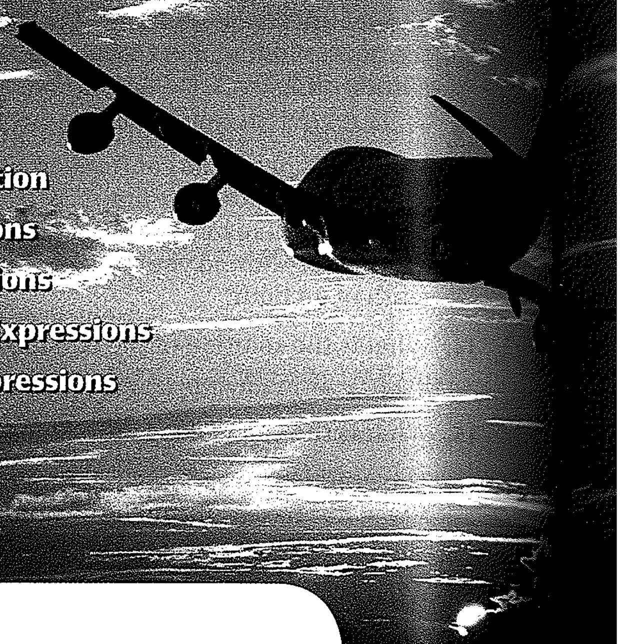
Alg 2

8 Rational Functions



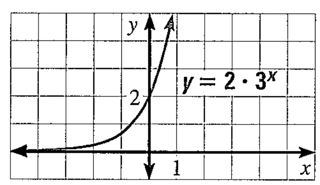
Before

In previous chapters, you learned the following skills, which you'll use in Chapter 8: writing direct variation equations, factoring polynomials, and performing polynomial operations.

Prerequisite Skills

VOCABULARY CHECK

1. The **asymptote** of the graph at the right is _?_.
2. Two variables x and y show **direct variation** provided _?_ where a is a nonzero constant.
3. An **extraneous solution** of a transformed equation is not an actual _?_ of the original equation.



SKILLS CHECK

The variables x and y vary directly. Write an equation that relates x and y. Then find the value of y when $x = -2$. *(Review p. 107 for 8.1.)*

4. $x = 2, y = 8$
5. $x = -1, y = 4$
6. $x = 12, y = 2$

Factor the polynomial completely. *(Review pp. 252, 353 for 8.4, 8.5.)*

7. $x^2 - 11x - 26$
8. $2x^3 - 4x^2 + 2x$
9. $6x^4 - 4x^3 - 24x + 16$

Perform the indicated operation. *(Review p. 346 for 8.4, 8.5.)*

10. $(3x^2 - 6) + (7x^2 - x)$
11. $(-2x^2 + 6) - (x^2 - x)$
12. $(x + 2)(x - 9)^2$

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Now

In Chapter 8, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 602. You will also use the key vocabulary listed below.

Big Ideas

1. **Graphing rational functions**
2. **Performing operations with rational expressions**
3. **Solving rational equations**

KEY VOCABULARY

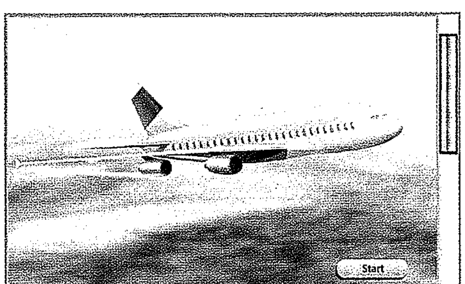
- inverse variation, *p. 551*
- constant of variation, *p. 551*
- joint variation, *p. 553*
- rational function, *p. 558*
- simplified form of a rational expression, *p. 573*
- complex fraction, *p. 584*
- cross multiplying, *p. 589*

Why?

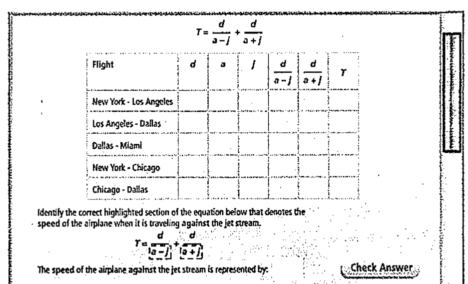
You can use rational functions to model real-life situations. For example, you can model the time it takes to travel across the United States and back in an airplane.

Animated Algebra

The animation illustrated below for Exercise 41 on page 587 helps you answer this question: How does the time required to fly from New York to Los Angeles and back depend on the speeds of the airplane and the jet stream?



The winds of the jet stream affect the overall speed of an airplane.



Examine how flying with the wind or against the wind affects the flight time.

Animated Algebra at classzone.com

Other animations for Chapter 8: pages 554, 559, 568, and 602

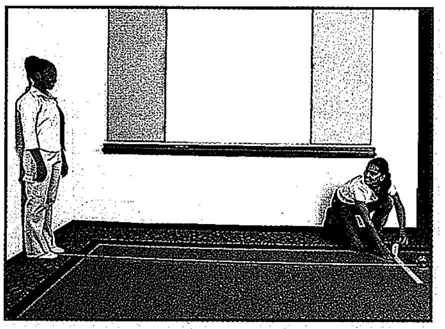
8.1 Investigating Inverse Variation

MATERIALS • tape measure or meter stick • centimeter ruler • masking tape

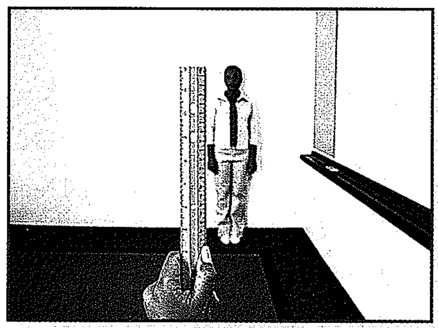
QUESTION **How can you model data that show inverse variation?**

EXPLORE **Collect and record data**

STEP 1 ***Mark distances***
Work with a partner. Have your partner stand against a wall. Place the end of the tape measure against the wall between your partner's feet. Use tape to mark off distances from 3 meters to 9 meters away from the wall.



STEP 2 ***Measure apparent height***
Face your partner, with your toes touching the 3 meter mark. Hold a centimeter ruler at arm's length and line up the "0" end of the ruler with the top of your partner's head. Measure the apparent height of your partner to the nearest centimeter.



STEP 3 ***Repeat for other distances***
Repeat Step 2 for each marked distance and record your results in a table like the one shown.

Distance (m), x	3	4	5	6	7	8	9
Apparent height (cm), y	?	?	?	?	?	?	?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Does apparent height vary directly with distance? *Justify* your answer mathematically.
2. Find the product $x \cdot y$ for each ordered pair in the table. What do you notice?
3. Based on your results from Exercise 2, write an equation relating distance and apparent height.
4. Use your equation to predict your partner's apparent height at an unmeasured distance. Then test your prediction by measuring your partner's apparent height at that distance. How close was your prediction?

8.1 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS14 for Exs. 15, 21, and 39
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 30, 35, and 41

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If z varies directly with the product of x and y, then z is said to vary _?_ with x and y.

2. ★ **WRITING** *Describe* how to tell whether a set of data pairs (x, y) shows inverse variation.

EXAMPLE 1 on p. 551 for Exs. 3–11

DETERMINING VARIATION Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

3. $xy = \frac{1}{5}$
4. $y = x + 4$
5. $\frac{y}{x} = 8$
6. $4x = y$
7. $y = \frac{2}{x}$
8. $x + y = 6$
9. $8y = x$
10. $xy = 12$

11. ★ **MULTIPLE CHOICE** Which equation represents inverse variation?

(A) $y = 4x$ (B) $y = x - 1$ (C) $xy = 5$ (D) $\frac{y}{7} = x$

EXAMPLE 2 on p. 551 for Exs. 12–19

USING INVERSE VARIATION The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = 3$.

12. $x = 5, y = -4$
13. $x = 1, y = 9$
14. $x = -3, y = 8$
15. $x = 7, y = 2$
16. $x = \frac{3}{4}, y = 28$
17. $x = -4, y = -\frac{5}{4}$
18. $x = -12, y = -\frac{1}{6}$
19. $x = \frac{5}{3}, y = -7$

EXAMPLE 4 on p. 553 for Exs. 20–23

INTERPRETING DATA Determine whether x and y show *direct variation*, *inverse variation*, or *neither*.

20.

x	y
1.5	40
2.5	24
4	15
7.5	8
10	6

21.

x	y
12	132
18	198
23	253
29	319
34	374

22.

x	y
4	16
5	11
6.2	10
7	9
11	6

23.

x	y
4	21
6	14
8	10.5
8.4	10
12	7

EXAMPLE 5 on p. 554 for Exs. 24–30

USING JOINT VARIATION Write an equation relating x, y, and z given that z varies jointly with x and y. Then find z when $x = -4$ and $y = 5$.

24. $x = 2, y = -6, z = 24$
25. $x = 8, y = 6, z = 12$
26. $x = -\frac{1}{4}, y = -3, z = 15$
27. $x = 6, y = -7, z = -3$
28. $x = 9, y = -2, z = 6$
29. $x = 5, y = -3, z = 75$

30. ★ **MULTIPLE CHOICE** Suppose z varies jointly with x and y, and $z = -36$ when $x = -3$ and $y = -4$. What is the constant of variation?

(A) -3 (B) -2 (C) 3 (D) 12

EXAMPLE 6
on p. 554
for Exs. 31–33

WRITING EQUATIONS Write an equation for the given relationship.

31. x varies directly with y and inversely with z.

32. y varies jointly with x and the square of z.

33. w varies inversely with y and jointly with x and z.

34. ERROR ANALYSIS A variable z varies jointly with x and the cube of y and inversely with the square root of w. *Describe* and correct the error in writing an equation relating the variables.

$$z = \frac{a\sqrt{w}}{xy^3}$$ ✗

35. ★ OPEN-ENDED MATH Let $f(x)$ represent a direct variation function, $g(x)$ represent an inverse variation function, and $h(x)$ be the sum of $f(x)$ and $g(x)$. Write possible functions $f(x)$ and $g(x)$ so that $h(2) = 5$.

36. CHALLENGE Suppose x varies inversely with y and y varies inversely with z. How does x vary with z? *Justify* your answer algebraically.

PROBLEM SOLVING

EXAMPLES 3 and 4
on pp. 552–553
for Exs. 37–39

37. DIGITAL CAMERAS The number n of photos your digital camera can store varies inversely with the average size s (in megapixels) of the photos. Your digital camera can store 54 photos when the average photo size is 1.92 megapixels. Write a model that gives n as a function of s. How many photos can your camera store when the average photo size is 3.87 megapixels?

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38. ELECTRONICS The table below compares the current I (in milliamps) with the resistance R (in ohms) for several electrical circuits. Write a model that gives R as a function of I. Then predict R when $I = 34$ milliamps.

Current (milliamps), I	7.4	8.9	12.1	17.9
Resistance (ohms), R	1200	1000	750	500

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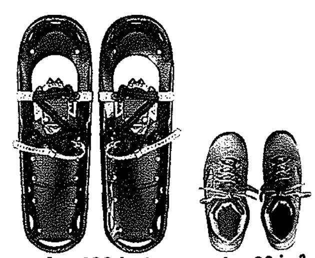
39. SNOWSHOES When you stand on snow, the average pressure P (in pounds per square inch) that you exert on the snow varies inversely with the total area A (in square inches) of the soles of your footwear. Suppose the pressure is 0.43 pound per square inch when you wear the snowshoes shown. Write an equation that gives P as a function of A. Then find the pressure if you wear the boots shown.



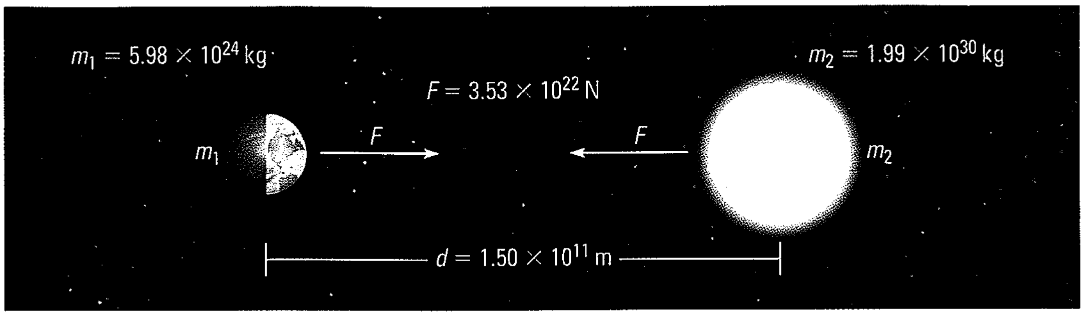
$A = 400$ in.^{2} $A = 60$ in.^{2}

40. MULTI-STEP PROBLEM A piano string's frequency f (in hertz) varies directly with the square root of the string's tension T (in Newtons) and inversely with both the string's length L and diameter d (each in centimeters).

a. The middle C note has a frequency of 262 Hz. The string producing this note has a tension of 670 N, a length of 62 cm, and a diameter of 0.1025 cm. Write an equation relating f, T, L, and d.

b. Find the frequency of the note produced by a string with a tension of 1629 N, a length of 201.6 cm, and a diameter of 0.49 cm.

41. ★ **EXTENDED RESPONSE** The *law of universal gravitation* states that the gravitational force F (in Newtons) between two objects varies jointly with their masses m_1 and m_2 (in kilograms) and inversely with the square of the distance d (in meters) between the two objects. The constant of variation is denoted by G and is called the *universal gravitational constant.*

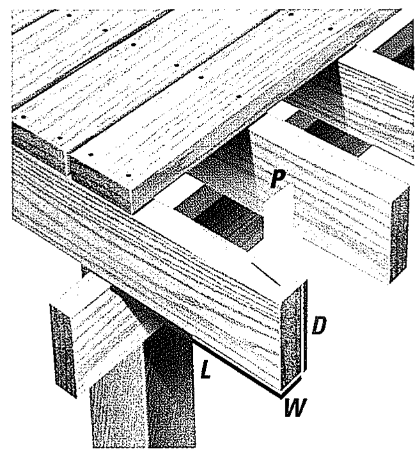


a. **Model** Write an equation that gives F in terms of m_1, m_2, d and G.

b. **Approximate** Use the information above about Earth and the Sun to approximate the universal gravitational constant G.

c. **Reasoning** *Explain* what happens to the gravitational force as the masses of the two objects increase and the distance between them is held constant. *Explain* what happens to the gravitational force as the masses of the two objects are held constant and the distance between them increases.

42. **CHALLENGE** The load P (in pounds) that can be safely supported by a horizontal beam varies jointly with the beam's width W and the square of its depth D, and inversely with its unsupported length L.



a. How does P change when the width and length of the beam are doubled?

b. How does P change when the width and depth of the beam are doubled?

c. How does P change when all three dimensions are doubled?

d. *Describe* several ways a beam can be modified if the safe load it is required to support is increased by a factor of 4.

MIXED REVIEW

PREVIEW
Prepare for Lesson 8.2 in Exs. 43–51.

Graph the function.

43. $y = x^2 + 8x - 20$ *(p. 236)*

44. $y = -x^2 + 4x + 3$ *(p. 236)*

45. $f(x) = x^3 + 1$ *(p. 337)*

46. $g(x) = -\sqrt{x}$ *(p. 446)*

47. $y = \sqrt{x + 4} - 1$ *(p. 446)*

48. $y = \frac{1}{2}\sqrt{x - 1} + 1$ *(p. 446)*

49. $y = -3^x$ *(p. 478)*

50. $h(x) = 3^{x+1} + 2$ *(p. 478)*

51. $y = \left(\frac{3}{4}\right)^{x-1}$ *(p. 486)*

Solve the equation. Check for extraneous solutions. *(p. 452)*

52. $\sqrt{x} = 14$

53. $2\sqrt{x} - 5 = 45$

54. $(x + 3)^{5/3} = 32$

55. $x - 6 = \sqrt{3x}$

56. $\sqrt{2x} = x - 4$

57. $\sqrt{4x - 3} = \sqrt{2x + 13}$

✓ **GUIDED PRACTICE** for Examples 3 and 4

Graph the function. State the domain and range.

4. $y = \frac{x-1}{x+3}$
5. $y = \frac{2x+1}{4x-2}$
6. $f(x) = \frac{-3x+2}{-x-1}$
7. **WHAT IF?** In Example 4, how do the function and graph change if the cost of the 3-D printer is $21,000?

8.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS14 for Exs. 5, 21, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 35, 40, and 41

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The function $y = \frac{7}{x+4} + 3$ has a(n) __?__ of all real numbers except 3 and a(n) __?__ of all real numbers except −4.

2. ★ **WRITING** Is $f(x) = \frac{-3x+5}{2^x+1}$ a rational function? *Explain* your answer.

EXAMPLE 1 on p. 558 for Exs. 3–10

GRAPHING FUNCTIONS **Graph the function. Compare the graph with the graph of $y = \frac{1}{x}$.**

3. $y = \frac{3}{x}$
4. $y = \frac{10}{x}$
5. $y = \frac{-5}{x}$
6. $y = \frac{-0.5}{x}$
7. $y = \frac{0.1}{x}$
8. $f(x) = \frac{15}{x}$
9. $g(x) = \frac{-6}{x}$
10. $h(x) = \frac{-3}{x}$

EXAMPLE 2 on p. 559 for Exs. 11–23

GRAPHING FUNCTIONS **Graph the function. State the domain and range.**

11. $y = \frac{4}{x} + 3$
12. $y = \frac{3}{x} - 2$
13. $y = \frac{6}{x-1}$
14. $f(x) = \frac{1}{x+2}$
15. $y = \frac{-5}{x} - 7$
16. $y = \frac{-6}{x} + 4$
17. $y = \frac{-3}{x+2}$
18. $g(x) = \frac{-2}{x-7}$
19. $y = \frac{-4}{x+4} + 3$
20. $y = \frac{10}{x+7} - 5$
21. $y = \frac{-3}{x-4} - 1$
22. $h(x) = \frac{11}{x-9} + 9$

23. ★ **MULTIPLE CHOICE** What are the asymptotes of the graph of $y = \frac{3}{x+8} - 3$?

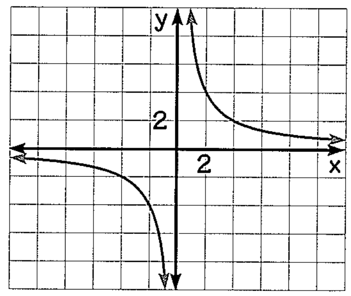
Ⓐ $x = 8, y = 3$ Ⓑ $x = 8, y = -3$ Ⓒ $x = -8, y = 3$ Ⓓ $x = -8, y = -3$

24. **GRAPHING CALCULATOR** Consider the function $y = \frac{a}{x-h} + k$ where $a = 1$, $h = 3$, and $k = -2$. Predict the effect on the functions graph of each change in a, h, or k described in parts (a)–(c). Use a graphing calculator to check your prediction by graphing the original and revised functions in the same coordinate plane.

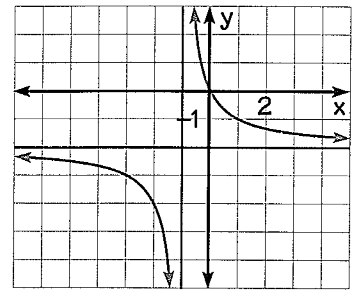
a. a changes to −3 **b.** h changes to −1 **c.** k changes to 2

ERROR ANALYSIS ***Describe*** **and correct the error in the graph.**

25. $y = \frac{-8}{x}$



26. $y = \frac{2}{x-1} - 2$



EXAMPLE 3 on p. 560 for Exs. 27–34

GRAPHING FUNCTIONS **Graph the function. State the domain and range.**

27. $y = \frac{x+4}{x-3}$

28. $y = \frac{x-1}{x+5}$

29. $y = \frac{x+6}{4x-8}$

30. $y = \frac{8x+3}{2x-6}$

31. $y = \frac{-5x+2}{4x+5}$

32. $f(x) = \frac{6x-1}{3x-1}$

33. $g(x) = \frac{5x}{2x+3}$

34. $h(x) = \frac{5x+3}{-x+10}$

35. ★ **OPEN-ENDED MATH** Write a rational function such that the domain is all real numbers except −8 and the range is all real numbers except 3.

36. **CHALLENGE** Show that the equation $f(x) = \frac{a}{x-h} + k$ represents a rational function by writing the right side as a quotient of polynomials.

PROBLEM SOLVING

EXAMPLE 4 on p. 560 for Exs. 37–38

37. **INTERNET SERVICE** An Internet service provider charges a \$50 installation fee and a monthly fee of \$43. Write and graph an equation that gives the average cost per month as a function of the number of months of service. After how many months will the average cost be \$53?

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38. **ROCK CLIMBING GYM** To join a rock climbing gym, you must pay an initial fee of \$100 and a monthly fee of \$59. Write and graph an equation that gives the average cost per month as a function of the number of months of membership. After how many months will the average cost be \$69?

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39. ◆ **MULTIPLE REPRESENTATIONS** The time t (in seconds) it takes for sound to travel 1 kilometer can be modeled by $t = \frac{1000}{0.6T + 331}$ where T is the air temperature (in degrees Celsius).

a. **Evaluating a Function** How long does it take for sound to travel 5 kilometers when the air temperature is 25°C? *Explain.*

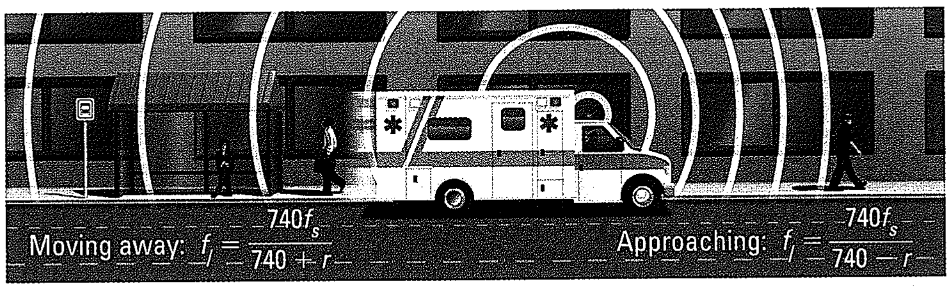
b. **Drawing a Graph** Suppose you are 1 kilometer from a lightning strike, and it takes 3 seconds to hear the thunder. Graph the given function, and use the graph to estimate the air temperature.

40. ★ **SHORT RESPONSE** A business is studying the cost to remove a pollutant from the ground at its site. The function $y = \frac{15x}{1.1 - x}$ models the estimated cost y (in thousands of dollars) to remove x percent (expressed as a decimal) of the pollutant.

a. Graph the function. *Describe* a reasonable domain and range.

b. How much does it cost to remove 20% of the pollutant? 40% of the pollutant? 80% of the pollutant? Does doubling the percent of the pollutant removed double the cost? *Explain.*

41. ★ **EXTENDED RESPONSE** The *Doppler effect* occurs when the source of a sound is moving relative to a listener, so that the frequency f_l (in hertz) heard by the listener is different from the frequency f_s (in hertz) at the source. The frequency heard depends on whether the sound source is approaching or moving away from the listener. In both equations below, r is the speed (in miles per hour) of the sound source.



a. An ambulance siren has a frequency of 2000 hertz. Write two equations modeling the frequencies you hear when the ambulance is approaching and when the ambulance is moving away.

b. Graph the equations from part (a) using the domain $0 \le r \le 60$.

c. For any speed r, how does the frequency heard for an approaching sound source compare with the frequency heard when the source moves away?

42. **CHALLENGE** A sailboat travels at a speed of 10 knots for 3 hours. It then uses a motor for power, which increases its speed to 15 knots. Write and graph an equation giving the boat's average speed s (in knots) for the entire trip as a function of the time t (in hours) that it uses the motor for power.

MIXED REVIEW

PREVIEW
Prepare for Lesson 8.3 in Exs. 43–48.

Factor the expression. *(p. 252)*

43. $m^2 + 18m + 65$
44. $p^2 + 15p + 56$
45. $q^2 - 49$
46. $r^2 - 20r + 100$
47. $x^2 - 4x - 21$
48. $z^2 - 9z + 20$

Graph the polynomial function. *(p. 337)*

49. $f(x) = 2x^4$
50. $f(x) = x^3 + 5$
51. $f(x) = x^5 - 1$
52. $f(x) = 3x^4 + 2$
53. $f(x) = -x^6$
54. $f(x) = -2x^3 + 3$

Simplify the expression. *(p. 492)*

55. $e^5 \cdot e^{-9}$
56. $3e^6 \cdot e^x$
57. $e^x \cdot e^{3x+2}$

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8.2 Graph Rational Functions

QUESTION **How can you use a graphing calculator to graph rational functions?**

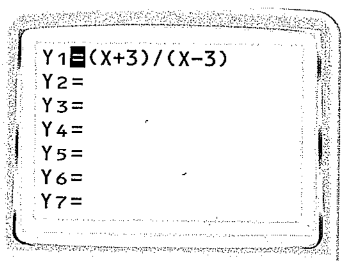
Most graphing calculators have two graphing modes: *connected* mode and *dot* mode. *Connected* mode displays the graph of a rational function as a smooth curve, while *dot* mode displays the graph as a series of dots.

EXAMPLE **Graph a rational function**

Graph $y = \frac{x+3}{x-3}$.

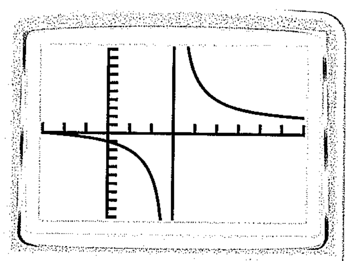
STEP 1 ***Enter function***

Enter the rational function, using parentheses.



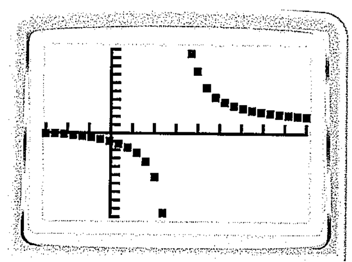
STEP 2 ***Use connected mode***

Graph the function in *connected* mode.



STEP 3 ***Use dot mode***

Graph the function in *dot* mode.



The graph in Step 2 includes a vertical line at approximately $x = 3$. This line is *not* part of the graph. It is simply the graphing calculator's attempt at connecting the two branches of the graph.

PRACTICE

Use a graphing calculator to graph the rational function. Choose a viewing window that displays the important characteristics of the graph.

1. $y = \frac{5}{x} + 2$
2. $y = 7 - \frac{3}{x}$
3. $y = 4 + \frac{2}{x-5}$
4. $y = \frac{6}{x+1} + 2$
5. $y = \frac{7}{2x+8}$
6. $y = \frac{9-2x}{x-3}$
7. $f(x) = \frac{x-4}{x+2}$
8. $g(x) = \frac{5x-2}{3x+9}$
9. **SKATEBOARDING** You are trying to decide whether it is worth joining a skate park. It costs $100 to join and then $4 for each visit. Write a function that gives the average cost y per visit after x visits. Graph the function. What happens to the average cost as the number of visits increases? What are a reasonable domain and range for the function?

8.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS15 for Exs. 7, 15, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 14, 24, and 35

◆ = **MULTIPLE REPRESENTATIONS** Ex. 33

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The graph of a rational function f has no __?__ when the degree of the function's numerator is greater than the degree of its denominator.

2. ★ **WRITING** Let $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials with no common factors other than ± 1. *Describe* how to find the x-intercepts and the vertical asymptotes of the graph of f.

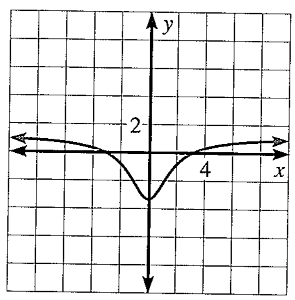
EXAMPLES 1, 2, and 3 on pp. 565–566 for Exs. 3–23

MATCHING GRAPHS **Match the function with its graph.**

3. $y = \frac{-10}{x^2 - 9}$

4. $y = \frac{x^2 - 10}{x^2 + 3}$

5. $y = \frac{x^3}{x^2 - 4}$

A.


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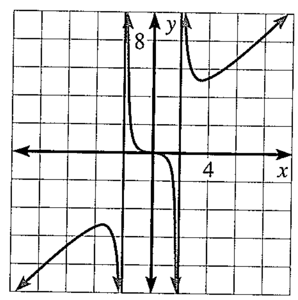
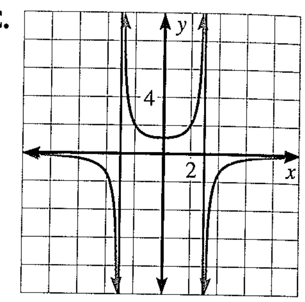
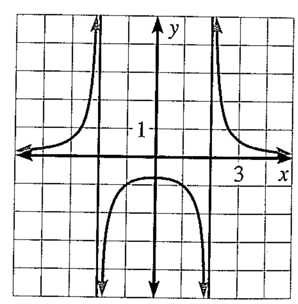


C.


6. ★ **MULTIPLE CHOICE** The graph of which function is shown?

Ⓐ $y = \frac{3}{x^2 - 4}$ Ⓑ $y = \frac{3x^2}{x^2 - 4}$

Ⓒ $y = \frac{x^2 - 4}{3x^2}$ Ⓓ $y = \frac{x^3}{x^2 - 4}$



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ANALYZING GRAPHS **Identify the x-intercept(s) and vertical asymptote(s) of the graph of the function.**

7. $y = \frac{5}{x^2 - 1}$

8. $y = \frac{x + 1}{x^2 + 5}$

9. $f(x) = \frac{x^2 + 9}{x^2 - 2x - 15}$

10. $y = \frac{x^2 - 7x - 60}{x + 3}$

11. $y = \frac{x^3 + 27}{3x^2 + x}$

12. $g(x) = \frac{2x^2 - 3x - 20}{x^2 + 1}$

13. **ERROR ANALYSIS** *Describe* and correct the error in finding the vertical asymptote(s) of $f(x) = \frac{x - 2}{x^2 - 8x + 7}$.

> The vertical asymptote occurs at the zero of the numerator $x - 2$. So, the vertical asymptote is $x = 2$. ✗

14. ★ **MULTIPLE CHOICE** What is the horizontal asymptote of the graph of the function $y = \frac{4x^2 - 21x + 5}{x^2 - 12}$?

Ⓐ $y = 0$ Ⓑ $y = \frac{1}{4}$ Ⓒ $y = 4$ Ⓓ $y = 4x$

GRAPHING FUNCTIONS Graph the function.

15. $y = \frac{2x}{x^2 - 1}$

16. $y = \frac{8}{x^2 - x - 6}$

17. $f(x) = \frac{x^2 - 9}{2x^2 + 1}$

18. $y = \frac{x - 4}{x^2 - 3x}$

19. $y = \frac{x^2 + 11x + 18}{2x + 1}$

20. $g(x) = \frac{x^3 - 8}{6 - x^2}$

21. $y = \frac{x^2 + 3}{2x^3}$

22. $y = \frac{x^2 - 5x - 36}{3x}$

23. $h(x) = \frac{3x^2 + 10x - 8}{x^2 + 4}$

24. ★ **OPEN-ENDED MATH** Write two different rational functions whose graphs have the same end behavior as the graph of $y = 3x^2$.

GRAPHING CALCULATOR Use a graphing calculator to find the range of the rational function.

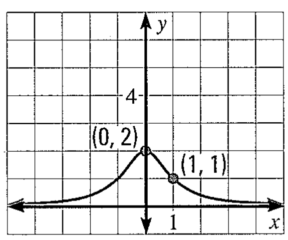
25. $y = \frac{15}{x^2 + 2}$

26. $y = \frac{3x^2}{x^2 - 9}$

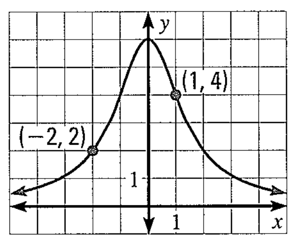
27. $y = \frac{x^2 - 2x}{2x + 3}$

CHALLENGE The graph of a function of the form $f(x) = \frac{a}{x^2 + b}$ is shown. Find the values of *a* and *b*.

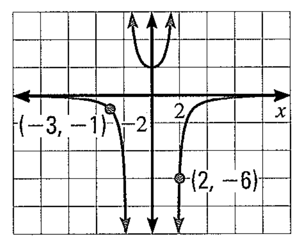
28.



29.



30.

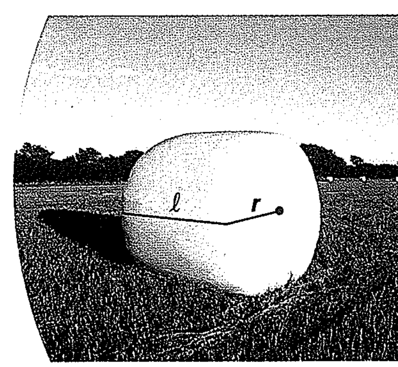


PROBLEM SOLVING

EXAMPLE 4
on p. 567
for Exs. 31–32

GRAPHING CALCULATOR You may wish to use a graphing calculator to complete the following Problem Solving exercises.

31. **AGRICULTURE** A farmer makes cylindrical bales of hay that have a volume of 100 cubic feet. Each bale is to be wrapped in plastic to keep the hay dry.



a. Using the formula for the volume of a cylinder, write an equation that gives the length ℓ of a bale in terms of the radius r.

b. Write a function that gives the surface area of a bale in terms of only the radius r.

c. Find the dimensions of a bale that has the given volume and uses the least amount of plastic possible when the bale is wrapped.

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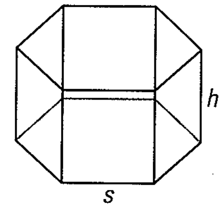
32. AQUARIUM DESIGN A manufacturer is designing an aquarium whose base is a regular hexagon. The aquarium should have a volume of 24 cubic feet and use the least amount of material possible. Let s be the length (in feet) of a side of the base, and let h be the height (in feet).



a. Write an equation that gives h in terms of s. (*Hint:* The volume of the aquarium is given by $V = \frac{3\sqrt{3}}{2}s^2h$.)

b. Find the dimensions s and h that minimize the amount of material used. (*Hint:* The surface area of the aquarium is given by $S = \frac{3\sqrt{3}}{2}s^2 + 6sh$.)

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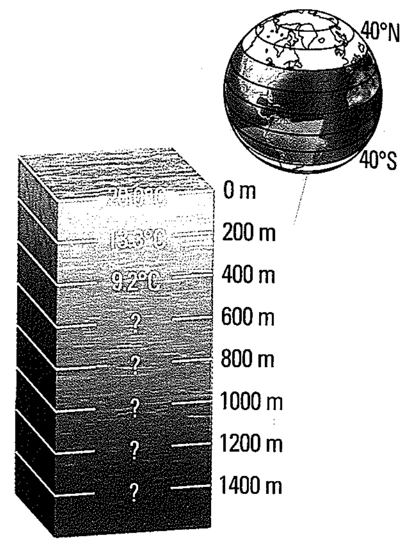
33. **MULTIPLE REPRESENTATIONS** The mean temperature T (in degrees Celsius) of the Atlantic Ocean between latitudes 40°N and 40°S can be modeled by



$$T = \frac{17{,}800d + 20{,}000}{3d^2 + 740d + 1000}$$

where d is the depth (in meters).

a. Making a Table Make a table of values showing the mean temperature for depths from 1000 meters to 1300 meters in 50 meter intervals.

b. Using a Graph Graph the model. Use your graph to estimate the depth at which the mean temperature is 4°C.

34. MULTI-STEP PROBLEM From 1993 to 2002, the number n (in billions) of shares of stock sold on the New York Stock Exchange can be modeled by

$$n = \frac{1054t + 6204}{-6.62t + 100}$$

where t is the number of years since 1993.

a. Graph the model.

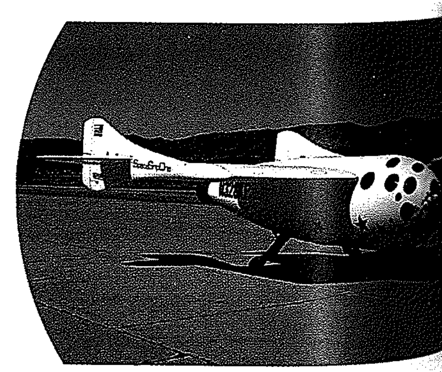
b. *Describe* the general trends shown by the graph.

c. Estimate the year when the number of shares of stock sold was first greater than 100 billion.

35. ★ EXTENDED RESPONSE The acceleration due to gravity g (in meters per second squared) changes as altitude changes and is given by the function

$$g = \frac{3.99 \times 10^{14}}{h^2 + (1.28 \times 10^7)h + (4.07 \times 10^{13})}$$

where h is the altitude (in meters) above sea level.



This spacecraft reached an altitude of 112 km in 2004.

a. Graph Graph the function.

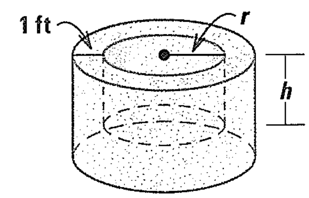
b. Apply A mountaineer is climbing to a height of 8000 meters. What is the value of g at this altitude?

c. Apply A spacecraft reaches an altitude of 112 kilometers above Earth. What is the value of g at this altitude?

d. Explain *Describe* what happens to the value of g as altitude increases.

○ = **WORKED-OUT SOLUTIONS on p. WS1**　★ = **STANDARDIZED TEST PRACTICE**　◆ = **MULTIPLE REPRESENTATIONS**

36. **CHALLENGE** You need to build a cylindrical water tank using 100 cubic feet of concrete. The sides and the base of the tank must be 1 foot thick.



a. Write an equation that gives the tank's inner height h in terms of its inner radius r.

b. Write an equation that gives the volume V of water that the tank can hold as a function of r.

c. Graph the equation from part (b). What values of r and h maximize the tank's capacity?

MIXED REVIEW

PREVIEW
Prepare for Lesson 8.4 in Exs. 37–45.

Factor the expression.

37. $x^2 - 64$ *(p. 252)*
38. $x^2 - 8x - 48$ *(p. 252)*
39. $18x^2 - 37x - 20$ *(p. 259)*
40. $12x^2 - 15x - 18$ *(p. 259)*
41. $5x^2 + 22x - 30$ *(p. 259)*
42. $5x^3 + 40$ *(p. 353)*
43. $x^3 - 4x^2 + 8x - 32$ *(p. 353)*
44. $x^3 + 2x^2 - 35x$ *(p. 353)*
45. $x^5 - 9x^3 - 36x$ *(p. 353)*

Simplify the expression. Tell which properties of exponents you used. *(p. 330)*

46. $\dfrac{x^5y}{x^2y^4}$
47. $\dfrac{48x^{-1}y^4}{6x^2y^3}$
48. $\left(\dfrac{x^2y^4}{xy^5}\right)^2$
49. $\left(\dfrac{72x^3y^{-1}}{12x^{-1}y^2}\right)^{-1}$
50. $\dfrac{6x^{-2}y^2}{36xy^{-3}}$
51. $\left(\dfrac{x^5y^4}{x^7y^8}\right)^{-2}$
52. $\left(\dfrac{90x^3y^{-1}}{18x^{-1}y^{-2}}\right)^2$
53. $\left(\dfrac{xy^6}{x^2y^5}\right)^3$

QUIZ for Lessons 8.1–8.3

The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = -4$. *(p. 551)*

1. $x = 8, y = 3$
2. $x = 2, y = -9$
3. $x = -5, y = \frac{8}{3}$
4. $x = -\frac{1}{4}, y = -32$

Graph the function.

5. $y = \dfrac{3}{2x}$ *(p. 558)*
6. $y = \dfrac{4}{x-2} + 1$ *(p. 558)*
7. $f(x) = \dfrac{-2x}{3x-6}$ *(p. 558)*
8. $y = \dfrac{-8}{x^2-1}$ *(p. 565)*
9. $y = \dfrac{x^2-6}{x^2+2}$ *(p. 565)*
10. $g(x) = \dfrac{x^3-8}{2x^2}$ *(p. 565)*

11. **SOFTBALL** A pitcher throws 16 strikes in her first 38 pitches. The table shows how the pitcher's strike percentage changes if she throws x consecutive strikes after the first 38 pitches. Write a rational function for the strike percentage in terms of x. Graph the function. How many consecutive strikes must the pitcher throw to reach a strike percentage of 0.60? *(p. 558)*

x	Total strikes	Total pitches	Strike percentage
0	16	38	0.42
5	21	43	0.49
10	26	48	0.54
x	$x + 16$	$x + 38$	?

Lessons 8.1–8.3

1. **MULTI-STEP PROBLEM** Your family buys a photo printer to print out digital pictures. The printer costs $200. The ink and paper cost about $.60 for each photo you print.
   a. Write an equation that gives the average cost of a printed photo as a function of the number of photos printed.
   b. Graph the function from part (a). Use the graph to estimate the number of photos you have to print before the average cost drops to $10 per printed photo.
   c. What happens to the average cost as the number of photos printed increases?

2. **MULTI-STEP PROBLEM** A food manufacturer wants to find the most efficient packaging for a canister of oatmeal with a volume of 1663 cubic centimeters.
   a. Use the formula for the volume of a cylinder, $V = \pi r^2 h$, to write an equation that gives the height h of a possible canister in terms of its radius r.
   b. Write an equation that gives the canister's surface area S in terms of its radius r by substituting the expression for h from part (a) into the formula for the surface area of a cylinder, $S = 2\pi r^2 + 2\pi rh$.
   c. Graph the equation from part (b) using a graphing calculator. What are the dimensions r and h of the canister that uses the least material possible?

3. **SHORT RESPONSE** The table shows the number y of candy boxes a manufacturer sells each month at different prices x (in dollars). Do the data show inverse variation? *Explain* your reasoning.

x	5	6	7	8
y	800	667	571	500

4. **OPEN-ENDED** Write a rational function whose domain is all real numbers except $x = 2$ and $x = 3$.

5. **GRIDDED ANSWER** The variables x and y vary inversely, and $y = 4$ when $x = 5$. What is the value of y when $x = 10$?

6. **EXTENDED RESPONSE** The body mass index b of a person varies directly with the person's weight w and inversely with the square of the person's height h. A person who is 1.6 meters tall and weighs 51.2 kilograms has a body mass index of 20.
   a. Write an equation that relates b, w, and h.
   b. Use the equation from part (a) to complete the table below.

Body mass index	Height (m)	Weight (kg)
20	?	45
?	1.4	41
19	1.5	?

   c. Suppose two people weigh the same, but the height of one person is 10% greater than the height of the other person. *Compare* the body mass indices of the two people.

7. **GRIDDED ANSWER** The intensity I of a sound (in watts per square meter) varies inversely with the square of the distance d (in meters) from the source of the sound. At a distance of 1 meter from the stage, the sound intensity of a rock concert is about 10 watts per square meter. What is the intensity of the sound you hear if you are 15 meters from the stage?

8. **SHORT RESPONSE** The value M (in dollars) of a motorcycle t years after it was purchased new can be estimated using the function
   $M(t) = \frac{3500}{t} + 500$ where $t \geq 1$.
   a. Estimate the motorcycle's value 5 years after it was purchased.
   b. What does the value of the motorcycle approach as time passes? *Explain.*

EXAMPLE 7 Divide a rational expression by a polynomial

Divide: $\dfrac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x)$

$$\frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x) = \frac{6x^2 + x - 15}{4x^2} \cdot \frac{1}{3x^2 + 5x}$$ **Multiply by reciprocal.**

$$= \frac{(3x + 5)(2x - 3)}{4x^2} \cdot \frac{1}{x(3x + 5)}$$ **Factor.**

$$= \frac{\cancel{(3x + 5)}(2x - 3)}{4x^2(x)\cancel{(3x + 5)}}$$ **Divide out common factors.**

$$= \frac{2x - 3}{4x^3}$$ **Simplified form**

✓ GUIDED PRACTICE for Examples 6 and 7

Divide the expressions. Simplify the result.

11. $\dfrac{4x}{5x - 20} \div \dfrac{x^2 - 2x}{x^2 - 6x + 8}$

12. $\dfrac{2x^2 + 3x - 5}{6x} \div (2x^2 + 5x)$

8.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS15 for Exs. 7, 25, and 49

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 20, 23, 50, and 52

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: To divide one rational expression by another, multiply the first rational expression by the _?_ of the second rational expression.

2. ★ **WRITING** How do you know when a rational expression is simplified?

EXAMPLE 1 on p. 573 for Exs. 3–20

REASONING Match the rational expression with its simplified form.

3. $\dfrac{x^2 - 9x + 14}{x^2 - 5x - 14}$

4. $\dfrac{x^2 - 4}{x^2 + 9x + 14}$

5. $\dfrac{x^2 + 5x - 14}{x^2 - 4x + 4}$

A. $\dfrac{x - 2}{x + 7}$

B. $\dfrac{x - 2}{x + 2}$

C. $\dfrac{x + 7}{x - 2}$

SIMPLIFYING Simplify the rational expression, if possible.

6. $\dfrac{4x^2}{20x^2 - 12x}$

(7.) $\dfrac{x^2 - x - 20}{x^2 + 2x - 15}$

8. $\dfrac{x^2 + 2x - 24}{x^2 + 7x + 6}$

9. $\dfrac{x^2 - 11x + 24}{x^2 - 3x - 40}$

10. $\dfrac{x^2 + 4x + 4}{x^2 - 5x + 4}$

11. $\dfrac{2x^2 + 2x - 4}{x^2 - 5x - 14}$

12. $\dfrac{x - 4}{x^3 - 64}$

13. $\dfrac{x^2 - 36}{x^2 + 12x + 36}$

14. $\dfrac{3x^3 + 6x^2 + 12x}{x^3 - 8}$

15. $\dfrac{8x^2 + 10x - 3}{6x^2 + 13x + 6}$

16. $\dfrac{5x^2 + 18x - 8}{10x^2 - x - 2}$

17. $\dfrac{x^3 - 5x^2 - 3x + 15}{x^2 - 8x + 15}$

ERROR ANALYSIS ***Describe*** **and correct the error in simplifying the rational expression.**

18. $\dfrac{\cancel{x^2} + 16x - 80}{\cancel{x^2} - 16} = \dfrac{16x - 80}{-16} = -x + 5$ ✗

19. $\dfrac{x^2 + \overset{2}{\cancel{16}}x + \overset{3}{\cancel{48}}}{x^2 + \underset{1}{\cancel{8}}x + \underset{1}{\cancel{16}}} = \dfrac{x^2 + 2x + 3}{x^2 + x + 1}$ ✗

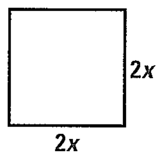
20. ★ **MULTIPLE CHOICE** Which rational expression is in simplified form?

Ⓐ $\dfrac{x^2 - x - 6}{x^2 + 3x + 2}$ Ⓑ $\dfrac{x^2 + 6x + 8}{x^2 + 2x - 3}$ Ⓒ $\dfrac{x^2 - 6x + 9}{x^2 - 2x - 3}$ Ⓓ $\dfrac{x^2 + 3x - 4}{x^2 + x - 2}$

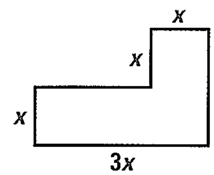
EXAMPLE 2
on p. 574
for Exs. 21–23

GEOMETRY **A farmer wants to fence in the field shown. Write a simplified rational expression for the ratio of the field's perimeter to its area.**

21.



22.



23. ★ **SHORT RESPONSE** Which of the fields in Exercises 21 and 22 has the lower fencing cost per unit of area? *Explain.*

EXAMPLES 3, 4, and 5
on pp. 575–576
for Exs. 24–33

MULTIPLYING **Multiply the expressions. Simplify the result.**

24. $\dfrac{5x^3y}{x^2y^2} \cdot \dfrac{y^3}{15x^2}$

(25.) $\dfrac{48x^5y^3}{y^4} \cdot \dfrac{x^2y}{6x^3y^2}$

26. $\dfrac{x(x-3)}{x-2} \cdot \dfrac{(x+3)(x-2)}{x}$

27. $\dfrac{4(x+5)}{x^2} \cdot \dfrac{x(x+1)}{2(x+5)}$

28. $\dfrac{3x-12}{x+5} \cdot \dfrac{x+6}{2x-8}$

29. $\dfrac{x+5}{4x-16} \cdot \dfrac{2x^2-32}{x^2-25}$

30. $\dfrac{x^2+3x-4}{x^2+4x+4} \cdot \dfrac{2x^2+4x}{x^2-4x+3}$

31. $\dfrac{x^2-3x-10}{x^2-2x-15} \cdot (x^2+10x+21)$

32. $\dfrac{x^2+5x-36}{x^2-49} \cdot (x^2-11x+28)$

33. $\dfrac{4x^2+20x}{x^3+4x^2} \cdot (x^2+8x+16)$

EXAMPLES 6 and 7
on pp. 576–577
for Exs. 34–43

DIVIDING **Divide the expressions. Simplify the result.**

34. $\dfrac{5x^2y^3}{x^7} \div \dfrac{30xy^4}{y^3}$

35. $\dfrac{8x^2y^2z}{xz^3} \div \dfrac{10xy}{x^4z}$

36. $\dfrac{(x+3)(x-2)}{x(x+1)} \div \dfrac{x+3}{x}$

37. $\dfrac{8x^2}{x+4} \div \dfrac{x}{2(x-4)}$

38. $\dfrac{x^2-6x-27}{2x^2+2x} \div \dfrac{x^2-14x+45}{x^2}$

39. $\dfrac{x^2-4x-5}{x+5} \div (x^2+6x+5)$

40. $\dfrac{3x^2+13x+4}{x^2-4} \div \dfrac{4x+16}{x+2}$

41. $\dfrac{x^2-x-2}{x^2+4x-5} \div \dfrac{x-2}{5x+25}$

42. $\dfrac{x^2-8x+15}{x^2+4x} \div (x^2-x-20)$

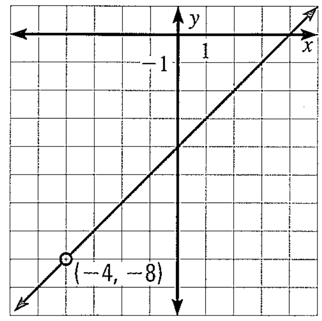
43. $\dfrac{x^2+12x+32}{6x+42} \div \dfrac{x^2+4x}{x^2-49}$

POINT DISCONTINUITY In Exercises 44–46, use the following information.

The graph of a rational function can have a hole in it, called a *point discontinuity*, where the function is undefined. An example is shown below.

$$y = \frac{x^2 - 16}{x + 4} = \frac{(x+4)(x-4)}{x+4} = x - 4$$

The graph of $y = \frac{x^2 - 16}{x + 4}$ is the same as the graph of $y = x - 4$ except that there is a hole at $(-4, -8)$ because the rational function is not defined when $x = -4$.



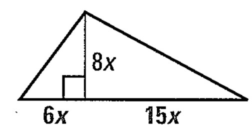
Graph the rational function. Use an open circle for a point discontinuity.

44. $y = \frac{x^2 + 10x + 21}{x + 3}$

45. $y = \frac{x^2 - 36}{x - 6}$

46. $y = \frac{2x^2 - x - 10}{x + 2}$

47. CHALLENGE Find the ratio of the perimeter to the area of the triangle shown at the right.



PROBLEM SOLVING

EXAMPLE 2 on p. 574 for Exs. 48, 50–52

48. GEOMETRY Find the ratio of the volume of the square pyramid to the volume of the inscribed cone. Write your answer in simplified form.

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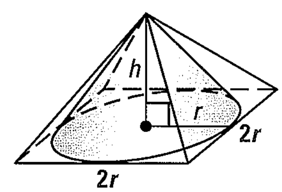

49. ENTERTAINMENT From 1992 to 2002, the gross ticket sales S (in millions of dollars) to Broadway shows and the total attendance A (in millions) at the shows can be modeled by

$$S = \frac{-6420t + 292{,}000}{6.02t^2 - 125t + 1000} \quad \text{and} \quad A = \frac{-407t + 7220}{5.92t^2 - 131t + 1000}$$

where t is the number of years since 1992. Write a model for the *average* dollar amount a person paid per ticket as a function of the year. What was the average amount a person paid per ticket in 1999?

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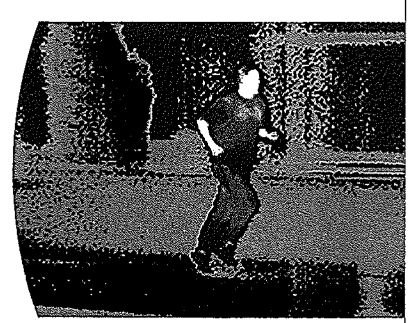
50. ★ SHORT RESPONSE Almost all of the energy generated by a long-distance runner is released in the form of heat. For a runner with height H and speed V, the rate h_g of heat generated and the rate h_r of heat released can be modeled by $h_g = k_1H^3V^2$ and $h_r = k_2H^2$ where k_1 and k_2 are constants.

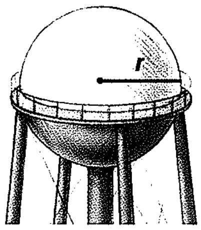
a. Write the ratio of heat generated to heat released. Simplify the expression.

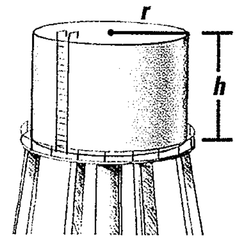
b. When the ratio of heat generated to heat released equals 1, how is speed related to height? Does a taller or shorter runner have the advantage? *Explain.*



Thermogram of runner

51. MULTI-STEP PROBLEM A manufacturer is comparing two designs for a water tower: a sphere and a cylinder. Both designs have the same volume and the same radius.





a. Show that the height h of the cylindrical tank is $\frac{4}{3}r$.

b. Write an expression for the surface area of each tank in terms of r.

c. Find the ratio of the surface area of the spherical tank to the surface area of the cylindrical tank. *Explain* what the ratio tells you about which water tower would take less material to build.

52. ★ EXTENDED RESPONSE The surface area S and the volume V of a cylindrical can are given by $S = 2\pi r^2 + 2\pi rh$ and $V = \pi r^2 h$ where r is the radius and h is the height.

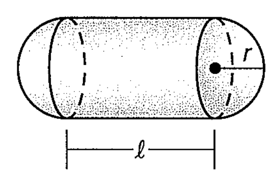
a. Model Write and simplify an expression for the efficiency ratio $\frac{S}{V}$.

b. Calculate Find the efficiency ratio for each can listed in the table.

	Soup can	Coffee can	Paint can
Height, *h*	10.2 cm	15.9 cm	19.4 cm
Radius, *r*	3.4 cm	7.8 cm	8.4 cm

c. Compare Rank the three cans in part (b) according to efficiency. *Explain* your ranking.

53. CHALLENGE A fuel storage container is shaped like a cylinder with a hemisphere on each end, as shown. The length of the cylinder is ℓ and the radius of each hemisphere is r. Show that the ratio of the surface area to the volume of the container is $\frac{6(2r + \ell)}{r(4r + 3\ell)}$.



MIXED REVIEW

PREVIEW Prepare for Lesson 8.5 in Exs. 54–59.

Find the greatest common factor and the least common multiple of the pair of numbers. *(p. 978)*

54. 48, 80 **55.** 120, 155 **56.** 38, 95

57. 52, 91 **58.** 66, 154 **59.** 360, 450

Find the product. *(p. 346)*

60. $x(x^2 + 4x - 7)$ **61.** $(x + 9)(x - 5)$ **62.** $(x + 11)(x - 7)$

63. $(x + 2)(x^2 - 6x + 10)$ **64.** $(3x - 7)(x^2 - 5x)$ **65.** $(x + 5)(x^3 + 8x^2)$

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Keystrokes

8.4 Verify Operations with Rational Expressions

QUESTION **How can you use a graphing calculator to verify the results of operations on rational expressions?**

EXAMPLE **Check a simplified rational expression in two ways**

Simplify $\frac{x^2 - x - 12}{x^2 - 9x + 20}$. Then verify the result numerically and graphically.

STEP 1 ***Simplify expression***

Simplify the rational expression by factoring the numerator and denominator, then dividing out common factors.

$$\frac{x^2 - x - 12}{x^2 - 9x + 20} = \frac{\cancel{(x-4)}(x+3)}{\cancel{(x-4)}(x-5)} = \frac{x+3}{x-5}$$

STEP 2 ***Enter expressions***

Enter the original expression as y_1 and the simplified result as y_2. Use the *thick* graph style for y_2.

```
\Y1=(X2-X-12)/
  (X2-9X+20)
\Y2=(X+3)/(X-5)
\Y3=
\Y4=
\Y5=
\Y6=
```

Remember to use parentheses correctly.

STEP 3 ***Display table***

Use the *table* feature to examine corresponding values of the two expressions.

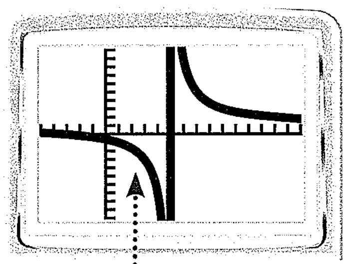
X	Y1	Y2
1	-1	-1
2	-1.667	-1.667
3	-3	-3
4	ERROR	-7
5	ERROR	ERROR

X=1

The values of y_1 and y_2 are the same, except that y_1 is undefined when $x = 4$ and $x = 5$, and y_2 is undefined only when $x = 5$.

STEP 4 ***Display graphs***

Put your calculator in *connected* mode. Display the graphs in an appropriate viewing window.



By using the *thick* graph style for y_2, you can see the graph of y_2 being drawn over the graph of y_1. So, the graphs coincide.

PRACTICE

Simplify the expression. Verify your result numerically and graphically.

1. $\frac{x^2 - 5x}{x^2 - 7x + 10}$
2. $\frac{3x^2 + 6x}{x^2 - 2x - 8}$
3. $\frac{x^2 + 5x + 4}{x^2 + x - 12}$

Perform the indicated operation and simplify. Verify your result numerically and graphically.

4. $\frac{x+3}{5x^2} \cdot \frac{x-1}{x+3}$
5. $\frac{4x^2 - 8x}{5x + 15} \div \frac{x-2}{x+3}$
6. $\frac{x^2 - 3x - 10}{x^2 + 3x + 3} \cdot \frac{x^2 + 2x - 3}{x^2 + x - 2}$

8.5 EXERCISES

HOMEWORK KEY ◯ = **WORKED-OUT SOLUTIONS** on p. WS15 for Exs. 5, 17, and 43

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 26, 37, and 44

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A fraction that contains a fraction in its numerator or denominator is called a(n) _?_.

2. ★ **WRITING** *Explain* how to add rational expressions with unlike denominators.

EXAMPLE 1 on p. 582 for Exs. 3–8

LIKE DENOMINATORS **Perform the indicated operation and simplify.**

3. $\frac{15}{4x} + \frac{5}{4x}$
4. $\frac{x}{16x^2} - \frac{4}{16x^2}$
5. $\frac{9}{x+1} - \frac{2x}{x+1}$
6. $\frac{3x^2}{x-8} + \frac{6x}{x-8}$
7. $\frac{5x}{x+3} + \frac{15}{x+3}$
8. $\frac{4x^2}{2x-1} - \frac{1}{2x-1}$

EXAMPLE 2 on p. 583 for Exs. 9–15

FINDING LCMS **Find the least common multiple of the polynomials.**

9. $3x$ and $3(x-2)$
10. $2x^2$ and $4x + 12$
11. $2x$ and $2x(x-5)$
12. $24x^2$ and $8x^2 - 16x$
13. $x^2 - 25$, x, and $x - 5$
14. $9x^2 - 16$ and $3x^2 - 2x - 8$

15. ★ **MULTIPLE CHOICE** What is the least common multiple of the polynomials $3x^2 - 9x$ and $6x^2$?

Ⓐ $3x(x-3)$ Ⓑ $6x^2$ Ⓒ $6x(x-3)$ Ⓓ $6x^2(x-3)$

EXAMPLES 3 and 4 on pp. 583–584 for Exs. 16–26

UNLIKE DENOMINATORS **Perform the indicated operation and simplify.**

16. $\frac{12}{5x} + \frac{7}{6x}$
17. $\frac{8}{3x^2} - \frac{5}{4x}$
18. $\frac{x-4}{5x} - \frac{12}{5(x-4)}$
19. $\frac{12}{x^2+5x-24} + \frac{3}{x-3}$
20. $\frac{3}{x+4} - \frac{1}{x+6}$
21. $\frac{9}{x-3} + \frac{2x}{x+1}$
22. $\frac{x+4}{x^2-4} - \frac{15}{x-2}$
23. $\frac{-15x}{x^2-8x+16} + \frac{12}{x-4}$
24. $\frac{x^2-5}{x^2+5x-14} - \frac{x+3}{x+7}$

25. **ERROR ANALYSIS** *Describe* and correct the error in adding the rational expressions.

$$\frac{x}{x+2} + \frac{4}{x-5} = \frac{x+4}{(x+2)(x-5)} \quad ✗$$

26. ★ **MULTIPLE CHOICE** Which expression is equivalent to $\frac{2x}{x+4} - \frac{x^2+4}{x^2-16}$?

Ⓐ $\frac{1}{x+4}$ Ⓑ $\frac{(x+2)(x-2)}{(x+4)(x-4)}$ Ⓒ $\frac{x^2-8x-4}{(x+4)(x-4)}$ Ⓓ $\frac{3x^2-8x+4}{(x+4)(x-4)}$

UNLIKE DENOMINATORS **Perform the indicated operation(s) and simplify.**

27. $\frac{x}{x^2-9} + \frac{x+1}{x^2+6x+9}$
28. $\frac{x+3}{x^2-2x-8} - \frac{x-5}{x^2-12x+32}$
29. $\frac{x+2}{x-4} + \frac{2}{x} + \frac{5x}{3x-1}$
30. $\frac{x+3}{x^2-25} - \frac{x-1}{x-5} + \frac{3}{x+3}$

EXAMPLES 5 and 6
on p. 585
for Exs. 31–36

SIMPLIFYING COMPLEX FRACTIONS **Simplify the complex fraction.**

31. $\dfrac{\frac{x}{3} - 6}{10 + \frac{4}{x}}$

32. $\dfrac{15 - \frac{2}{x}}{\frac{x}{5} + 4}$

33. $\dfrac{\frac{16}{x-2}}{\frac{4}{x+1} + \frac{6}{x}}$

34. $\dfrac{\frac{1}{2x-5} - \frac{7}{8x-20}}{\frac{x}{2x-5}}$

35. $\dfrac{\frac{3}{x-2} - \frac{6}{x^2-4}}{\frac{3}{x+2} + \frac{1}{x-2}}$

36. $\dfrac{\frac{1}{3x^2-3}}{\frac{5}{x+1} - \frac{x+4}{x^2-3x-4}}$

37. ★ **OPEN-ENDED MATH** Write two different complex fractions that each simplify to $\frac{x-3}{x+4}$.

CHALLENGE **Simplify the complex fraction.**

38. $\dfrac{\frac{1}{x} - \frac{x}{x^{-1}+1}}{\frac{5}{x}}$

39. $\dfrac{\frac{3-2x}{x^3}}{\frac{2}{x^2} - \frac{1}{x^3+x^2}}$

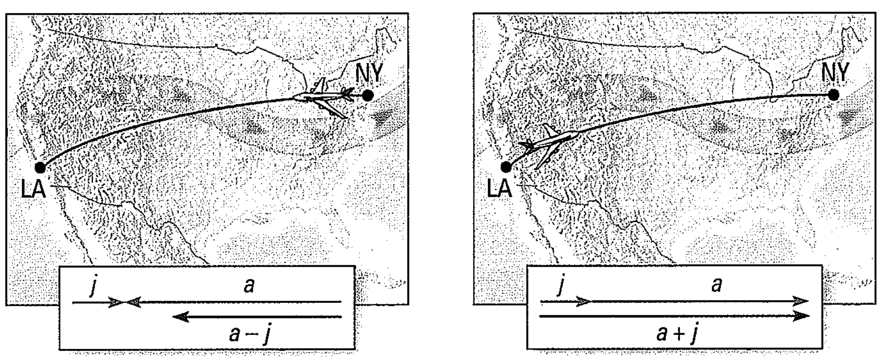
40. $\dfrac{3x^{-2} + (2x-1)^{-1}}{\frac{6}{x^{-1}+2} + 3x^{-1}}$

PROBLEM SOLVING

EXAMPLE 3
on p. 583
for Ex. 41

41. **JET STREAM** The total time T (in hours) needed to fly from New York to Los Angeles and back (ignoring layovers) can be modeled by the equation in the diagram, where d is the distance each way (in miles), a is the average airplane speed (in miles per hour), and j is the average speed of the jet stream (in miles per hour).

$$T = \frac{d}{a-j} + \frac{d}{a+j}$$

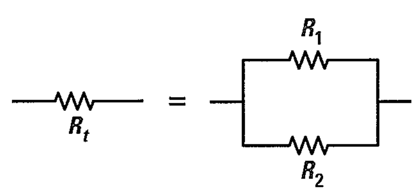


Rewrite the equation so that the right side is simplified. Then find the total time if $d = 2468$ miles, $a = 510$ mi/h, and $j = 115$ mi/h.

Animated Algebra at classzone.com

EXAMPLES 5 and 6
on p. 585
for Exs. 42–43

42. **ELECTRONICS** If two resistors in a parallel circuit have resistances R_1 and R_2 (both in ohms), then the total resistance R_t (in ohms) is given by the equation shown. Simplify the complex fraction. Then find the total resistance if $R_1 = 2000$ ohms and $R_2 = 5600$ ohms.



$$R_t = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

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43. **CAR LOANS** If you borrow P dollars to buy a car and agree to repay the loan over t years at a monthly interest rate of i (expressed as a decimal), then your monthly payment M is given by either formula below.

Formula 1: $M = \dfrac{Pi}{1 - \left(\dfrac{1}{1+i}\right)^{12t}}$ **Formula 2:** $M = \dfrac{Pi(1+i)^{12t}}{(1+i)^{12t} - 1}$

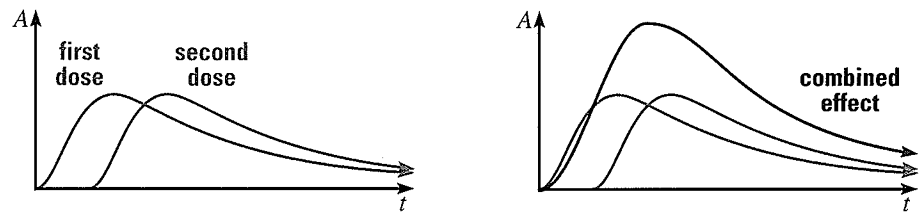
a. Show that the formulas are equivalent by simplifying the first formula.

b. Find your monthly payment if you borrow \$15,500 at a monthly interest rate of 0.5% and repay the loan over 4 years.

44. ★ **EXTENDED RESPONSE** The amount A (in milligrams) of aspirin in a person's bloodstream can be modeled by

$$A = \frac{391t^2 + 0.112}{0.218t^4 + 0.991t^2 + 1}$$

where t is the time (in hours) after one dose is taken.



a. Graph the equation using a graphing calculator.

b. A second dose of the drug is taken 1 hour after the first dose. Write an equation to model the amount of the second dose in the bloodstream.

c. Write and graph a model for the *total* amount of aspirin in the bloodstream after the second dose is taken.

d. About how long after the second dose has been taken is the greatest amount of aspirin in the bloodstream?

45. **CHALLENGE** Find the next two expressions in the pattern shown. Then simplify all five expressions. What value do the expressions approach?

$$1 + \frac{1}{2 + \frac{1}{2}},\ 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}},\ 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}},\ \dots$$

MIXED REVIEW

PREVIEW
Prepare for Lesson 8.6 in Exs. 46–54.

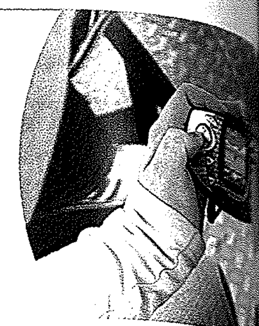
Solve the equation.

46. $\frac{1}{3}x + 4 = 15$ *(p. 18)*
47. $2x = -\frac{5}{8}x - 18$ *(p. 18)*
48. $12x + 7 = \frac{14}{3}x$ *(p. 18)*
49. $x^2 + 9x - 36 = 0$ *(p. 252)*
50. $3x^2 + x - 14 = 0$ *(p. 259)*
51. $4x(x - 5) = 4x - 35$ *(p. 259)*
52. $6x^2 - 25 = x^2$ *(p. 266)*
53. $4(x - 2)^2 = 144$ *(p. 266)*
54. $3(x + 5)^2 - 10 = 182$ *(p. 266)*

Graph the function.

55. $y = 4^x$ *(p. 478)*
56. $y = -2 \cdot 3^x$ *(p. 478)*
57. $f(x) = \frac{2}{3} \cdot 2^x$ *(p. 478)*
58. $y = 4\left(\frac{1}{2}\right)^x$ *(p. 486)*
59. $y = -3\left(\frac{1}{4}\right)^x$ *(p. 486)*
60. $g(x) = 5\left(\frac{3}{8}\right)^x$ *(p. 486)*

EXAMPLE 6 Solve a rational equation given a function



VIDEO GAME SALES From 1995 through 2003, the annual sales S (in billions of dollars) of entertainment software can be modeled by

$$S(t) = \frac{848t^2 + 3220}{115t^2 + 1000}, \quad 0 \le t \le 8$$

where t is the number of years since 1995. For which year were the total sales of entertainment software about $5.3 billion?

ANOTHER WAY
For alternative methods for solving the problem in Example 6, turn to page 596 for the **Problem Solving Workshop**.

Solution

$S(t) = \frac{848t^2 + 3220}{115t^2 + 1000}$	**Write given function.**
$5.3 = \frac{848t^2 + 3220}{115t^2 + 1000}$	**Substitute 5.3 for $S(t)$.**
$5.3(115t^2 + 1000) = 848t^2 + 3220$	**Multiply each side by $115t^2 + 1000$.**
$609.5t^2 + 5300 = 848t^2 + 3220$	**Simplify.**
$5300 = 238.5t^2 + 3220$	**Subtract $609.5t^2$ from each side.**
$2080 = 238.5t^2$	**Subtract 3220 from each side.**
$8.72 \approx t^2$	**Divide each side by 238.5.**
$\pm 2.95 \approx t$	**Take square roots of each side.**

Because -2.95 is not in the domain $(0 \le t \le 8)$, the only solution is 2.95.

▶ So, the total sales of entertainment software were about $5.3 billion about 3 years after 1995, or in 1998.

✓ **GUIDED PRACTICE** for Example 6

11. **WHAT IF?** Use the information in Example 6 to determine in which year the total sales of entertainment software were about $4.5 billion.

8.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS15 for Exs. 5, 15, and 35
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 28, 29, 34, and 36

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: When you write $\frac{x}{3} = \frac{x+2}{5}$ as $5x = 3(x + 2)$, you are __?__.

2. ★ **WRITING** A student solved the equation $\frac{5}{x-4} = \frac{x}{x-4}$ and got the solutions 4 and 5. Which, if either, of these is extraneous? *Explain.*

3. **REASONING** *Describe* how you can use a graph to determine if an apparent solution of a rational equation is extraneous.

EXAMPLE 1 on p. 589 for Exs. 4–13

CROSS MULTIPLYING Solve the equation by cross multiplying. Check for extraneous solutions.

4. $\frac{4}{2x} = \frac{5}{x+6}$

5. $\frac{9}{3x} = \frac{4}{x+2}$

6. $\frac{6}{x-1} = \frac{9}{x+1}$

7. $\frac{8}{3x-2} = \frac{2}{x-1}$

8. $\frac{x}{x+1} = \frac{3}{x+1}$

9. $\frac{x-3}{x+5} = \frac{x}{x+2}$

10. $\frac{x}{x^2-2} = \frac{-1}{x}$

11. $\frac{4(x-4)}{x^2+2x-8} = \frac{4}{x+4}$

12. $\frac{9}{x^2-6x+9} = \frac{3x}{x^2-3x}$

13. ★ **MULTIPLE CHOICE** What is the solution of $\frac{3}{x+2} = \frac{6}{x-1}$?

Ⓐ −5 Ⓑ −4 Ⓒ −1 Ⓓ 4

EXAMPLES 3, 4, and 5 on pp. 590–591 for Exs. 14–27

LEAST COMMON DENOMINATOR Solve the equation by using the LCD. Check for extraneous solutions.

14. $\frac{4}{x} + x = 5$

15. $\frac{2}{3x} + \frac{1}{6} = \frac{4}{3x}$

16. $\frac{5}{x} - 2 = \frac{2}{x+3}$

17. $\frac{1}{2x} + \frac{3}{x+7} = \frac{-1}{x}$

18. $\frac{1}{x-2} + 2 = \frac{3x}{x+2}$

19. $\frac{5}{x^2+x-6} = 2 + \frac{x-3}{x-2}$

20. $\frac{x+1}{x+6} + \frac{1}{x} = \frac{2x+1}{x+6}$

21. $\frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3}$

22. $\frac{6x}{x+4} + 4 = \frac{2x+2}{x-1}$

23. $\frac{10}{x} + 3 = \frac{x+9}{x-4}$

24. $\frac{18}{x^2-3x} - \frac{6}{x-3} = \frac{5}{x}$

25. $\frac{x+3}{x-3} + \frac{x}{x-5} = \frac{x+5}{x-5}$

ERROR ANALYSIS *Describe* and correct the error in the first step of solving the equation.

26.

$$\frac{3}{2x} + \frac{4}{x^2} = 1$$

$$3x^2 + 8x = 1$$

✗

27.

$$\frac{5}{x} + \frac{23}{6} = \frac{45}{x}$$

$$\frac{28}{x+6} = \frac{45}{x}$$

✗

28. ★ **MULTIPLE CHOICE** What is (are) the solution(s) of $\frac{2}{x-3} = \frac{1}{x^2-2x-3}$?

Ⓐ $-3, -\frac{1}{2}$ Ⓑ $-\frac{1}{2}, 3$ Ⓒ $-\frac{1}{2}$ Ⓓ 3

29. ★ **OPEN-ENDED MATH** Give an example of a rational equation that you would solve using cross multiplication. Then give an example of a rational equation that you would solve by multiplying each side by the LCD of the fractions.

CHALLENGE In Exercises 30–32, *a* is a nonzero real number. Tell whether the algebraic statement is *always true, sometimes true,* or *never true. Explain* your answer.

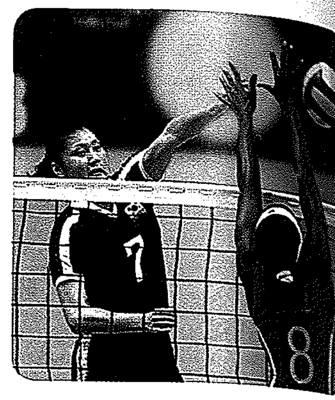
30. For the equation $\frac{1}{x-a} = \frac{x}{x-a}$, $x = a$ is an extraneous solution.

31. The equation $\frac{3}{x-a} = \frac{x}{x-a}$ has exactly one solution.

32. The equation $\frac{1}{x-a} = \frac{2}{x+a} + \frac{2a}{x^2-a^2}$ has no solution.

EXAMPLE 2 on p. 589 for Exs. 33–34

33. VOLLEYBALL So far in your volleyball match, you have put into play 37 of the 44 serves you have attempted. Solve the equation $\frac{90}{100} = \frac{37 + x}{44 + x}$ to find the number of consecutive serves you need to put into play in order to raise your service percentage to 90%.



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34. ★ EXTENDED RESPONSE A speed skater travels 9 kilometers in the same amount of time that it takes a second skater to travel 8 kilometers. The first skater travels 4.38 kilometers per hour faster than the second skater.

a. Use the verbal model below to write an equation that relates the skating times of the skaters.

$$\frac{\text{Distance for skater 1}}{\text{Skater 1 speed}} = \frac{\text{Distance for skater 2}}{\text{Skater 2 speed}}$$

b. Solve the equation in part (a) to find the speeds of both skaters.

c. How long did the skaters skate? *Explain* your answer.

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EXAMPLE 6 on p. 592 for Ex. 35

35. MUSIC INDUSTRY From 1994 through 2003, the number n (in millions) of CDs shipped can be modeled by

$$n = \frac{635t^2 - 7350t + 27{,}200}{t^2 - 11.5t + 39.4}, \quad 0 \le t \le 9$$

where t is the number of years since 1994. During which year was the total number of CDs shipped about 720 million?

36. ★ EXTENDED RESPONSE You can paint a room in 8 hours. Working together, you and your friend can paint the room in just 5 hours.

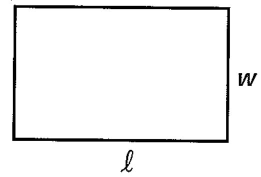
a. Let t be the time (in hours) your friend would take to paint the room when working alone. Copy and complete the table.

	Work Rate •	Time =	Work Done
You	$\frac{1 \text{ room}}{8 \text{ hours}}$	5 hours	?
Friend	?	5 hours	?

b. What is the sum of the expressions in the table's last column? *Explain.*

c. Write and solve an equation to find how long your friend would take to paint the room when working alone. *Explain* your answer.

37. GEOMETRY *Golden rectangles* are rectangles for which the ratio of the width w to the length ℓ is equal to the ratio of ℓ to $\ell + w$. The ratio of the length to the width for these rectangles is called the *golden ratio*. Find the value of the golden ratio using a rectangle with a width of 1 unit.



○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

38. **CHALLENGE** Let x be the number of years since 1998, let $g(x)$ be the average monthly bill (in dollars) for mobile phone users in the United States, and let $h(x)$ be the average number of minutes used by U.S. mobile phone users. Then $g(x)$ and $h(x)$ are as given below.

$$g(x) = -0.27x^3 + 1.40x^2 + 1.05x + 39.4$$

$$h(x) = -8.25x^3 + 53.1x^2 - 7.82x + 138$$

a. Write a rational function $f(x)$ that gives the average price per minute x years after 1998.

b. Find the average price per minute in 1998.

c. In what year did the average price per minute fall to 11 cents?

MIXED REVIEW

Graph the function.

39. $y = -2x + 7$ *(p. 89)*
40. $y = x^2 - 8x + 21$ *(p. 236)*
41. $f(x) = x^3 - 3$ *(p. 337)*
42. $y = -\sqrt{x - 4} + 1$ *(p. 446)*
43. $y = \log 4x$ *(p. 499)*
44. $g(x) = \frac{2}{x + 3} + 6$ *(p. 558)*

PREVIEW
Prepare for Lesson 9.1 in Exs. 45–52.

Simplify the expression. *(p. 266)*

45. $\sqrt{52}$
46. $\sqrt{24}$
47. $\sqrt{125}$
48. $\sqrt{252}$
49. $\sqrt{8} \cdot \sqrt{90}$
50. $\sqrt{5} \cdot \sqrt{80}$
51. $\sqrt{\frac{8}{20}}$
52. $\sqrt{\frac{60}{9}}$

QUIZ for Lessons 8.4–8.6

Perform the indicated operation and simplify. *(p. 573)*

1. $\frac{x^2 - 2x - 24}{x^2 + 3x - 10} \cdot \frac{3x^2 - 6x}{x^3 + 4x^2}$
2. $\frac{x^2 - 10x + 16}{x^2 - 1} \cdot (x - 1)$
3. $\frac{x^2 + 9x + 20}{x^2 - 11x + 28} \div \frac{x^2 + 8x + 15}{x^2 - 3x - 4}$
4. $\frac{x^2 + 12x + 36}{x^2 - 8x + 12} \div (x^2 - 36)$

Perform the indicated operation and simplify. *(p. 582)*

5. $\frac{1}{x + 4} + \frac{1}{x - 4}$
6. $\frac{4x + 3}{x^2 - 16} + \frac{2}{x - 4}$
7. $\frac{4}{x + 5} - \frac{6x - 1}{x^2 + 10x + 25}$

Solve the equation. Check for extraneous solutions. *(p. 589)*

8. $\frac{x - 4}{x - 1} = \frac{10}{x + 7}$
9. $\frac{x - 4}{x - 2} - \frac{2x - 1}{x - 2} = 2$
10. $\frac{3x + 6}{x^2 - 4} = \frac{x + 1}{x - 2}$
11. $\frac{5}{x} + \frac{x + 1}{x + 2} = \frac{2x + 9}{x + 2}$
12. $\frac{x - 3}{x + 2} = \frac{x - 1}{3x - 1}$
13. $\frac{x - 1}{x} + \frac{2x - 1}{x + 3} = \frac{x + 6}{x + 3}$

14. **BATTING AVERAGE** So far this baseball season, you have gotten a hit 12 times out of 60 at-bats. Solve the equation $0.360 = \frac{12 + x}{60 + x}$ to find the number of consecutive hits you have to get to raise your batting average to 0.360. *(p. 589)*

PROBLEM SOLVING WORKSHOP
LESSON 8.6

Using ALTERNATIVE METHODS

Another Way to Solve Example 6, page 592

MULTIPLE REPRESENTATIONS In Example 6 on page 592, you solved a rational equation algebraically. You can also solve rational equations using tables and graphs.

PROBLEM

VIDEO GAME SALES From 1995 through 2003, the annual sales S (in billions of dollars) of entertainment software can be modeled by

$$S(t) = \frac{848t^2 + 3220}{115t^2 + 1000}, \quad 0 \le t \le 8$$

where t is the number of years since 1995. For which year were the total sales of entertainment software about \$5.3 billion?

METHOD 1

Using a Table The problem requires solving the following rational equation:

$$5.3 = \frac{848t^2 + 3220}{115t^2 + 1000}$$

One way to solve this equation is to make a table of values. You can use a graphing calculator to make the table.

STEP 1 **Enter** the function $y = \frac{848x^2 + 3220}{115x^2 + 1000}$ into a graphing calculator.

```
Y1=(848X²+3220)/
(115X²+1000)
Y2=
Y3=
Y4=
Y5=
Y6=
```

STEP 2 **Set up** a table of values for the function. Start the table at zero so that the first several x-values in the table are in the domain of the function. The step value (ΔTbl) should represent one entire year.

```
TABLE SETUP
 TblStart=0
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

STEP 3 **Create** the table of values. You can see that $y \approx 5.3$ when $x = 3$.

X	Y1
0	3.22
1	3.6484
2	4.5288
3	5.3327
4	5.9113

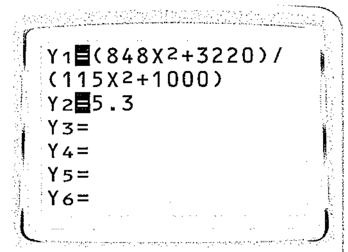
X=3

▶ Because $x = 3$ represents the number of years after 1995, total sales of entertainment software were about \$5.3 billion in 1998.

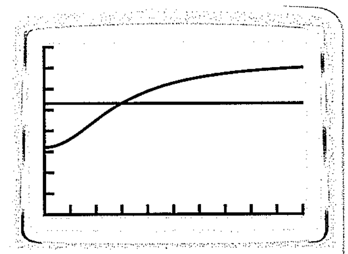
METHOD 2

Using a Graph You can also use a graph to solve $5.3 = \dfrac{848t^2 + 3220}{115t^2 + 1000}$.

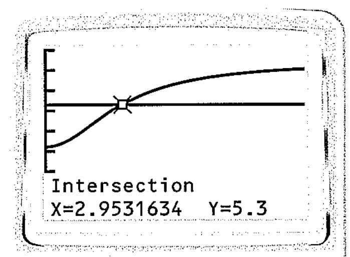
STEP 1 **Enter** the functions $y = \dfrac{848x^2 + 3220}{115x^2 + 1000}$ and $y = 5.3$ into a graphing calculator.



STEP 2 **Graph** the functions. Adjust the viewing window so that it shows the point in the first quadrant where the graphs intersect.



STEP 3 **Find** the intersection point of the graphs using the calculator's *intersect* feature. The graphs intersect at about (3.0, 5.3).



▶ Total sales of entertainment software were about $5.3 billion 3 years after 1995, or in the year 1998.

PRACTICE

RATIONAL EQUATIONS Solve the equation using a table and using a graph.

1. $\dfrac{80x^2 + 300}{15x^2 + 200} = 4.2$

2. $\dfrac{5x + 5}{x^2 + 4} = 2$

3. $\dfrac{9x + 2}{x - 5} = 20.75$

4. $\dfrac{6x^2}{2x - 3} = 18$

5. $\dfrac{14x^2 + 60}{5x^2 + 7} = 3.5$

6. **WHAT IF?** In the problem on page 596, suppose you want to find the year when total sales of entertainment software were $4.5 billion. Find this year using a table and using a graph.

7. **DIVING** The recommended percent p of oxygen (by volume) in the air that a diver breathes is given by $p = \dfrac{660}{d + 33}$ where d is the depth (in feet) of the diver.
   a. At what depth is air containing 5% oxygen recommended? Use a table to find the answer.
   b. At what depth is air containing 10% oxygen recommended? Use a graph to find the answer.

Extension Solve Rational Inequalities

Use after Lesson 8.6

GOAL Find solutions of rational inequalities.

In Lesson 8.6, you solved rational equations. You can also solve rational inequalities using tables, graphs, or algebraic methods.

EXAMPLE 1 Solve a rational inequality using a table

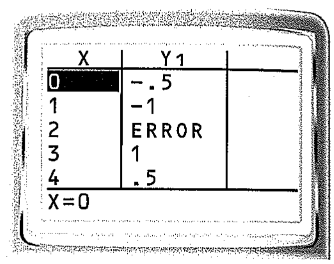
Use a table to solve $\frac{x^2 - 2x + 1}{x - 2} > x$.

Solution

Subtract x from each side of the inequality so that 0 is on one side.

$$\frac{x^2 - 2x + 1}{x - 2} - x > 0 \qquad \textbf{Subtract } x \textbf{ from each side.}$$

Enter $y = \frac{x^2 - 2x + 1}{x - 2} - x$ into a graphing calculator.



Use the *table* feature to find values of x for which y is positive.

The value of y is undefined when $x = 2$ and appears to be positive when $x > 2$. Use a smaller step value for x to convince yourself of this.

▶ The solution is $x > 2$.

EXAMPLE 2 Solve a rational inequality by graphing

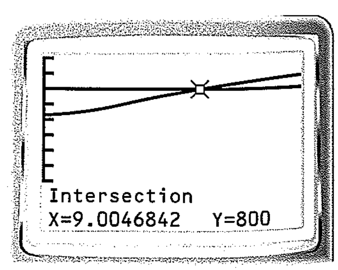
From 1990 to 2001, the number d (in thousands) of doctors in the United States can be modeled by the function $d = \frac{966t^2 + 50{,}300}{t^2 + 79.7}$ where t is the number of years since 1990. When were there fewer than 800,000 doctors?

Solution

The problem requires solving this inequality:

$$\frac{966t^2 + 50{,}300}{t^2 + 79.7} < 800$$

Enter $y_1 = \frac{966x^2 + 50{,}300}{x^2 + 79.7}$ and $y_2 = 800$ into a graphing calculator.



Graph the functions and use the *intersect* feature. The graph of y_1 lies below the graph of y_2 when $0 \le x \le 9$.

▶ In the years 1990–1999, there were fewer than 800,000 doctors.

EXAMPLE 3 Solve a rational inequality algebraically

Solve $\frac{6}{x-2} \geq -4$ algebraically.

Solution

STEP 1 **Rewrite** the inequality so that one side is 0. Then write the other side as a simplified rational expression.

AVOID ERRORS
Do not multiply each side of an inequality by an expression involving x if the expression can take on both positive and negative values.

$\frac{6}{x-2} \geq -4$ **Write original inequality.**

$\frac{6}{x-2} + 4 \geq 0$ **Add 4 to each side.**

$\frac{6 + 4(x-2)}{x-2} \geq 0$ **Write left side as a single fraction.**

$\frac{4x-2}{x-2} \geq 0$ **Simplify.**

STEP 2 **Identify** the *critical x-values*, which are the x-values that make the numerator or denominator equal to 0.

Numerator equal to 0:

$4x - 2 = 0$

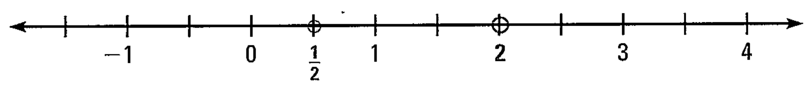
$x = \frac{1}{2}$

Denominator equal to 0:

$x - 2 = 0$

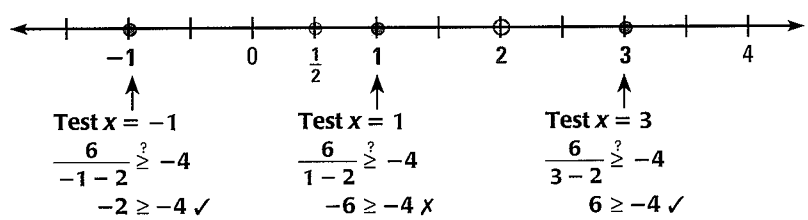
$x = 2$

So, the critical x-values are $x = \frac{1}{2}$ and $x = 2$.

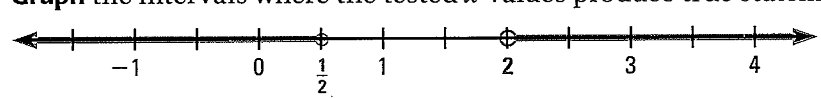


The critical x-values divide the number line into three intervals. Note that $x = \frac{1}{2}$ will be included in the solution, but $x = 2$ will not because it results in division by zero.

STEP 3 **Test** an x-value in each interval to see if it satisfies the original inequality. If it does, *every* x-value in the interval will satisfy the inequality. If it does not, *no* x-value in the interval will satisfy the inequality.



STEP 4 **Graph** the intervals where the tested x-values produce true statements.



STEP 5 **Write** inequalities to describe the solution.

▶ The solution is $x \leq \frac{1}{2}$ or $x > 2$.

PRACTICE

EXAMPLE 1 on p. 598 for Exs. 1–6

Use a table to solve the inequality.

1. $\frac{5}{x-2} < 0$
2. $\frac{x-5}{x+3} > 1$
3. $\frac{x^2-3x+2}{x-3} < x$
4. $\frac{10}{x+2} > 0$
5. $\frac{-2x-3}{x-4} > 0$
6. $\frac{x^2-4x+8}{x-1} < x$

EXAMPLE 2 on p. 598 for Exs. 7–12

Use a graph to solve the inequality.

7. $-\frac{4}{x+5} < 0$
8. $\frac{4}{x-3} < 0$
9. $\frac{8}{x^2+1} \geq 4$
10. $\frac{20}{x^2+1} < 2$
11. $\frac{3x+2}{x-1} < -2$
12. $\frac{3x+2}{x-1} > x$

EXAMPLE 3 on p. 599 for Exs. 13–18

Solve the inequality algebraically.

13. $\frac{3}{x+2} > 0$
14. $-\frac{1}{x+5} \leq -2$
15. $\frac{2}{x+2} > \frac{1}{x+3}$
16. $\frac{5}{x-4} < \frac{1}{x+4}$
17. $\frac{5}{x+3} \geq \frac{4}{x+2}$
18. $\frac{2}{x+6} > \frac{-3}{x-3}$

19. **EGG PRODUCTION** From 1994 to 2002, the total number E (in billions) of eggs produced in the United States can be modeled by

$$E = \frac{-3680}{t-50}, \quad 0 \leq t \leq 8$$

where t is the number of years since 1994. For what years was the number of eggs produced greater than 80 billion?

20. **PHONE COSTS** One phone company advertises a flat rate of $.07 per minute for long-distance calls. Your long-distance plan charges $5.00 per month plus a rate of $.05 per minute. How many minutes do you have to talk each month so that your average cost is less than $.07 per minute?

21. **SATELLITE TV** You subscribe to a satellite television service. The monthly cost for programming is $43, and there is a one-time installation fee of $50. The average monthly cost c of the service is given by $c = \frac{43t+50}{t}$ where t is the time (in months) that you have subscribed to the service. For what subscription times is the average monthly cost at most $47? Solve the problem using a table and using a graph.

22. **FUNDRAISER** Your school is publishing a wildlife calendar to raise money for a local charity. The total cost of using the photos in the calendar is $710. In addition to this one-time charge, the unit cost of printing each calendar is $4.50.
    a. The school wants the average cost per calendar to be below $10. Write a rational inequality relating the average cost per calendar to the desired cost per calendar.
    b. Solve the inequality from part (a) by graphing. How many calendars need to be printed to bring the average cost per calendar below $10?
    c. Suppose the school wanted to have the average cost per calendar be below $6. How many calendars would then need to be printed?

Lessons 8.4–8.6

1. **MULTI-STEP PROBLEM** A cyclist travels 50 miles from her home to a state park at a speed of s miles per hour. On the return trip, she increases her speed by 5 miles per hour.

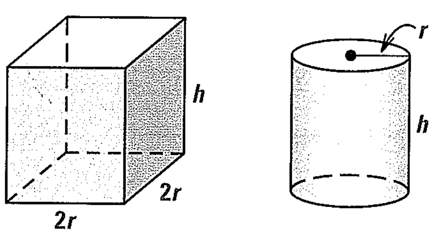
   a. Write an expression in terms of s for the time the cyclist takes to travel from her home to the state park.

   b. Write an expression in terms of s for the time the cyclist takes to return home from the state park.

   c. Write an expression in simplified form for the *total* time of the cyclist's round trip.

2. **SHORT RESPONSE** The speed of a river's current is 3 miles per hour. You travel 2 miles with the current and then return to where you started in a total time of 1.25 hours. What is your speed in still water?

3. **SHORT RESPONSE** A manufacturer of instant rice is considering two different styles of packaging. One is a rectangular container with a square base. The other is a cylinder.



   a. Find the ratio of surface area to volume for each container.

   b. Using the ratios, what can you determine about the efficiencies of the containers?

4. **OPEN-ENDED** Write two rational expressions $r(x)$ and $s(x)$ such that $r(x)$ and $s(x)$ each contain a quadratic polynomial and $r(x) \cdot s(x) = \frac{x-3}{x+4}$.

5. **MULTI-STEP PROBLEM** Brass is an alloy composed of 55% copper and 45% zinc by weight. You have 25 ounces of copper, and you want to determine how many ounces of zinc you need to make brass.

   a. Let x be the number of ounces of zinc you need. Write a verbal model and then a rational equation that you can use to find x.

   b. Solve the equation from part (a) to find the number of ounces of zinc you need to make brass.

   c. Consider the more general case where you have c ounces of copper. In terms of c, how many ounces of zinc must be added to make brass?

6. **EXTENDED RESPONSE** A car travels 120 miles in the same amount of time that it takes a truck to travel 100 miles. The car travels 10 miles per hour faster than the truck.

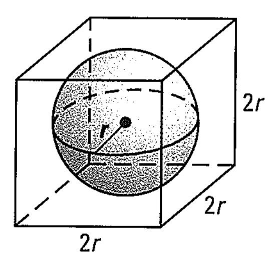
   a. Use the verbal model below to write an equation that relates the speeds of the vehicles.

$$\frac{\text{Distance for car}}{\text{Speed of car}} = \frac{\text{Distance for truck}}{\text{Speed of truck}}$$

   b. Solve the equation from part (a) to find the speeds of the car and the truck.

   c. How much time did the vehicles spend traveling? *Explain* your answer.

7. **GRIDDED ANSWER** Find the ratio of the volume of the sphere to the volume of the cube in the diagram below.



Use the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere and the formula $V = s^3$ for the volume of a cube where r is the radius of the sphere and s is the side length of the cube. Write your answer as a decimal rounded to the nearest hundredth.

8 CHAPTER SUMMARY

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BIG IDEAS — For Your Notebook

Big Idea 1

Graphing Rational Functions

Use the following steps to graph $f(x) = \frac{p(x)}{q(x)} = \frac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}$

where $p(x)$ and $q(x)$ have no common factors other than ± 1.

STEP 1 **Plot** the x-intercepts. The x-intercepts are the real zeros of $p(x)$.

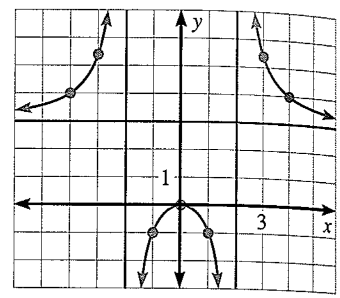
STEP 2 **Draw** the vertical asymptote(s). A vertical asymptote occurs at each real zero of $q(x)$.

STEP 3 **Draw** the horizontal asymptote, if it exists.

If $m < n$, $y = 0$ is a horizontal asymptote.

If $m = n$, $y = \frac{a_m}{b_n}$ is a horizontal asymptote.

If $m > n$, there is no horizontal asymptote.



$y = \frac{3x^2}{x^2 - 4}$

STEP 4 **Plot** several points on both sides of each vertical asymptote.

Big Idea 2

Performing Operations with Rational Expressions

Operation	Example
Simplify Divide out common factors from the numerator and denominator.	$\frac{x^2 + 3x}{x^2 + 8x + 15} = \frac{x\cancel{(x+3)}}{(x+5)\cancel{(x+3)}} = \frac{x}{x+5}$
Multiply Multiply numerators and denominators. Then simplify.	$\frac{x}{15} \cdot \frac{3}{x^2 + 7x} = \frac{3x}{15x(x+7)} = \frac{1}{5(x+7)}$
Divide Multiply the first expression by the reciprocal of the second expression. Then simplify.	$\frac{x^2}{3x+1} \div \frac{1}{6x+2} = \frac{x^2}{3x+1} \cdot \frac{2(3x+1)}{1} = 2x^2$
Add or Subtract Write the expressions with like denominators. Then add or subtract the numerators over the common denominator. Lastly, simplify.	$\frac{5}{x} + \frac{x}{x+2} = \frac{5(x+2)}{x(x+2)} + \frac{x^2}{x(x+2)} = \frac{x^2 + 5x + 10}{x(x+2)}$

Big Idea 3

Solving Rational Equations

Solve $\frac{x}{x+1} + \frac{2}{x+4} = 1$.

STEP 1 **Find** the LCD. LCD is $(x+1)(x+4)$.

STEP 2 **Multiply** each side of the equation by the LCD. $x(x+4) + 2(x+1) = (x+1)(x+4)$

STEP 3 **Solve** the resulting equation.

$$x^2 + 4x + 2x + 2 = x^2 + 5x + 4$$
$$6x + 2 = 5x + 4$$
$$x = 2$$

8 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

- inverse variation, *p. 551*
- constant of variation, *p. 551*
- joint variation, *p. 553*
- rational function, *p. 558*
- simplified form of a rational expression, *p. 573*
- complex fraction, *p. 584*
- cross multiplying, *p. 589*

VOCABULARY EXERCISES

1. Copy and complete: If two variables x and y are related by an equation of the form $y = \frac{a}{x}$ where $a \neq 0$, then x and y show _?_.
2. Suppose z varies jointly with x and y. What can you say about $\frac{z}{xy}$?
3. Copy and complete: A function of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$ is called a(n) _?_.
4. Give two examples of a complex fraction.
5. Copy and complete: When you rewrite the equation $\frac{3}{x} = \frac{2}{x-1}$ as $3(x - 1) = 2x$, you are _?_.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 8.

8.1 Model Inverse and Joint Variation

pp. 551–557

EXAMPLE

The variables x and y vary inversely, and $y = 12$ when $x = 3$. Write an equation that relates x and y. Then find y when $x = -4$.

$y = \frac{a}{x}$ **Write general equation for inverse variation.**

$12 = \frac{a}{3}$ **Substitute 12 for y and 3 for x.**

$36 = a$ **Solve for a.**

▶ The inverse variation equation is $y = \frac{36}{x}$. When $x = -4$, $y = \frac{36}{-4} = -9$.

EXERCISES

EXAMPLE 2 on p. 551 for Exs. 6–9

The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = -3$.

6. $x = 1, y = 5$ **7.** $x = -4, y = -6$ **8.** $x = \frac{5}{2}, y = 18$ **9.** $x = -12, y = \frac{2}{3}$

8 CHAPTER REVIEW

8.2 Graph Simple Rational Functions

pp. 558–563

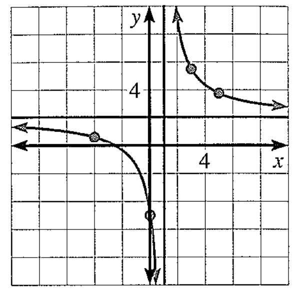
EXAMPLE

Graph $y = \frac{2x+5}{x-1}$. State the domain and range.

STEP 1 **Draw** the asymptotes. Solve $x - 1 = 0$ for x to find the vertical asymptote $x = 1$. The horizontal asymptote is the line $y = \frac{2}{1} = 2$.

STEP 2 **Plot** points to the left and to the right of the vertical asymptote.

STEP 3 **Draw** the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



▶ The domain is all real numbers except 1. The range is all real numbers except 2.

EXERCISES

EXAMPLES 2 and 3 on pp. 559–560 for Exs. 10–12

Graph the function. State the domain and range.

10. $y = \frac{4}{x-3}$

11. $y = \frac{1}{x+5} + 2$

12. $f(x) = \frac{3x-2}{x-4}$

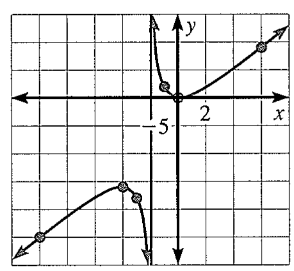
8.3 Graph General Rational Functions

pp. 565–571

EXAMPLE

Graph $y = \frac{2x^2}{x+2}$.

- The numerator has 0 as its only zero, so the graph has an x-intercept at (0, 0).
- The denominator has −2 as its only zero, so the graph has a vertical asymptote at $x = -2$.
- The degree of the numerator (2) is greater than the degree of the denominator (1). So, there is no horizontal asymptote. The graph has the same end behavior as the graph of $y = \frac{2}{1}x^{2-1} = 2x$.



EXERCISES

EXAMPLES 1, 2, and 3 on pp. 565–566 for Exs. 13–18

Graph the function.

13. $y = \frac{5}{x^2+1}$

14. $y = \frac{4x^2}{x-1}$

15. $h(x) = \frac{6x^2}{x-2}$

16. $y = \frac{-8}{x^2+3}$

17. $y = \frac{x^2+6}{x^2-3x-40}$

18. $g(x) = \frac{x^2-1}{x+4}$

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Chapter Review Practice

8.4 Multiply and Divide Rational Expressions

pp. 573–580

EXAMPLE

Divide: $\frac{3x+27}{6x-48} \div \frac{x^2+9x}{x^2-4x-32}$

$$\frac{3x+27}{6x-48} \div \frac{x^2+9x}{x^2-4x-32} = \frac{3x+27}{6x-48} \cdot \frac{x^2-4x-32}{x^2+9x}$$ **Multiply by reciprocal.**

$$= \frac{3(x+9)}{6(x-8)} \cdot \frac{(x+4)(x-8)}{x(x+9)}$$ **Factor.**

$$= \frac{\cancel{3}\cancel{(x+9)}(x+4)\cancel{(x-8)}}{2\cancel{(3)}\cancel{(x-8)}(x)\cancel{(x+9)}}$$ **Divide out common factors.**

$$= \frac{x+4}{2x}$$ **Simplified form**

EXERCISES

EXAMPLES 3, 4, 6, and 7 on pp. 575–577 for Exs. 19–22

Perform the indicated operation. Simplify the result.

19. $\frac{80x^4}{y^3} \cdot \frac{xy}{5x^2}$

20. $\frac{x-3}{2x-8} \cdot \frac{6x^2-96}{x^2-9}$

21. $\frac{16x^2-8x+1}{x^3-7x^2+12x} \div \frac{20x^2-5x}{15x^3}$

22. $\frac{x^2-13x+40}{x^2-2x-15} \div (x^2-5x-24)$

8.5 Add and Subtract Rational Expressions

pp. 582–588

EXAMPLE

Add: $\frac{x}{6x+24} + \frac{x+2}{x^2+9x+20}$

The denominators factor as $6(x+4)$ and $(x+4)(x+5)$, so the LCD is $6(x+4)(x+5)$. Use this result to rewrite each expression with a common denominator, and then add.

$$\frac{x}{6x+24} + \frac{x+2}{x^2+9x+20} = \frac{x}{6(x+4)} + \frac{x+2}{(x+4)(x+5)}$$

$$= \frac{x}{6(x+4)} \cdot \frac{x+5}{x+5} + \frac{x+2}{(x+4)(x+5)} \cdot \frac{6}{6}$$

$$= \frac{x^2+5x}{6(x+4)(x+5)} + \frac{6x+12}{6(x+4)(x+5)}$$

$$= \frac{x^2+11x+12}{6(x+4)(x+5)}$$

EXERCISES

EXAMPLES 3 and 4 on pp. 583–584 for Exs. 23–25

Perform the indicated operation and simplify.

23. $\frac{5}{6(x+3)} + \frac{x+4}{2x}$

24. $\frac{5x}{x+8} + \frac{4x-9}{x^2+5x-24}$

25. $\frac{x+2}{x^2+4x+3} - \frac{5x}{x^2-9}$

8 CHAPTER REVIEW

8.6 Solve Rational Equations

pp. 589–595

EXAMPLE

Solve: $\frac{3x}{x+1} + \frac{6}{2x} = \frac{7}{x}$

The least common denominator is $2x(x + 1)$.

$\frac{3x}{x+1} + \frac{6}{2x} = \frac{7}{x}$	**Write original equation.**
$2x(x+1)\left(\frac{3x}{x+1} + \frac{6}{2x}\right) = 2x(x+1) \cdot \frac{7}{x}$	**Multiply each side by the LCD, $2x(x + 1)$.**
$2x(3x) + 6(x + 1) = 2(x + 1)(7)$	**Simplify.**
$6x^2 + 6x + 6 = 14x + 14$	**Simplify.**
$6x^2 - 8x - 8 = 0$	**Write in standard form.**
$3x^2 - 4x - 4 = 0$	**Divide each side by 2.**
$(3x + 2)(x - 2) = 0$	**Factor.**
$3x + 2 = 0$ or $x - 2 = 0$	**Zero product property**
$x = -\frac{2}{3}$ or $x = 2$	**Solve for x.**

▶ The solutions are $-\frac{2}{3}$ and 2. Check these in the original equation to make sure neither solution is extraneous.

EXERCISES

EXAMPLES 1, 4, and 5 on pp. 589–591 for Exs. 26–36

Solve the equation by cross multiplying. Check your solution(s).

26. $\frac{2x}{9} = \frac{2}{x}$

27. $\frac{5}{x} = \frac{7}{x+2}$

28. $\frac{x-1}{4} = \frac{3x}{9}$

29. $\frac{2}{x+2} = \frac{6}{2x+5}$

30. $\frac{x+12}{3} = \frac{2x+3}{x+2}$

31. $\frac{2x}{x+4} = \frac{-3x}{4x-3}$

Solve the equation by using the LCD. Check for extraneous solutions.

32. $\frac{5}{2} + \frac{3}{x} = 3$

33. $\frac{8(x-1)}{x^2-4} = \frac{4}{x+2}$

34. $\frac{3x}{x+1} = \frac{12}{x^2-1} + 2$

35. $\frac{2(x+7)}{x+4} - 2 = \frac{2x+20}{2x+8}$

36. BASKETBALL So far this season, a basketball player has made 60 of 75 free-throw attempts.

a. Write a rational expression that represents the player's free-throw percentage (expressed as a decimal) if she makes her next x free throws.

b. How many consecutive free throws must the player make in order to raise her free-throw percentage to at least 82%?

8 CHAPTER TEST

The variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when $x = 4$.

1. $x = 5, y = 2$
2. $x = -2, y = 8$
3. $x = \frac{3}{2}, y = 10$
4. $x = 3, y = 6$
5. $x = -4, y = \frac{7}{2}$
6. $x = \frac{3}{4}, y = \frac{5}{8}$

Graph the function. State the domain and range.

7. $y = \frac{2}{x + 5} - 3$
8. $y = \frac{-1}{x - 4} - 1$
9. $f(x) = \frac{6 - x}{2x + 1}$

Graph the function.

10. $y = \frac{4}{x^2 + 2}$
11. $y = \frac{x^2 - 4}{x^2 + 8x + 15}$
12. $g(x) = \frac{x^2 + 3}{2x - 1}$

Find the least common multiple of the polynomials.

13. $(x - 3)(x + 5)$ and $x(x + 5)$
14. $4x^2(x - 2)$ and $8x(x + 2)$
15. $x^2 - 4x$ and $x^2 - 2x - 8$
16. $2x + 6$ and $x^3 + 10x^2 + 21x$

Perform the indicated operation and simplify.

17. $\frac{3x^2y}{4x^3y^5} \div \frac{6y^2}{2xy^3}$
18. $\frac{x^2 - 3x - 4}{x^2 - 3x - 18} \cdot \frac{x - 6}{x + 1}$
19. $\frac{x^2 - 8x + 15}{x^2 + 12x + 32} \cdot \frac{x + 4}{x^2 - 25}$
20. $\frac{x^2 - 11x + 28}{x^2 + 5x + 4} \div (x^2 - 16)$
21. $\frac{3x}{x + 5} - \frac{4x + 1}{x + 5}$
22. $\frac{4}{x - 3} + \frac{2}{x + 6}$
23. $\frac{3x}{x^2 + x - 12} - \frac{6}{x + 4}$
24. $\frac{4}{x + 5} + \frac{2x}{x^2 - 25}$

Solve the equation. Check for extraneous solutions.

25. $\frac{3}{x + 2} = \frac{x - 3}{2x + 4}$
26. $\frac{1}{x + 6} + \frac{x + 1}{x} = \frac{13}{x + 6}$
27. $\frac{x - 2}{x - 1} = \frac{x + 2}{x + 4}$

28. **SOUND INTENSITY** The intensity I of a sound varies inversely with the square of the distance r from the source of the sound. Write an equation relating I, r, and a constant a.

29. **CABLE TV** You have subscribed to a cable television service. The cable company charges you a one-time installation fee of \$30 and a monthly fee of \$50. Write and graph a model that gives the average cost per month as a function of the number of months you have subscribed to the service. After how many months will the average cost be \$56?

30. **WEB HOSTING** You are building a new website for your school. A company that hosts websites offers a dedicated server for a \$50 setup fee plus a monthly fee of \$99. How many months would you need to use this service in order for your average monthly cost to fall to \$100?