

3.7 Finding the Determinate

- * Used to solve systems of equations (like graph, sub & elim)
- & Area of triangles

The Determinant of:

2x2 Matrix

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

3x3 Matrix

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aet + bfg + cdh) - (gec + hfa + idb)$$

Example - Find the det

1) $\begin{bmatrix} 3 & 6 \\ 2 & -4 \end{bmatrix}$

$$\begin{aligned} & 3(-4) - 2(6) \\ & -12 - 12 \\ & = -24 \end{aligned}$$

2) $\begin{bmatrix} 7 & -2 \\ 4 & 5 \end{bmatrix}$

$$\begin{aligned} & 35 + 8 \\ & = 43 \end{aligned}$$

3) $\begin{bmatrix} -1 & 5 \\ 0 & 3 \end{bmatrix}$

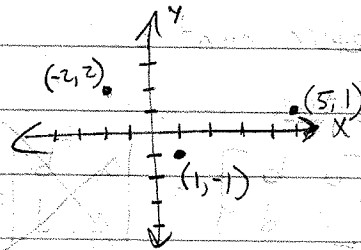
$$\begin{aligned} & -3 - 0 \\ & = -3 \end{aligned}$$

4) $\begin{bmatrix} 2 & -1 & -3 \\ 4 & 1 & 0 \\ 3 & -4 & -2 \end{bmatrix}$

$$\begin{aligned} & = [2(1)(-2) + (-1)(0)(3) + (-3)(4)(-4)] - [(-3)(1)(3) + (-1)(4)(-2) + (2)(0)(-4)] \\ & = (-4 + 0 + 48) - (-9 + 8 + 0) \\ & = 44 - -1 \\ & = 45 \end{aligned}$$

Area of D

$$\text{Area} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$



Plug in coordinates into matrix
Mult by $\frac{1}{2}$ or $-\frac{1}{2}$ to get positive value

$$A = \frac{1}{2} \begin{vmatrix} -2 & 2 & 1 \\ 5 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = (-2+2-5) - (1+2+10) = -5 - 13 = -\frac{1}{2}(-18) = 9$$

Cramer's Rule - Solve System of Equations

For $ax + by = e$
 $cx + dy = f$ the coefficient matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

The solution is $x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A}$ and $y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$

Example

$$4x + 3y = -2$$

$$x + 5y = -9$$

$$A = \begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} \quad \det A = 20 - 3 = 17$$

$$x = \frac{\begin{vmatrix} -2 & 3 \\ -9 & 5 \end{vmatrix}}{17} = \frac{-10 + 27}{17} = \frac{17}{17} = 1$$

$$y = \frac{\begin{vmatrix} 4 & -2 \\ 1 & -9 \end{vmatrix}}{17} = \frac{-36 + 2}{17} = \frac{-34}{17} = -2$$

$(1, -2)$

Homework
p207: 3, 7, 12, 23,
29, 30, 41,
try 40, 42