6 Rational Exponents and Radical Functions

Before

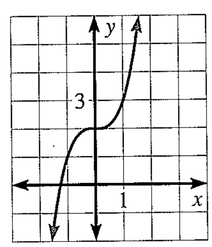
In previous chapters, you learned the following skills, which you'll use in Chapter 6: simplifying expressions involving exponents, rewriting equations, and graphing polynomial functions.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. The **square roots** of 81 are _?_ and _?_.
2. In the expression 2^5, the **exponent** is _?_.
3. For the polynomial function whose graph is shown, the sign of the **leading coefficient** is _?_.



SKILLS CHECK

Simplify the expression. ***(Review p. 330 for 6.2.)***

4. $\dfrac{5x^2y}{15x^3y^{-1}}$
5. $\dfrac{32x^{-3}y^4}{24x^{-3}y^{-2}} \cdot \dfrac{3x}{9y}$
6. $(2x^5y^{-3})^{-3}$

Solve the equation for *y*. ***(Review p. 26 for 6.4.)***

7. $-2x - 5y = 10$
8. $x - \frac{1}{3}y = -1$
9. $8x - 4xy = 3$

Graph the polynomial function. ***(Review p. 337 for 6.5.)***

10. $f(x) = x^3 - 4x + 6$
11. $f(x) = -x^5 + 7x^2 + 2$
12. $f(x) = x^4 - 4x^2 + x$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 6, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 465. You will also use the key vocabulary listed below.

Big Ideas

1. **Using rational exponents**
2. **Performing function operations and finding inverse functions**
3. **Graphing radical functions and solving radical equations**

KEY VOCABULARY

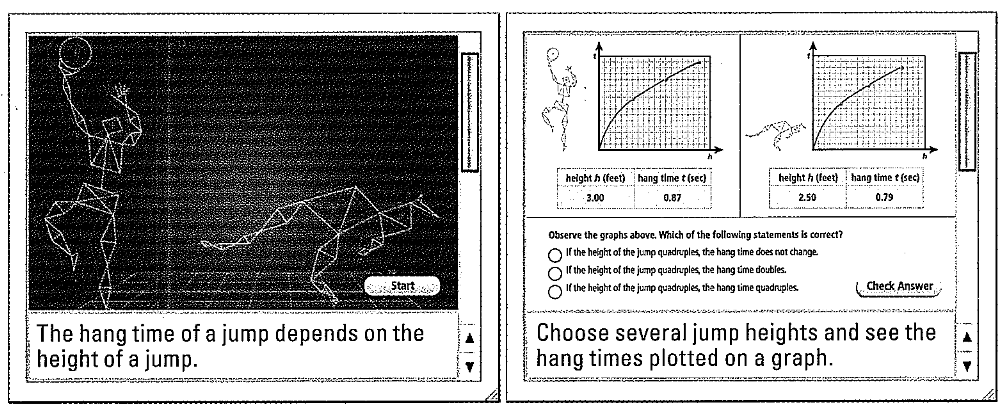
- *n*th root of *a*, *p. 414*
- index of a radical, *p. 414*
- simplest form of a radical, *p. 422*
- like radicals, *p. 422*
- power function, *p. 428*
- composition, *p. 430*
- inverse relation, *p. 438*
- inverse function, *p. 438*
- radical function, *p. 446*
- radical equation, *p. 452*

Why?

You can use a radical function to model the time you are suspended in the air during a jump. For example, the hang time of a basketball player can be modeled by a radical function.

Animated Algebra

The animation illustrated below for Exercise 60 on page 458 helps you answer this question: What is the relationship between the height of a jump and the time the jumper is suspended in air?



The hang time of a jump depends on the height of a jump.

Choose several jump heights and see the hang times plotted on a graph.

Animated **Algebra** at classzone.com

Other animations for Chapter 6: pages 431, 444, 448, and 465

6.1 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 9, 25, and 63
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 33, 46, 47, and 65

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In the expression $\sqrt[4]{10{,}000}$, the number 4 is called the _?_.

2. ★ **WRITING** *Explain* how the sign of a determines the number of real fourth roots of a and the number of real fifth roots of a.

EXAMPLE 1 on p. 414 for Exs. 3–20

MATCHING EXPRESSIONS Match the expression in rational exponent notation with the equivalent expression in radical notation.

3. $2^{1/3}$ 4. $2^{3/2}$ 5. $2^{2/3}$ 6. $2^{1/2}$

A. $(\sqrt{2})^3$ B. $\sqrt{2}$ C. $\sqrt[3]{2}$ D. $(\sqrt[3]{2})^2$

USING RATIONAL EXPONENT NOTATION Rewrite the expression using rational exponent notation.

7. $\sqrt[3]{12}$ 8. $\sqrt[5]{8}$ (9.) $(\sqrt[3]{10})^7$ 10. $(\sqrt[8]{15})^3$

USING RADICAL NOTATION Rewrite the expression using radical notation.

11. $5^{1/4}$ 12. $7^{1/3}$ 13. $14^{2/5}$ 14. $21^{9/4}$

FINDING *N*TH ROOTS Find the indicated real *n*th root(s) of *a*.

15. $n = 2, a = 64$ 16. $n = 3, a = -27$ 17. $n = 4, a = 0$
18. $n = 3, a = 343$ 19. $n = 4, a = -16$ 20. $n = 5, a = -32$

EXAMPLE 2 on p. 415 for Exs. 21–33

EVALUATING EXPRESSIONS Evaluate the expression without using a calculator.

21. $\sqrt[6]{64}$ 22. $8^{1/3}$ 23. $16^{3/2}$ 24. $\sqrt[3]{-125}$
(25.) $27^{2/3}$ 26. $(-243)^{1/5}$ 27. $(\sqrt[3]{8})^{-2}$ 28. $(\sqrt[3]{-64})^4$
29. $(\sqrt[4]{16})^{-7}$ 30. $25^{3/2}$ 31. $64^{-2/3}$ 32. $\frac{1}{81^{-3/4}}$

33. ★ **MULTIPLE CHOICE** What is the value of $128^{5/7}$?
(A) 8 (B) 16 (C) 32 (D) 64

EXAMPLE 3 on p. 415 for Exs. 34–46

APPROXIMATING ROOTS Evaluate the expression using a calculator. Round the result to two decimals places when appropriate.

34. $\sqrt[5]{32{,}768}$ 35. $\sqrt[7]{1695}$ 36. $\sqrt[9]{-230}$ 37. $85^{1/6}$
38. $25^{-1/3}$ 39. $20{,}736^{1/4}$ 40. $(\sqrt[4]{187})^3$ 41. $(\sqrt{6})^{-5}$
42. $(\sqrt[5]{-8})^8$ 43. $86^{-5/6}$ 44. $1974^{2/7}$ 45. $\frac{1}{(-17)^{3/5}}$

46. ★ **MULTIPLE CHOICE** Which expression has the greatest value?
(A) $27^{3/5}$ (B) $5^{3/2}$ (C) $\sqrt[3]{81}$ (D) $(\sqrt[3]{2})^8$

47. ★ **OPEN-ENDED MATH** Write two different expressions of the form $a^{1/n}$ that equal 3, where a is a real number and n is an integer greater than 1.

EXAMPLE 4
on p. 416
for Exs. 48–58

ERROR ANALYSIS ***Describe*** **and correct the error in solving the equation.**

48.

$x^3 = 27$

$x = \sqrt[3]{27}$

$x = 9$

49.

$x^4 = 81$

$x = \sqrt[4]{81}$

$x = 3$

SOLVING EQUATIONS **Solve the equation. Round the result to two decimal places when appropriate.**

50. $x^3 = 125$

51. $5x^3 = 1080$

52. $x^6 + 36 = 100$

53. $(x - 5)^4 = 256$

54. $x^5 = -48$

55. $7x^4 = 56$

56. $x^3 + 40 = 25$

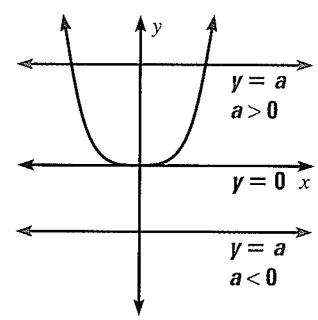
57. $(x + 10)^5 = 70$

58. $x^6 - 34 = 181$

59. **CHALLENGE** The general shape of the graph of $y = x^n$, where n is a positive *even* integer, is shown in red.

a. *Explain* how the graph justifies the results in the Key Concept box on page 414 when n is a positive *even* integer.

b. Draw a similar graph that justifies the results in the Key Concept box when n is a positive *odd* integer.



PROBLEM SOLVING

EXAMPLE 5
on p. 416
for Exs. 60–65

60. **SHOT PUT** The shot used in men's shot put has a volume of about 905 cubic centimeters. Find the radius of the shot. (*Hint:* Use the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere.)

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61. **BOWLING** A bowling ball has a surface area of about 232 square inches. Find the radius of the bowling ball. (*Hint:* Use the formula $S = 4\pi r^2$ for the surface area of a sphere.)

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62. **INFLATION** If the average price of an item increases from p_1 to p_2 over a period of n years, the annual rate of inflation r (expressed as a decimal) is given by $r = \left(\frac{p_2}{p_1}\right)^{1/n} - 1$. Find the rate of inflation for each item in the table. Write each answer as a percent rounded to the nearest tenth.

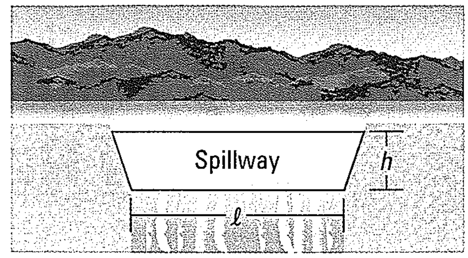
Item	Price in 1950	Price in 1990
Butter (lb)	\$.7420	\$2.195
Chicken (lb)	\$.4430	\$1.087
Eggs (dozen)	\$.6710	\$1.356
Sugar (lb)	\$.0936	\$.4560

63. **MULTI-STEP PROBLEM** The power p (in horsepower) used by a fan with rotational speed s (in revolutions per minute) can be modeled by the formula $p = ks^3$ for some constant k. A certain fan uses 1.2 horsepower when its speed is 1700 revolutions per minute. First find the value of k for this fan. Then find the speed of the fan if it uses 1.5 horsepower.

○ = **WORKED-OUT SOLUTIONS on p. WS1**

★ = **STANDARDIZED TEST PRACTICE**

64. **WATER RATE** A *weir* is a dam that is built across a river to regulate the flow of water. The flow rate Q (in cubic feet per second) can be calculated using the formula $Q = 3.367\ell h^{3/2}$ where ℓ is the length (in feet) of the bottom of the spillway and h is the depth (in feet) of the water on the spillway. Determine the flow rate of a weir with a spillway that is 20 feet long and has a water depth of 5 feet.



65. ★ **EXTENDED RESPONSE** Some games use dice in the shape of regular polyhedra. You are designing dice and want them all to have the same volume as a cube with an edge length of 16 millimeters.

Name	Tetrahedron 	Octahedron 	Dodecahedron 	Icosahedron 
Number of faces	4	8	12	20
Volume formula	$V = 0.118x^3$	$V = 0.471x^3$	$V = 7.663x^3$	$V = 2.182x^3$

a. Find the volume of a cube with an edge length of 16 millimeters.

b. Find the edge length x for each of the polyhedra shown in the table.

c. Does the polyhedron with the greatest number of faces have the smallest edge length? *Explain.*

66. **CHALLENGE** The mass of the particles that a river can transport is proportional to the sixth power of the speed of the river. A certain river normally flows at a speed of 1 meter per second. What must its speed be in order to transport particles that are twice as massive as usual? 10 times as massive? 100 times as massive?

MIXED REVIEW

Evaluate the expressions for the given values of x and y. *(p. 10)*

67. $\dfrac{x + 3y}{x - y}$ when $x = 3$ and $y = 5$

68. $\dfrac{4x - y}{x - 2y}$ when $x = 6$ and $y = -2$

Find all the zeros of the function.

69. $f(x) = x^2 - 2x - 35$ *(p. 252)*

70. $f(x) = x^2 - 8x + 25$ *(p. 292)*

71. $f(x) = x^3 - 8x^2 + 4x - 32$ *(p. 379)*

72. $f(x) = x^3 + 4x^2 + 25x + 100$ *(p. 379)*

73. $f(x) = x^4 - 3x^3 - 31x^2 + 63x + 90$ *(p. 379)*

74. $f(x) = x^4 + 10x^3 + 25x^2 - 36$ *(p. 379)*

PREVIEW Prepare for Lesson 6.2 in Exs. 75–82.

Simplify the expression. Tell which properties of exponents you used. *(p. 330)*

75. $\dfrac{x^{-4}}{x^3}$

76. $(x^4)^{-3}$

77. $(3x^2y)^{-3}$

78. $4x^0y^{-4}$

79. $x^6 \cdot x^{-2}$

80. $\left(\dfrac{x^3}{y^{-2}}\right)^2$

81. $\dfrac{4x^3y^6}{10x^5y^{-3}}$

82. $\dfrac{3x}{x^3y^2} \cdot \dfrac{y^4}{9x^{-2}}$

6.2 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 5, 27, and 85

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 51, 69, 86, and 89

SKILL PRACTICE

1. **VOCABULARY** Are $2\sqrt{5}$ and $2\sqrt[3]{5}$ like radicals? *Explain* why or why not.

2. ★ **WRITING** Under what conditions is a radical expression in simplest form?

EXAMPLE 1 on p. 420 for Exs. 3–14

PROPERTIES OF RATIONAL EXPONENTS **Simplify the expression.**

3. $5^{3/2} \cdot 5^{1/2}$
4. $(6^{2/3})^{1/2}$
5. $3^{1/4} \cdot 27^{1/4}$
6. $\dfrac{9}{9^{-4/5}}$
7. $\dfrac{80^{1/4}}{5^{-1/4}}$
8. $\left(\dfrac{7^3}{4^3}\right)^{-1/3}$
9. $\dfrac{11^{2/5}}{11^{4/5}}$
10. $(12^{3/5} \cdot 8^{3/5})^5$
11. $\dfrac{120^{-2/5} \cdot 120^{2/5}}{7^{-3/4}}$
12. $\dfrac{64^{5/9} \cdot 64^{2/9}}{4^{3/4}}$
13. $(16^{5/9} \cdot 5^{7/9})^{-3}$
14. $\dfrac{13^{3/7}}{13^{5/7}}$

EXAMPLE 3 on p. 421 for Exs. 15–22

PROPERTIES OF RADICALS **Simplify the expression.**

15. $\sqrt{20} \cdot \sqrt{5}$
16. $\sqrt[3]{16} \cdot \sqrt[3]{4}$
17. $\sqrt[4]{8} \cdot \sqrt[4]{8}$
18. $(\sqrt[3]{3} \cdot \sqrt[4]{3})^{12}$
19. $\dfrac{\sqrt[5]{64}}{\sqrt[5]{2}}$
20. $\dfrac{\sqrt{3}}{\sqrt{75}}$
21. $\dfrac{\sqrt[4]{36} \cdot \sqrt[4]{9}}{\sqrt[4]{4}}$
22. $\dfrac{\sqrt[4]{8} \cdot \sqrt[4]{16}}{\sqrt[8]{2} \cdot \sqrt[8]{3}}$

EXAMPLE 4 on p. 422 for Exs. 23–31

23. ★ **MULTIPLE CHOICE** What is the simplest form of the expression $3\sqrt[4]{32} \cdot (-6\sqrt[4]{5})$?

Ⓐ $\sqrt[4]{10}$ Ⓑ $-18\sqrt[4]{10}$ Ⓒ $-36\sqrt[4]{10}$ Ⓓ $36\sqrt[8]{10}$

SIMPLEST FORM **Write the expression in simplest form.**

24. $\sqrt{72}$
25. $\sqrt[6]{256}$
26. $\sqrt[3]{108} \cdot \sqrt[3]{4}$
27. $5\sqrt[4]{64} \cdot 2\sqrt[4]{8}$
28. $\sqrt[3]{\dfrac{1}{6}}$
29. $\dfrac{3}{\sqrt[4]{144}}$
30. $\sqrt[6]{\dfrac{81}{4}}$
31. $\dfrac{\sqrt[3]{9}}{\sqrt[5]{27}}$

EXAMPLE 5 on p. 422 for Exs. 32–41

COMBINING RADICALS AND ROOTS **Simplify the expression.**

32. $2\sqrt[6]{3} + 7\sqrt[6]{3}$
33. $\frac{3}{5}\sqrt[3]{5} - \frac{1}{5}\sqrt[3]{5}$
34. $25\sqrt[5]{2} - 15\sqrt[5]{2}$
35. $\frac{1}{8}\sqrt[4]{7} + \frac{3}{8}\sqrt[4]{7}$
36. $6\sqrt[3]{5} + 4\sqrt[3]{625}$
37. $-6\sqrt[7]{2} + 2\sqrt[7]{256}$
38. $12\sqrt[4]{2} - 7\sqrt[4]{512}$
39. $2\sqrt[4]{1250} - 8\sqrt[4]{32}$
40. $5\sqrt[3]{48} - \sqrt[3]{750}$

ERROR ANALYSIS *Describe* **and correct the error in simplifying the expression.**

41.
$$2\sqrt[3]{10} + 6\sqrt[3]{5} = (2+6)\sqrt[3]{15}$$
$$= 8\sqrt[3]{15} \quad \times$$

42.
$$\sqrt[3]{\frac{x}{y^2}} = \sqrt[3]{\frac{x}{y^2 \cdot y}} = \sqrt[3]{\frac{x}{y^3}}$$
$$= \frac{\sqrt[3]{x}}{y} \quad \times$$

EXAMPLE 6
on p. 423
for Exs. 43–51

VARIABLE EXPRESSIONS Simplify the expression. Assume all variables are positive.

43. $x^{1/4} \cdot x^{1/3}$

44. $(y^4)^{1/6}$

45. $\sqrt[4]{81x^4}$

46. $\dfrac{2}{x^{-3/2}}$

47. $\dfrac{x^{2/5}y}{xy^{-1/3}}$

48. $\sqrt[3]{\dfrac{x^{15}}{y^6}}$

49. $\left(\sqrt[3]{x^2} \cdot \sqrt[6]{x^4}\right)^{-3}$

50. $\dfrac{\sqrt[3]{x} \cdot \sqrt{x^5}}{\sqrt{25x^{16}}}$

51. ★ **OPEN-ENDED MATH** Write two variable expressions with noninteger exponents whose quotient is $x^{3/4}$.

EXAMPLE 7
on p. 423
for Exs. 52–59

SIMPLEST FORM Write the expression in simplest form. Assume all variables are positive.

52. $\sqrt{49x^5}$

53. $\sqrt[4]{12x^2y^6z^{12}}$

54. $\sqrt[3]{4x^3y^5} \cdot \sqrt[3]{12y^2}$

55. $\sqrt{x^2yz^3} \cdot \sqrt{x^3z^5}$

56. $\dfrac{-3}{\sqrt[5]{x^6}}$

57. $\sqrt[3]{\dfrac{x^3}{y^4}}$

58. $\sqrt{\dfrac{20x^3y^2}{9xz^3}}$

59. $\dfrac{\sqrt[4]{x^6}}{\sqrt[7]{x^5}}$

EXAMPLE 8
on p. 423
for Exs. 60–65

COMBINING VARIABLE EXPRESSIONS Perform the indicated operation. Assume all variables are positive.

60. $3\sqrt[5]{x} + 9\sqrt[5]{x}$

61. $\frac{3}{4}y^{3/2} - \frac{1}{4}y^{3/2}$

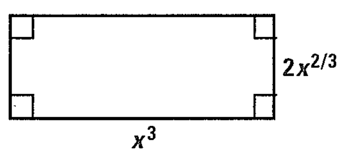
62. $-7\sqrt[3]{y} + 16\sqrt[3]{y}$

63. $(x^4y)^{1/2} + (xy^{1/4})^2$

64. $x\sqrt{9x^3} - 2\sqrt{x^5}$

65. $y\sqrt[4]{32x^6} + \sqrt[4]{162x^2y^4}$

GEOMETRY Find simplified expressions for the perimeter and area of the given figure.

66.


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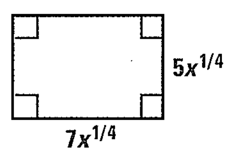

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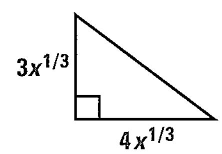

69. ★ **MULTIPLE CHOICE** What is the simplified form of $-\frac{1}{6}\sqrt{4x} - \frac{1}{6}\sqrt{9x}$?

(A) $-\frac{1}{3}\sqrt{x}$ (B) $-\frac{1}{3}\sqrt{36x}$ (C) $-\frac{5}{6}\sqrt{x}$ (D) $-\frac{5}{6}\sqrt{36x}$

DECIMAL EXPONENTS Simplify the expression. Assume all variables are positive.

70. $x^{0.5} \cdot x^2$

71. $y^{-0.6} \cdot y^{-6}$

72. $(x^6y^2)^{-0.75}$

73. $\dfrac{x^{0.3}}{x^{1.5}}$

74. $(x^5y^{-3})^{-0.25}$

75. $\dfrac{y^{-0.5}}{y^{0.8}}$

76. $10x^{0.6} + (4x^{0.3})^2$

77. $15z^{0.3} - (2z^{0.1})^3$

IRRATIONAL EXPONENTS The properties in this lesson can also be applied to irrational exponents. Simplify the expression. Assume all variables are positive.

78. $\dfrac{x^{5\sqrt{3}}}{x^{2\sqrt{3}}}$

79. $(x^{\sqrt{2}})^{\sqrt{3}}$

80. $\left(\dfrac{x^{\pi}}{x^{\pi/3}}\right)^2$

81. $x^2y^{\sqrt{2}} + 3x^2y^{\sqrt{2}}$

82. **CHALLENGE** Solve the equation using the properties of rational exponents.

a. $\dfrac{3}{9^x} = 243$

b. $2^x \cdot 2^{x+1} = \dfrac{1}{16}$

c. $(4^x)^{x+2} = 64$

PROBLEM SOLVING

EXAMPLE 2 on p. 421 for Exs. 83–84

83. **BIOLOGY** Look back at Example 2 on page 421. Use the model $S = km^{2/3}$ to approximate the surface area of the mammal given its mass.

    a. Bat: 32 grams

    b. Human: 59 kilograms

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84. **AIRPLANE VELOCITY** The velocity v (in feet per second) of a jet can be approximated by the model

$$v = 8.8\sqrt{\frac{L}{A}}$$

where A is the area of the wings (in square feet) and L is the lift (in Newtons). Find the velocity of a jet with a wing area of 5.5×10^3 square feet and a lift of 1.4×10^7 Newtons.

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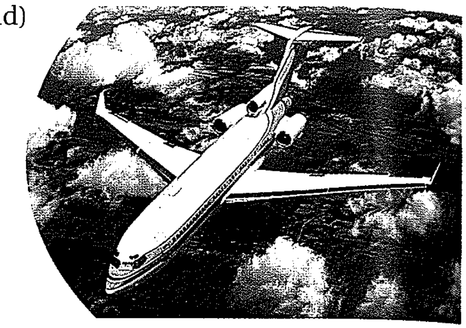


85. **PINHOLE CAMERA** The optimum diameter d (in millimeters) of the pinhole in a pinhole camera can be modeled by

$$d = 1.9[(5.5 \times 10^{-4})\ell]^{1/2}$$

where ℓ is the length of the camera box (in millimeters). Find the optimum pinhole diameter for a camera box with a length of 10 centimeters.

86. ★ **SHORT RESPONSE** Show that the hypotenuse of an isosceles right triangle with legs of length x is $x\sqrt{2}$.

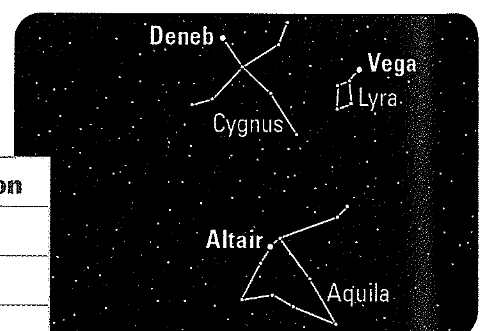
87. **STAR MAGNITUDE** The *apparent magnitude* of a star is a number that indicates how faint the star is in relation to other stars. The expression $\frac{2.512^{m_1}}{2.512^{m_2}}$ tells how many times fainter a star with magnitude m_1 is than a star with magnitude m_2.

    a. How many times fainter is Altair than Vega?

    b. How many times fainter is Deneb than Altair?

    c. How many times fainter is Deneb than Vega?

Star	Apparent magnitude	Constellation
Vega	0.03	Lyra
Altair	0.77	Aquila
Deneb	1.25	Cygnus



88. **PHYSICAL SCIENCE** The maximum horizontal distance d that an object can travel when launched at an optimum angle of projection is given by

$$d = \frac{v_0\sqrt{(v_0)^2 + 2gh_0}}{g}$$

where h_0 is the object's initial height, v_0 is its initial speed, and g is the acceleration due to gravity. Simplify the model when $h_0 = 0$.

○ = **WORKED-OUT SOLUTIONS on p. WS1**

★ = **STANDARDIZED TEST PRACTICE**

89. ★ **EXTENDED RESPONSE** You have filled two round balloons with water. One balloon contains twice as much water as the other balloon.

a. Solve the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$, for r.

b. Substitute the expression for r from part (a) into the formula for the surface area of a sphere, $S = 4\pi r^2$. Simplify to show that $S = (4\pi)^{1/3}(3V)^{2/3}$.

c. *Compare* the surface areas of the two water balloons using the formula from part (b).

90. **CHALLENGE** Substitute different combinations of odd and even positive integers for m and n in the expression $\sqrt[n]{x^m}$. If x is not always positive, when is absolute value needed in simplifying the expression?

MIXED REVIEW

Solve the inequality.

91. $x - 7 \geq 15$ *(p. 41)*
92. $10x + 7 < -4x + 9$ *(p. 41)*
93. $3x \leq -6x - 20$ *(p. 41)*
94. $x^2 + 7x + 10 > 0$ *(p. 300)*
95. $-x^2 + 4x \geq -32$ *(p. 300)*
96. $6x^2 + x - 7 < 5$ *(p. 300)*

Let $f(x) = x^3 - 2x^2 - x - 3$. Evaluate the function at the given value. *(p. 337)*

97. $f(3)$
98. $f(-3)$
99. $f(5)$
100. $f(-4)$

PREVIEW
Prepare for Lesson 6.3 in Exs. 101–106.

Perform the indicated operation.

101. $(12x^2 + 2x) - (-8x^3 + 5x^2 - 9x)$ *(p. 346)*
102. $(35x^3 - 14) + (-15x^2 + 7x + 20)$ *(p. 346)*
103. $18x^2(x + 4)$ *(p. 346)*
104. $(8x - 3)^2$ *(p. 346)*
105. $(x - 4)(x + 1)(x + 2)$ *(p. 346)*
106. $(x^3 + x^2 - 7x - 15) \div (x - 3)$ *(p. 362)*

QUIZ *for Lessons 6.1–6.2*

Evaluate the expression without using a calculator. *(p. 414)*

1. $36^{3/2}$
2. $64^{-2/3}$
3. $-(625^{3/4})$
4. $(-32)^{2/5}$

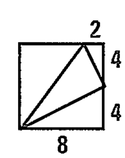
Solve the equation. Round your answer to two decimal places when appropriate. *(p. 414)*

5. $x^4 = 20$
6. $x^5 = -10$
7. $x^6 + 5 = 26$
8. $(x + 3)^3 = -16$

Simplify the expression. Assume all variables are positive. *(p. 420)*

9. $\sqrt[4]{32} \cdot \sqrt[4]{8}$
10. $(\sqrt{10} \cdot \sqrt[3]{10})^8$
11. $(x^6y^4)^{1/8} + 2(x^{1/3}y^{1/4})^2$
12. $\dfrac{3\sqrt{7^3} + 4\sqrt{7^3}}{\sqrt{7^5}}$
13. $\dfrac{2\sqrt{x} \cdot \sqrt{x^3}}{\sqrt{64x^{15}}}$
14. $y^2\sqrt[5]{64x^6} - 6\sqrt[5]{2x^6y^{10}}$

15. **GEOMETRY** Find a radical expression for the perimeter of the red triangle inscribed in the square shown to the right. Simplify the expression. *(p. 420)*



6.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 3, 13, and 45

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 38, 39, and 44

◆ = **MULTIPLE REPRESENTATIONS** Ex. 46

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The function $h(x) = g(f(x))$ is called the _?_ of the function g with the function f.

2. ★ **WRITING** Tell whether the sum of two power functions is *sometimes*, *always*, or *never* a power function. *Explain* your reasoning.

EXAMPLE 1 on p. 428 for Exs. 3–11

ADD AND SUBTRACT FUNCTIONS **Let $f(x) = -3x^{1/3} + 4x^{1/2}$ and $g(x) = 5x^{1/3} + 4x^{1/2}$. Perform the indicated operation and state the domain.**

3. $f(x) + g(x)$	4. $g(x) + f(x)$	5. $f(x) + f(x)$	6. $g(x) + g(x)$
7. $f(x) - g(x)$	8. $g(x) - f(x)$	9. $f(x) - f(x)$	10. $g(x) - g(x)$

11. ★ **MULTIPLE CHOICE** What is $f(x) + g(x)$ if $f(x) = -7x^{2/3} - 1$ and $g(x) = 2x^{2/3} + 6$?

(A) $5x^{2/3} - 5$　(B) $-5x^{2/3} + 5$　(C) $9x^{2/3} + 7$　(D) $-9x^{2/3} - 7$

EXAMPLE 2 on p. 429 for Exs. 12–19

MULTIPLY AND DIVIDE FUNCTIONS **Let $f(x) = 4x^{2/3}$ and $g(x) = 5x^{1/2}$. Perform the indicated operation and state the domain.**

12. $f(x) \cdot g(x)$	13. $g(x) \cdot f(x)$	14. $f(x) \cdot f(x)$	15. $g(x) \cdot g(x)$
16. $\frac{f(x)}{g(x)}$	17. $\frac{g(x)}{f(x)}$	18. $\frac{f(x)}{f(x)}$	19. $\frac{g(x)}{g(x)}$

EXAMPLE 4 on p. 430 for Exs. 20–27

EVALUATE COMPOSITIONS OF FUNCTIONS **Let $f(x) = 3x + 2$, $g(x) = -x^2$, and $h(x) = \frac{x-2}{5}$. Find the indicated value.**

20. $f(g(-3))$	21. $g(f(2))$	22. $h(f(-9))$	23. $g(h(8))$
24. $h(g(5))$	25. $f(f(7))$	26. $h(h(-4))$	27. $g(g(-5))$

EXAMPLE 5 on p. 430 for Exs. 28–38

FIND COMPOSITIONS OF FUNCTIONS **Let $f(x) = 3x^{-1}$, $g(x) = 2x - 7$, and $h(x) = \frac{x+4}{3}$. Perform the indicated operation and state the domain.**

28. $f(g(x))$	29. $g(f(x))$	30. $h(f(x))$	31. $g(h(x))$
32. $h(g(x))$	33. $f(f(x))$	34. $h(h(x))$	35. $g(g(x))$

ERROR ANALYSIS **Let $f(x) = x^2 - 3$ and $g(x) = 4x$. *Describe* and correct the error in the composition.**

36.
$$\begin{aligned} f(g(x)) &= f(4x) \\ &= (x^2 - 3)(4x) \\ &= 4x^3 - 12x \end{aligned}$$
✗

37.
$$\begin{aligned} g(f(x)) &= g(x^2 - 3) \\ &= 4x^2 - 3 \end{aligned}$$
✗

38. ★ **MULTIPLE CHOICE** What is $g(f(x))$ if $f(x) = 7x^2$ and $g(x) = 3x^{-2}$?

Ⓐ $\frac{3}{49x^4}$ Ⓑ 21 Ⓒ $21x^4$ Ⓓ $\frac{7}{9x^4}$

39. ★ **OPEN-ENDED MATH** Find two different functions f and g such that $f(g(x)) = g(f(x))$.

CHALLENGE Find functions f and g such that $f(g(x)) = h(x)$, $g(x) \neq x$, and $f(x) \neq x$.

40. $h(x) = \sqrt[3]{x + 2}$

41. $h(x) = \frac{4}{3x^2 + 7}$

42. $h(x) = |2x + 9|$

PROBLEM SOLVING

EXAMPLE 3
on p. 429
for Exs. 43, 46

43. **BIOLOGY** For a mammal that weighs w grams, the volume b (in milliliters) of air breathed in and the volume d (in milliliters) of "dead space" (the portion of the lungs not filled with air) can be modeled by:

$$b(w) = 0.007w \qquad d(w) = 0.002w$$

The breathing rate r (in breaths per minute) of a mammal that weighs w grams can be modeled by:

$$r(w) = \frac{1.1w^{0.734}}{b(w) - d(w)}$$

Simplify $r(w)$ and calculate the breathing rate for body weights of 6.5 grams, 300 grams, and 70,000 grams.

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EXAMPLE 6
on p. 431
for Exs. 44–45

44. ★ **SHORT RESPONSE** The cost (in dollars) of producing x sneakers in a factory is given by $C(x) = 60x + 750$. The number of sneakers produced in t hours is given by $x(t) = 50t$. Find $C(x(t))$. Evaluate $C(x(5))$ and explain what this number represents.

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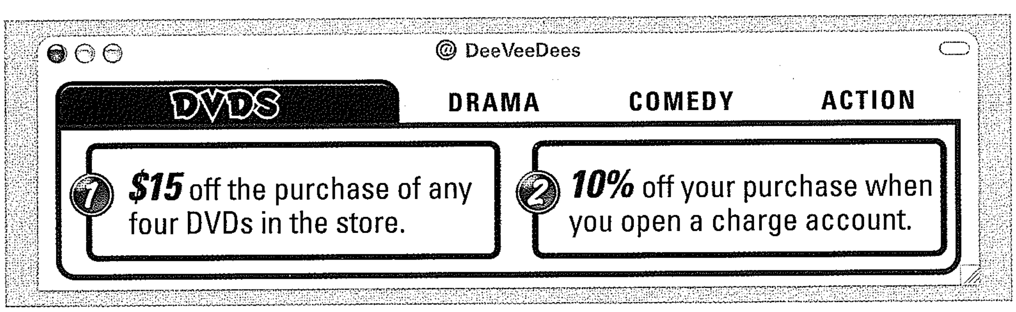
45. **MULTI-STEP PROBLEM** An online movie store is having a sale. You decide to open a charge account and buy four DVDs.

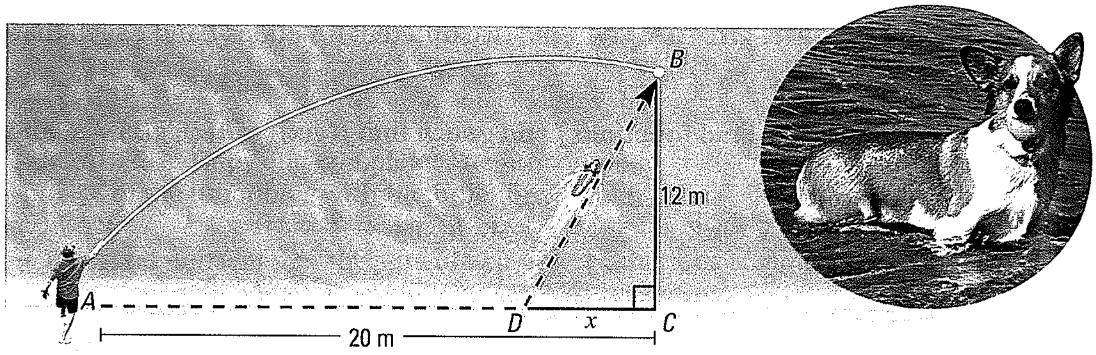


a. Use composition of functions to find the sale price of \$85 worth of DVDs when the \$15 discount is applied before the 10% discount.

b. Use composition of functions to find the sale price of \$85 worth of DVDs when the 10% discount is applied before the \$15 discount.

c. Which order of discounts gives you a better deal? *Explain.*

46. **MULTIPLE REPRESENTATIONS** A mathematician at a lake throws a tennis ball from point A along the water's edge to point B in the water, as shown. His dog, Elvis, first runs along the beach from point A to point D and then swims to fetch the ball at point B.



a. **Using a Diagram** Elvis's running speed is about 6.4 meters per second. Write a function $r(x)$ for the time he spends running from point A to point D. Elvis's swimming speed is about 0.9 meter per second. Write a function $s(x)$ for the time he spends swimming from point D to point B.

b. **Writing a Function** Write a function $t(x)$ that represents the total time Elvis spends traveling from point A to point D to point B.

c. **Using a Graph** Use a graphing calculator to graph $t(x)$. Find the value of x that minimizes $t(x)$. *Explain* the meaning of this value.

47. **CHALLENGE** To approximate the square root of a number n, the Babylonians used a method that involves starting with an initial guess x and calculating a sequence of values that approaches the exact answer. Their method was based on the function shown at the right.

$$f(x) = \frac{x + \frac{n}{x}}{2}$$

a. Let $n = 2$, and choose $x = 1$ as an initial guess for $\sqrt{n} = \sqrt{2}$. Calculate $f(x)$, $f(f(x))$, $f(f(f(x)))$, and $f(f(f(f(x))))$.

b. How many times do you need to compose the function in order for the result to approximate $\sqrt{2}$ to three decimal places? six decimal places?

MIXED REVIEW

PREVIEW
Prepare for Lesson 6.4 in Exs. 48–53.

Solve the equation for y. ***(p. 26)***

48. $y - 2x = 12$
49. $3x - 2y = 10$
50. $x = -3y + 9$
51. $3x - 4y = 7$
52. $x - y = 12$
53. $ax + by = c$

Graph the ordered pairs in a coordinate plane. ***(p. 72)***

54. $(-5, 2), (1, 3), (2, -5), (3, 1)$
55. $(4, 5), (5, -4), (-4, 5), (5, 4)$
56. $(-2, 2), (2, 1), (2, -2), (1, 2)$
57. $(5, 9), (9, -5), (-5, 9), (9, 5)$

Graph the function.

58. $y = 3x - 5$ ***(p. 89)***
59. $y = 7x + 4$ ***(p. 89)***
60. $f(x) = -4x - 6$ ***(p. 89)***
61. $f(x) = -3x + 9$ ***(p. 89)***
62. $y = x^2 - x - 2$ ***(p. 236)***
63. $y = 3x^2 + 20x - 7$ ***(p. 236)***
64. $y = -2x^2 + 8x + 1$ ***(p. 236)***
65. $f(x) = (x - 3)^2 - 4$ ***(p. 245)***
66. $y = (x + 4)^2 - 6$ ***(p. 245)***

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Keystrokes

6.3 Use Operations with Functions

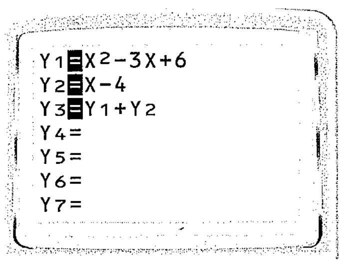
QUESTION **How can you use a graphing calculator to perform operations with functions?**

EXAMPLE **Perform function operations**

Let $f(x) = x^2 - 3x + 6$ and $g(x) = x - 4$. Find $f(4) + g(4)$ and $f(g(-2))$.

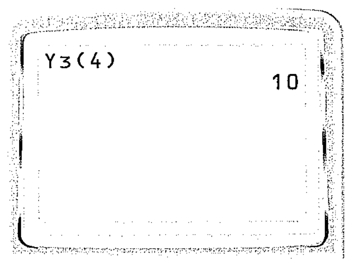
STEP 1 ***Form sum***

Enter $y_1 = x^2 - 3x + 6$ and $y_2 = x - 4$. The sum can be entered as $y_3 = y_1 + y_2$. To do so, press VARS, choose the Y-Vars menu, and select Function.



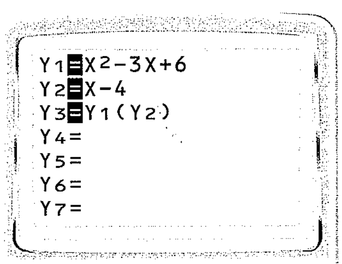
STEP 2 ***Evaluate sum***

On the home screen, enter $y_3(4)$ and press ENTER. The screen shows that $y_3(4) = 10$, so $f(4) + g(4) = 10$.



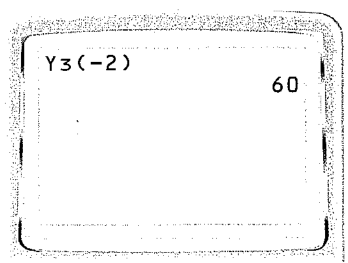
STEP 3 ***Form composition***

The composition $f(g(x))$ can be entered as $y_3 = y_1(y_2)$.



STEP 4 ***Evaluate composition***

On the home screen, enter $y_3(-2)$ and press ENTER. The screen shows that $y_3(-2) = 60$, so $f(g(-2)) = 60$.



PRACTICE

Use a graphing calculator and the functions f and g to find the indicated value.

1. $f(x) = x^3 + 5x - 3$, $g(x) = -3x^2 - x$: $g(7) + f(7)$
2. $f(x) = x^{1/3}$, $g(x) = 9x$: $\dfrac{f(-8)}{g(-8)}$
3. $f(x) = 5x^3 - 3x^2$, $g(x) = -2x^2 - 5$: $g(2) - f(2)$
4. $f(x) = 2x^2 + 7x - 2$, $g(x) = x - 6$: $f(g(5))$

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
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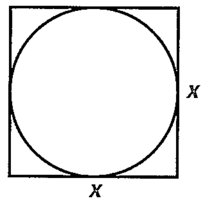
Lessons 6.1–6.3

1. **MULTI-STEP PROBLEM** A circle is inscribed in a square, as shown.



   a. Write a function $s(x)$ for the area of the square.

   b. Write a function $c(x)$ for the area of the circle.

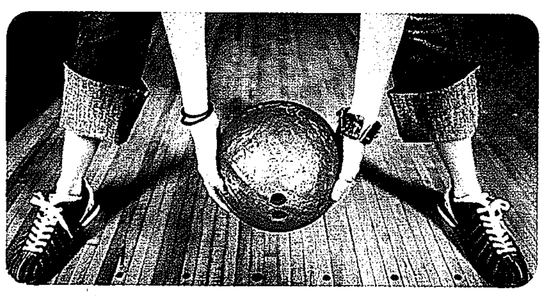
   c. Write and simplify a function $r(x)$ for the area of the shaded region.

2. **MULTI-STEP PROBLEM** The formula for the volume V of a sphere in terms of its surface area S is $V = 3^{-1}(4\pi)^{-1/2}(S^3)^{1/2}$.

   a. Simplify the right side of the formula.

   b. A candlepin bowling ball has a surface area of about 79 square inches. What is its volume?

   c. A 10-pin bowling ball has a surface area of about 232 square inches. What is its volume?



Ten-pin bowling ball

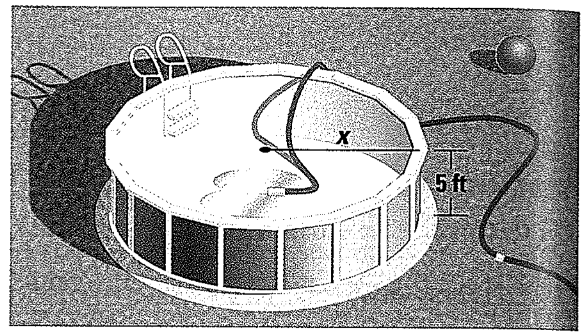
   d. *Compare* the surface areas and volumes of the two bowling balls.

3. **SHORT RESPONSE** You are working as a sales representative for a clothing manufacturer. You are paid an annual salary plus a bonus of 3% of your sales over \$100,000. Consider these two functions:

$$f(x) = x - 100{,}000 \qquad g(x) = 0.03x$$

Which composition, $f(g(x))$ or $g(f(x))$, represents your bonus if $x > 100{,}000$? *Explain.*

4. **EXTENDED RESPONSE** A cylindrical above-ground pool has a height of 5 feet and a radius of x feet.



   a. Write an equation that gives the volume V of the pool as a function of the radius x. Use 3.14 for π.

   b. You use a hose to fill the pool with water. Water flows from the hose at a rate of 128 cubic feet per hour. After 8.8 hours the pool is half full. Write an equation that you can use to find the radius x of the pool.

   c. What is the radius of the pool?

   d. A second hose that outputs 104 cubic feet of water per hour is added after the pool is half full. Find the total number of hours it will take to fill $\frac{4}{5}$ of the pool. Your answer should include the 8.8 hours it took to fill the bottom half of the pool in part (b).

5. **OPEN-ENDED** Find two different functions $f(x)$ such that $f(f(x)) = x$.

6. **SHORT RESPONSE** *Describe* the steps you would use to simplify this expression:

$$\left(\frac{16^{1/2}}{4^{1/2}}\right)^5$$

Is there another set of steps you could use to simplify the expression? *Explain* your reasoning.

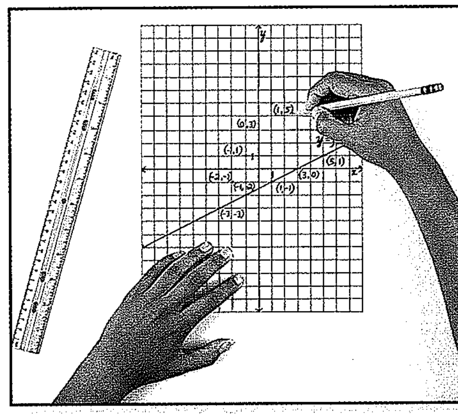
7. **GRIDDED ANSWER** The volume of a sphere is 900 cubic inches. Use the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$, to find the radius r to the nearest tenth of an inch. Use 3.14 for π.

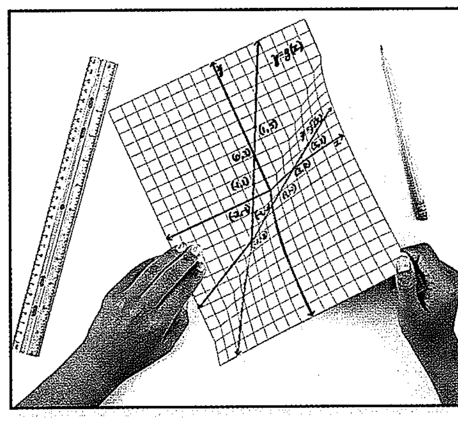
6.4 Exploring Inverse Functions

MATERIALS • graph paper • straightedge

QUESTION **How are a function and its *inverse* related?**

EXPLORE **Find the inverse of $f(x) = \frac{x-3}{2}$**





STEP 1 ***Graph function*** Choose values of x and find the corresponding values of $y = f(x)$. Plot the points and draw the line that passes through them.

STEP 2 ***Interchange coordinates*** Interchange the x- and y-coordinates of the ordered pairs found in Step 1. Plot the new points and draw the line that passes through them.

STEP 3 ***Write equation*** Write an equation of the line from Step 2. Call this function g.

STEP 4 ***Compare graphs*** Fold your graph paper so that the graphs of f and g coincide. How are the graphs geometrically related?

STEP 5 ***Describe functions*** In words, f is the function that subtracts 3 from x and then divides the result by 2. Describe the function g in words.

STEP 6 ***Find compositions*** Predict what the compositions $f(g(x))$ and $g(f(x))$ will be. Confirm your predictions by finding $f(g(x))$ and $g(f(x))$.

The functions f and g are called *inverses* of each other.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

Complete Exercises 1–3 for each function below.

$f(x) = 3x + 2$ $\qquad$ $f(x) = \frac{x-1}{6}$ $\qquad$ $f(x) = 4 - \frac{3}{2}x$

1. Complete Steps 1–3 above to find the inverse of the function.
2. Complete Step 4. How can you graph the inverse of a function without first finding ordered pairs (x, y)?
3. Complete Steps 5 and 6. How can you test to see if the function you found in Exercise 1 is indeed the inverse of the original function?

EXAMPLE 7 Use an inverse power model to make a prediction

Use the inverse power model from Example 6 to predict the year when the average ticket price will reach $58.

Solution

$t = \left(\frac{P}{35}\right)^{5.2}$ **Write inverse power model.**

$= \left(\frac{58}{35}\right)^{5.2}$ **Substitute 58 for *P*.**

≈ 14 **Use a calculator.**

▶ You can predict that the average ticket price will reach $58 about 14 years after 1995, or in 2009.

✓ **GUIDED PRACTICE** for Examples 6 and 7

11. **TICKET PRICES** The average price P (in dollars) for a Major League Baseball ticket can be modeled by $P = 10.7t^{0.272}$ where t is the number of years since 1995. Write the inverse model. Then use the inverse to predict the year when the average ticket price will reach $25.

6.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 7, 15, and 49

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, 21, 28, and 48

SKILL PRACTICE

1. **VOCABULARY** State the definition of an inverse relation.

2. ★ **WRITING** *Explain* how to determine whether a function g is an inverse of f.

EXAMPLE 1 on p. 438 for Exs. 3–13

INVERSE RELATIONS Find an equation for the inverse relation.

3. $y = 4x - 1$
4. $y = -2x + 5$
5. $y = 7x - 6$
6. $y = 10x - 28$
7. $y = 12x + 7$
8. $y = -18x - 5$
9. $y = 5x + \frac{1}{3}$
10. $y = -\frac{2}{3}x + 2$
11. $y = -\frac{3}{5}x + \frac{7}{5}$

ERROR ANALYSIS ***Describe*** **and correct the error in finding the inverse of the relation.**

12.

$y = 6x - 11$

$x = 6y - 11$

$x + 11 = 6y$

$\frac{x}{6} + 11 = y$

✗

13.

$y = -x + 3$

$-x = y + 3$

$-x - 3 = y$

✗

14. ★ **OPEN-ENDED MATH** Write a function f such that the graph of f^{-1} is a line with a slope of 3.

EXAMPLE 2
on p. 439
for Exs. 15–21

VERIFYING INVERSE FUNCTIONS Verify that f and g are inverse functions.

15. $f(x) = x + 4, g(x) = x - 4$

16. $f(x) = 2x + 3, g(x) = \frac{1}{2}x - \frac{3}{2}$

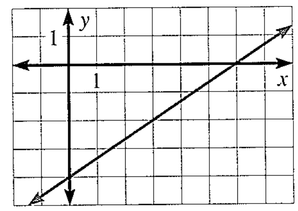
17. $f(x) = \frac{1}{4}x^3, g(x) = (4x)^{1/3}$

18. $f(x) = \frac{1}{5}x - 1, g(x) = 5x + 5$

19. $f(x) = 4x + 9, g(x) = \frac{1}{4}x - \frac{9}{4}$

20. $f(x) = 5x^2 - 2, x \geq 0; g(x) = \left(\frac{x+2}{5}\right)^{1/2}$

21. ★ **MULTIPLE CHOICE** What is the inverse of the function whose graph is shown?



Ⓐ $g(x) = \frac{3}{2}x - 6$ Ⓑ $g(x) = \frac{3}{2}x + 6$

Ⓒ $g(x) = \frac{2}{3}x - 6$ Ⓓ $g(x) = \frac{3}{2}x + 12$

EXAMPLE 4
on p. 440
for Exs. 22–28

INVERSES OF POWER FUNCTIONS Find the inverse of the power function.

22. $f(x) = x^7$

23. $f(x) = 4x^4, x \geq 0$

24. $f(x) = -10x^6, x \leq 0$

25. $f(x) = 32x^5$

26. $f(x) = -\frac{2}{5}x^3$

27. $f(x) = \frac{16}{25}x^2, x \leq 0$

28. ★ **MULTIPLE CHOICE** What is the inverse of $f(x) = -\frac{1}{64}x^3$?

Ⓐ $g(x) = -4x^3$ Ⓑ $g(x) = 4\sqrt[3]{x}$ Ⓒ $g(x) = -4\sqrt[3]{x}$ Ⓓ $g(x) = \sqrt[3]{-4x}$

EXAMPLE 5
on p. 441
for Exs. 29–43

HORIZONTAL LINE TEST Graph the function f. Then use the graph to determine whether the inverse of f is a function.

29. $f(x) = 3x + 1$

30. $f(x) = -x - 5$

31. $f(x) = \frac{1}{4}x^2 - 1$

32. $f(x) = -6x^2, x \geq 0$

33. $f(x) = \frac{1}{3}x^3$

34. $f(x) = x^3 - 2$

35. $f(x) = (x - 4)(x + 1)$

36. $f(x) = |x| + 4$

37. $f(x) = 4x^4 - 5x^2 - 6$

INVERSES OF NONLINEAR FUNCTIONS Find the inverse of the function.

38. $f(x) = \frac{3}{2}x^4, x \geq 0$

39. $f(x) = x^3 - 2$

40. $f(x) = \frac{3}{4}x^5 + 5$

41. $f(x) = -\frac{2}{5}x^6 + 8, x \leq 0$

42. $f(x) = \frac{2x^3 - 6}{9}$

43. $f(x) = x^4 - 9, x \geq 0$

44. **REASONING** Determine whether the statement is *true* or *false*. *Explain* your reasoning.

    a. If $f(x) = x^n$ where n is a positive even integer, then the inverse of f is a function.

    b. If $f(x) = x^n$ where n is a positive odd integer, then the inverse of f is a function.

45. **CHALLENGE** Show that the inverse of any linear function $f(x) = mx + b$, where $m \neq 0$, is also a linear function. Give the slope and y-intercept of the graph of f^{-1} in terms of m and b.

PROBLEM SOLVING

EXAMPLE 3
on p. 439
for Exs. 46–48

46. EXCHANGE RATES The *euro* is the unit of currency for the European Union. On a certain day, the number E of euros that could be obtained for D dollars was given by this function:

$$E = 0.81419D$$

Find the inverse of the function. Then use the inverse to find the number of dollars that could be obtained for 250 euros on that day.

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47. MULTI-STEP PROBLEM When calibrating a spring scale, you need to know how far the spring stretches for various weights. Hooke's law states that the length a spring stretches is proportional to the weight attached to it. A model for one scale is $\ell = 0.5w + 3$ where ℓ is the total length (in inches) of the stretched spring and w is the weight (in pounds) of the object.

a. Find the inverse of the given model.

b. If you place a weight on the scale and the spring stretches to a total length of 6.5 inches, how heavy is the weight?

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48. ★ EXTENDED RESPONSE At the start of a dog sled race in Anchorage, Alaska, the temperature was 5°C. By the end of the race, the temperature was −10°C. The formula for converting temperatures from degrees Fahrenheit F to degrees Celsius C is $C = \frac{5}{9}(F - 32)$.

a. Find the inverse of the given model. *Describe* what information you can obtain from the inverse.

b. Find the Fahrenheit temperatures at the start and end of the race.

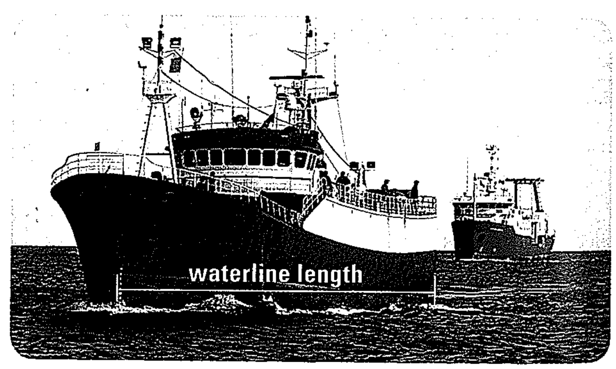
c. Use a graphing calculator to graph the original function and its inverse. Find the temperature that is the same on both temperature scales.

EXAMPLES 6 and 7
on pp. 441–442
for Exs. 49–50

(49.) BOAT SPEED The maximum hull speed v (in knots) of a boat with a displacement hull can be approximated by

$$v = 1.34\sqrt{\ell}$$

where ℓ is the length (in feet) of the boat's waterline. Find the inverse of the model. Then find the waterline length needed to achieve a maximum speed of 7.5 knots.



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50. BIOLOGY The body surface area A (in square meters) of a person with a mass of 60 kilograms can be approximated by the model

$$A = 0.2195h^{0.3964}$$

where h is the person's height (in centimeters). Find the inverse of the model. Then estimate the height of a 60 kilogram person who has a body surface area of 1.6 square meters.

○ = WORKED-OUT SOLUTIONS on p. WS1   ★ = STANDARDIZED TEST PRACTICE

51. **CHALLENGE** Consider the function $g(x) = -x$.

a. Graph $g(x) = -x$ and explain why it is its own inverse. Also verify that $g(x) = g^{-1}(x)$ algebraically.

b. Graph other linear functions that are their own inverses. Write equations of the lines you graphed.

c. Use your results from part (b) to write a general equation describing the family of linear functions that are their own inverses.

MIXED REVIEW

PREVIEW Prepare for Lesson 6.5 in Exs. 52–57.

Graph the function.

52. $y = |x| - 2$ *(p. 123)*
53. $y = |x + 1| + 3$ *(p. 123)*
54. $y = -3|x|$ *(p. 123)*
55. $y = 3(x - 4)(x + 2)$ *(p. 245)*
56. $y = -2(x + 3)^2 - 4$ *(p. 245)*
57. $y = \frac{3}{2}(x - 1)^2 + 7$ *(p. 245)*

Solve the system of linear equations. *(p. 160)*

58. $3x - 4y = 24$
$x + 2y = 2$

59. $2x - 4y = 13$
$4x - 5y = 8$

60. $7x - 12y = -22$
$25x + 8y = 14$

Solve the equation. *(p. 353)*

61. $6x^3 - 54x = 0$
62. $2x^3 - 8x^2 + 8x = 0$
63. $16x^3 = -250$
64. $x^3 - 3x^2 + 8x = 24$
65. $4x^3 + 8x^2 - 25x - 50 = 0$
66. $12x^4 - 7x^2 - 45 = 0$

QUIZ *for Lessons 6.3–6.4*

Let $f(x) = 4x^2 - x$ and $g(x) = 2x^2$. Perform the indicated operation and state the domain. *(p. 428)*

1. $f(x) + g(x)$
2. $g(x) - f(x)$
3. $f(x) \cdot g(x)$
4. $\frac{f(x)}{g(x)}$
5. $f(g(x))$
6. $g(f(x))$
7. $f(f(x))$
8. $g(g(x))$

Verify that f and g are inverse functions. *(p. 438)*

9. $f(x) = x - 9, g(x) = x + 9$
10. $f(x) = 5x^3, g(x) = \sqrt[3]{\frac{x}{5}}$
11. $f(x) = -\frac{3}{2}x + \frac{1}{4}, g(x) = -\frac{2}{3}x + \frac{1}{6}$
12. $f(x) = 6x^2 + 1, x \geq 0; g(x) = \left(\frac{x-1}{6}\right)^{1/2}$

Find the inverse of the function. *(p. 438)*

13. $f(x) = -\frac{1}{3}x + 5$
14. $f(x) = x^2 - 16, x \geq 0$
15. $f(x) = -\frac{2}{9}x^5$
16. $f(x) = 5x + 12$
17. $f(x) = -3x^3 - 4$
18. $f(x) = 9x^4 - 49, x \leq 0$

19. **GASOLINE COSTS** The cost (in dollars) of g gallons of gasoline can be modeled by $C(g) = 2.15g$. The amount of gasoline used by a car can be modeled by $g(d) = 0.02d$ where d is the distance (in miles) that the car has been driven. Find $C(g(d))$ and $C(g(400))$. What does $C(g(400))$ represent? *(p. 428)*

✓ GUIDED PRACTICE for Examples 4 and 5

Graph the function. Then state the domain and range.

6. $y = -4\sqrt{x} + 2$ **7.** $y = 2\sqrt{x+1}$ **8.** $f(x) = \frac{1}{2}\sqrt{x-3} - 1$

9. $y = 2\sqrt[3]{x-4}$ **10.** $y = \sqrt[3]{x} - 5$ **11.** $g(x) = -\sqrt[3]{x+2} - 3$

6.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 11, 17, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 25, 27, and 37

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

SKILL PRACTICE

1. VOCABULARY Copy and complete: Square root functions and cube root functions are examples of __?__ functions.

2. ★ WRITING The graph of $y = \sqrt{x}$ is the graph of $y = a\sqrt{x-h} + k$ with $a = 1$, $h = 0$, and $k = 0$. Predict how the graph of $y = \sqrt{x}$ will change if:

a. $a = -3$ **b.** $h = 2$ **c.** $k = 4$

EXAMPLE 1 on p. 446 for Exs. 3–9

SQUARE ROOT FUNCTIONS Graph the function. Then state the domain and range.

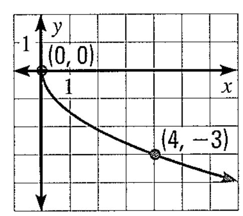
3. $y = -4\sqrt{x}$ **4.** $f(x) = \frac{1}{2}\sqrt{x}$ **5.** $y = -\frac{4}{5}\sqrt{x}$

6. $y = -6\sqrt{x}$ **7.** $y = 5\sqrt{x}$ **8.** $g(x) = 9\sqrt{x}$

9. ★ MULTIPLE CHOICE The graph of which function is shown?

Ⓐ $y = \frac{3}{4}\sqrt{x}$ Ⓑ $y = -\frac{3}{4}\sqrt{x}$

Ⓒ $y = \frac{3}{2}\sqrt{x}$ Ⓓ $y = -\frac{3}{2}\sqrt{x}$



EXAMPLE 2 on p. 447 for Exs. 10–15

CUBE ROOT FUNCTIONS Graph the function. Then state the domain and range.

10. $y = \frac{1}{4}\sqrt[3]{x}$ **(11.)** $y = 2\sqrt[3]{x}$ **12.** $f(x) = -5\sqrt[3]{x}$

13. $h(x) = -\frac{1}{7}\sqrt[3]{x}$ **14.** $g(x) = 6\sqrt[3]{x}$ **15.** $y = \frac{7}{9}\sqrt[3]{x}$

EXAMPLES 4 and 5 on p. 448 for Exs. 16–24

RADICAL FUNCTIONS Graph the function. Then state the domain and range.

16. $f(x) = 2\sqrt{x-1} + 3$ **(17.)** $y = (x+1)^{1/2} + 8$ **18.** $y = -4\sqrt{x-5} + 1$

19. $y = \frac{3}{4}x^{1/3} - 1$ **20.** $y = -2\sqrt[3]{x+5} + 5$ **21.** $h(x) = -3\sqrt[3]{x+7} - 6$

22. $y = -\sqrt{x-4} - 7$ **23.** $g(x) = -\frac{1}{3}\sqrt[3]{x} - 6$ **24.** $y = 4\sqrt[3]{x-4} + 5$

25. ★ SHORT RESPONSE *Explain* why there are limitations on the domain and range of the function $y = \sqrt{x-5} + 4$.

26. **ERROR ANALYSIS** A student tried to explain how the graphs of $y = -2\sqrt[3]{x}$ and $y = -2\sqrt[3]{x+1} - 3$ are related. *Describe* and correct the error.

> The graph of $y = -2\sqrt[3]{x+1} - 3$ is the graph of $y = -2\sqrt[3]{x}$ translated right 1 unit and down 3 units.

27. ★ **MULTIPLE CHOICE** If the graph of $y = 3\sqrt[3]{x}$ is shifted left 2 units, what is the equation of the translated graph?

Ⓐ $y = 3\sqrt[3]{x-2}$ Ⓑ $y = 3\sqrt[3]{x} - 2$ Ⓒ $y = 3\sqrt[3]{x+2}$ Ⓓ $y = 3\sqrt[3]{x} + 2$

REASONING Find the domain and range of the function without graphing. *Explain* how you found your answers.

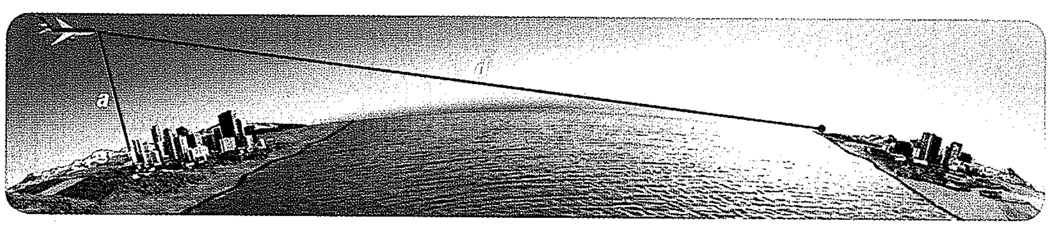
28. $y = \sqrt{x+5}$
29. $y = \sqrt{x-12}$
30. $y = \frac{1}{3}\sqrt{x} - 4$
31. $y = \frac{1}{2}\sqrt[3]{x+7}$
32. $g(x) = \sqrt[3]{x+7}$
33. $f(x) = \frac{1}{4}\sqrt{x-3} + 6$

34. **CHALLENGE** Graph $y = \sqrt[4]{x}$, $y = \sqrt[5]{x}$, $y = \sqrt[6]{x}$, and $y = \sqrt[7]{x}$ on a graphing calculator. Make generalizations about the graph of $y = \sqrt[n]{x}$ when n is even and when n is odd.

PROBLEM SOLVING

EXAMPLE 3 on p. 447 for Exs. 35–36

35. **INDIRECT MEASUREMENT** The distance d (in miles) that a pilot can see to the horizon can be modeled by $d = 1.22\sqrt{a}$ where a is the plane's altitude (in feet above sea level). Graph the model on a graphing calculator. Then determine at what altitude the pilot can see 8 miles.



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36. **PENDULUMS** Use the model $T = 1.11\sqrt{\ell}$ for the period of a pendulum from Example 3 on page 447.
    a. Find the period of a pendulum with a length of 2 feet.
    b. Find the length of a pendulum with a period of 2 seconds.

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37. ★ **SHORT RESPONSE** The speed v (in meters per second) of sound waves in air depends on the temperature K (in kelvins) and can be modeled by:

$$v = 331.5\sqrt{\frac{K}{273.15}},\ K \geq 0$$

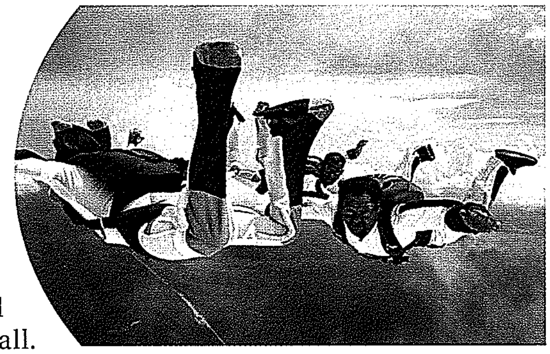
a. Kelvin temperature K is related to Celsius temperature C by the formula $K = 273.15 + C$. Write an equation that gives the speed v of sound waves in air as a function of the temperature C in degrees Celsius.
b. What are a reasonable domain and range for the function from part (a)?

38. DRAG RACING For a given total weight, the speed of a car at the end of a drag race is a function of the car's power. For a car with a total weight of 3500 pounds, the speed s (in miles per hour) can be modeled by $s = 14.8\sqrt[3]{p}$ where p is the power (in horsepower). Graph the model. Then determine the power of a 3500 pound car that reaches a speed of 200 miles per hour.

39. MULTIPLE REPRESENTATIONS Under certain conditions, a skydiver's terminal velocity v_t (in feet per second) is given by

$$v_t = 33.7\sqrt{\frac{W}{A}}$$

where W is the weight of the skydiver (in pounds) and A is the skydiver's cross-sectional surface area (in square feet). Note that skydivers can vary their cross-sectional surface area by changing positions as they fall.



a. Writing an Equation Write an equation that gives v_t as a function of A for a skydiver who weighs 165 pounds.

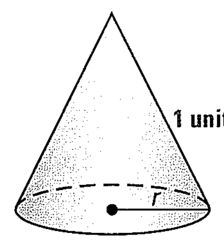
b. Making a Table Make a table of values for the equation from part (a).

c. Drawing a Graph Use your table to graph the equation.

40. CHALLENGE The surface area S of a right circular cone with a slant height of 1 unit is given by $S = \pi r + \pi r^2$ where r is the cone's radius.

a. Use completing the square to show the following:

$$r = \frac{1}{\sqrt{\pi}}\sqrt{S + \frac{\pi}{4}} - \frac{1}{2}$$



b. Graph the equation from part (a) using a graphing calculator.

c. Find the radius of a right circular cone with a slant height of 1 unit and a surface area of $\frac{3\pi}{4}$ square units.

MIXED REVIEW

PREVIEW Prepare for Lesson 6.6 in Exs. 41–46.

Solve the equation.

41. $\frac{1}{5}(x + 8)^2 = 3$ *(p. 266)*

42. $9(x - 3)^2 + 22 = 130$ *(p. 266)*

43. $7x^2 - 20 = 36$ *(p. 266)*

44. $x^2 - 14x + 37 = 0$ *(p. 284)*

45. $x^2 - 40x + 383 = 0$ *(p. 284)*

46. $x^2 - 22x + 97 = 0$ *(p. 284)*

Find all the zeros of the polynomial function. *(p. 379)*

47. $f(x) = x^4 + 5x^3 - x^2 - 5x$

48. $f(x) = x^4 - 3x^3 - 27x^2 - 13x + 42$

49. $f(x) = x^4 + x^3 + 2x^2 + 4x - 8$

50. $f(x) = x^4 + 6x^3 + 14x^2 + 54x + 45$

Let $f(x) = 5x^{2/3}$ and $g(x) = 3x^{1/2}$. Perform the indicated operation. *(p. 428)*

51. $\frac{f(x)}{g(x)}$

52. $f(x) \cdot f(x)$

53. $g(x) \cdot g(x)$

54. $f(x) \cdot g(x)$

55. $f(g(x))$

56. $g(f(x))$

57. $f(f(x))$

58. $g(g(x))$

6.6 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 5, 13, and 59

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 22, 43, 44, 59, and 60

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: When you solve an equation algebraically, an apparent solution that must be rejected because it does not satisfy the original equation is called a(n) __?__ solution.

2. ★ **WRITING** A student was asked to solve $\sqrt{3x-1} - \sqrt{9x-5} = 0$. His first step was to square each side. While trying to isolate x, he gave up in frustration. What could the student have done to avoid this situation?

EXAMPLE 1 on p. 452 for Exs. 3–21

EQUATIONS WITH SQUARE ROOTS **Solve the equation. Check your solution.**

3. $\sqrt{5x+1} = 6$
4. $\sqrt{3x+10} = 8$
5. $\sqrt{9x} + 11 = 14$
6. $\sqrt{2x} - \frac{2}{3} = 0$
7. $-2\sqrt{24x} + 13 = -11$
8. $8\sqrt{10x} - 7 = 9$
9. $\sqrt{x-25} + 3 = 5$
10. $-4\sqrt{x} - 6 = -20$
11. $\sqrt{-2x+3} - 2 = 10$

12. ★ **MULTIPLE CHOICE** What is the solution of $\sqrt{8x+3} = 3$?

Ⓐ $-\frac{3}{4}$ Ⓑ 0 Ⓒ $\frac{3}{4}$ Ⓓ $\frac{9}{8}$

EQUATIONS WITH CUBE ROOTS **Solve the equation. Check your solution.**

13. $\sqrt[3]{x} - 10 = -3$
14. $\sqrt[3]{x-16} = 2$
15. $\sqrt[3]{12x} - 13 = -7$
16. $3\sqrt[3]{16x} - 7 = 17$
17. $-5\sqrt[3]{8x} + 12 = -8$
18. $\sqrt[3]{4x+5} = \frac{1}{2}$
19. $\sqrt[3]{x-3} + 2 = 4$
20. $\sqrt[3]{4x+2} - 6 = -10$
21. $-4\sqrt[3]{x+10} + 3 = 15$

22. ★ **OPEN-ENDED MATH** Write a radical equation of the form $\sqrt[3]{ax+b} = c$ that has -3 as a solution. *Explain* the method you used to find your equation.

EXAMPLES 3 and 4 on pp. 453–454 for Exs. 23–33

EQUATIONS WITH RATIONAL EXPONENTS **Solve the equation. Check your solution.**

23. $2x^{3/2} = 16$
24. $\frac{1}{2}x^{5/2} = 16$
25. $9x^{3/5} = 72$
26. $(16x)^{3/4} + 44 = 556$
27. $\frac{1}{7}(x+9)^{3/2} = 49$
28. $(x-5)^{5/3} - 73 = 170$
29. $\left(\frac{1}{3}x - 11\right)^{1/2} = 5$
30. $(5x-19)^{5/6} = 32$
31. $(3x+5)^{7/3} + 22 = 150$

ERROR ANALYSIS ***Describe*** **and correct the error in solving the equation.**

32.
$$\sqrt[3]{x} + 2 = 4$$
$$(\sqrt[3]{x} + 2)^3 = 4^3$$
$$x + 8 = 64$$
$$x = 56$$
✗

33.
$$(x+7)^{1/2} = 5$$
$$[(x+7)^{1/2}]^2 = 5$$
$$x + 7 = 5$$
$$x = -2$$
✗

EXAMPLE 5
on p. 454
for Exs. 34–44

SOLVING RADICAL EQUATIONS **Solve the equation. Check for extraneous solutions.**

34. $x - 6 = \sqrt{3x}$

35. $x - 10 = \sqrt{9x}$

36. $x = \sqrt{16x + 225}$

37. $\sqrt{21x + 1} = x + 5$

38. $\sqrt{44 - 2x} = x - 10$

39. $\sqrt{x^2 + 4} = x + 5$

40. $x - 2 = \sqrt{\frac{3}{2}x - 2}$

41. $\sqrt[4]{3 - 8x^2} = 2x$

42. $\sqrt[3]{8x^3 - 1} = 2x - 1$

43. ★ **MULTIPLE CHOICE** What is (are) the solution(s) of $\sqrt{32x - 64} = 2x$?

Ⓐ 4 Ⓑ −16 Ⓒ 4, −16 Ⓓ 1, 3

44. ★ **SHORT RESPONSE** *Explain* how you can tell that $\sqrt{x + 4} = -5$ has no solution without solving it.

EXAMPLE 6
on p. 455
for Exs. 45–52

EQUATIONS WITH TWO RADICALS **Solve the equation. Check for extraneous solutions.**

45. $\sqrt{4x + 1} = \sqrt{x + 10}$

46. $\sqrt[3]{12x - 5} - \sqrt[3]{8x + 15} = 0$

47. $\sqrt{3x - 8} + 1 = \sqrt{x + 5}$

48. $\sqrt{\frac{2}{3}x - 4} = \sqrt{\frac{2}{5}x - 7}$

49. $\sqrt{x + 2} = 2 - \sqrt{x}$

50. $\sqrt{2x + 3} + 2 = \sqrt{6x + 7}$

51. $\sqrt{2x + 5} = \sqrt{x + 2} + 1$

52. $\sqrt{5x + 6} + 3 = \sqrt{3x + 3} + 4$

SOLVING SYSTEMS **Solve the system of equations.**

53. $3\sqrt{x} + 5\sqrt{y} = 31$
$5\sqrt{x} - 5\sqrt{y} = -15$

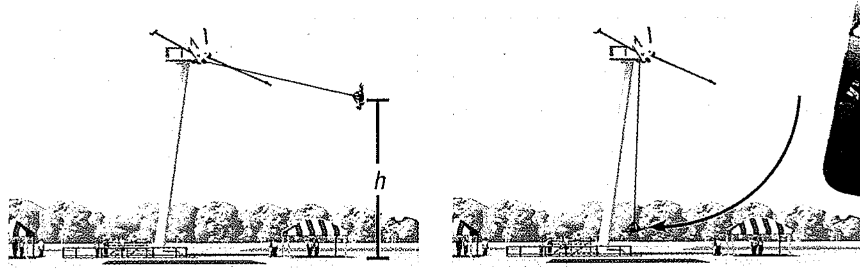
54. $5\sqrt{x} - 2\sqrt{y} = 4\sqrt{2}$
$2\sqrt{x} + 3\sqrt{y} = 13\sqrt{2}$

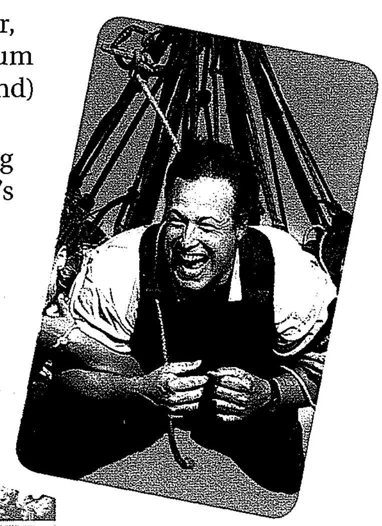
55. **CHALLENGE** Give an example of a radical equation that has two extraneous solutions.

PROBLEM SOLVING

EXAMPLE 2
on p. 453
for Exs. 56–57

56. **MAXIMUM SPEED** In an amusement park ride called the Sky Flyer, a rider suspended by a cable swings back and forth like a pendulum from a tall tower. A rider's maximum speed v (in meters per second) occurs at the bottom of each swing and can be approximated by $v = \sqrt{2gh}$ where h is the height (in meters) at the top of each swing and g is the acceleration due to gravity $(g \approx 9.8 \text{ m/sec}^2)$. If a rider's maximum speed was 15 meters per second, what was the rider's height at the top of the swing?





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57. **BURNING RATE** A burning candle has a radius of r inches and was initially h_0 inches tall. After t minutes, the height of the candle has been reduced to h inches. These quantities are related by the formula

$$r = \sqrt{\frac{kt}{\pi(h_0 - h)}}$$

where k is a constant. How long will it take for the entire candle to burn if its radius is 0.875 inch, its initial height is 6.5 inches, and $k = 0.04$?

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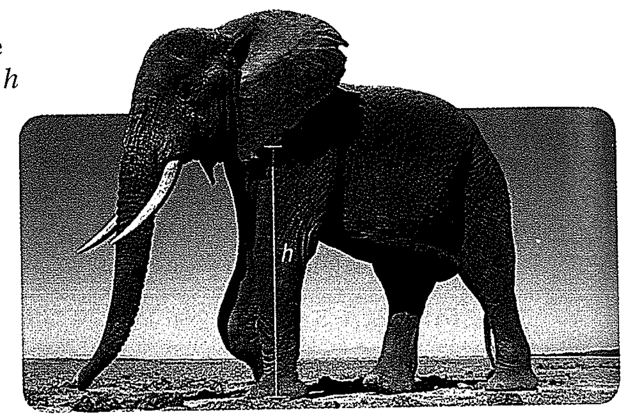
58. **CONSTRUCTION** The length ℓ (in inches) of a standard nail can be modeled by $\ell = 54d^{3/2}$ where d is the diameter (in inches) of the nail. What is the diameter of a standard nail that is 3 inches long?

59. ★ **SHORT RESPONSE** Biologists have discovered that the shoulder height h (in centimeters) of a male African elephant can be modeled by

$$h = 62.5\sqrt[3]{t} + 75.8$$

where t is the age (in years) of the elephant. *Compare* the ages of two elephants, one with a shoulder height of 150 centimeters and the other with a shoulder height of 250 centimeters.



60. ★ **EXTENDED RESPONSE** "Hang time" is the time you are suspended in the air during a jump. Your hang time t (in seconds) is given by the function $t = 0.5\sqrt{h}$ where h is the height of the jump (in feet). A basketball player jumps and has a hang time of 0.81 second. A kangaroo jumps and has a hang time of 1.12 seconds.

   a. **Solve** Find the heights that the basketball player and the kangaroo jumped.

   b. **Calculate** Double the hang times of the basketball player and the kangaroo and calculate the corresponding heights of each jump.

   c. **Interpret** If the hang time doubles, does the height of the jump double? *Explain.*

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61. **MULTI-STEP PROBLEM** The Beaufort wind scale was devised to measure wind speed. The Beaufort numbers B, which range from 0 to 12, can be modeled by

$$B = 1.69\sqrt{s + 4.25} - 3.55$$

where s is the speed (in miles per hour) of the wind.

   a. Find the wind speed that corresponds to the Beaufort number $B = 0$.

   b. Find the wind speed that corresponds to the Beaufort number $B = 12$.

   c. Write an inequality that describes the range of wind speeds represented by the Beaufort model.

Beaufort Wind Scale

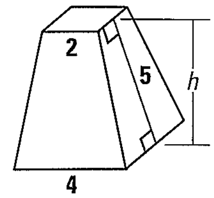
Beaufort number	Force of wind
0	Calm
3	Gentle breeze
6	Strong breeze
9	Strong gale
12	Hurricane

○ = **WORKED-OUT SOLUTIONS on p. WS1**

★ = **STANDARDIZED TEST PRACTICE**

62. **CHALLENGE** You are trying to determine a truncated pyramid's height, which cannot be measured directly. The height h and slant height ℓ of the truncated pyramid are related by the formula shown below.

$$\ell = \sqrt{h^2 + \frac{1}{4}(b_2 - b_1)^2}$$



In the given formula, b_1 and b_2 are the side lengths of the upper and lower bases of the pyramid, respectively. If $\ell = 5$, $b_1 = 2$, and $b_2 = 4$, what is the height of the pyramid?

MIXED REVIEW

PREVIEW
Prepare for Lesson 7.1 in Exs. 63–68.

Evaluate the expression. Tell which properties of exponents you used. *(p. 330)*

63. $4^3 \cdot 4^2$
64. $(3^{-2})^3$
65. $(-5)(-5)^{-4}$
66. $(10^{-3})^{-1}$
67. $8^{-4} \cdot 8^3$
68. $6^0 \cdot 6^4 \cdot 6^{-4}$

Graph the function. *(p. 337)*

69. $f(x) = -x^3$
70. $f(x) = x^4 - 9$
71. $f(x) = x^3 + 2$
72. $f(x) = x^4 - 8x^2 - 48$
73. $f(x) = -\frac{1}{3}x^3 + x$
74. $f(x) = x^5 - 2x - 4$

Evaluate the expression without using a calculator. *(p. 414)*

75. $16^{3/2}$
76. $\frac{1}{8^{-5/3}}$
77. $-256^{1/4}$
78. $4^{-5/2}$
79. $3125^{-3/5}$
80. $\frac{1}{27^{4/3}}$

QUIZ *for Lessons 6.5–6.6*

Graph the function. Then state the domain and range. *(p. 446)*

1. $y = 4\sqrt{x}$
2. $y = \sqrt{x} + 3$
3. $g(x) = \sqrt{x + 2} - 5$
4. $y = -\frac{1}{2}\sqrt[3]{x}$
5. $f(x) = \sqrt[3]{x} - 4$
6. $y = \sqrt[3]{x - 3} + 2$

Solve the equation. Check for extraneous solutions. *(p. 452)*

7. $\sqrt{6x + 15} = 9$
8. $\frac{1}{4}(7x + 8)^{3/2} = 54$
9. $\sqrt[3]{3x + 5} + 2 = 5$
10. $x - 3 = \sqrt{10x - 54}$
11. $\sqrt{4x - 4} = \sqrt{5x - 1} - 1$
12. $\sqrt[3]{\frac{4}{5}x - 9} = \sqrt[3]{x - 6}$

13. **ASTRONOMY** According to Kepler's third law of planetary motion, the function $P = 0.199a^{3/2}$ relates a planet's orbital period P (in days) to the length a (in millions of kilometers) of the orbit's minor axis. The orbital period of Mars is about 1.88 years. What is the length of the orbit's minor axis? *(p. 452)*

PROBLEM SOLVING WORKSHOP
LESSON 6.6

Using ALTERNATIVE METHODS

Another Way to Solve Example 2, page 453

MULTIPLE REPRESENTATIONS In Example 2 on page 453, you solved a radical equation algebraically. You can also solve a radical equation using a table or a graph.

PROBLEM

WIND VELOCITY In a hurricane, the mean sustained wind velocity v (in meters per second) is given by

$$v(p) = 6.3\sqrt{1013 - p}$$

where p is the air pressure (in millibars) at the center of the hurricane. Estimate the air pressure at the center of a hurricane when the mean sustained wind velocity is 54.5 meters per second.

METHOD 1

Using a Table The problem requires solving the radical equation $6.3\sqrt{1013 - p} = 54.5$. One way to solve this equation is to make a table of values. You can use a graphing calculator to make the table.

STEP 1 **Enter** the function $y = 6.3\sqrt{1013 - x}$ into a graphing calculator. Note that x represents air pressure and y represents wind velocity. Set up a table to display x-values starting at 900 and increasing in increments of 10.

```
Y1=6.3*√(1013-X)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

```
TABLE SETUP
  TblStart=900
  ΔTbl=10
Indpnt: Auto Ask
Depend: Auto Ask
```

STEP 2 **Make** a table of values for the function. The first table below shows that $y = 54.5$ between $x = 930$ and $x = 940$. To approximate x more precisely, set up the table to display x-values starting at 930 and increasing in increments of 1. The second table below shows that $y = 54.5$ between $x = 938$ and $x = 939$.

X	Y1
900	66.97
910	63.938
920	60.755
930	57.396
940	53.827

X=930

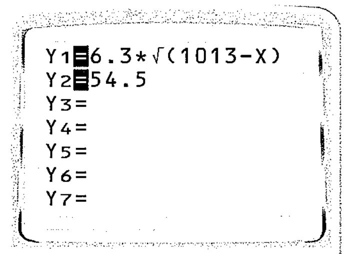
X	Y1
935	55.64
936	55.282
937	54.922
938	54.56
939	54.195

X=938

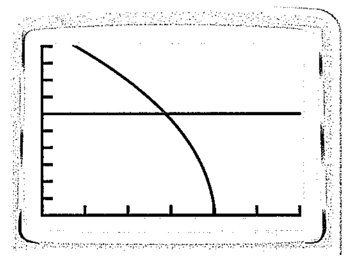
▶ The mean sustained wind velocity is 54.5 meters per second when the air pressure is between 938 and 939 millibars.

METHOD 2 **Using a Graph** You can also use a graph to solve the equation $6.3\sqrt{1013 - p} = 54.5$.

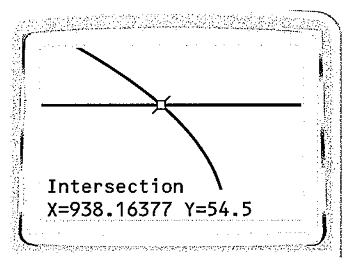
STEP 1 **Enter** the functions $y = 6.3\sqrt{1013 - x}$ and $y = 54.5$ into a graphing calculator.



STEP 2 **Graph** the functions from Step 1. Adjust the viewing window so that it shows the interval $800 \le x \le 1100$ with a scale of 50 and the interval $25 \le y \le 75$ with a scale of 5.



STEP 3 **Find** the intersection point of the two graphs using the *intersect* feature. The graphs intersect at about (938, 54.5).



▶ The mean sustained wind velocity is 54.5 meters per second when the air pressure is about 938 millibars.

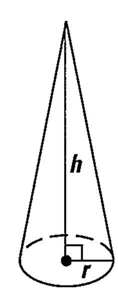
PRACTICE

SOLVING EQUATIONS **Solve the radical equation using a table and using a graph.**

1. $\sqrt{25 - x} = 8$
2. $2.3\sqrt{x - 1} = 11.5$
3. $4.3\sqrt{x - 7} = 30$
4. $6\sqrt{2 - 7x} - 1.2 = 22.8$
5. **ROCKETS** A model rocket is launched 25 feet from you. When the rocket is at height h, the distance d between you and the rocket is given by $d = \sqrt{625 + h^2}$ where h and d are measured in feet. What is the rocket's height when the distance between you and the rocket is 100 feet?
6. **WHAT IF?** In the problem on page 460, what is the air pressure at the center of a hurricane when the mean sustained wind velocity is 25 meters per second?
7. **GEOMETRY** The lateral surface area L of a right circular cone is given by

$$L = \pi r\sqrt{r^2 + h^2}$$

where r is the radius and h is the height. Find the height of a right circular cone with a radius of 7.5 centimeters and a lateral surface area of 900 square centimeters.



Extension

Use after Lesson 6.6

Solve Radical Inequalities

GOAL Solve radical inequalities by using tables and graphs.

In Chapter 4, you learned how to use tables and graphs to solve quadratic inequalities. You can also use tables and graphs to solve radical inequalities.

EXAMPLE 1 Solve a radical inequality using a table

Use a table to solve $3\sqrt{x} - 1 \le 11$.

Solution

STEP 1 **Enter** the function $y = 3\sqrt{x} - 1$ into a graphing calculator.

```
Y1=3*√(X)-1
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

STEP 2 **Set up** the table to display x-values starting at 0 and increasing in increments of 1.

```
TABLE SETUP
 TblStart=0
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

STEP 3 **Make** the table of values for $y = 3\sqrt{x} - 1$. Scroll through the table to find the x-value for which $y = 11$. This x-value is 16. It appears that $3\sqrt{x} - 1 \le 11$ when $x \le 16$.

X	Y1
13	9.8167
14	10.225
15	10.619
16	11
17	11.369

X=16

STEP 4 **Check** the domain of $y = 3\sqrt{x} - 1$. The domain is $x \ge 0$, so the solutions of $3\sqrt{x} - 1 \le 11$ cannot be negative. (This is indicated by the word ERROR next to the negative x-values.)

X	Y1
-3	ERROR
-2	ERROR
-1	ERROR
0	-1
1	2

X=-3

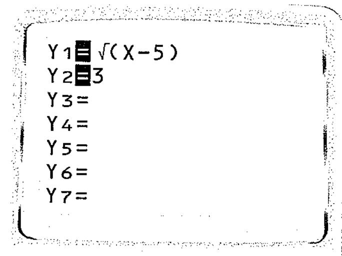
▶ The solution of the inequality is $x \le 16$ *and* $x \ge 0$, which you can write as $0 \le x \le 16$.

EXAMPLE 2 Solve a radical inequality using a graph

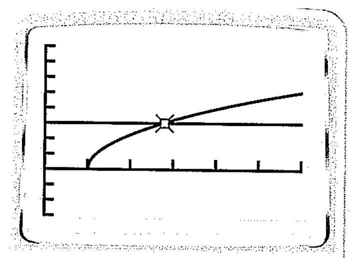
Use a graph to solve $\sqrt{x-5} > 3$.

Solution

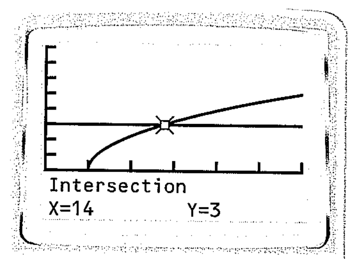
STEP 1 **Enter** the functions $y = \sqrt{x-5}$ and $y = 3$ into a graphing calculator.



STEP 2 **Graph** the functions from Step 1. Adjust the viewing window so that the x-axis shows $0 \le x \le 30$ with a scale of 5 and the y-axis shows $-3 \le y \le 8$ with a scale of 1.



STEP 3 **Identity** the x-values for which the graph of $y = \sqrt{x-5}$ lies above the graph of $y = 3$. You can use the *intersect* feature to show that the graphs intersect when $x = 14$. The graph of $y = \sqrt{x-5}$ lies above the graph of $y = 3$ when $x > 14$.



▶ The solution of the inequality is $x > 14$.

INTERPRET DOMAIN
In Example 2, note that the domain of $y = \sqrt{x-5}$ is $x \ge 5$. Therefore, the domain does not affect the solution.

PRACTICE

EXAMPLE 1
on p. 462
for Exs. 1–6

Use a table to solve the inequality.

1. $2\sqrt{x} - 5 \ge 3$
2. $\sqrt{x-4} \le 5$
3. $4\sqrt{x} + 1 \le 9$
4. $\sqrt{x+7} \ge 3$
5. $\sqrt{x} + \sqrt{x+3} \ge 3$
6. $\sqrt{x} + \sqrt{x-5} \le 5$

EXAMPLE 2
on p. 463
for Exs. 7–12

Use a graph to solve the inequality.

7. $2\sqrt{x} + 3 \le 8$
8. $\sqrt{x+3} \ge 2.6$
9. $7\sqrt{x} + 1 < 9$
10. $4\sqrt{3x-7} > 7.8$
11. $\sqrt{x} - \sqrt{x+5} < -1$
12. $\sqrt{x+2} + \sqrt{x-1} \le 9$

13. **SAILBOAT RACE** In order to compete in the America's Cup sailboat race, a boat must satisfy the rule

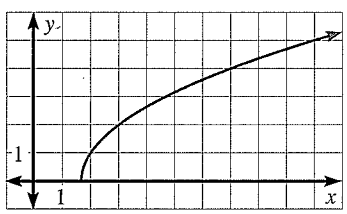
$$\ell + 1.25\sqrt{s} - 9.8\sqrt[3]{d} \le 16$$

where ℓ is the length (in meters) of the boat, s is the area (in square meters) of the sails, and d is the volume (in cubic meters) of water displaced by the boat. A boat has a length of 20 meters and displaces 27 cubic meters of water. What is the maximum allowable value for s?

Lessons 6.4–6.6

1. **MULTI-STEP PROBLEM** A manager at a clothing store is determining the retail prices of items so that they can be tagged and placed on the sales floor. The equation that the manager uses is $R = C + MC$ where R is the retail price, C is the cost that the store pays for the item, and M is the percent (expressed as a decimal) that the item is marked up.
   - **a.** The markup for women's athletic shoes is 40%. Write a function that gives the retail price R in terms of the cost C.
   - **b.** Find the inverse of the function from part (a).
   - **c.** Use the inverse function to find the cost of a pair of women's athletic shoes that has a retail price of \$60.

2. **SHORT RESPONSE** The graph of $y = \sqrt{3x - 5}$ is shown below. Solve the equation $\sqrt{3x - 5} = 4$. *Explain* how you can use the graph of $y = \sqrt{3x - 5}$ to verify that your solution is correct.



3. **OPEN-ENDED** Write a radical equation whose only solution is -5.

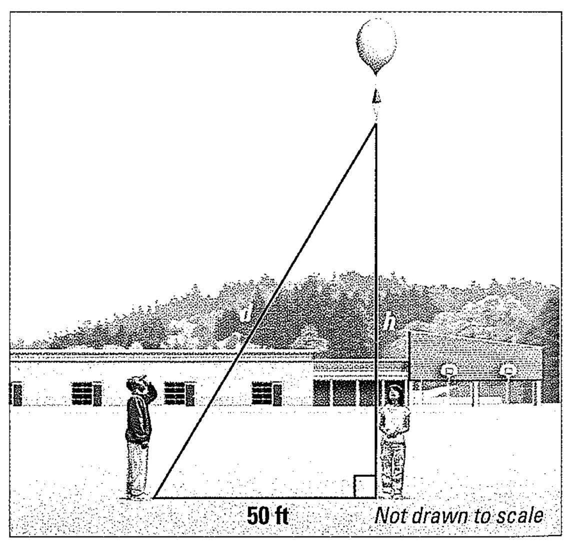
4. **EXTENDED RESPONSE** On a certain day, the function that gives Swedish kronor in terms of U.S. dollars is $k = 7.463d$ where k represents kronor and d represents U.S. dollars.
   - **a.** Find the inverse function.
   - **b.** How many U.S. dollars do you receive for 25 kronor?
   - **c.** Express what the inverse function means in words.

5. **SHORT RESPONSE** Find a square root function that passes through the points $(-14, 0)$ and $(-13, 1)$. Are there other square root functions that pass through these points? *Explain.*

6. **OPEN-ENDED** Write two functions whose graphs are translations of the graph of $y = \sqrt{x}$. The first function should have a domain of $x \geq 4$. The second function should have a range of $y \geq -2$.

7. **SHORT RESPONSE** An object is launched upward from ground level and reaches a maximum height of h feet. The initial velocity v (in feet per second) of the object is given by the function $v = 8\sqrt{h}$.
   - **a.** Find the inverse function.
   - **b.** Write a problem that can be solved using the inverse function. Show how to solve the problem.

8. **GRIDDED ANSWER** Your friend releases a weather balloon 50 feet from you. When the balloon is at height h, the distance d between you and the balloon is given by $d = \sqrt{2500 + h^2}$ where h and d are measured in feet. To the nearest foot, what is the height of the balloon when the distance between you and the balloon is 100 feet?



9. **SHORT RESPONSE** You drop a pebble into a calm pond, causing ripples of concentric circles. The radius r (in feet) of the outer ripple is given by $r(t) = 6t$ where t is the time (in seconds) after the pebble hits the water. The area A (in square feet) of the outer ripple is given by $A(r) = \pi r^2$. Find $A(r(t))$ and evaluate $A(r(2))$. What does $A(r(2))$ represent?

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BIG IDEAS — For Your Notebook

Big Idea 1

Using Rational Exponents

The following are properties of rational exponents. Let a and b be real numbers and let m and n be rational numbers.

Property	Example
$a^m \cdot a^n = a^{m+n}$	$4^{5/2} \cdot 4^{1/2} = 4^3 = 64$
$(a^m)^n = a^{mn}$	$(2^8)^{1/4} = 2^2 = 4$
$(ab)^m = a^m b^m$	$(25 \cdot 4)^{1/2} = 25^{1/2} \cdot 4^{1/2} = 5 \cdot 2 = 10$
$a^{-m} = \frac{1}{a^m}, a \neq 0$	$8^{-1/3} = \frac{1}{8^{1/3}} = \frac{1}{2}$
$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{9^{5/8}}{9^{1/8}} = 9^{4/8} = 9^{1/2} = 3$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{16}{81}\right)^{1/4} = \frac{16^{1/4}}{81^{1/4}} = \frac{2}{3}$

Big Idea 2

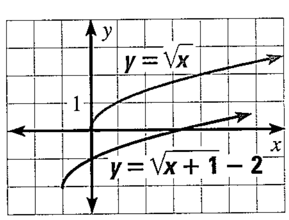
Performing Function Operations and Finding Inverse Functions

Operation	Definition	Example: $f(x) = 2x, g(x) = x - 5$
Addition	$h(x) = f(x) + g(x)$	$h(x) = 2x + (x - 5) = 3x - 5$
Subtraction	$h(x) = f(x) - g(x)$	$h(x) = 2x - (x - 5) = x + 5$
Multiplication	$h(x) = f(x) \cdot g(x)$	$h(x) = 2x(x - 5) = 2x^2 - 10x$
Division	$h(x) = \frac{f(x)}{g(x)}$	$h(x) = \frac{2x}{x - 5}$
Composition	$h(x) = g(f(x))$	$h(x) = 2x - 5$
Inverse	$h(x) = g^{-1}(x)$	$h(x) = x + 5$

Big Idea 3

Graphing Radical Functions and Solving Radical Equations

To **graph** radical functions, use the graph of the parent functions. For example, to graph $y = \sqrt{x + 1} - 2$, translate the graph of $y = \sqrt{x}$ left 1 unit and down 2 units.



To **solve** a radical equation, first isolate the radical. Then raise each side of the equation to the same power and solve the polynomial equation.

$\sqrt{2x - 5} - 3 = 2$	**Write equation.**
$\sqrt{2x - 5} = 5$	**Isolate radical.**
$(\sqrt{2x - 5})^2 = 5^2$	**Square each side.**
$2x - 5 = 25$	**Simplify.**
$x = 15$	**Solve.**

6 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- nth root of a, *p. 414*
- index of a radical, *p. 414*
- simplest form of a radical, *p. 422*
- like radicals, *p. 422*
- power function, *p. 428*
- composition, *p. 430*
- inverse relation, *p. 438*
- inverse function, *p. 438*
- radical function, *p. 446*
- radical equation, *p. 452*

VOCABULARY EXERCISES

1. Copy and complete: The index of the radical $\sqrt[4]{7}$ is _?_.
2. List two different pairs of like radicals.
3. Copy and complete: A(n) _?_ function has the form $y = ax^b$ where a is a real number and b is a rational number.
4. **WRITING** *Explain* how the graph of a function and the graph of its inverse are related.
5. **WRITING** *Explain* how to use the horizontal line test to determine whether the inverse of a function f is also a function.
6. **WRITING** *Describe* how the graph of $y = \sqrt[3]{x - 4} + 5$ is related to the graph of the parent function $y = \sqrt[3]{x}$.
7. **REASONING** A student began solving the equation $x^{2/3} = 5$ by cubing each side. What will the student have to do next? What could the student have done to solve the equation in just one step?

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 6.

6.1 Evaluate nth Roots and Use Rational Exponents *pp. 414–419*

EXAMPLE

Evaluate the expression.

a. $\left(\sqrt[4]{16}\right)^5 = 2^5 = 32$

b. $27^{-4/3} = \dfrac{1}{27^{4/3}} = \dfrac{1}{(27^{1/3})^4} = \dfrac{1}{3^4} = \dfrac{1}{81}$

EXERCISES

EXAMPLE 2 on p. 415 for Exs. 8–15

Evaluate the expression without using a calculator.

8. $81^{1/4}$ **9.** $0^{1/3}$ **10.** $\sqrt[3]{-64}$ **11.** $\sqrt[3]{125}$

12. $256^{3/4}$ **13.** $27^{-2/3}$ **14.** $\left(\sqrt[3]{8}\right)^7$ **15.** $\dfrac{1}{\left(\sqrt[5]{-32}\right)^{-3}}$

Glossary
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Chapter Review Practice

6.2 Apply Properties of Rational Exponents *pp. 420–427*

EXAMPLE

Write the expression in simplest form. Assume all variables are positive.

a. $\sqrt[3]{48} = \sqrt[3]{8 \cdot 6} = \sqrt[3]{8} \cdot \sqrt[3]{6} = 2\sqrt[3]{6}$

b. $\left(\frac{x^4}{y^8}\right)^{1/2} = \frac{(x^4)^{1/2}}{(y^8)^{1/2}} = \frac{x^{4 \cdot 1/2}}{y^{8 \cdot 1/2}} = \frac{x^2}{y^4}$

EXAMPLES 4, 6, and 7 on pp. 422–423 for Exs. 16–19

EXERCISES

Write the expression in simplest form. Assume all variables are positive.

16. $\sqrt[3]{80}$ **17.** $(3^4 \cdot 5^4)^{-1/4}$ **18.** $(25a^{10}b^{16})^{1/2}$ **19.** $\sqrt{\frac{18x^5y^4}{49xz^3}}$

6.3 Perform Function Operations and Composition *pp. 428–434*

EXAMPLE

Let $f(x) = 3x^2 + 1$ and $g(x) = x + 4$. Perform the indicated operation.

a. $f(x) + g(x) = (3x^2 + 1) + (x + 4) = 3x^2 + x + 5$

b. $f(x) \cdot g(x) = (3x^2 + 1)(x + 4) = 3x^3 + 12x^2 + x + 4$

c. $f(g(x)) = f(x + 4) = 3(x + 4)^2 + 1 = 3(x^2 + 8x + 16) + 1 = 3x^2 + 24x + 49$

EXAMPLES 1, 2, and 5 on pp. 428–430 for Exs. 20–23

EXERCISES

Let $f(x) = 4x - 6$ and $g(x) = x + 8$. Perform the indicated operation.

20. $f(x) + g(x)$ **21.** $f(x) - g(x)$ **22.** $f(x) \cdot g(x)$ **23.** $f(g(x))$

6.4 Use Inverse Functions *pp. 438–445*

EXAMPLE

Find the inverse of the function $y = 3x + 7$.

$y = 3x + 7$	**Write original function.**
$x = 3y + 7$	**Switch *x* and *y*.**
$x - 7 = 3y$	**Subtract 7 from each side.**
$\frac{1}{3}x - \frac{7}{3} = y$	**Divide each side by 3.**

EXAMPLES 1, 4, and 5 on pp. 438–441 for Exs. 24–26

EXERCISES

Find the inverse of the function.

24. $y = \frac{1}{3}x + 4$ **25.** $y = 4x^2 + 9, x \geq 0$ **26.** $f(x) = x^3 - 4$

9

6 CHAPTER REVIEW

6.5 Graph Square Root and Cube Root Functions

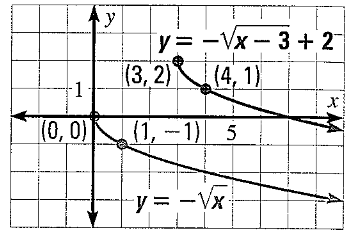
pp. 446–451

EXAMPLE

Graph $y = -\sqrt{x-3} + 2$.

Sketch the graph of $y = -\sqrt{x}$. Notice that it begins at the origin and passes through the point (1, −1).

For $y = -\sqrt{x-3} + 2$, $h = 3$, and $k = 2$. So, shift the graph of $y = -\sqrt{x}$ right 3 units and up 2 units. The resulting graph begins at the point (3, 2) and passes through the point (4, 1).

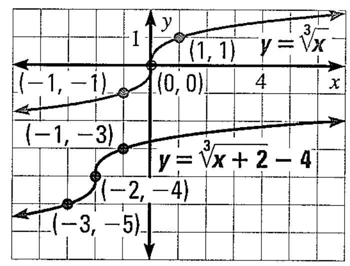


EXAMPLE

Graph $y = \sqrt[3]{x+2} - 4$.

Sketch the graph of $y = \sqrt[3]{x}$. Notice that it passes through the points (−1, −1), (0, 0), and (1, 1).

For $y = \sqrt[3]{x+2} - 4$, $h = -2$ and $k = -4$. So, shift the graph of $y = \sqrt[3]{x}$ left 2 units and down 4 units. The resulting graph passes through the points (−3, −5), (−2, −4), and (−1, −3).



EXAMPLES 4 and 5 on p. 448 for Exs. 27–29

EXERCISES

Graph the function. Then state the domain and range.

27. $y = \sqrt{x+3} + 5$

28. $y = 3\sqrt{x+1} - 4$

29. $y = \sqrt[3]{x-4} - 5$

6.6 Solve Radical Equations

pp. 452–459

EXAMPLE

Solve $\sqrt{4x+9} = 5$.

$\sqrt{4x+9} = 5$	**Write original equation.**
$(\sqrt{4x+9})^2 = 5^2$	**Square each side to eliminate the radical.**
$4x + 9 = 25$	**Simplify.**
$4x = 16$	**Subtract 9 from each side.**
$x = 4$	**Divide each side by 4.**

CHECK Check $x = 4$ in the original equation.

$\sqrt{4x+9} = \sqrt{4(4)+9} = \sqrt{25} = 5$ ✓

EXAMPLES 1, 3, and 5 on pp. 452–454 for Exs. 30–32

EXERCISES

Solve the equation. Check for extraneous solutions.

30. $\sqrt[3]{5x-4} = 2$

31. $3x^{3/4} = 24$

32. $\sqrt{x^2 - 10} = \sqrt{3x}$

6 CHAPTER TEST

Evaluate the expression without using a calculator.

1. $-125^{1/3}$
2. $32^{1/5}$
3. $\sqrt[4]{81}$
4. $\sqrt[3]{27}$
5. $8^{5/3}$
6. $16^{-3/2}$
7. $(\sqrt[3]{-27})^2$
8. $(\sqrt[3]{64})^{-4}$

Write the expression in simplest form. Assume all variables are positive.

9. $\sqrt[3]{88}$
10. $\sqrt[5]{16} \cdot \sqrt[5]{8}$
11. $\sqrt{\frac{12}{49}}$
12. $\frac{\sqrt[3]{24}}{\sqrt[3]{9}}$
13. $\sqrt[3]{64x^4y^2}$
14. $\sqrt[4]{2x^6y^8z}$
15. $\sqrt[5]{\frac{x^6}{y^4}}$
16. $\sqrt{\frac{75x^5y^6}{36xz^5}}$

Let $f(x) = 2x + 9$ and $g(x) = 3x - 1$. Perform the indicated operation and state the domain.

17. $f(x) + g(x)$
18. $f(x) - g(x)$
19. $f(x) \cdot g(x)$
20. $\frac{f(x)}{g(x)}$
21. $f(g(x))$
22. $g(f(x))$
23. $f(f(x))$
24. $g(g(x))$

Find the inverse of the function.

25. $y = -2x + 5$
26. $y = \frac{1}{3}x + 4$
27. $f(x) = 5x - 12$
28. $y = \frac{1}{2}x^4, x \geq 0$
29. $f(x) = x^3 + 5$
30. $f(x) = -2x^3 + 1$

Graph the function. Then state the domain and range.

31. $y = -6\sqrt[3]{x}$
32. $y = \sqrt{x - 4} - 2$
33. $f(x) = -\sqrt[3]{x + 3} + 4$

Solve the equation. Check for extraneous solutions.

34. $\sqrt{3x + 7} = 4$
35. $\sqrt{3x} - \sqrt{x + 6} = 0$
36. $x - 3 = \sqrt{x - 1}$

37. **KINETIC ENERGY** The kinetic energy E (in joules) of a 1250 kilogram compact car is given by the equation $E = 625s^2$ where s is the speed of the car (in meters per second).
    a. Write an inverse model that gives the speed of the car as a function of its kinetic energy.
    b. Use the inverse model to find the speed of the car if its kinetic energy is 120,000 joules. Give the speed in kilometers per hour.
    c. If the kinetic energy doubles, will the speed double? *Explain* why or why not.

38. **BOWLING SCORES** In bowling, a *handicap* is a change in score to adjust for differences in players' abilities. You belong to a bowling league in which each bowler's handicap h is determined by his or her average a using this formula:

$$h = 0.9(200 - a)$$

If a bowler's average is over 200, the handicap is 0. Find the inverse of the model. Then find your average if your handicap is 36.