5.1 Properties of Exponents

① Products of Powers

$$2^3 \cdot 2^4 = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2) = 2^7 \text{ or } 2^{3+4} = 2^7$$

$$4^3 \cdot 12^7 = 4^3 \cdot 12^7 \quad \text{don't have same bases}$$

So $a^m \cdot a^n = a^{m+n}$ ← Add exponents when same base

② Power of a Power

$$(5^3)^2 = 5^3 \cdot 5^3 = 5^6 \text{ or } 5^{3 \cdot 2} = 5^6$$

So $(a^m)^n = a^{m \cdot n}$ ← multiply exponents

③ Power of a Product

$$(2^3 \cdot 7^1)^3 = 2^3 7^1 \cdot 2^3 7^1 \cdot 2^3 7^1 = 2^9 7^3 \text{ or}$$

$$(2^3 \cdot 7^1)^3 \quad \text{distribute the 3}$$

$$2^{3 \cdot 3} \cdot 7^{1 \cdot 3} = 2^9 7^3$$

So $(a^n \cdot b)^m = a^{n \cdot m} b^{n \cdot m}$ ← distribute the exponent to the exponents

④ Quotient of Powers

$$\frac{4^5}{4^2} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4} = \frac{4^3}{1} = 4^3 \text{ or } 4^{5-2} = 4^3$$

$$\frac{4^3}{4^7} = \frac{4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \frac{1}{4^4} = \frac{1}{4^4} \text{ or } \frac{1}{4^{7-3}} = \frac{1}{4^4}$$

$$\frac{a^m}{a^n} = \frac{a^{m-n}}{\text{good wins}} = a^{m-n} \quad \text{or} \quad \frac{1}{a^{n-m}}_{\text{evil wins}}$$

$$\text{If } m > n \qquad\qquad \text{If } n > m$$