__ point of the function's graph.

2. ★ **WRITING** *Explain* what a local maximum of a function is and how it may be different from the maximum value of the function.

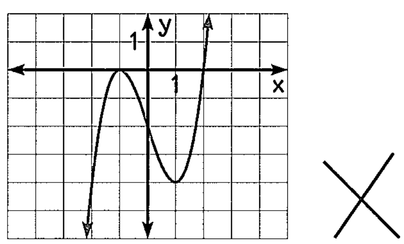
EXAMPLE 1 on p. 387 for Exs. 3–14

GRAPHING POLYNOMIAL FUNCTIONS Graph the function.

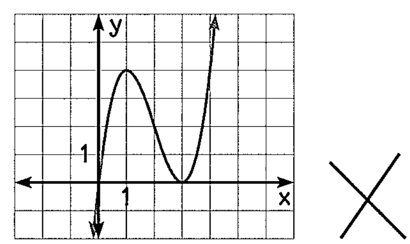
3. $f(x) = (x-2)^2(x+1)$
4. $f(x) = (x+1)^2(x-1)(x-3)$
5. $g(x) = \frac{1}{3}(x-5)(x+2)(x-3)$
6. $h(x) = \frac{1}{12}(x+4)(x+8)(x-1)$
7. $h(x) = 4(x+1)(x+2)(x-1)$
8. $f(x) = 0.2(x-4)^2(x+1)^2$
9. $f(x) = 2(x+2)^2(x+4)^2$
10. $h(x) = 5(x-1)(x-2)(x-3)$
11. $g(x) = (x-3)(x^2+x+1)$
12. $h(x) = (x-4)(2x^2-2x+1)$

ERROR ANALYSIS ***Describe*** **and correct the error in graphing *f*.**

13. $f(x) = (x+2)(x-1)^2$

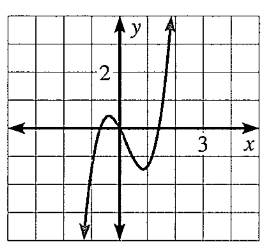


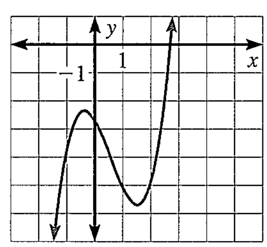
14. $f(x) = x(x-3)^3$

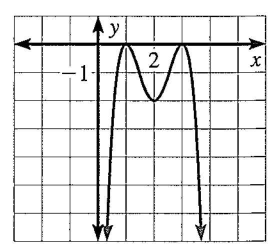


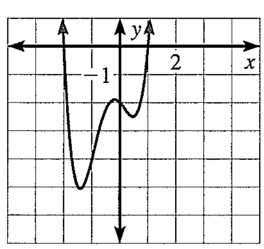
EXAMPLE 2 on p. 388 for Exs. 15–30

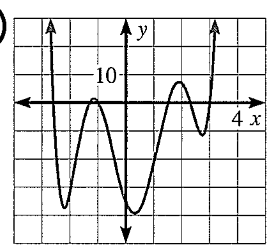
ANALYZING GRAPHS Estimate the coordinates of each turning point and state whether each corresponds to a local maximum or a local minimum. Then estimate all real zeros and determine the least degree the function can have.

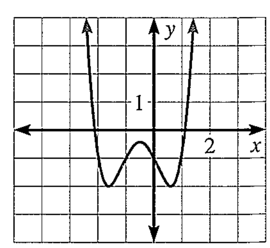
15. 

16. 

17. 

18. 

19. 

20. 

21. ★ **MULTIPLE CHOICE** Which point is a local maximum of the function $f(x) = 0.25(x + 2)(x - 1)^2$?

Ⓐ $(-2, 0)$ Ⓑ $(-1, 1)$ Ⓒ $(1, 0)$ Ⓓ $(2, 1)$

GRAPHING CALCULATOR Use a graphing calculator to graph the polynomial function. Identify the x-intercepts and the points where the local maximums and local minimums occur.

22. $f(x) = 2x^3 + 8x^2 - 3$
23. $g(x) = 0.5x^3 - 2x + 2.5$
24. $h(x) = -x^4 + 3x$
25. $f(x) = x^5 - 4x^3 + x^2 + 2$
26. $g(x) = x^4 - 3x^2 + x$
27. $h(x) = x^4 - 5x^3 + 2x^2 + x - 3$
28. $h(x) = x^5 + 2x^2 - 17x - 4$
29. $g(x) = 0.7x^4 - 8x^3 + 5x$

30. ★ **MULTIPLE CHOICE** What is a turning point of the graph of the function $g(x) = x^4 - 9x^2 + 4x + 12$?

Ⓐ $(-3, 0)$ Ⓑ $(-1, 0)$ Ⓒ $(0, 12)$ Ⓓ $(2, 0)$

31. **REASONING** Why is the adjective *local* used to describe the maximums and minimums of cubic functions but not quadratic functions?

32. ★ **SHORT RESPONSE** Does a cubic function *always*, *sometimes*, or *never* have a turning point? *Justify* your answer.

33. ★ **OPEN-ENDED MATH** Write a cubic function, a quartic function, and a fifth-degree function whose graphs have x-intercepts only at $x = -2$, 0, and 4.

DOMAIN AND RANGE Graph the function. Then identify its domain and range.

34. $f(x) = x(x - 3)^2$
35. $f(x) = x^2(x - 2)(x - 4)(x - 5)$
36. $f(x) = (x + 1)^3(x - 1)$
37. $f(x) = (x + 2)(x + 1)(x - 1)^2(x - 2)^2$

38. **CHALLENGE** In general, what can you say about the domain and range of odd-degree polynomial functions? What can you say about the domain and range of even-degree polynomial functions?

PROBLEM SOLVING

EXAMPLE 3
on p. 389
for Exs. 39–40

In Exercises 39 and 40, assume that the box is constructed using the method illustrated in Example 3 on page 389.

39. **POSTCARDS** Marcie wants to make a box to hold her postcard collection from a piece of cardboard that is 10 inches by 18 inches. What are the dimensions of the box with the maximum volume? What is the maximum volume of the box?

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40. **COIN COLLECTION** Jorge is making a box for his coin collection from a piece of cardboard that is 30 centimeters by 40 centimeters. What are the dimensions of the box with the maximum volume? What is the maximum volume of the box?

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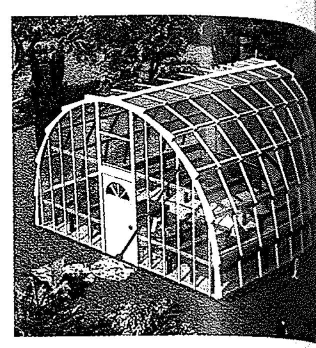
41. **SWIMMING** For a swimmer doing the breaststroke, the function

$$S = -241t^7 + 1060t^6 - 1870t^5 + 1650t^4 - 737t^3 + 144t^2 - 2.43t$$

models the swimmer's speed S (in meters per second) during one complete stroke, where t is the number of seconds since the start of the stroke. Graph the function. According to the model, at what time during the stroke is the swimmer going the fastest?

42. **MULTIPLE REPRESENTATIONS** You have 600 square feet of material for building a greenhouse that is shaped like half a cylinder.



a. Writing an Expression The surface area S of the greenhouse is given by $S = \pi r^2 + \pi r\ell$. Substitute 600 for S and then write an expression for ℓ in terms of r.

b. Writing a Function The volume V of the greenhouse is given by $V = \frac{1}{2}\pi r^2 \ell$. Write an equation that gives V as a polynomial function of r alone.

c. Graphing a Function Graph the volume function from part (b). What are the dimensions r and ℓ that maximize the volume of the greenhouse? What is the maximum volume?

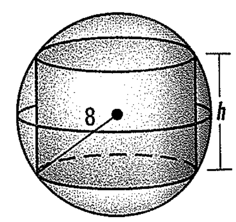
43. ★ **EXTENDED RESPONSE** From 1960 to 2001, the number of students S (in thousands) enrolled in public schools in the United States can be modeled by $S = 1.64x^3 - 102x^2 + 1710x + 36{,}300$ where x is the number of years since 1960.

a. Graph the function.

b. Identify any turning points on the domain $0 \le x \le 41$. What real-life meaning do these points have?

c. What is the range of the function for the domain $0 \le x \le 41$?

44. **CHALLENGE** A cylinder is inscribed in a sphere of radius 8. Write an equation for the volume of the cylinder as a function of h. Find the value of h that maximizes the volume of the inscribed cylinder. What is the maximum volume of the cylinder?



MIXED REVIEW

The variables x and y vary directly. Write an equation that relates x and y. *(p. 107)*

45. $x = 1, y = 5$

46. $x = -2, y = 8$

47. $x = 3, y = -5$

48. $x = 4, y = 6$

49. $x = 5, y = -2$

50. $x = -12, y = -4$

PREVIEW
Prepare for Lesson 5.9 in Exs. 51–54.

Write a quadratic function whose graph has the given characteristics. *(p. 309)*

51. vertex: (5, 4); passes through: (3, 12)

52. vertex: (−4, −6); passes through: (2, 3)

53. passes through: (−3, 12), (−2, 3), (1, 0)

54. passes through: (−2, 19), (2, −5), (4, −11)

Simplify the expression. Tell which properties of exponents you used. *(p. 330)*

55. $(3^2x^3)^5$

56. $(x^2y^4)^{-1}$

57. $(xy^3)(x^{-2}y)^{-3}$

58. $-4x^{-3}y^0$

59. $\frac{x^6}{x^{-2}}$

60. $\frac{3x^2y}{12xy^{-1}}$

61. $\frac{8xy}{7x^4} \cdot \frac{7x^5y}{4y^2}$

62. $\left(\frac{5x^3y^7}{25x^2y^4}\right)^3$

EXTRA PRACTICE for Lesson 5.8, p. 1014 **ONLINE QUIZ** at classzone.com

5.9 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS11 for Exs. 9, 15, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 10, 22, 23, and 28

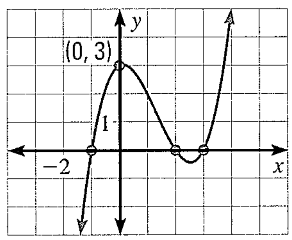
SKILL PRACTICE

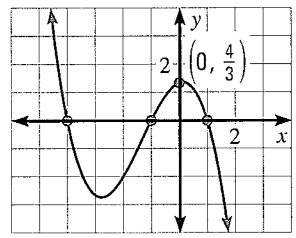
1. **VOCABULARY** Copy and complete: When the x-values in a data set are equally spaced, the differences of consecutive y-values are called _?_.

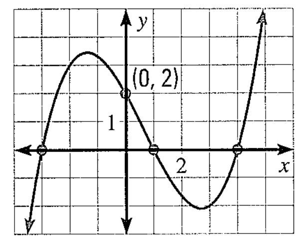
2. ★ **WRITING** *Describe* first-order differences and second-order differences.

EXAMPLE 1 on p. 393 for Exs. 3–11

WRITING CUBIC FUNCTIONS **Write the cubic function whose graph is shown.**

3. 

4. 

5. 

CUBIC MODELS **Write a cubic function whose graph passes through the points.**

6. $(-3, 0), (-1, 10), (0, 0), (4, 0)$
7. $(-2, 0), (-1, 0), (0, -8), (2, 0)$
8. $(-3, 0), (1, 0), (3, 2), (4, 0)$
9. $(-5, 0), (0, 0), (1, -12), (6, 0)$

10. ★ **MULTIPLE CHOICE** Which cubic function's graph passes through the points $(-3, 0), (-1, 0), (3, 0)$, and $(0, 3)$?

Ⓐ $f(x) = (x - 3)(x + 3)(x - 1)$

Ⓑ $f(x) = -\frac{1}{3}(x - 3)(x + 3)(x + 1)$

Ⓒ $f(x) = -2(x - 3)(x + 3)(x - 1)$

Ⓓ $f(x) = (x - 3)(x + 3)(x + 1)$

11. **ERROR ANALYSIS** A student tried to write a cubic function whose graph has x-intercepts -1, 2, and 5, and passes through $(1, 3)$. *Describe* and correct the error in the student's calculation of the leading coefficient a.

$$1 = a(3 + 1)(3 - 2)(3 - 5)$$
$$1 = -8a$$
$$-\frac{1}{8} = a$$

EXAMPLE 2 on p. 394 for Exs. 12–17

FINDING FINITE DIFFERENCES **Show that the nth-order differences for the given function of degree n are nonzero and constant.**

12. $f(x) = 5x^3 - 10$
13. $f(x) = -2x^2 + 5x$
14. $f(x) = x^4 - 3x^2 + 2$
15. $f(x) = 4x^2 - 9x + 2$
16. $f(x) = x^3 - 4x^2 - x + 1$
17. $f(x) = 2x^5 - 3x^2 + x$

EXAMPLE 3 on p. 395 for Exs. 18–21

FINDING A MODEL **Use finite differences and a system of equations to find a polynomial function that fits the data in the table.**

18.

x	1	2	3	4	5	6
$f(x)$	0	−3	−8	−15	−24	−35

19.

x	1	2	3	4	5	6
$f(x)$	11	14	9	−4	−25	−54

20.

x	1	2	3	4	5	6
$f(x)$	−12	−14	−10	6	40	98

21.

x	1	2	3	4	5	6
$f(x)$	5	14	27	41	53	60

22. ★ **OPEN-ENDED MATH** Write two different cubic functions whose graphs pass through the points $(-3, 0)$, $(-1, 0)$, and $(2, 6)$.

23. ★ **SHORT RESPONSE** How many points do you need to determine a quartic function? a quintic (fifth-degree) function? *Justify* your answers.

24. **CHALLENGE** Substitute the expressions $k, k + 1, k + 2, \ldots, k + 5$ for x in the function $f(x) = ax^3 + bx^2 + cx + d$ to generate six equally-spaced ordered pairs. Then show that third-order differences are constant.

PROBLEM SOLVING

EXAMPLE 3
on p. 395
for Ex. 25

25. **GEOMETRY** Find a polynomial function that gives the number of diagonals d of a polygon with n sides.

Number of sides, *n*	3	4	5	6	7	8
Number of diagonals, *d*	0	2	5	9	14	20

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EXAMPLE 4
on p. 396
for Exs. 26–28

26. **AVIATION** The table shows the number of active pilots (in thousands) with airline transport licenses in the United States for the years 1997 to 2004. Use a graphing calculator to find a polynomial model for the data.

Years since 1997, *t*	0	1	2	3	4	5	6	7
Transport pilots, *p*	131	135	138	142	145	145	144	145

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27. **MULTI-STEP PROBLEM** The table shows the average U.S. movie ticket price (in dollars) for various years from 1983 to 2003.

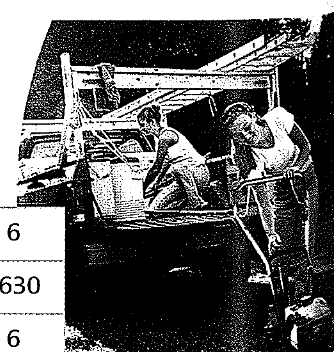
Years since 1983, *t*	0	4	8	12	16	20
Movie ticket price, *m*	3.15	3.91	4.21	4.35	5.08	6.03

a. Use a graphing calculator to find a polynomial model for the data.

b. Estimate the average U.S. movie ticket price in 2010.

c. In which year was the average U.S. movie ticket price about \$4.50?

28. ★ **SHORT RESPONSE** Based on data collected from friends, you estimate the cumulative profits (in dollars) after each of six months for two potential businesses. Find a polynomial function that models the profit for each business. Which business will yield the greatest long-term profit? Why?



Yard work	Month, t	1	2	3	4	5	6
	Profit, p	30	210	410	680	1070	1630
Pet care	Month, t	1	2	3	4	5	6
	Profit, p	30	50	220	540	1010	1630

29. **GEOMETRY** The maximum number of regions R into which space can be divided by n intersecting spheres is given by $R(n) = \frac{1}{3}n^3 - n^2 + \frac{8}{3}n$. Show that this function has constant third-order differences.

30. **CHALLENGE** A cylindrical cake is divided into the maximum number of pieces p by c planes. When $c = 1, 2, 3, 4, 5$, and 6 the values of $p(c)$ are 2, 4, 8, 15, 26, and 42 respectively. What is the maximum number of pieces into which the cake can be divided when it is cut by 8 planes?

MIXED REVIEW

Draw a scatter plot of the data and approximate the best-fitting line. ***(p. 113)***

31.

x	0.5	1	2	2.5	4	5
y	5.5	5	3.5	2	0.5	0.5

32.

x	−4	−2	0	1	2	2.5	3
y	−2	−1	0.5	2	2	3	3

Factor the polynomial.

33. $x^2 - 19x + 48$ ***(p. 252)***
34. $18x^2 + 30x - 12$ ***(p. 259)***
35. $64x^2 - 144x + 81$ ***(p. 259)***
36. $18x^3 + 33x^2 - 30x$ ***(p. 353)***
37. $64x^3 + 27$ ***(p. 353)***
38. $3x^5 - 66x^3 - 225x$ ***(p. 353)***

PREVIEW
Prepare for Lesson 6.1 in Exs. 39–44.

Solve the equation. ***(p. 266)***

39. $5x^2 = 10$
40. $24x^2 = 6$
41. $9x^2 + 2 = 6$
42. $7x^2 - 4 = 8$
43. $-x^2 + 16 = 5x^2 - 12$
44. $4x^2 + 3 = -4x^2 + 15$

QUIZ *for Lessons 5.7–5.9*

Find all zeros of the polynomial function. ***(p. 379)***

1. $f(x) = x^3 - 4x^2 - 11x + 30$
2. $f(x) = 2x^4 - 2x^3 - 49x^2 + 9x + 180$

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros. ***(p. 379)***

3. $-4, -1, 2$
4. $4, 1 + i$
5. $-3, 5, 7 + \sqrt{2}$
6. $1, -2i, 3 - \sqrt{6}$

Graph the function. ***(p. 387)***

7. $f(x) = -(x - 3)(x - 2)(x + 2)$
8. $f(x) = 3(x - 1)(x + 1)(x - 4)$
9. $f(x) = x(x - 4)(x - 1)(x + 2)$
10. $f(x) = (x - 3)(x + 2)^2(x + 3)^2$

Write a cubic function whose graph passes through the given points. ***(p. 393)***

11. $(-5, 0), (-2, 0), (1, 9), (2, 0)$
12. $(-1, 0), (0, 16), (2, 0), (4, 0)$

13. **DRIVE-INS** The table shows the number of U.S. drive-in movie theaters for the years 1995 to 2002. Find a polynomial model that fits the data. ***(p. 393)***

Years since 1995, t	0	1	2	3	4	5	6	7
Drive-in movie theaters, D	848	826	815	750	737	667	663	634

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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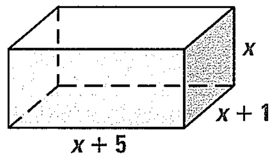
Lessons 5.6–5.9

1. **MULTI-STEP PROBLEM** The volume of the rectangular prism shown is 180 cubic inches.



   a. Write a polynomial equation that you can use to find the value of x.

   b. Identify the possible rational solutions of the equation in part (a).

   c. Use synthetic division to find a rational solution of the equation. Show that no other real solutions exist.

   d. What are the dimensions of the prism?

2. **MULTI-STEP PROBLEM** You want to make an open box from a piece of cardboard to hold your school supplies. The box will be formed using the method described in Example 3 on page 389. The original piece of cardboard is 20 inches by 30 inches.

   a. Write a polynomial function for the volume of the box.

   b. Graph the function in part (a).

   c. What are the dimensions of the box with the maximum volume?

   d. What is the maximum volume of the box?

3. **GRIDDED ANSWER** From 1980 to 2002, the number R (in thousands) of retirees receiving Social Security benefits can be modeled by the function

$$R = 0.629t^3 - 27.8t^2 + 744t + 19{,}600$$

   where t is the number of years since 1980. In which year was the number of retirees receiving Social Security benefits about 26,900,000?

4. **OPEN-ENDED** Write a polynomial function with real coefficients that has degree 4 and zeros -2, 1, and $4 - i$.

5. **OPEN-ENDED** Write a polynomial function with rational coefficients that has 16 *possible* rational zeros according to the rational zero theorem, but has no actual rational zeros.

6. **EXTENDED RESPONSE** You are making a sculpture that is a pyramid with a square base. You want the height of the pyramid to be 4 inches less than the length of a side of the base. You want the volume of the sculpture to be 200 cubic inches.

   a. Let x represent the length (in inches) of a side of the sculpture's base. Draw a diagram of the sculpture, and label the dimensions in terms of x.

   b. Write a function that gives the volume V of the sculpture in terms of x.

   c. Graph the function in part (b). Use the graph to estimate the value of x for the sculpture.

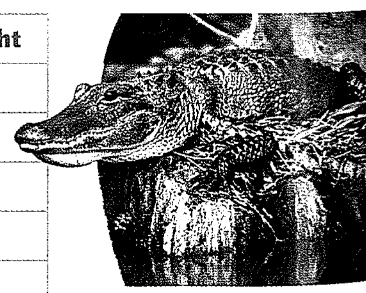
   d. Write and solve an equation to find the value of x. *Compare* your answer with your estimate from part (c). What are the dimensions of the sculpture?

7. **SHORT RESPONSE** Your friend has started a golf caddying business. The table shows the profit p (in dollars) of the business in the first 5 months. Use finite differences to find a polynomial model for the data. Then use the model to predict the profit when $t = 7$.

Month, t	1	2	3	4	5
Profit, p	4	2	6	22	56

8. **SHORT RESPONSE** The table shows the average relationship between length (in inches) and weight (in pounds) for an alligator as it grows. Use a graphing calculator to find a polynomial model for the data.

Length	Weight
12	0.2
24	0.7
36	8.6
48	17.7
54	28.0
60	39.6
66	45.4
72	49.6



5 CHAPTER SUMMARY

Animated Algebra
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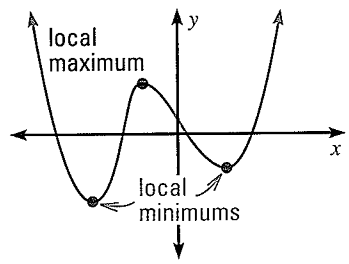
BIG IDEAS — *For Your Notebook*

Big Idea 1

Graphing Polynomial Functions



The end behavior of the graph of $f(x)$ is

$f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

so $f(x)$ is of even degree and has a positive leading coefficient.

The graph has 3 turning points, so the degree of $f(x)$ is *at least* 4 and $f(x)$ has *at least* 4 zeros.

Big Idea 2

Performing Operations with Polynomials

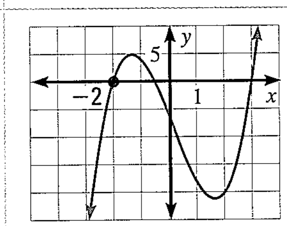
You can add, subtract, multiply, and divide polynomials. You can also factor polynomials using any combination of the methods below.

Factoring method	Example
General trinomial	$6x^2 - 7x - 3 = (3x + 1)(2x - 3)$
Perfect square trinomial	$x^2 + 10x + 25 = (x + 5)^2$
Difference of two squares	$x^2 - 49 = (x + 7)(x - 7)$
Common monomial factor	$15x^3 + 9x^2 = 3x^2(5x + 3)$
Sum or difference of two cubes	$8x^3 - 27 = (2x - 3)(4x^2 + 6x + 9)$
Factor by grouping	$x^3 - 5x^2 + 9x - 45 = x^2(x - 5) + 9(x - 5) = (x^2 + 9)(x - 5)$

Big Idea 3

Solving Polynomial Equations and Finding Zeros

The terms *zero, factor, solution,* and *x-intercept* are closely related. Consider the function $f(x) = 2x^3 - x^2 - 13x - 6$.

-2 is a **zero** of f.	$f(\mathbf{-2}) = 2(\mathbf{-2})^3 - (\mathbf{-2})^2 - 13(\mathbf{-2}) - 6 = 0$
$x + 2$ is a **factor** of $f(x)$.	$2x^3 - x^2 - 13x - 6 = (\mathbf{x + 2})(x - 3)(2x + 1)$
$x = -2$ is a **solution** of the equation $f(x) = 0$.	$2(\mathbf{-2})^3 - (\mathbf{-2})^2 - 13(\mathbf{-2}) - 6 = 0$
-2 is an ***x*-intercept** of the graph of f.	

5 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- scientific notation, *p. 331*
- polynomial, *p. 337*
- polynomial function, *p. 337*
- leading coefficient, *p. 337*
- degree, *p. 337*
- constant term, *p. 337*
- standard form of a polynomial function, *p. 337*
- synthetic substitution, *p. 338*
- end behavior, *p. 339*
- factored completely, *p. 353*
- factor by grouping, *p. 354*
- quadratic form, *p. 355*
- polynomial long division, *p. 362*
- synthetic division, *p. 363*
- repeated solution, *p. 379*
- local maximum, *p. 388*
- local minimum, *p. 388*
- finite differences, *p. 393*

VOCABULARY EXERCISES

1. Copy and complete: At each of its turning points, the graph of a polynomial function has a(n) _?_ or a(n) _?_.

2. **WRITING** *Explain* how you can tell whether a solution of a polynomial equation is a repeated solution when the equation is written in factored form.

3. **WRITING** *Explain* how you can tell whether a number is expressed in scientific notation.

4. Let f be a fourth-degree polynomial function with four distinct real zeros. How many turning points does the graph of f have?

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 5.

5.1 Use Properties of Exponents *pp. 330–335*

EXAMPLE

Simplify the expression.

$(x^2y^3)^3x^4 = (x^2)^3(y^3)^3x^4$ **Power of a product property**

$= x^6y^9x^4$ **Power of a power property**

$= x^{6+4}y^9$ **Product of powers property**

$= x^{10}y^9$ **Simplify exponent.**

EXERCISES

EXAMPLES 1, 2, 3, and 4 on pp. 330–332 for Exs. 5–12

Evaluate or simplify the expression. Tell which properties of exponents you used.

5. $2^2 \cdot 2^5$
6. $(3^2)^{-3}(3^3)$
7. $(x^{-2}y^5)^2$
8. $(3x^4y^{-2})^{-3}$
9. $\left(\frac{3}{4}\right)^{-2}$
10. $\frac{8 \times 10^7}{2 \times 10^3}$
11. $\left(\frac{x^2}{y^{-2}}\right)^{-4}$
12. $\frac{2x^{-6}y^5}{16x^3y^{-2}}$

5.2 Evaluate and Graph Polynomial Functions *pp. 337–344*

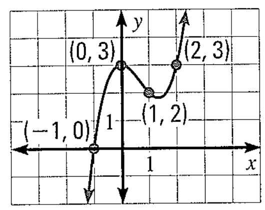
EXAMPLE

Graph the polynomial function $f(x) = x^3 - 2x^2 + 3$.

Make a table of values.

x	−2	−1	0	1	2	3
$f(x)$	−13	0	3	2	3	12

Plot the points, connect the points with a smooth curve, and check the end behavior.



The degree is odd and the leading coefficient is positive, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

EXERCISES

EXAMPLES 5 and 6 on p. 340 for Exs. 13–16

Graph the polynomial function.

13. $f(x) = -x^4$

14. $f(x) = x^3 - 4$

15. $f(x) = x^3 + 2x + 3$

16. FISH CONSUMPTION From 1990 to 2002, the amount of fish F (in millions of pounds) caught for human consumption in the United States can be modeled by

$$F = -0.907t^4 + 28.0t^3 - 258t^2 + 902t + 12{,}700$$

where t is the number of years since 1990. Graph the function. Use the graph to estimate the year when the amount of fish caught first was greater than 14.5 *billion* pounds.

5.3 Add, Subtract, and Multiply Polynomials *pp. 346–352*

EXAMPLE

Perform the indicated operation.

a. $(3x^3 - 6x^2 - 7x + 5) + (x^3 + 8x + 3) = 3x^3 + x^3 - 6x^2 - 7x + 8x + 5 + 3$

$= 4x^3 - 6x^2 + x + 8$

b. $(x - 4)(2x^2 - 7x + 5) = (x - 4)2x^2 - (x - 4)7x + (x - 4)5$

$= 2x^3 - 8x^2 - 7x^2 + 28x + 5x - 20$

$= 2x^3 - 15x^2 + 33x - 20$

EXERCISES

EXAMPLES 1, 2, 4, and 5 on pp. 346–348 for Exs. 17–20

Perform the indicated operation.

17. $(5x^3 - x + 3) + (x^3 - 9x^2 + 4x)$

18. $(x^3 + 4x^2 - 5x) - (4x^3 + x^2 - 7)$

19. $(x - 6)(5x^2 + x - 8)$

20. $(x - 4)(x + 7)(5x - 1)$

5 CHAPTER REVIEW

5.4 Factor and Solve Polynomial Equations

pp. 353–359

EXAMPLE

Factor the polynomial completely.

a. $x^3 + 125 = x^3 + 5^3 = (x + 5)(x^2 - 5x + 25)$	Sum of two cubes
b. $x^3 + 5x^2 - 9x - 45 = x^2(x + 5) - 9(x + 5)$	Factor by grouping.
$= (x^2 - 9)(x + 5)$	Distributive property
$= (x + 3)(x - 3)(x + 5)$	Difference of two squares
c. $3x^6 + 12x^4 - 96x^2 = 3x^2(x^4 + 4x^2 - 32)$	Factor common monomial.
$= 3x^2(x^2 - 4)(x^2 + 8)$	Factor trinomial in quadratic form.
$= 3x^2(x + 2)(x - 2)(x^2 + 8)$	Difference of two squares

EXERCISES

EXAMPLES 2, 3, 4, and 6 on pp. 354–356 for Exs. 21–24

Factor the polynomial completely.

21. $64x^3 - 8$ **22.** $2x^5 - 12x^3 + 10x$ **23.** $2x^3 - 7x^2 - 8x + 28$

24. SCULPTURE You have 240 cubic inches of clay with which to make a sculpture shaped as a rectangular prism. You want the width to be 4 inches less than the length and the height to be 2 inches more than 3 times the length. What should the dimensions of the sculpture be?

5.5 Apply the Remainder and Factor Theorems

pp. 362–368

EXAMPLE

Divide $f(x) = 4x^4 + 29x^3 + 4x^2 - 14x + 37$ by $x + 7$.

Rewrite the divisor in the form $x - k$. Because $x + 7 = x - (-7)$, $k = -7$.

−7	4	29	4	−14	37
		−28	−7	21	−49
	4	1	−3	7	−12

So, $\frac{4x^4 + 29x^3 + 4x^2 - 14x + 37}{x + 7} = 4x^3 + x^2 - 3x + 7 - \frac{12}{x + 7}$.

EXERCISES

EXAMPLES 1, 3, and 4 on pp. 362–364 for Exs. 25–32

Divide.

25. $(x^3 - 3x^2 - x - 10) \div (x^2 + 3x - 1)$ **26.** $(4x^4 - 17x^2 + 9x - 18) \div (2x^2 - 2)$

27. $(2x^3 - 11x^2 + 13x - 44) \div (x - 5)$ **28.** $(5x^4 + 2x^2 - 15x + 10) \div (x + 2)$

Given polynomial $f(x)$ and a factor of $f(x)$, factor $f(x)$ completely.

29. $f(x) = x^3 - 5x^2 - 2x + 24$; $x + 2$ **30.** $f(x) = x^3 - 11x^2 + 14x + 80$; $x - 8$

31. $f(x) = 9x^3 - 9x^2 - 4x + 4$; $x - 1$ **32.** $f(x) = 2x^3 + 7x^2 - 33x - 18$; $x + 6$

5.6 Find Rational Zeros

pp. 370–377

EXAMPLE

Find all real zeros of $f(x) = x^3 + 6x^2 + 5x - 12$.

The leading coefficient is 1 and the constant term is −12.

Possible rational zeros: $x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$

Test these zeros using synthetic division. Test $x = 1$:

$$\begin{array}{r|rrrr} 1 & 1 & 6 & 5 & -12 \\ & & 1 & 7 & 12 \\ \hline & 1 & 7 & 12 & 0 \end{array}$$

← **1 is a zero.**

You can write $f(x) = (x - 1)(x^2 + 7x + 12)$. Factor the trinomial.

$f(x) = (x - 1)(x^2 + 7x + 12) = (x - 1)(x + 3)(x + 4)$

The zeros of f are 1, −3, and −4.

EXAMPLES 2 and 3 on pp. 371–372 for Exs. 33–34

EXERCISES

Find all real zeros of the function.

33. $f(x) = x^3 - 4x^2 - 11x + 30$

34. $f(x) = 2x^4 - x^3 - 42x^2 + 16x + 160$

5.7 Apply the Fundamental Theorem of Algebra

pp. 379–386

EXAMPLE

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and −4 and $5 + \sqrt{2}$ as zeros.

Because $5 + \sqrt{2}$ is a zero, $5 - \sqrt{2}$ must also be a zero.

$f(x) = (x + 4)[x - (5 + \sqrt{2})][x - (5 - \sqrt{2})]$ **Write $f(x)$ in factored form.**

$= (x + 4)[(x - 5) - \sqrt{2}][(x - 5) + \sqrt{2}]$ **Regroup terms.**

$= (x + 4)[(x - 5)^2 - 2]$ **Multiply.**

$= x^3 - 6x^2 - 17x + 92$ **Multiply.**

EXAMPLES 3 and 6 on pp. 381–383 for Exs. 35–38

EXERCISES

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.

35. −4, 1, 5

36. −1, −1, 6, $3i$

37. 2, 7, $3 - \sqrt{5}$

38. ECONOMICS For the 15 years that a computer store has been open, its annual revenue R (in millions of dollars) can be modeled by

$$R = -0.0040t^4 + 0.088t^3 - 0.36t^2 - 0.55t + 5.8$$

where t is the number of years since the store opened. In what year was the revenue first greater than \$7 million?

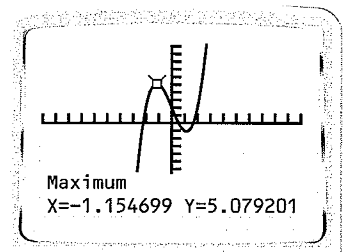
5 CHAPTER REVIEW

5.8 Analyze Graphs of Polynomial Functions *pp. 387–392*

EXAMPLE

Graph the function $f(x) = x^3 - 4x + 2$. Identify the x-intercepts and the points where the local maximums and local minimums occur.

Use a graphing calculator to graph the function.



Notice that the graph has three x-intercepts and two turning points. You can use the graphing calculator's *zero, maximum,* and *minimum* features to approximate the coordinates of the points.

The x-intercepts of the graph are about -2.21, 0.54, and 1.68. The function has a local maximum at $(-1.15, 5.08)$ and a local minimum at $(1.15, -1.08)$.

EXERCISES

EXAMPLE 2 on p. 388 for Exs. 39–40

Use a graphing calculator to graph the function. Identify the x-intercepts and the points where the local maximums and local minimums occur.

39. $f(x) = -2x^3 - 3x^2 - 1$

40. $f(x) = x^4 + 3x^3 - x^2 - 8x + 2$

5.9 Write Polynomial Functions and Models *pp. 393–399*

EXAMPLE

Use finite differences and a system of equations to find a polynomial function that fits the data.

x	1	2	3	4	5	6
$f(x)$	1	9	23	43	69	101

```
f(1) f(2) f(3) f(4) f(5) f(6)
 1    9    23   43   69   101     Write function values for equally-spaced x-values.
   8    14   20   26   32         First-order differences
     6     6    6    6            Second-order differences
```

Because the second-order differences are constant, the data can be represented by a function of the form $f(x) = ax^2 + bx + c$. By substituting the first 3 data points into the function, you obtain a system of 3 linear equations in 3 variables.

$a(1)^2 + b(1) + c = 1 \rightarrow a + b + c = 1$

$a(2)^2 + b(2) + c = 9 \rightarrow 4a + 2b + c = 9$

$a(3)^2 + b(3) + c = 23 \rightarrow 9a + 3b + c = 23$

Solve the system. The solution is $(3, -1, -1)$, so $f(x) = 3x^2 - x - 1$.

EXERCISES

EXAMPLE 3 on p. 395 for Ex. 41

41. Use finite differences to find a polynomial function that fits the data.

x	1	2	3	4	5	6
$f(x)$	−6	−21	−40	−57	−66	−61

5 CHAPTER TEST

Simplify the expression. Tell which properties of exponents you used.

1. $x^3 \cdot x^2 \cdot x^{-4}$ **2.** $(2x^{-2}y^3)^{-5}$ **3.** $\left(\frac{x^{-4}}{y^2}\right)^{-2}$ **4.** $\frac{3(xy)^3}{27x - 5y^3}$

Graph the polynomial function.

5. $f(x) = -x^3$ **6.** $f(x) = x^4 - 2x^2 - 5x + 1$ **7.** $f(x) = x^5 - x^4 - 9$

Perform the indicated operation.

8. $(2x^3 + 5x^2 - 7x + 4) + (x^3 - 3x^2 - 4x)$
9. $(3x^3 - 4x^2 + 3x - 5) - (x^2 + 4x - 8)$
10. $(3x - 2)(x^2 + 4x - 7)$
11. $(3x - 5)^3$
12. $(3x^3 - 14x^2 + 16x - 22) \div (x - 4)$
13. $(6x^4 + 7x^2 + 4x - 17) \div (3x^2 - 3x + 2)$

Factor the polynomial completely.

14. $8x^3 + 27$ **15.** $x^4 + 5x^2 - 6$ **16.** $x^3 - 3x^2 - 4x + 12$

Find all real zeros of the function.

17. $f(x) = x^3 + x^2 - 22x - 40$ **18.** $f(x) = 4x^4 - 8x^3 - 19x^2 + 23x - 6$

Write a polynomial function *f* of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.

19. $-1, 3, 4$ **20.** $6, 2i$ **21.** $-3, -1, 1 - \sqrt{5}$ **22.** $1 + 3i, 4 + \sqrt{10}$

Use a graphing calculator to graph the function. Identify the *x*-intercepts and the points where the local maximums and local minimums occur.

23. $f(x) = x^3 - 5x^2 + 3x + 4$ **24.** $f(x) = x^4 + 3x^3 - x^2 - 6x + 2$

Use finite differences and a system of equations to find a polynomial function that fits the data in the table.

25.

x	1	2	3	4	5	6
f(*x*)	3	1	1	3	7	13

26.

x	1	2	3	4	5	6
f(*x*)	0	−7	−4	21	80	185

27. GROSS DOMESTIC PRODUCT In 2003, the gross domestic product (GDP) of the United States was about 1.099×10^{13} dollars. The population of the U.S. in 2003 was about 2.91×10^8. What was the per capita GDP in 2003?

28. TELEVISION From 1980 to 2002, the number T (in millions) of households in the United States with televisions and the percent P of those households with VCRs can be modeled by

$$T = 1.22x + 76.9 \quad \text{and} \quad P = -0.205x^2 + 8.36x + 1.98$$

where x is the number of years since 1980. Write a polynomial model for the total number of U.S. households with both televisions and VCRs.

29. GEOMETRY A rectangular prism has edges of lengths x, $x + 2$, and $2x - 3$ inches. The volume of the prism is 1040 cubic inches. Write a polynomial equation that models the prism's volume. What are the prism's dimensions?