12 Surface Area and Volume of Solids

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 12: properties of similar polygons, areas and perimeters of two-dimensional figures, and right triangle trigonometry.

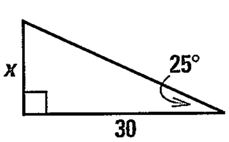
Prerequisite Skills

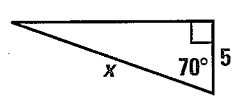
VOCABULARY CHECK

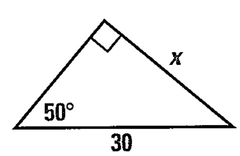
1. Copy and complete: The area of a regular polygon is given by the formula $A =$ _?_.
2. *Explain* what it means for two polygons to be similar.

SKILLS AND ALGEBRA CHECK

Use trigonometry to find the value of *x*. ***(Review pp. 466, 473 for 12.2–12.5.)***

3.


4.


5.


Find the circumference and area of the circle with the given dimension. ***(Review pp. 746, 755 for 12.2–12.5.)***

6. $r = 2$ m
7. $d = 3$ in.
8. $r = 2\sqrt{5}$ cm

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 12, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 856. You will also use the key vocabulary listed below.

Big Ideas

1. **Exploring solids and their properties**
2. **Solving problems using surface area and volume**
3. **Connecting similarity to solids**

KEY VOCABULARY

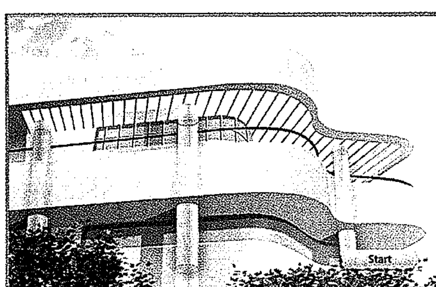
- polyhedron, *p. 794*
  face, edge, vertex
- Platonic solids, *p. 796*
- cross section, *p. 797*
- prism, *p. 803*
- surface area, *p. 803*
- lateral area, *p. 803*
- net, *p. 803*
- right prism, *p. 804*
- oblique prism, *p. 804*
- cylinder, *p. 805*
- right cylinder, *p. 805*
- pyramid, *p. 810*
- regular pyramid, *p. 810*
- cone, *p. 812*
- right cone, *p. 812*
- volume, *p. 819*
- sphere, *p. 838*
- great circle, *p. 839*
- hemisphere, *p. 839*
- similar solids, *p. 847*

Why?

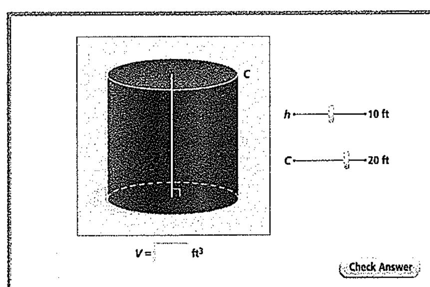
Knowing how to use surface area and volume formulas can help you solve problems in three dimensions. For example, you can use a formula to find the volume of a column in a building.

Animated Geometry

The animation illustrated below for Exercise 31 on page 825 helps you answer this question: What is the volume of the column?



You can use the height and circumference of a column to find its volume.



Drag the sliders to change the height and circumference of the cylinder.

Animated Geometry at classzone.com

Other animations for Chapter 12: pages 795, 805, 821, 833, 841, and 852

12.1 Investigate Solids

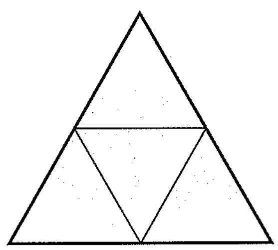
MATERIALS • poster board • scissors • tape • straightedge

QUESTION **What solids can be made using congruent regular polygons?**

Platonic solids, named after the Greek philosopher Plato (427 B.C.–347 B.C.), are solids that have the same congruent regular polygon as each *face,* or side of the solid.

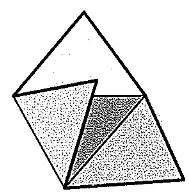
EXPLORE 1 Make a solid using four equilateral triangles

STEP 1



Make a net Copy the full-sized triangle from page 793 on poster board to make a template. Trace the triangle four times to make a *net* like the one shown.

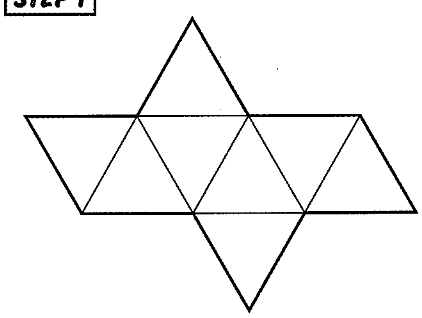
STEP 2



Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each *vertex*?

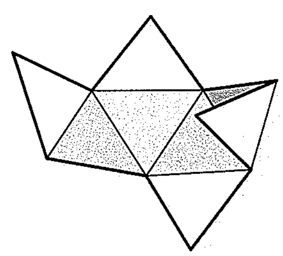
EXPLORE 2 Make a solid using eight equilateral triangles

STEP 1



Make a net Trace your triangle template from Explore 1 eight times to make a net like the one shown.

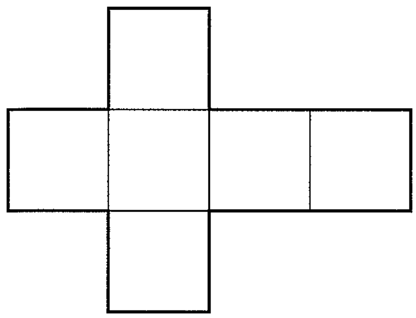
STEP 2



Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each vertex?

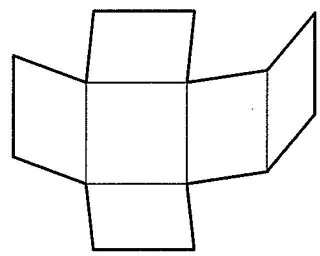
EXPLORE 3 Make a solid using six squares

STEP 1



Make a net Copy the full-sized square from the bottom of the page on poster board to make a template. Trace the square six times to make a net like the one shown.

STEP 2

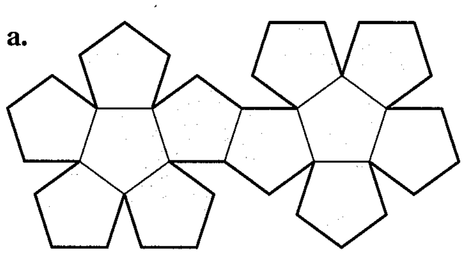


Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each vertex?

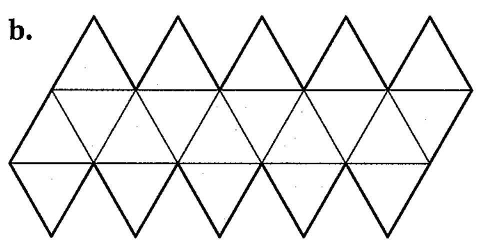
DRAW CONCLUSIONS Use your observations to complete these exercises

1. The two other convex solids that you can make using congruent, regular faces are shown below. For each of these solids, how many faces meet at each vertex?

   a.

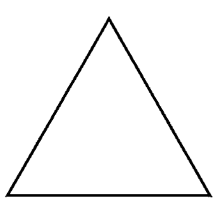
   

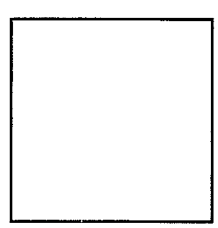
   b.

2. *Explain* why it is not possible to make a solid that has six congruent equilateral triangles meeting at each vertex.
3. *Explain* why it is not possible to make a solid that has three congruent regular hexagons meeting at each vertex.
4. Count the number of vertices V, edges E, and faces F for each solid you made. Make a conjecture about the relationship between the sum $F + V$ and the value of E.

Templates:





12.1 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS16 for Exs. 11, 25, and 35
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 28, 30, 31, 39, and 41

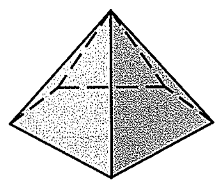
SKILL PRACTICE

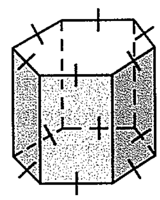
1. **VOCABULARY** Name the five Platonic solids and give the number of faces for each.

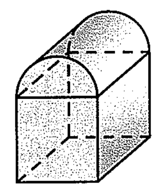
2. ★ **WRITING** State Euler's Theorem in words.

EXAMPLE 1 on p. 795 for Exs. 3–10

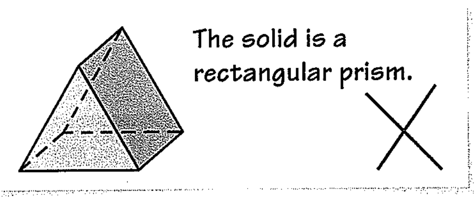
IDENTIFYING POLYHEDRA **Determine whether the solid is a polyhedron. If it is, name the polyhedron. *Explain* your reasoning.**

3.


4.


5.


6. **ERROR ANALYSIS** *Describe* and correct the error in identifying the solid.



SKETCHING POLYHEDRA **Sketch the polyhedron.**

7. Rectangular prism
8. Triangular prism
9. Square pyramid
10. Pentagonal pyramid

EXAMPLES 2 and 3 on pp. 796–797 for Exs. 11–24

APPLYING EULER'S THEOREM **Use Euler's Theorem to find the value of n.**

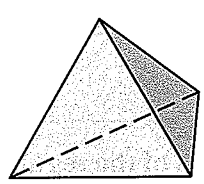
11. Faces: n
Vertices: 12
Edges: 18

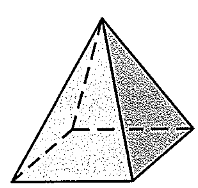
12. Faces: 5
Vertices: n
Edges: 8

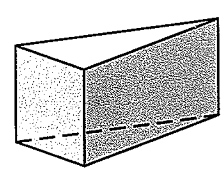
13. Faces: 10
Vertices: 16
Edges: n

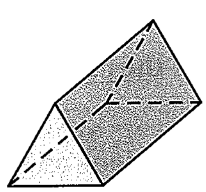
14. Faces: n
Vertices: 12
Edges: 30

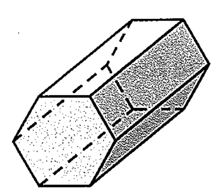
APPLYING EULER'S THEOREM **Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.**

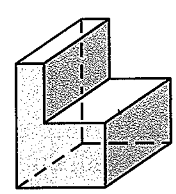
15.


16.


17.


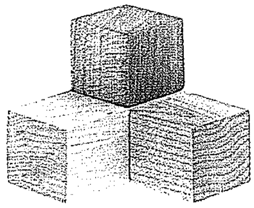
18.


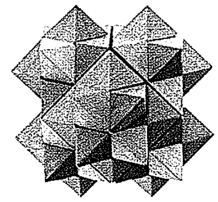
19.


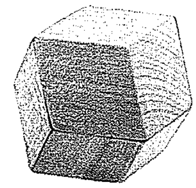
20.


21. ★ **WRITING** *Explain* why a cube is also called a regular hexahedron.

PUZZLES **Determine whether the solid puzzle is *convex* or *concave*.**

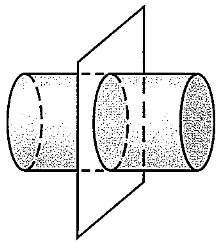
22. 

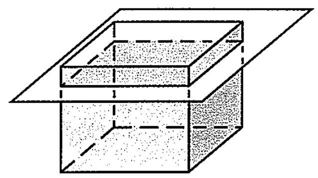
23. 

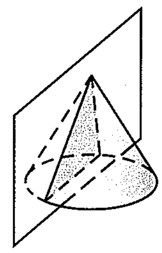
24. 

EXAMPLE 4
on p. 797
for Exs. 25–28

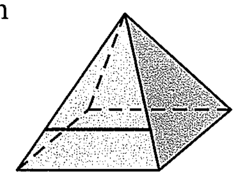
CROSS SECTIONS **Draw and *describe* the cross section formed by the intersection of the plane and the solid.**

25. 

26. 

27. 

28. ★ **MULTIPLE CHOICE** What is the shape of the cross section formed by the plane parallel to the base that intersects the red line drawn on the square pyramid?



Ⓐ Square Ⓑ Triangle

Ⓒ Kite Ⓓ Trapezoid

29. **ERROR ANALYSIS** *Describe* and correct the error in determining that a tetrahedron has 4 faces, 4 edges, and 6 vertices.

30. ★ **MULTIPLE CHOICE** Which two solids have the same number of faces?

Ⓐ A triangular prism and a rectangular prism

Ⓑ A triangular pyramid and a rectangular prism

Ⓒ A triangular prism and a square pyramid

Ⓓ A triangular pyramid and a square pyramid

31. ★ **MULTIPLE CHOICE** How many faces, vertices, and edges does an octagonal prism have?

Ⓐ 8 faces, 6 vertices, and 12 edges

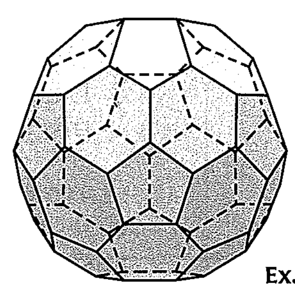
Ⓑ 8 faces, 12 vertices, and 18 edges

Ⓒ 10 faces, 12 vertices, and 20 edges

Ⓓ 10 faces, 16 vertices, and 24 edges

32. **EULER'S THEOREM** The solid shown has 32 faces and 90 edges. How many vertices does the solid have? *Explain* your reasoning.

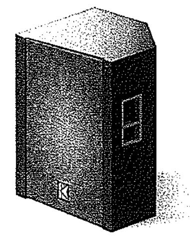
33. **CHALLENGE** *Describe* how a plane can intersect a cube to form a hexagonal cross section.



Ex. 32

EXAMPLE 2
on p. 796
for Exs. 34–35

34. **MUSIC** The speaker shown at the right has 7 faces. Two faces are pentagons and 5 faces are rectangles.

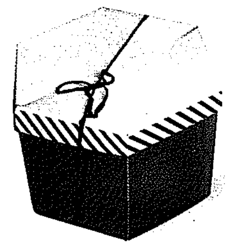


a. Find the number of vertices.

b. Use Euler's Theorem to determine how many edges the speaker has.

@HomeTutor for problem solving help at classzone.com

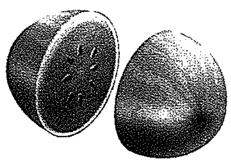
(35.) **CRAFT BOXES** The box shown at the right is a hexagonal prism. It has 8 faces. Two faces are hexagons and 6 faces are squares. Count the edges and vertices. Use Euler's Theorem to check your answer.



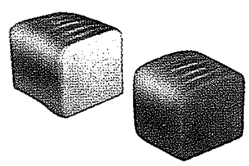
@HomeTutor for problem solving help at classzone.com

FOOD ***Describe*** **the shape that is formed by the cut made in the food shown.**

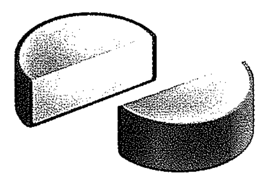
36. Watermelon



37. Bread

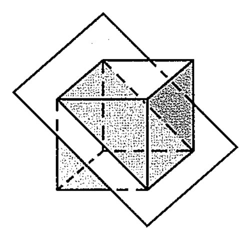


38. Cheese



39. ★ **SHORT RESPONSE** Name a polyhedron that has 4 vertices and 6 edges. Can you draw a polyhedron that has 4 vertices, 6 edges, and a different number of faces? *Explain* your reasoning.

40. **MULTI-STEP PROBLEM** The figure at the right shows a plane intersecting a cube through four of its vertices. An edge length of the cube is 6 inches.

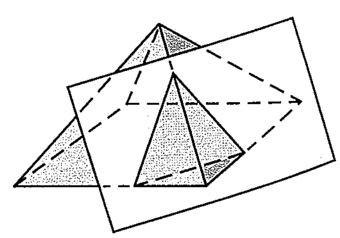


a. *Describe* the shape formed by the cross section.

b. What is the perimeter of the cross section?

c. What is the area of the cross section?

41. ★ **EXTENDED RESPONSE** Use the diagram of the square pyramid intersected by a plane.



a. *Describe* the shape of the cross section shown.

b. Can a plane intersect the pyramid at a point? If so, sketch the intersection.

c. *Describe* the shape of the cross section when the pyramid is sliced by a plane parallel to its base.

d. Is it possible to have a pentagon as a cross section of this pyramid? If so, draw the cross section.

42. **PLATONIC SOLIDS** Make a table of the number of faces, vertices, and edges for the five Platonic solids. Use Euler's Theorem to check each answer.

○ = **WORKED-OUT SOLUTIONS on p. WS1**   ★ = **STANDARDIZED TEST PRACTICE**

REASONING **Is it possible for a cross section of a cube to have the given shape? If yes, *describe* or sketch how the plane intersects the cube.**

43. Circle

44. Pentagon

45. Rhombus

46. Isosceles triangle

47. Regular hexagon

48. Scalene triangle

49. CUBE *Explain* how the numbers of faces, vertices, and edges of a cube change when you cut off each feature.

a. A corner **b.** An edge **c.** A face **d.** 3 corners

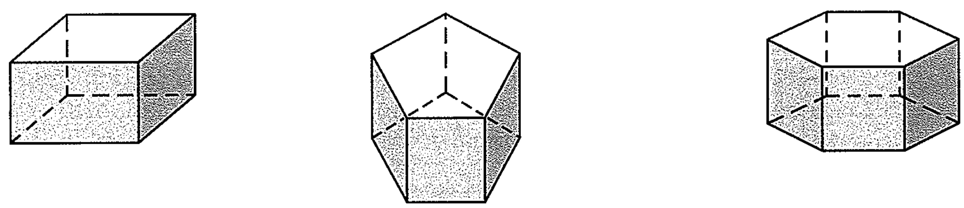
50. TETRAHEDRON *Explain* how the numbers of faces, vertices, and edges of a regular tetrahedron change when you cut off each feature.

a. A corner **b.** An edge **c.** A face **d.** 2 edges

51. CHALLENGE The *angle defect D* at a vertex of a polyhedron is defined as follows:

$D = 360° - $ (sum of all angle measures at the vertex)

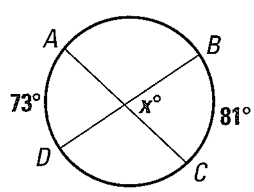
Verify that for the figures with regular bases below, $DV = 720°$ where V is the number of vertices.



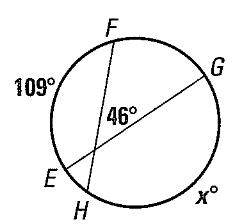
MIXED REVIEW

Find the value of x. ***(p. 680)***

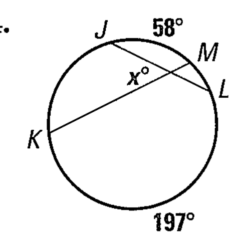
52.



53.



54.



PREVIEW
Prepare for Lesson 12.2 in Exs. 55–60.

Use the given radius r or diameter d to find the circumference and area of the circle. Round your answers to two decimal places. ***(p. 755)***

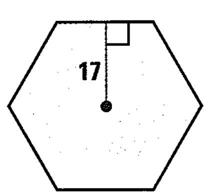
55. $r = 11$ cm

56. $d = 28$ in.

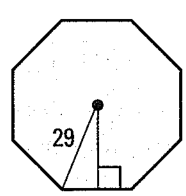
57. $d = 15$ ft

Find the perimeter and area of the regular polygon. Round your answers to two decimal places. ***(p. 762)***

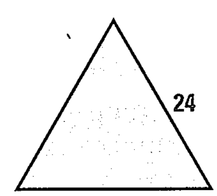
58.



59.



60.



12.2 Investigate Surface Area

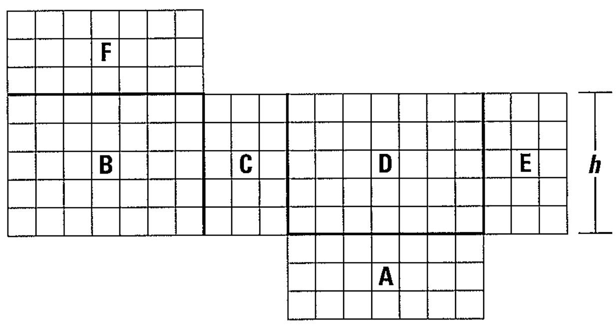
MATERIALS • graph paper • scissors • tape

QUESTION **How can you find the surface area of a polyhedron?**

A *net* is a pattern that can be folded to form a polyhedron. To find the *surface area* of a polyhedron, you can find the area of its net.

EXPLORE **Create a polyhedron using a net**

STEP 1 ***Draw a net*** Copy the net below on graph paper. Be sure to label the sections of the net.



STEP 2 ***Create a polyhedron*** Cut out the net and fold it along the black lines to form a polyhedron. Tape the edges together. Describe the polyhedron. Is it regular? Is it convex?

STEP 3 ***Find surface area*** The *surface area* of a polyhedron is the sum of the areas of its faces. Find the surface area of the polyhedron you just made. (Each square on the graph paper measures 1 unit by 1 unit.)

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Lay the net flat again and find the following measures.

   A: the area of Rectangle A

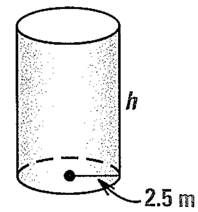
   P: the perimeter of Rectangle A

   h: the height of Rectangles B, C, D, and E

2. Use the values from Exercise 1 to find $2A + Ph$. *Compare* this value to the surface area you found in Step 3 above. What do you notice?
3. Make a conjecture about the surface area of a rectangular prism.
4. Use graph paper to draw the net of another rectangular prism. Fold the net to make sure that it forms a rectangular prism. Use your conjecture from Exercise 3 to calculate the surface area of the prism.

EXAMPLE 4 Find the height of a cylinder

Find the height of the right cylinder shown, which has a surface area of 157.08 square meters.



Solution

Substitute known values in the formula for the surface area of a right cylinder and solve for the height h.

$S = 2\pi r^2 + 2\pi rh$	**Surface area of a cylinder**
$157.08 = 2\pi(2.5)^2 + 2\pi(2.5)h$	**Substitute known values.**
$157.08 = 12.5\pi + 5\pi h$	**Simplify.**
$157.08 - 12.5\pi = 5\pi h$	**Subtract 12.5π from each side.**
$117.81 \approx 5\pi h$	**Simplify. Use a calculator.**
$7.5 \approx h$	**Divide each side by 5π.**

▶ The height of the cylinder is about 7.5 meters.

✓ **GUIDED PRACTICE** for Examples 3 and 4

3. Find the surface area of a right cylinder with height 18 centimeters and radius 10 centimeters. Round your answer to two decimal places.

4. Find the radius of a right cylinder with height 5 feet and surface area 208π square feet.

12.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS16 for Exs. 7, 9, and 23

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 24, 25, and 26

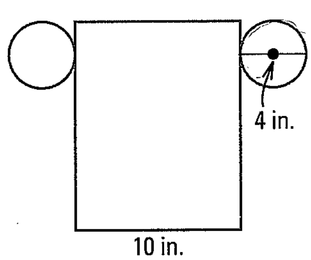
SKILL PRACTICE

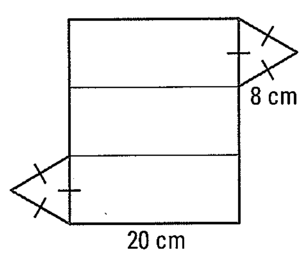
1. **VOCABULARY** Sketch a triangular prism. Identify its *bases*, *lateral faces*, and *lateral edges*.

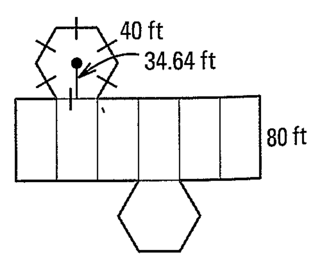
2. ★ **WRITING** *Explain* how the formula $S = 2B + Ph$ applies to finding the surface area of both a right prism and a right cylinder.

EXAMPLE 1 on p. 803 for Exs. 3–5

USING NETS Find the surface area of the solid formed by the net. Round your answer to two decimal places.

3. 

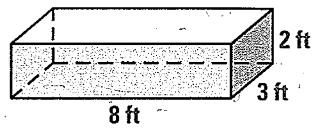
4. 

5. 

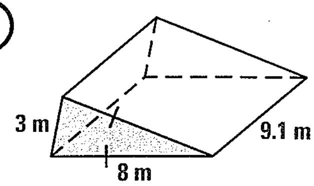
EXAMPLE 2
on p. 804
for Exs. 6–8

SURFACE AREA OF A PRISM **Find the surface area of the right prism. Round your answer to two decimal places.**

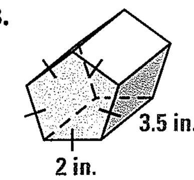
6.



7.



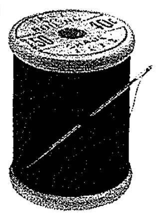
8.



EXAMPLE 3
on p. 805
for Exs. 9–12

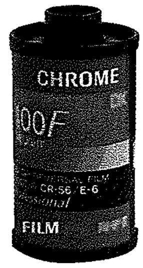
SURFACE AREA OF A CYLINDER **Find the surface area of the right cylinder using the given radius *r* and height *h*. Round your answer to two decimal places.**

9.



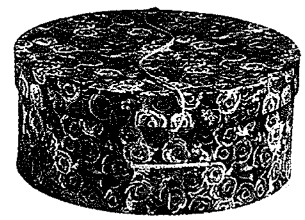
$r = 0.8$ in.
$h = 2$ in.

10.



$r = 12$ mm
$h = 40$ mm

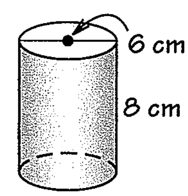
11.



$r = 8$ in.
$h = 8$ in.

12. ERROR ANALYSIS *Describe* and correct the error in finding the surface area of the right cylinder.

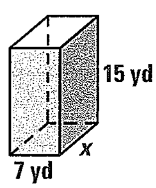
$$S = 2\pi(6^2) + 2\pi(6)(8)$$
$$= 2\pi(36) + 2\pi(48)$$
$$= 168\pi$$
$$\approx 528 \text{ cm}^2$$



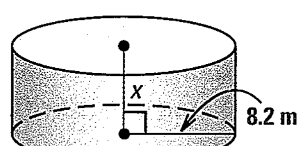
EXAMPLE 4
on p. 806
for Exs. 13–15

ALGEBRA **Solve for *x* given the surface area *S* of the right prism or right cylinder. Round your answer to two decimal places.**

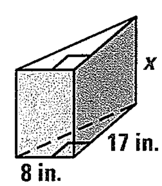
13. $S = 606 \text{ yd}^2$



14. $S = 1097 \text{ m}^2$



15. $S = 616 \text{ in.}^2$



16. SURFACE AREA OF A PRISM A triangular prism with a right triangular base has leg length 9 units and hypotenuse length 15 units. The height of the prism is 8 units. Sketch the prism and find its surface area.

17. ★ MULTIPLE CHOICE The length of each side of a cube is multiplied by 3. What is the change in the surface area of the cube?

(A) The surface area is 3 times the original surface area.

(B) The surface area is 6 times the original surface area.

(C) The surface area is 9 times the original surface area.

(D) The surface area is 27 times the original surface area.

18. SURFACE AREA OF A CYLINDER The radius and height of a right cylinder are each divided by $\sqrt{5}$. What is the change in surface area of the cylinder?

19. **SURFACE AREA OF A PRISM** Find the surface area of a right hexagonal prism with all edges measuring 10 inches.

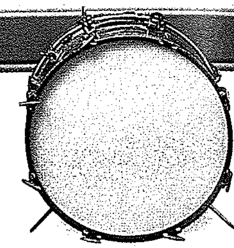
20. **HEIGHT OF A CYLINDER** Find the height of a cylinder with a surface area of 108π square meters. The radius of the cylinder is twice the height.

21. **CHALLENGE** The *diagonal* of a cube is a segment whose endpoints are vertices that are not on the same face. Find the surface area of a cube with diagonal length 8 units.

PROBLEM SOLVING

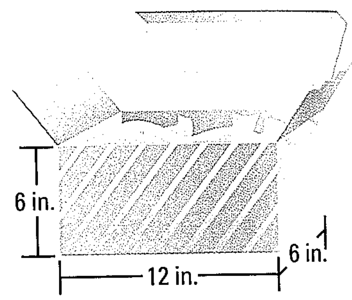
EXAMPLE 3 on p. 805 for Ex. 22

22. **BASS DRUM** A bass drum has a diameter of 20 inches and a depth of 8 inches. Find the surface area of the drum.



@HomeTutor for problem solving help at classzone.com

23. **GIFT BOX** An open gift box is shown at the right. When the gift box is closed, it has a length of 12 inches, a width of 6 inches, and a height of 6 inches.



   a. What is the minimum amount of wrapping paper needed to cover the closed gift box?

   b. Why is the area of the net of the box larger than the amount of paper found in part (a)?

   c. When wrapping the box, why would you want more paper than the amount found in part (a)?

@HomeTutor for problem solving help at classzone.com

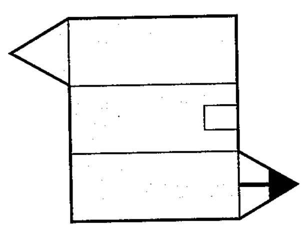
24. ★ **EXTENDED RESPONSE** A right cylinder has a radius of 4 feet and height of 10 feet.

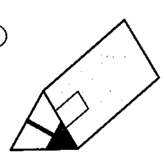
   a. Find the surface area of the cylinder.

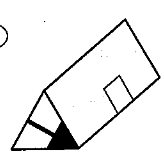
   b. Suppose you can either *double the radius* or *double the height*. Which do you think will create a greater surface area?

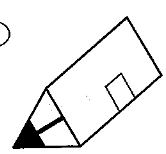
   c. Check your answer in part (b) by calculating the new surface areas.

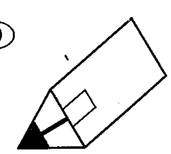
25. ★ **MULTIPLE CHOICE** Which three-dimensional figure does the net represent?



Ⓐ 

Ⓑ 

Ⓒ 

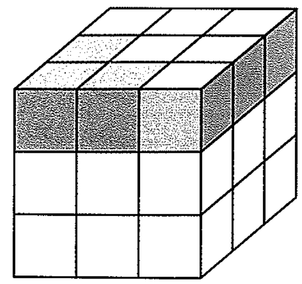
Ⓓ 

○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

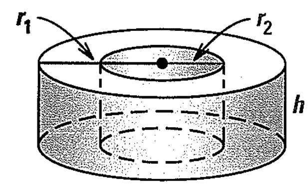
26. ★ **SHORT RESPONSE** A company makes two types of recycling bins. One type is a right rectangular prism with length 14 inches, width 12 inches, and height 36 inches. The other type is a right cylinder with radius 6 inches and height 36 inches. Both types of bins are missing a base, so the bins have one open end. Which bin requires more material to make? *Explain.*

27. **MULTI-STEP PROBLEM** Consider a cube that is built using 27 unit cubes as shown at the right.



 a. Find the surface area of the solid formed when the red unit cubes are removed from the solid shown.
 b. Find the surface area of the solid formed when the blue unit cubes are removed from the solid shown.
 c. Why are your answers different in parts (a) and (b)?

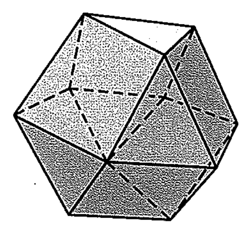
28. **SURFACE AREA OF A RING** The ring shown is a right cylinder of radius r_1 with a cylindrical hole of radius r_2. The ring has height h.



 a. Find the surface area of the ring if r_1 is 12 meters, r_2 is 6 meters, and h is 8 meters. Round your answer to two decimal places.
 b. Write a formula that can be used to find the surface area S of any cylindrical ring where $0 < r_2 < r_1$.

29. **DRAWING SOLIDS** A cube with edges 1 foot long has a cylindrical hole with diameter 4 inches drilled through one of its faces. The hole is drilled perpendicular to the face and goes completely through to the other side. Draw the figure and find its surface area.

30. **CHALLENGE** A cuboctahedron has 6 square faces and 8 equilateral triangle faces, as shown. A cuboctahedron can be made by slicing off the corners of a cube.



 a. Sketch a net for the cuboctahedron.
 b. Each edge of a cuboctahedron has a length of 5 millimeters. Find its surface area.

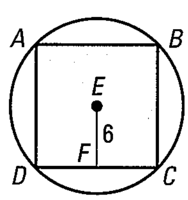
MIXED REVIEW

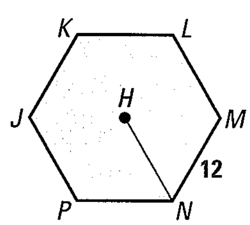
The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides. *(p. 507)*

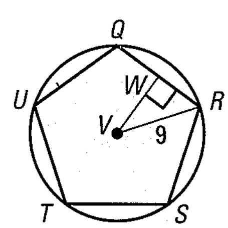
31. 1260° 32. 1080° 33. 720° 34. 1800°

PREVIEW
Prepare for Lesson 12.3 in Exs. 35–37.

Find the area of the regular polygon. *(p. 762)*

35.


36.


37.


12.3 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS17 for Exs. 7, 11, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, and 31

SKILL PRACTICE

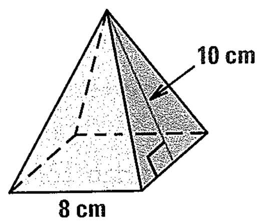
1. **VOCABULARY** Draw a regular square pyramid. Label its *height, slant height,* and *base.*

2. ★ **WRITING** *Compare* the height and slant height of a right cone.

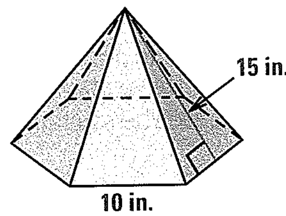
EXAMPLE 1 on p. 810 for Exs. 3–5

AREA OF A LATERAL FACE Find the area of each lateral face of the regular pyramid.

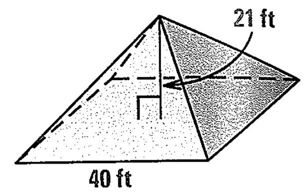
3.



4.



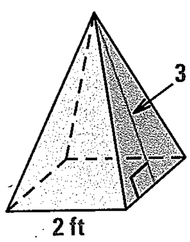
5.



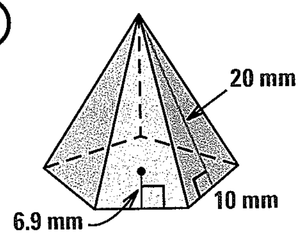
EXAMPLE 2 on p. 811 for Exs. 6–9

SURFACE AREA OF A PYRAMID Find the surface area of the regular pyramid. Round your answer to two decimal places.

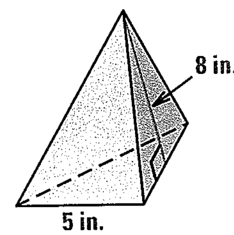
6.



(7.)



8.

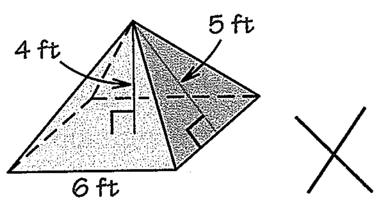


9. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the regular pyramid.

$S = B + \frac{1}{2}P\ell$

$= 6^2 + \frac{1}{2}(24)(4)$

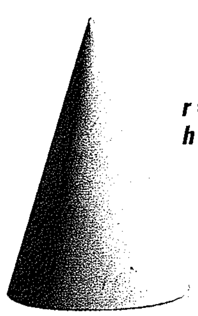
$= 84 \text{ ft}^2$



EXAMPLES 3 and 4 on p. 813 for Exs. 10–17

LATERAL AREA OF A CONE Find the lateral area of the right cone. Round your answer to two decimal places.

10.

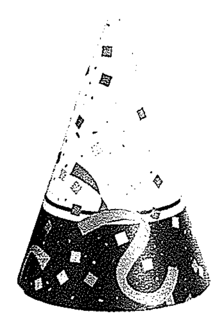


$r = 7.5$ cm
$h = 25$ cm

(11.)

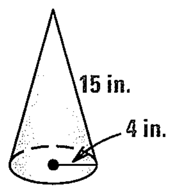
$r = 1$ in.
$h = 4$ in.

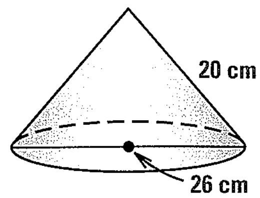
12.

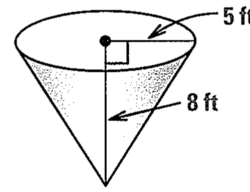


$d = 7$ in.
$h = 1$ ft

SURFACE AREA OF A CONE **Find the surface area of the right cone. Round your answer to two decimal places.**

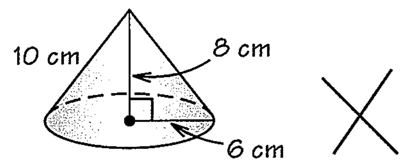
13. 

14. 

15. 

16. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the right cone.

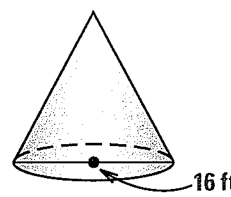
$$S = \pi(r^2) + \pi r^2 \ell$$
$$= \pi(36) + \pi(36)(10)$$
$$= 396\pi \text{ cm}^2$$



17. ★ **MULTIPLE CHOICE** The surface area of the right cone is 200π square feet. What is the slant height of the cone?

Ⓐ 10.5 ft Ⓑ 17 ft

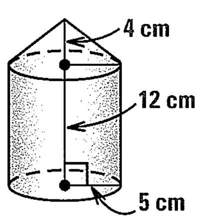
Ⓒ 23 ft Ⓓ 24 ft

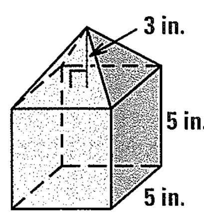


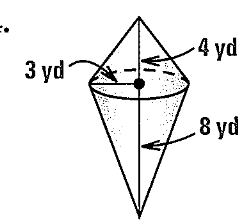
VISUAL REASONING **In Exercises 18–21, sketch the described solid and find its surface area. Round your answer to two decimal places.**

18. A right cone has a radius of 15 feet and a slant height of 20 feet.
19. A right cone has a diameter of 16 meters and a height of 30 meters.
20. A regular pyramid has a slant height of 24 inches. Its base is an equilateral triangle with a base edge length of 10 inches.
21. A regular pyramid has a hexagonal base with a base edge length of 6 centimeters and a slant height of 9 centimeters.

COMPOSITE SOLIDS **Find the surface area of the solid. The pyramids are regular and the cones are right. Round your answers to two decimal places, if necessary.**

22. 

23. 

24. 

25. **TETRAHEDRON** Find the surface area of a regular tetrahedron with edge length 4 centimeters.

26. **CHALLENGE** A right cone with a base of radius 4 inches and a regular pyramid with a square base both have a slant height of 5 inches. Both solids have the same surface area. Find the length of a base edge of the pyramid. Round your answer to the nearest hundredth of an inch.

Problem Solving

EXAMPLE 2
on p. 811
for Ex. 27

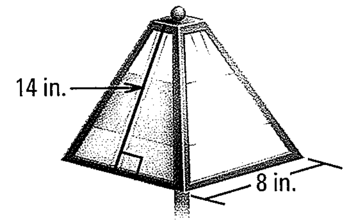
27. CANDLES A candle is in the shape of a regular square pyramid with base edge length 6 inches. Its height is 4 inches. Find its surface area.

@HomeTutor for problem solving help at classzone.com

28. LAMPSHADE A glass lampshade is shaped like a regular square pyramid.

a. Approximate the lateral area of the lampshade shown.

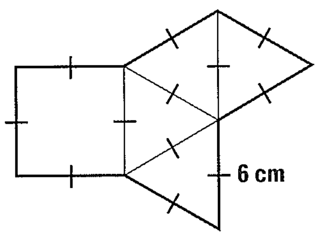
b. *Explain* why your answer to part (a) is not the exact lateral area.



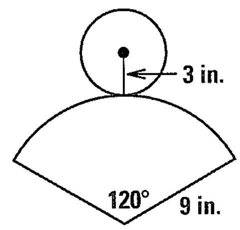
@HomeTutor for problem solving help at classzone.com

USING NETS **Name the figure that is represented by the net. Then find its surface area. Round your answer to two decimal places.**

(29.)



30.

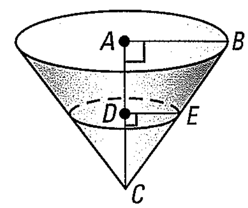


31. ★ SHORT RESPONSE In the figure, $AC = 4$, $AB = 3$, and $DC = 2$.

a. Prove $\triangle ABC \sim \triangle DEC$.

b. Find BC, DE, and EC.

c. Find the surface areas of the larger cone and the smaller cone in terms of π. *Compare* the surface areas using a percent.

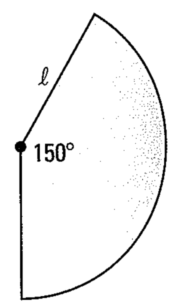


32. MULTI-STEP PROBLEM The sector shown can be rolled to form the lateral surface of a right cone. The lateral surface area of the cone is 20 square meters.

a. Write the formula for the area of a sector.

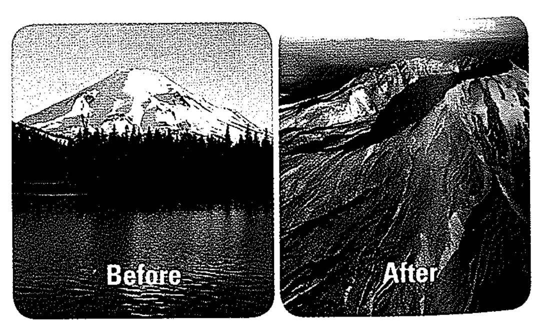
b. Use the formula in part (a) to find the slant height of the cone. *Explain* your reasoning.

c. Find the radius and height of the cone.

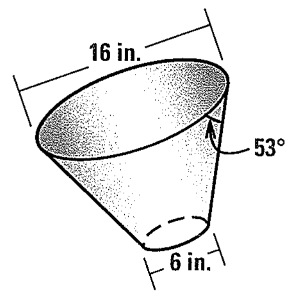


33. VOLCANOES Before 1980, Mount St. Helens was a conic volcano with a height from its base of about 1.08 miles and a base radius of about 3 miles. In 1980, the volcano erupted, reducing its height to about 0.83 mile.

Approximate the lateral area of the volcano after 1980. (*Hint:* The ratio of the radius of the destroyed cone-shaped top to its height is the same as the ratio of the radius of the original volcano to its height.)



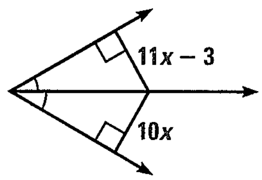
34. **CHALLENGE** An *Elizabethan collar* is used to prevent an animal from irritating a wound. The angle between the opening with a 16 inch diameter and the side of the collar is 53°. Find the surface area of the collar shown.



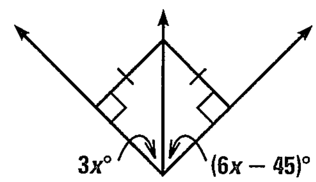
MIXED REVIEW

Find the value of x. ***(p. 310)***

35.



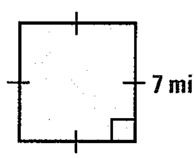
36.



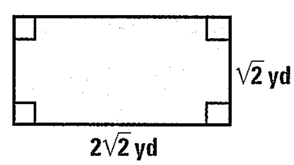
PREVIEW
Prepare for Lesson 12.4 in Exs. 37–39.

In Exercises 37–39, find the area of the polygon. ***(pp. 720, 730)***

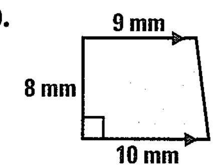
37.



38.



39.

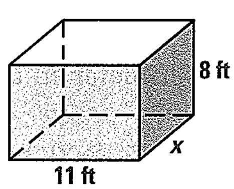


QUIZ *for Lessons 12.1–12.3*

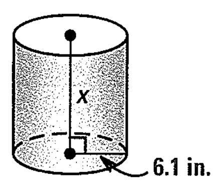
1. A polyhedron has 8 vertices and 12 edges. How many faces does the polyhedron have? ***(p. 794)***

Solve for x given the surface area S of the right prism or right cylinder. Round your answer to two decimal places. ***(p. 803)***

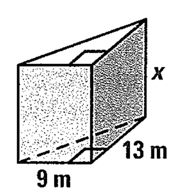
2. $S = 366 \text{ ft}^2$



3. $S = 717 \text{ in.}^2$

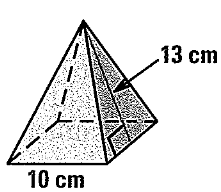


4. $S = 567 \text{ m}^2$

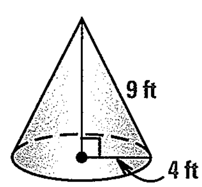


Find the surface area of the regular pyramid or right cone. Round your answer to two decimal places. ***(p. 810)***

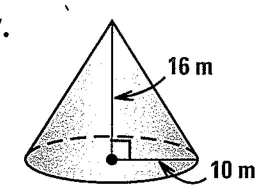
5.



6.



7.

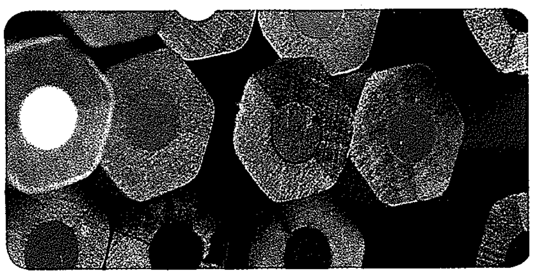


Lessons 12.1–12.3

1. **SHORT RESPONSE** Using Euler's Theorem, *explain* why it is not possible for a polyhedron to have 6 vertices and 7 edges.

2. **SHORT RESPONSE** *Describe* two methods of finding the surface area of a rectangular solid.

3. **EXTENDED RESPONSE** Some pencils are made from slats of wood that are machined into right regular hexagonal prisms.

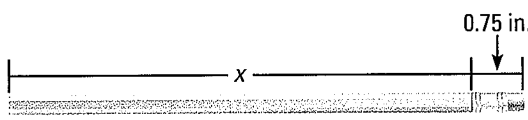


   a. The formula for the surface area of a new unshapened pencil without an eraser is

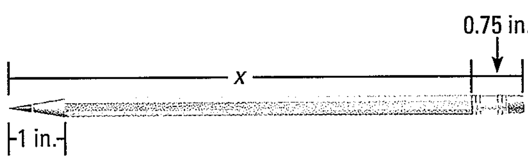
   $$S = 3\sqrt{3}r^2 + 6rh.$$

   Tell what each variable in this formula represents.

   b. After a pencil is painted, a metal band that holds an eraser is wrapped around one end. Write a formula for the surface area of the visible portion of the pencil, shown below.

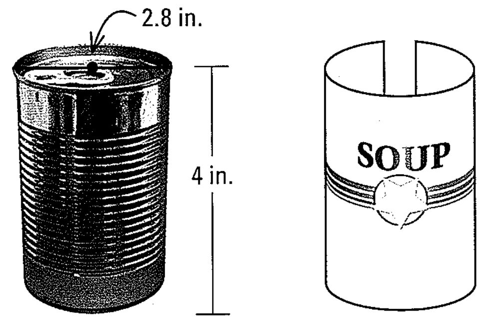


   c. After a pencil is sharpened, the end is shaped like a cone. Write a formula to find the surface area of the visible portion of the pencil, shown below.



   d. Use your formulas from parts (b) and (c) to write a formula for the difference of the surface areas of the two pencils. Define any variables in your formula.

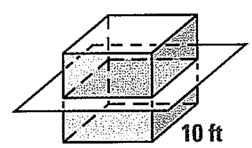
4. **GRIDDED ANSWER** The amount of paper needed for a soup can label is approximately equal to the lateral area of the can. Find the lateral area of the soup can in square inches. Round your answer to two decimal places.



5. **SHORT RESPONSE** If you know the diameter d and slant height ℓ of a right cone, how can you find the surface area of the cone?

6. **OPEN-ENDED** Identify an object in your school or home that is a rectangular prism. Measure its length, width, and height to the nearest quarter inch. Then approximate the surface area of the object.

7. **MULTI-STEP PROBLEM** The figure shows a plane intersecting a cube parallel to its base. The cube has a side length of 10 feet.



   a. Describe the shape formed by the cross section.

   b. Find the perimeter and area of the cross section.

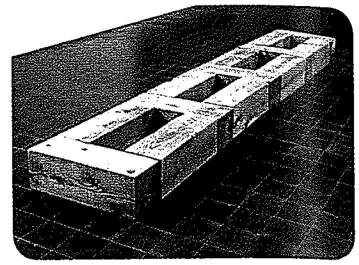
   c. When the cross section is cut along its diagonal, what kind of triangles are formed?

   d. Find the area of one of the triangles formed in part (c).

8. **SHORT RESPONSE** A cone has a base radius of $3x$ units and a height of $4x$ units. The surface area of the cone is 1944π square units. Find the value of x. *Explain* your steps.

EXAMPLE 5 Solve a real-world problem

SCULPTURE The sculpture is made up of 13 beams. In centimeters, suppose the dimensions of each beam are 30 by 30 by 90. Find its volume.



Romartyr Hamburg, 1989 © Carl Andre/ licensed by VAGA, NY

ANOTHER WAY
For alternative methods for solving the problem in Example 5, turn to page 826 for the **Problem Solving Workshop**.

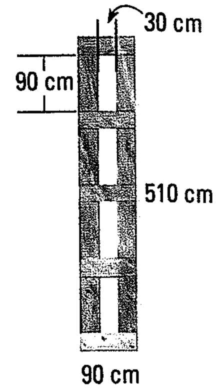
Solution

The area of the base B can be found by subtracting the area of the small rectangles from the area of the large rectangle.

B = Area of large rectangle − 4 • Area of small rectangle

$= 90 \cdot 510 - 4(30 \cdot 90)$

$= 35{,}100 \text{ cm}^2$



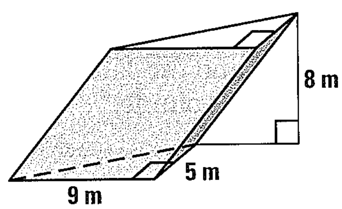
Use the formula for the volume of a prism.

$V = Bh$	**Formula for volume of a prism**
$= 35{,}100(30)$	**Substitute.**
$= 1{,}053{,}000 \text{ cm}^3$	**Simplify.**

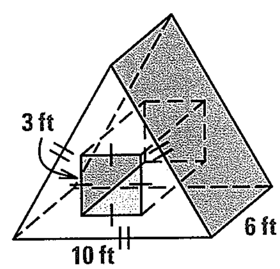
▶ The volume of the sculpture is $1{,}053{,}000 \text{ cm}^3$, or 1.053 m^3.

✓ **GUIDED PRACTICE** for Examples 4 and 5

4. Find the volume of the oblique prism shown below.



5. Find the volume of the solid shown below.



12.4 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS17 for Exs. 7, 11, and 29
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 3, 21, and 33

SKILL PRACTICE

1. **VOCABULARY** In what type of units is the volume of a solid measured?

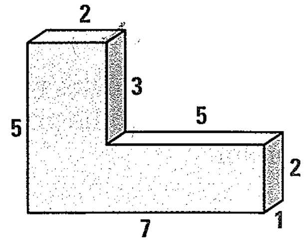
2. ★ **WRITING** Two solids have the same surface area. Do they have the same volume? *Explain* your reasoning.

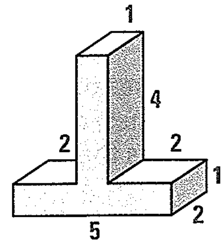
EXAMPLE 1
on p. 819
for Exs. 3–6

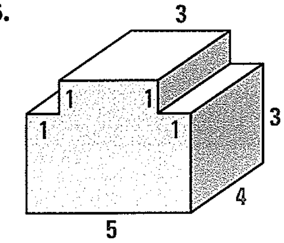
3. ★ **MULTIPLE CHOICE** How many 3 inch cubes can fit completely in a box that is 15 inches long, 9 inches wide, and 3 inches tall?

Ⓐ 15 Ⓑ 45 Ⓒ 135 Ⓓ 405

USING UNIT CUBES Find the volume of the solid by determining how many unit cubes are contained in the solid.

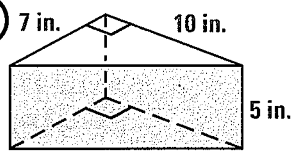
4.


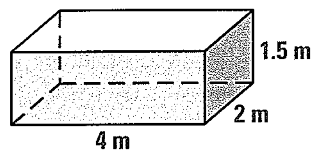
5.


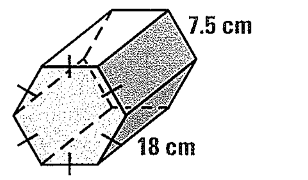
6.


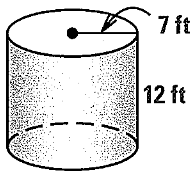
EXAMPLE 2 on p. 820 for Exs. 7–13

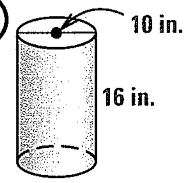
FINDING VOLUME Find the volume of the right prism or right cylinder. Round your answer to two decimal places.

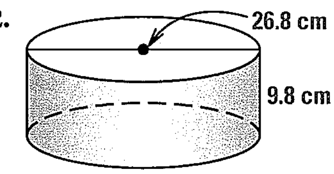
7.


8.


9.


10.


11.


12.


13. **ERROR ANALYSIS** *Describe* and correct the error in finding the volume of a right cylinder with radius 4 feet and height 3 feet.

$V = 2\pi rh$

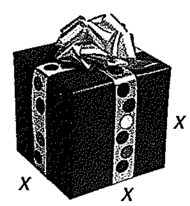
$= 2\pi(4)(3)$

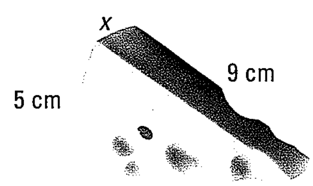
$= 24\pi \text{ ft}^3$

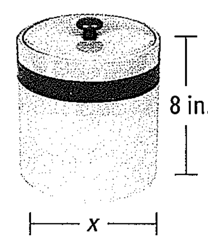
14. **FINDING VOLUME** Sketch a rectangular prism with height 3 feet, width 11 inches, and length 7 feet. Find its volume.

EXAMPLE 3 on p. 820 for Exs. 15–17

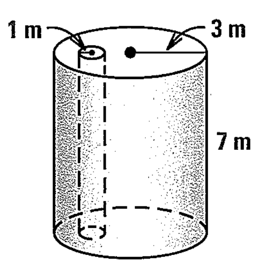
ALGEBRA Find the length x using the given volume V.

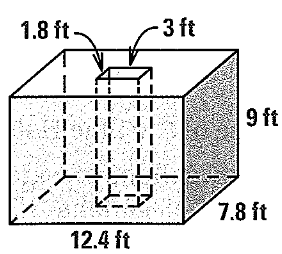
15. $V = 1000 \text{ in.}^3$


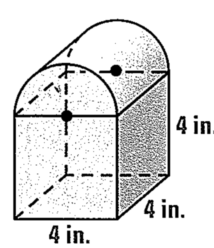
16. $V = 45 \text{ cm}^3$


17. $V = 128\pi \text{ in.}^3$


COMPOSITE SOLIDS Find the volume of the solid. The prisms and cylinders are right. Round your answer to two decimal places, if necessary.

18.


19.


20.


21. ★ **MULTIPLE CHOICE** What is the height of a cylinder with radius 4 feet and volume 64π cubic feet?

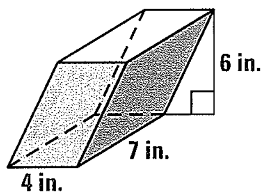
Ⓐ 4 feet Ⓑ 8 feet Ⓒ 16 feet Ⓓ 256 feet

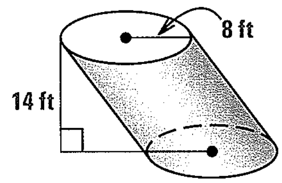
22. **FINDING HEIGHT** The bases of a right prism are right triangles with side lengths of 3 inches, 4 inches, and 5 inches. The volume of the prism is 96 cubic inches. What is the height of the prism?

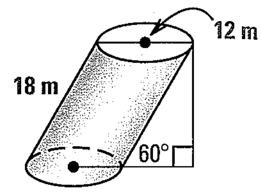
23. **FINDING DIAMETER** A cylinder has height 8 centimeters and volume 1005.5 cubic centimeters. What is the diameter of the cylinder?

EXAMPLE 4
on p. 821
for Exs. 24–26

VOLUME OF AN OBLIQUE SOLID Use Cavalieri's Principle to find the volume of the oblique prism or cylinder. Round your answer to two decimal places.

24.


25.


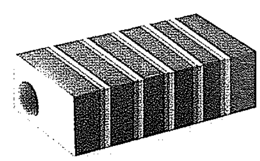
26.


27. **CHALLENGE** The bases of a right prism are rhombuses with diagonals 12 meters and 16 meters long. The height of the prism is 8 meters. Find the lateral area, surface area, and volume of the prism.

PROBLEM SOLVING

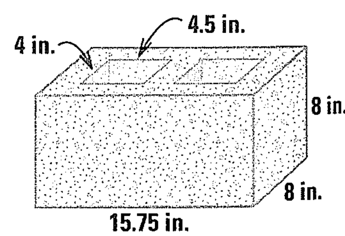
EXAMPLE 5
on p. 822
for Exs. 28–30

28. **JEWELRY** The bead at the right is a rectangular prism of length 17 millimeters, width 9 millimeters, and height 5 millimeters. A 3 millimeter wide hole is drilled through the smallest face. Find the volume of the bead.



@HomeTutor for problem solving help at classzone.com

(29.) **MULTI-STEP PROBLEM** In the concrete block shown, the holes are 8 inches deep.



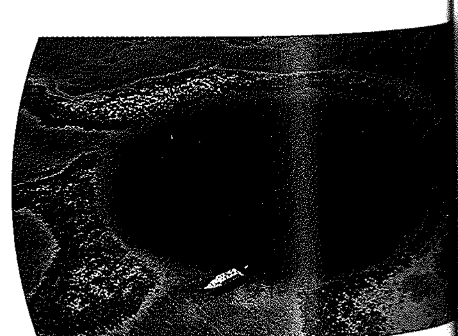
a. Find the volume of the block using the Volume Addition Postulate.

b. Find the volume of the block using the formula in Theorem 12.6.

c. *Compare* your answers in parts (a) and (b).

@HomeTutor for problem solving help at classzone.com

30. **OCEANOGRAPHY** The Blue Hole is a cylindrical trench located on Lighthouse Reef Atoll, an island off the coast of Central America. It is approximately 1000 feet wide and 400 feet deep.

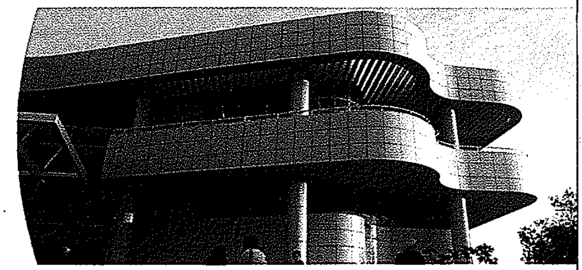


a. Find the volume of the Blue Hole.

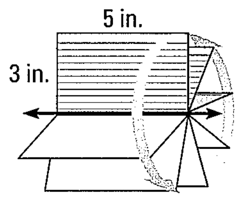
b. About how many gallons of water does the Blue Hole contain? ($1\ ft^3 = 7.48$ gallons)

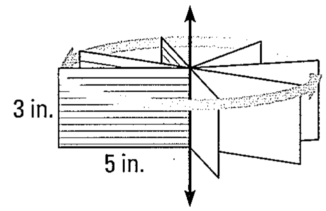
31. **ARCHITECTURE** A cylindrical column in the building shown has circumference 10 feet and height 20 feet. Find its volume. Round your answer to two decimal places.

Animated **Geometry** at classzone.com



32. **ROTATIONS** A 3 inch by 5 inch index card is rotated around a horizontal line and a vertical line to produce two different solids, as shown. Which solid has a greater volume? *Explain* your reasoning.





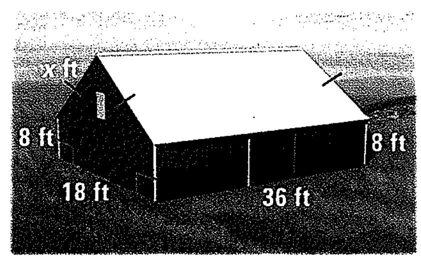
33. ★ **EXTENDED RESPONSE** An aquarium shaped like a rectangular prism has length 30 inches, width 10 inches, and height 20 inches.

a. **Calculate** You fill the aquarium $\frac{3}{4}$ full with water. What is the volume of the water?

b. **Interpret** When you submerge a rock in the aquarium, the water level rises 0.25 inch. Find the volume of the rock.

c. **Interpret** How many rocks of the same size as the rock in part (b) can you place in the aquarium before water spills out?

34. **CHALLENGE** A barn is in the shape of a pentagonal prism with the dimensions shown. The volume of the barn is 9072 cubic feet. Find the dimensions of each half of the roof.

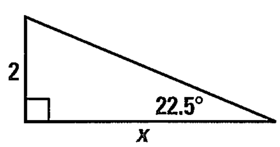


MIXED REVIEW

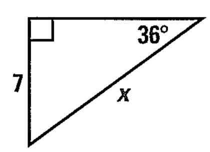
PREVIEW
Prepare for Lesson 12.5 in Exs. 35–40.

Find the value of x. Round your answer to two decimal places. *(pp. 466, 473)*

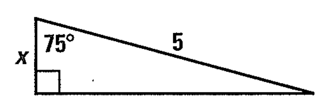
35.



36.



37.



Find the area of the figure described. Round your answer to two decimal places. *(pp. 755, 762)*

38. A circle with radius 9.5 inches

39. An equilateral triangle with perimeter 78 meters and apothem 7.5 meters

40. A regular pentagon with radius 10.6 inches

PROBLEM SOLVING WORKSHOP
LESSON 12.4

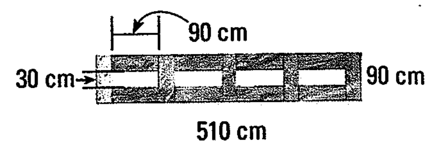
Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 822

MULTIPLE REPRESENTATIONS In Lesson 12.4, you used volume postulates and theorems to find volumes of prisms and cylinders. Now, you will learn two different ways to solve Example 5 on page 822.

PROBLEM

SCULPTURE The sculpture is made up of 13 beams. In centimeters, suppose the dimensions of each beam are 30 by 30 by 90. Find its volume.



METHOD 1

Finding Volume by Subtracting Empty Spaces One alternative approach is to compute the volume of the prism formed if the holes in the sculpture were filled. Then, to get the correct volume, you must subtract the volume of the four holes.

STEP 1 **Read** the problem. In centimeters, each beam measures 30 by 30 by 90.

The dimensions of the entire sculpture are 30 by 90 by $(4 \cdot 90 + 5 \cdot 30)$, or 30 by 90 by 510.

The dimensions of each hole are equal to the dimensions of one beam.

STEP 2 **Apply** the Volume Addition Postulate. The volume of the sculpture is equal to the volume of the larger prism minus 4 times the volume of a hole.

$$\begin{aligned}\text{Volume } V \text{ of sculpture} &= \text{Volume of larger prism} - \text{Volume of 4 holes}\\ &= 30 \cdot 90 \cdot 510 - 4(30 \cdot 30 \cdot 90)\\ &= 1{,}377{,}000 - 4 \cdot 81{,}000\\ &= 1{,}377{,}000 - 324{,}000\\ &= 1{,}053{,}000\end{aligned}$$

▸ The volume of the sculpture is 1,053,000 cubic centimeters, or 1.053 cubic meters.

STEP 3 **Check** page 822 to verify your new answer, and confirm that it is the same.

METHOD 2

Finding Volume of Pieces Another alternative approach is to use the dimensions of each beam.

STEP 1 **Look** at the sculpture. Notice that the sculpture consists of 13 beams, each with the same dimensions. Therefore, the volume of the sculpture will be 13 times the volume of one beam.

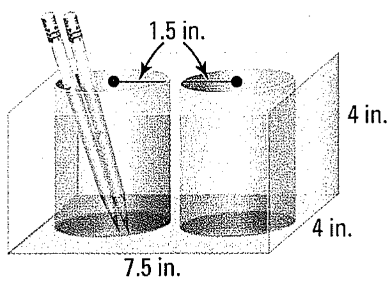
STEP 2 **Write** an expression for the volume of the sculpture and find the volume.

$$\begin{aligned} \text{Volume of sculpture} &= 13(\text{Volume of one beam}) \\ &= 13(30 \cdot 30 \cdot 90) \\ &= 13 \cdot 81{,}000 \\ &= 1{,}053{,}000 \end{aligned}$$

▶ The volume of the sculpture is 1,053,000 cm^3, or 1.053 m^3.

PRACTICE

1. **PENCIL HOLDER** The pencil holder has the dimensions shown.



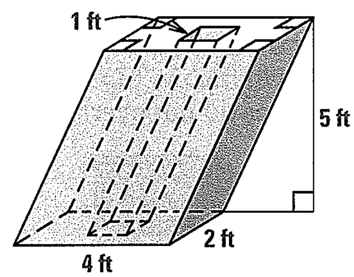
   a. Find its volume using the Volume Addition Postulate.

   b. Use its base area to find its volume.

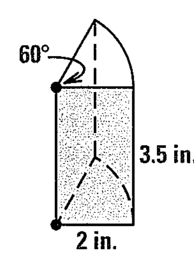
2. **ERROR ANALYSIS** A student solving Exercise 1 claims that the surface area is found by subtracting four times the base area of the cylinders from the surface area of the rectangular prism. *Describe* and correct the student's error.

3. **REASONING** You drill a circular hole of radius *r* through the base of a cylinder of radius *R*. Assume the hole is drilled completely through to the other base. You want the volume of the hole to be half the volume of the cylinder. Express *r* as a function of *R*.

4. **FINDING VOLUME** Find the volume of the solid shown below. Assume the hole has square cross sections.



5. **FINDING VOLUME** Find the volume of the solid shown to the right.



6. **SURFACE AREA** Refer to the diagram of the sculpture on page 826.

   a. *Describe* a method to find the surface area of the sculpture.

   b. *Explain* why adding the individual surface areas of the beams will give an incorrect result for the total surface area.

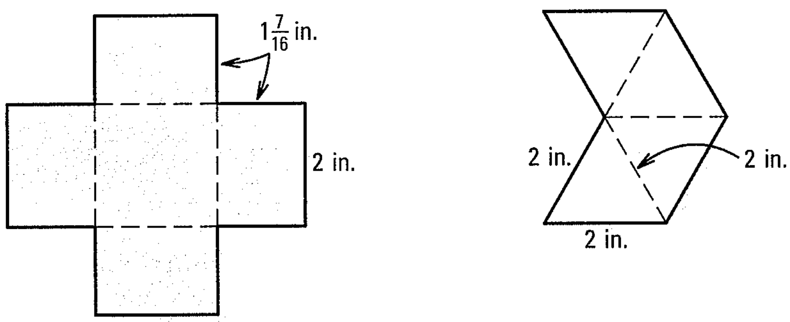
12.5 Investigate the Volume of a Pyramid

MATERIALS • ruler • poster board • scissors • tape • uncooked rice

QUESTION **How is the volume of a pyramid related to the volume of a prism with the same base and height?**

EXPLORE **Compare the volume of a prism and a pyramid using nets**

STEP 1 ***Draw nets*** Use a ruler to draw the two nets shown below on poster board. (Use $1\frac{7}{16}$ inches to approximate $\sqrt{2}$ inches.)



STEP 2 ***Create an open prism and an open pyramid*** Cut out the nets. Fold along the dotted lines to form an open prism and an open pyramid, as shown below. Tape each solid to hold it in place, making sure that the edges do not overlap.

STEP 3 ***Compare volumes*** Fill the pyramid with uncooked rice and pour it into the prism. Repeat this as many times as needed to fill the prism. How many times did you fill the pyramid? What does this tell you about the volume of the solids?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. *Compare* the area of the base of the pyramid to the area of the base of the prism. Placing the pyramid inside the prism will help. What do you notice?
2. *Compare* the heights of the solids. What do you notice?
3. Make a conjecture about the ratio of the volumes of the solids.
4. Use your conjecture to write a formula for the volume of a pyramid that uses the formula for the volume of a prism.

12.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS17 for Exs. 3, 17, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 18, and 35

◈ = **MULTIPLE REPRESENTATIONS** Ex. 39

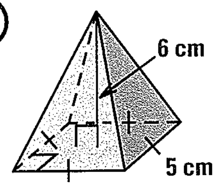
SKILL PRACTICE

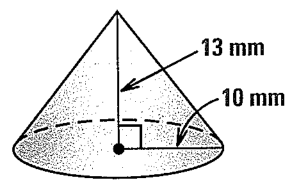
1. **VOCABULARY** *Explain* the difference between a *triangular prism* and a *triangular pyramid*. Draw an example of each.

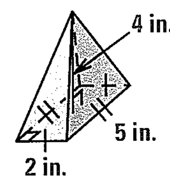
2. ★ **WRITING** *Compare* the volume of a square pyramid to the volume of a square prism with the same base and height as the pyramid.

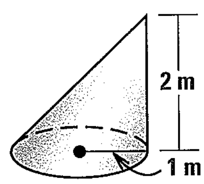
EXAMPLE 1 on p. 829 for Exs. 3–11

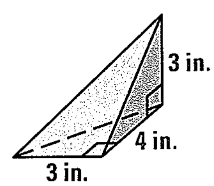
VOLUME OF A SOLID **Find the volume of the solid. Round your answer to two decimal places.**

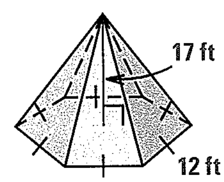
3.


4.


5.


6.


7.


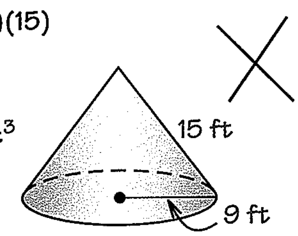
8.


ERROR ANALYSIS *Describe* **and correct the error in finding the volume of the right cone or pyramid.**

9. $V = \frac{1}{3}\pi(9^2)(15)$

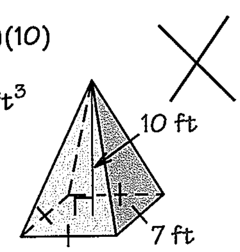
$= 405\pi$

$\approx 1272 \text{ ft}^3$



10. $V = \frac{1}{2}(49)(10)$

$= 245 \text{ ft}^3$

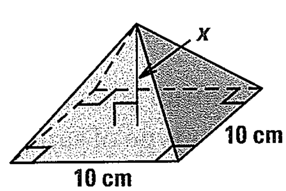


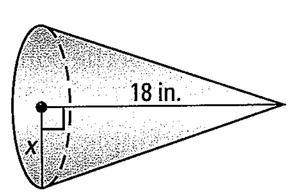
11. ★ **MULTIPLE CHOICE** The volume of a pyramid is 45 cubic feet and the height is 9 feet. What is the area of the base?

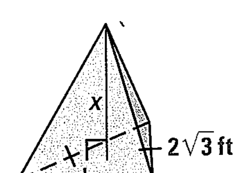
Ⓐ 3.87 ft^2 Ⓑ 5 ft^2 Ⓒ 10 ft^2 Ⓓ 15 ft^2

EXAMPLE 2 on p. 830 for Exs. 12–14

ALGEBRA **Find the value of *x*.**

12. Volume = 200 cm^3


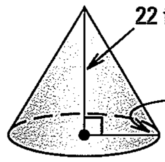
13. Volume = $216\pi \text{ in.}^3$


14. Volume = $7\sqrt{3} \text{ ft}^3$


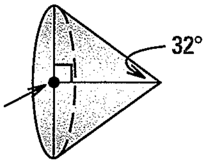
EXAMPLE 3
on p. 830
for Exs. 15–19

VOLUME OF A CONE Find the volume of the right cone. Round your answer to two decimal places.

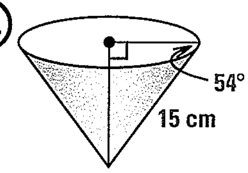
15.



16.



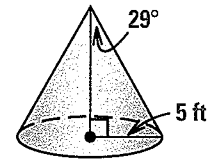
17.



18. ★ **MULTIPLE CHOICE** What is the approximate volume of the cone?

Ⓐ 47.23 ft^3 Ⓑ 236.15 ft^3

Ⓒ 269.92 ft^3 Ⓓ 354.21 ft^3

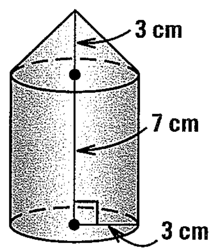


19. **HEIGHT OF A CONE** A cone with a diameter of 8 centimeters has volume 143.6 cubic centimeters. Find the height of the cone. Round your answer to two decimal places.

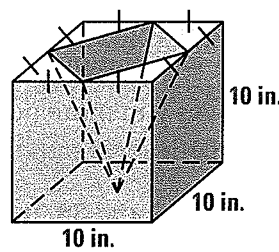
EXAMPLE 4
on p. 831
for Exs. 20–25

COMPOSITE SOLIDS Find the volume of the solid. The prisms, pyramids, and cones are right. Round your answer to two decimal places.

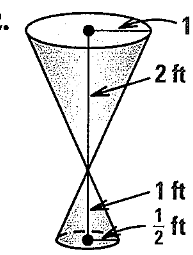
20.



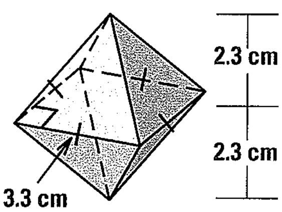
21.



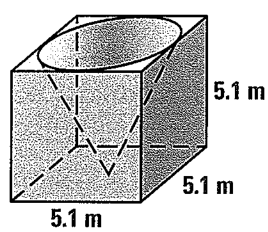
22.



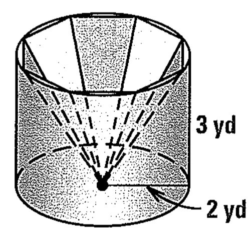
23.



24.

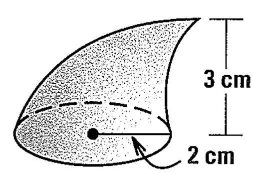


25.



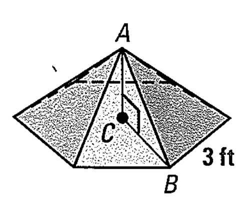
AnimatedGeometry at classzone.com

26. **FINDING VOLUME** The figure at the right is a cone that has been warped but whose cross sections still have the same area as a right cone with equal base area and height. Find the volume of this solid.



27. **FINDING VOLUME** Sketch a regular square pyramid with base edge length 5 meters inscribed in a cone with height 7 meters. Find the volume of the cone. *Explain* your reasoning.

28. **CHALLENGE** Find the volume of the regular hexagonal pyramid. Round your answer to the nearest hundredth of a cubic foot. In the diagram, $m\angle ABC = 35°$.



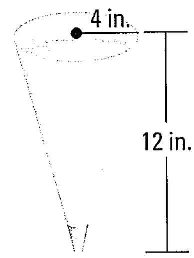
PROBLEM SOLVING

EXAMPLE 5
on p. 831
for Ex. 30

29. CAKE DECORATION A pastry bag filled with frosting has height 12 inches and radius 4 inches. A cake decorator can make 15 flowers using one bag of frosting.

a. How much frosting is in the pastry bag? Round your answer to the nearest cubic inch.

b. How many cubic inches of frosting are used to make each flower?



@HomeTutor for problem solving help at classzone.com

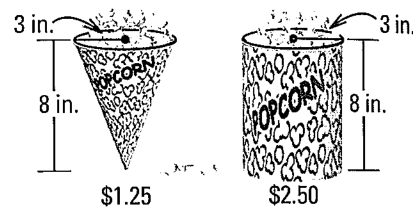
POPCORN A snack stand serves a small order of popcorn in a cone-shaped cup and a large order of popcorn in a cylindrical cup.

30. Find the volume of the small cup.

@HomeTutor for problem solving help at classzone.com

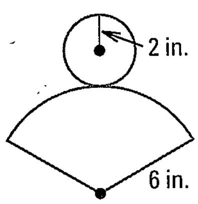
31. How many small cups of popcorn do you have to buy to equal the amount of popcorn in a large container? Do not perform any calculations. *Explain.*

32. Which container gives you more popcorn for your money? *Explain.*

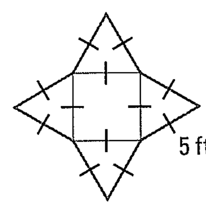


USING NETS In Exercises 33 and 34, use the net to sketch the solid. Then find the volume of the solid. Round your answer to two decimal places.

33.



34.



35. ★ EXTENDED RESPONSE A pyramid has height 10 feet and a square base with side length 7 feet.

a. How does the volume of the pyramid change if the base stays the same and the height is doubled?

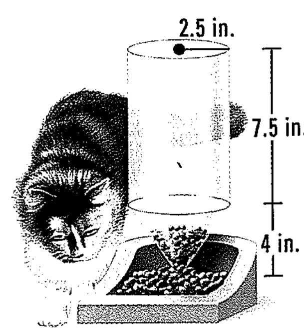
b. How does the volume of the pyramid change if the height stays the same and the side length of the base is doubled?

c. *Explain* why your answers to parts (a) and (b) are true for any height and side length.

36. AUTOMATIC FEEDER Assume the automatic pet feeder is a right cylinder on top of a right cone of the same radius. (1 cup = 14.4 in.^{3})

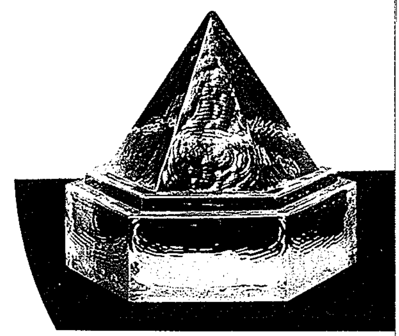
a. Calculate the amount of food in cups that can be placed in the feeder.

b. A cat eats one third of a cup of food, twice per day. How many days will the feeder have food without refilling it?



○ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE** ◆ = **MULTIPLE REPRESENTATIONS**

37. **NAUTICAL PRISMS** The nautical deck prism shown is composed of the following three solids: a regular hexagonal prism with edge length 3.5 inches and height 1.5 inches, a regular hexagonal prism with edge length 3.25 inches and height 0.25 inch, and a regular hexagonal pyramid with edge length 3 inches and height 3 inches. Find the volume of the deck prism.



38. **MULTI-STEP PROBLEM** Calculus can be used to show that the average value of r^2 of a circular cross section of a cone is $\frac{r_b^2}{3}$, where r_b is the radius of the base.

 a. Find the average area of a circular cross section of a cone whose base has radius R.

 b. Show that the volume of the cone can be expressed as follows:

 V_{cone} = (Average area of a circular cross section) • (Height of cone)

39. **MULTIPLE REPRESENTATIONS** Water flows into a reservoir shaped like a right cone at the rate of 1.8 cubic meters per minute. The height and diameter of the reservoir are equal.

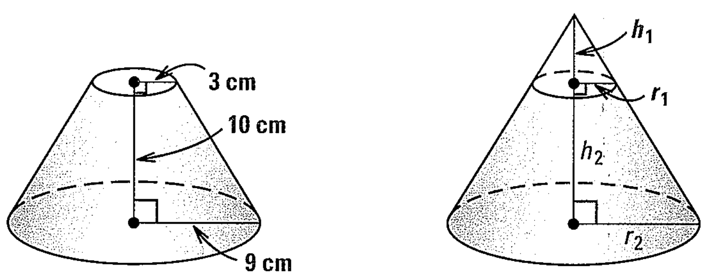
 a. Using Algebra As the water flows into the reservoir, the relationship $h = 2r$ is always true. Using this fact, show that $V = \frac{\pi h^3}{12}$.

 b. Making a Table Make a table that gives the height h of the water after 1, 2, 3, 4, and 5 minutes.

 c. Drawing a Graph Make a graph of height versus time. Is there a linear relationship between the height of the water and time? *Explain.*

FRUSTUM A frustum of a cone is the part of the cone that lies between the base and a plane parallel to the base, as shown. Use the information to complete Exercises 40 and 41.

One method for calculating the volume of a frustum is to add the areas of the two bases to their geometric mean, then multiply the result by $\frac{1}{3}$ the height.



40. Use the measurements in the diagram at the left above to calculate the volume of the frustum.

41. Complete parts (a) and (b) below to write a formula for the volume of a frustum that has bases with radii r_1 and r_2 and a height h_2.

 a. Use similar triangles to find the value of h_1 in terms of h_2, r_1, and r_2.

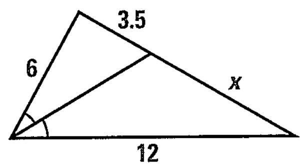
 b. Write a formula in terms of h_2, r_1, and r_2 for V_{frustum} = (Original volume) − (Removed volume).

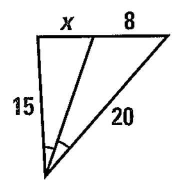
 c. Show that your formula in part (b) is equivalent to the formula involving geometric mean described above.

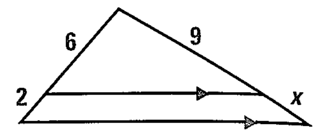
42. **CHALLENGE** A square pyramid is inscribed in a right cylinder so that the base of the pyramid is on a base of the cylinder, and the vertex of the pyramid is on the other base of the cylinder. The cylinder has radius 6 feet and height 12 feet. Find the volume of the pyramid. Round your answer to two decimal places.

MIXED REVIEW

In Exercises 43–45, find the value of x. ***(p. 397)***

43.


44.


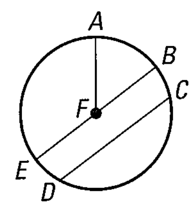
45.


PREVIEW
Prepare for Lesson 12.6 in Exs. 46–52.

46. Copy the diagram at the right. Name a radius, diameter, and chord. ***(p. 651)***

47. Name a minor arc of $\odot F$. ***(p. 659)***

48. Name a major arc of $\odot F$. ***(p. 659)***



Find the area of the circle with the given radius r, diameter d, or circumference C. ***(p. 755)***

49. $r = 3$ m

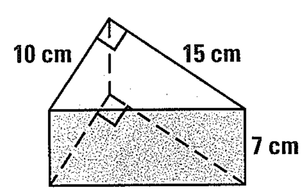
50. $d = 7$ mi

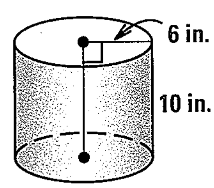
51. $r = 0.4$ cm

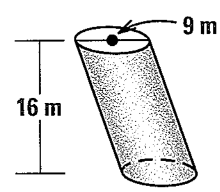
52. $C = 8\pi$ in.

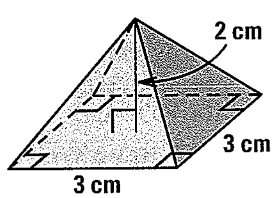
QUIZ for Lessons 12.4–12.5

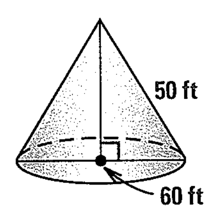
Find the volume of the figure. Round your answer to two decimal places, if necessary. ***(pp. 819, 829)***

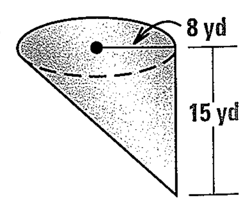
1.


2.


3.


4.


5.


6.


7. Suppose you fill up a cone-shaped cup with water. You then pour the water into a cylindrical cup with the same radius. Both cups have a height of 6 inches. Without doing any calculation, determine how high the water level will be in the cylindrical cup once all of the water is poured into it. *Explain* your reasoning. ***(p. 829)***

@HomeTutor
classzone.com
Keystrokes

12.5 Minimize Surface Area

MATERIALS • computer

QUESTION **How can you find the minimum surface area of a solid with a given volume?**

A manufacturer needs a cylindrical container with a volume of 72 cubic centimeters. You have been asked to find the dimensions of such a container so that it has a minimum surface area.

EXAMPLE **Use a spreadsheet**

STEP 1 ***Make a table*** Make a table with the four column headings shown in Step 4. The first column is for the given volume V. In cell A2, enter 72. In cell A3, enter the formula "=A2".

STEP 2 ***Enter radius*** The second column is for the radius r. Cell B2 stores the starting value for r. So, enter 2 into cell B2. In cell B3, use the formula "=B2 + 0.05" to increase r in increments of 0.05 centimeter.

STEP 3 ***Enter formula for height*** The third column is for the height. In cell C2, enter the formula "=A2/(PI()*B2^2)". *Note:* Your spreadsheet might use a different expression for π.

STEP 4 ***Enter formula for surface area*** The fourth column is for the surface area. In cell D2, enter the formula "=2*PI()*B2^2+2*PI()*B2*C2".

	A	B	C	D
1	Volume V	Radius r	Height= $V/(\pi r^2)$	Surface area $S=2\pi r^2+2\pi rh$
2	72.00	2.00	=A2/(PI()*B2^2)	=2*PI()*B2^2+2*PI()*B2*C2
3	=A2	=B2+0.05		

STEP 5 ***Create more rows*** Use the *Fill Down* feature to create more rows. Rows 3 and 4 of your spreadsheet should resemble the one below.

	A	B	C	D
...				
3	72.00	2.05	5.45	96.65
4	72.00	2.10	5.20	96.28

PRACTICE

1. From the data in your spreadsheet, which dimensions yield a minimum surface area for the given volume? *Explain* how you know.

2. **WHAT IF?** Find the dimensions that give the minimum surface area if the volume of a cylinder is instead 200π cubic centimeters.

12.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS18 for Exs. 3, 13, and 31
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 20, 28, 33, and 34

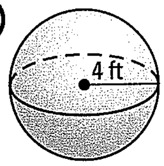
SKILL PRACTICE

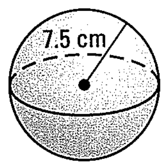
1. **VOCABULARY** What are the formulas for finding the surface area of a sphere and the volume of a sphere?

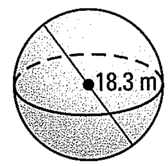
2. ★ **WRITING** When a plane intersects a sphere, what point in the sphere must the plane contain for the intersection to be a great circle? *Explain.*

EXAMPLE 1 on p. 839 for Exs. 3–5

FINDING SURFACE AREA Find the surface area of the sphere. Round your answer to two decimal places.

3.


4.


5.


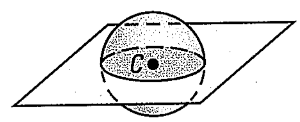
EXAMPLE 2 on p. 839 for Ex. 6

6. ★ **MULTIPLE CHOICE** What is the approximate radius of a sphere with surface area 32π square meters?

Ⓐ 2 meters Ⓑ 2.83 meters Ⓒ 4.90 meters Ⓓ 8 meters

EXAMPLE 3 on p. 840 for Exs. 7–11

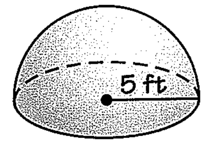
USING A GREAT CIRCLE In Exercises 7–9, use the sphere below. The center of the sphere is C and its circumference is 9.6π inches.



7. Find the radius of the sphere.
8. Find the diameter of the sphere.
9. Find the surface area of one hemisphere.

10. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of a hemisphere with radius 5 feet.

$$S = 4\pi r^2$$
$$= 4\pi(5)^2$$
$$= 100\pi$$
$$\approx 314.16 \text{ ft}^2$$



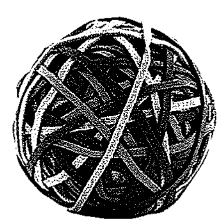
11. **GREAT CIRCLE** The circumference of a great circle of a sphere is 48.4π centimeters. What is the surface area of the sphere?

EXAMPLE 4 on p. 841 for Exs. 12–15

FINDING VOLUME Find the volume of the sphere using the given radius r or diameter d. Round your answer to two decimal places.

12. $r = 6$ in.


13. $r = 40$ mm


14. $d = 5$ cm


15. ERROR ANALYSIS *Describe* and correct the error in finding the volume of a sphere with diameter 16 feet.

$$V = \frac{4}{3}\pi r^2$$
$$= \frac{4}{3}\pi(8)^2$$
$$= 85.33\pi \approx 268.08 \text{ ft}^2$$

USING VOLUME In Exercises 16–18, find the radius of a sphere with the given volume *V*. Round your answers to two decimal places.

16. $V = 1436.76 \text{ m}^3$ **17.** $V = 91.95 \text{ cm}^3$ **18.** $V = 20{,}814.37 \text{ in.}^3$

19. FINDING A DIAMETER The volume of a sphere is 36π cubic feet. What is the diameter of the sphere?

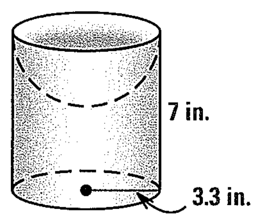
20. ★ MULTIPLE CHOICE Let *V* be the volume of a sphere, *S* be the surface area of the sphere, and *r* be the radius of the sphere. Which equation represents the relationship between these three measures?

Ⓐ $V = \frac{rS}{3}$ Ⓑ $V = \frac{r^2S}{3}$ Ⓒ $V = \frac{3}{2}rS$ Ⓓ $V = \frac{3}{2}r^2S$

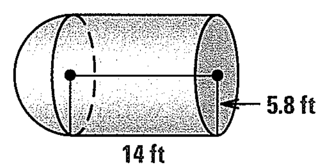
EXAMPLE 5 on p. 841 for Exs. 21–23

COMPOSITE SOLIDS Find the surface area and the volume of the solid. The cylinders and cones are right. Round your answers to two decimal places.

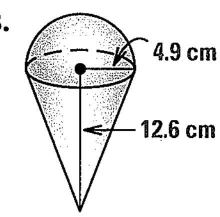
21.



22.



23.



USING A TABLE Copy and complete the table below. Leave your answers in terms of π.

	Radius of sphere	Circumference of great circle	Surface area of sphere	Volume of sphere
24.	10 ft	?	?	?
25.	?	26π in.	?	?
26.	?	?	$2500\pi \text{ cm}^2$	?
27.	?	?	?	$12{,}348\pi \text{ m}^3$

28. ★ MULTIPLE CHOICE A sphere is inscribed in a cube with volume 64 cubic centimeters. What is the surface area of the sphere?

Ⓐ $4\pi \text{ cm}^2$ Ⓑ $\frac{32}{3}\pi \text{ cm}^2$ Ⓒ $16\pi \text{ cm}^2$ Ⓓ $64\pi \text{ cm}^2$

29. CHALLENGE The volume of a right cylinder is the same as the volume of a sphere. The radius of the sphere is 1 inch.

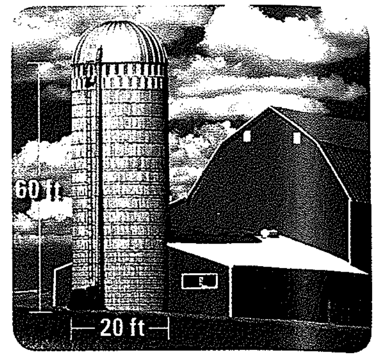
a. Give three possibilities for the dimensions of the cylinder.

b. Show that the surface area of the cylinder is sometimes greater than the surface area of the sphere.

PROBLEM SOLVING

EXAMPLE 5
on p. 841
for Ex. 30

30. **GRAIN SILO** A grain silo has the dimensions shown. The top of the silo is a hemispherical shape. Find the volume of the grain silo.



@HomeTutor for problem solving help at classzone.com

31. **GEOGRAPHY** The circumference of Earth is about 24,855 miles. Find the surface area of the Western Hemisphere of Earth.

@HomeTutor for problem solving help at classzone.com

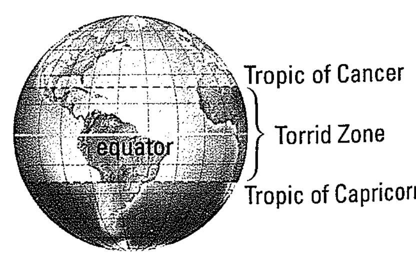
32. **MULTI-STEP PROBLEM** A ball has volume 1427.54 cubic centimeters.
   - **a.** Find the radius of the ball. Round your answer to two decimal places.
   - **b.** Find the surface area of the ball. Round your answer to two decimal places.

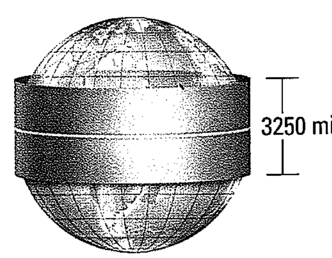
33. ★ **SHORT RESPONSE** Tennis balls are stored in a cylindrical container with height 8.625 inches and radius 1.43 inches.

   - **a.** The circumference of a tennis ball is 8 inches. Find the volume of a tennis ball.
   - **b.** There are 3 tennis balls in the container. Find the amount of space within the cylinder not taken up by the tennis balls.

34. ★ **EXTENDED RESPONSE** A partially filled balloon has circumference 27π centimeters. Assume the balloon is a sphere.
   - **a. Calculate** Find the volume of the balloon.
   - **b. Predict** Suppose you double the radius by increasing the air in the balloon. *Explain* what you expect to happen to the volume.
   - **c. Justify** Find the volume of the balloon with the radius doubled. Was your prediction from part (b) correct? What is the ratio of this volume to the original volume?

35. **GEOGRAPHY** The Torrid Zone on Earth is the area between the Tropic of Cancer and the Tropic of Capricorn, as shown. The distance between these two tropics is about 3250 miles. You can think of this distance as the height of a cylindrical belt around Earth at the equator, as shown.

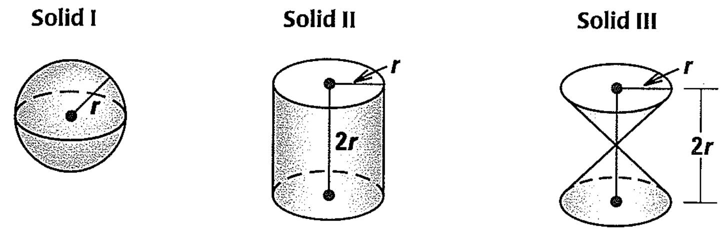




   - **a.** Estimate the surface area of the Torrid Zone and the surface area of Earth. (Earth's radius is about 3963 miles at the equator.)
   - **b.** A meteorite is equally likely to hit anywhere on Earth. Estimate the probability that a meteorite will land in the Torrid Zone.

○ = **WORKED-OUT SOLUTIONS on p. WS1**   ★ = **STANDARDIZED TEST PRACTICE**

36. **REASONING** List the following three solids in order from least to greatest (a) surface area and (b) volume.



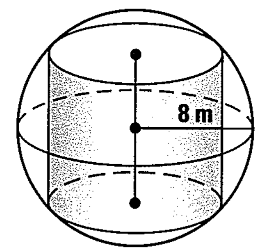
37. **ROTATION** A circle with diameter 18 inches is rotated about its diameter. Find the surface area and the volume of the solid formed.

38. **TECHNOLOGY** A cylinder with height $2x$ is inscribed in a sphere with radius 8 meters. The center of the sphere is the midpoint of the altitude that joins the centers of the bases of the cylinder.

a. Show that the volume V of the cylinder is $2\pi x(64 - x^2)$.

b. Use a graphing calculator to graph $V = 2\pi x(64 - x^2)$ for values of x between 0 and 8. Find the value of x that gives the maximum value of V.

c. Use the value for x from part (b) to find the maximum volume of the cylinder.



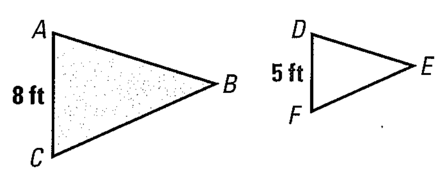
39. **CHALLENGE** A sphere with radius 2 centimeters is inscribed in a right cone with height 6 centimeters. Find the surface area and the volume of the cone.

MIXED REVIEW

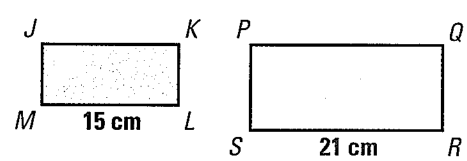
PREVIEW
Prepare for Lesson 12.7 in Exs. 40–41.

In Exercises 40 and 41, the polygons are similar. Find the ratio (red to blue) of their areas. Find the unknown area. Round your answer to two decimal places. *(p. 737)*

40. Area of $\triangle ABC = 42\text{ ft}^2$
Area of $\triangle DEF$ = ___?___

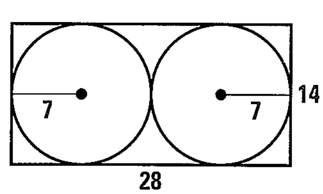


41. Area of $PQRS = 195\text{ cm}^2$
Area of $JKLM$ = ___?___

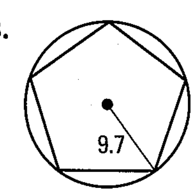


Find the probability that a randomly chosen point in the figure lies in the shaded region. *(p. 771)*

42.



43.



44. A cone is inscribed in a right cylinder with volume 330 cubic units. Find the volume of the cone. *(pp. 819, 829)*

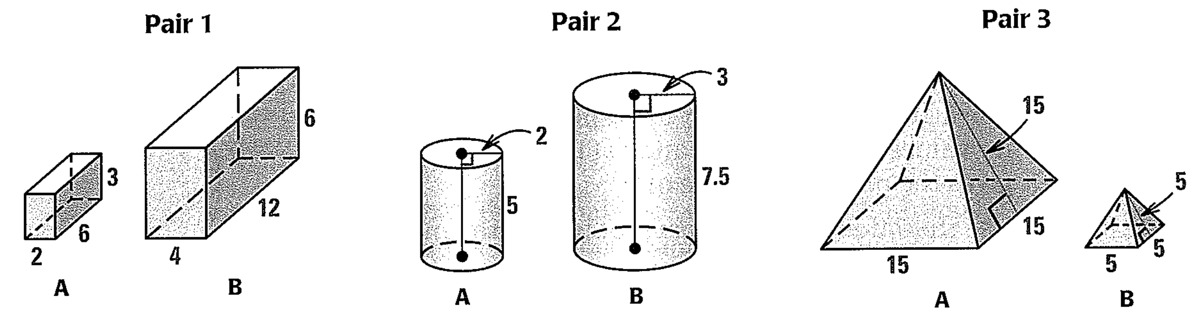
12.7 Investigate Similar Solids

MATERIALS • paper • pencil

QUESTION **How are the surface areas and volumes of similar solids related?**

EXPLORE **Compare the surface areas and volumes of similar solids**

The solids shown below are *similar.*



STEP 1 ***Make a table*** Copy and complete the table below.

	Scale factor of Solid A to Solid B	Surface area of Solid A, S_A	Surface area of Solid B, S_B	$\frac{S_A}{S_B}$
Pair 1	$\frac{1}{2}$	?	?	?
Pair 2	?	?	63π	?
Pair 3	?	?	?	$\frac{9}{1}$

STEP 2 ***Insert columns*** Insert columns for V_A, V_B, and $\frac{V_A}{V_B}$. Use the dimensions of the solids to find V_A, the volume of Solid A, and V_B, the volume of Solid B. Then find the ratio of these volumes.

STEP 3 ***Compare ratios*** Compare the ratios $\frac{S_A}{S_B}$ and $\frac{V_A}{V_B}$ to the scale factor.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Make a conjecture about how the surface areas and volumes of similar solids are related to the scale factor.
2. Use your conjecture to write a ratio of surface areas and volumes if the dimensions of two similar rectangular prisms are ℓ, w, h, and $k\ell$, kw, kh.

12.7 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS18 for Exs. 3, 9, and 27
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 16, 28, 31, and 33
◆ = **MULTIPLE REPRESENTATIONS** Ex. 34

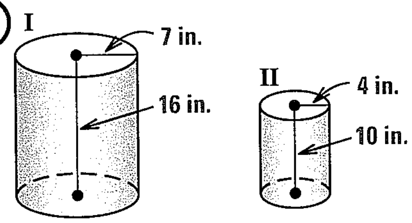
SKILL PRACTICE

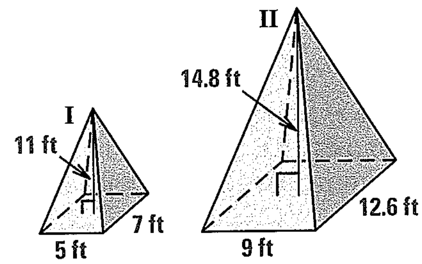
1. **VOCABULARY** What does it mean for two solids to be similar?

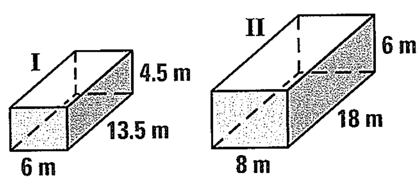
2. ★ **WRITING** How are the volumes of similar solids related?

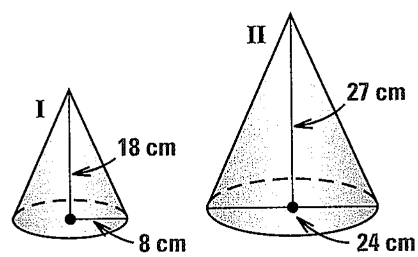
EXAMPLE 1 on p. 847 for Exs. 3–7

IDENTIFYING SIMILAR SOLIDS **Tell whether the pair of right solids is similar. *Explain* your reasoning.**

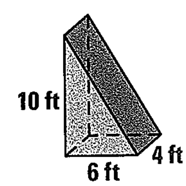
3.


4.


5.


6.


7. ★ **MULTIPLE CHOICE** Which set of dimensions corresponds to a triangular prism that is similar to the prism shown?

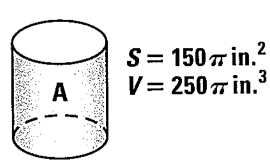


Ⓐ 2 feet by 1 foot by 5 feet
Ⓑ 4 feet by 2 feet by 8 feet
Ⓒ 9 feet by 6 feet by 20 feet
Ⓓ 15 feet by 10 feet by 25 feet

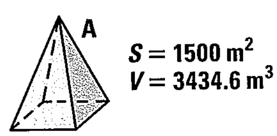
EXAMPLE 2 on p. 848 for Exs. 8–11

USING SCALE FACTOR **Solid A (shown) is similar to Solid B (not shown) with the given scale factor of A to B. Find the surface area and volume of Solid B.**

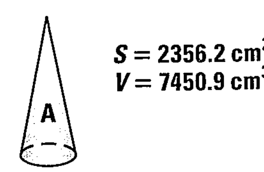
8. Scale factor of 1 : 2



9. Scale factor of 3 : 1



10. Scale factor of 5 : 2

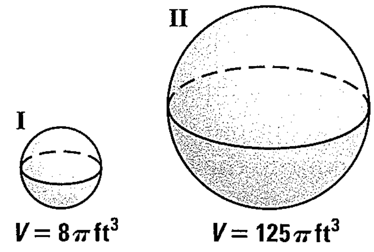


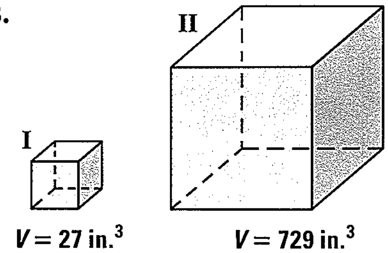
11. **ERROR ANALYSIS** The scale factor of two similar solids is 1 : 4. The volume of the smaller Solid A is 500π. *Describe* and correct the error in writing an equation to find the volume of the larger Solid B.

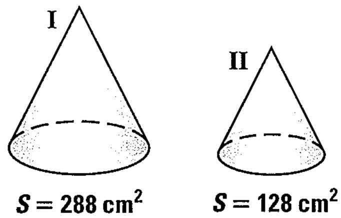
$$\frac{500\pi}{\text{Volume of B}} = \frac{1^2}{4^2}$$ ✗

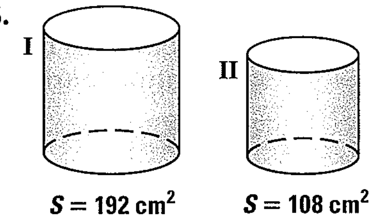
EXAMPLE 3
on p. 849
for Exs. 12–18

FINDING SCALE FACTOR In Exercises 12–15, Solid I is similar to Solid II. Find the scale factor of Solid I to Solid II.

12.


13.


14.


15.


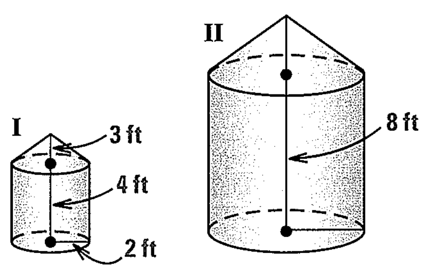
16. ★ **MULTIPLE CHOICE** The volumes of two similar cones are 8π and 27π. What is the ratio of the lateral areas of the cones?

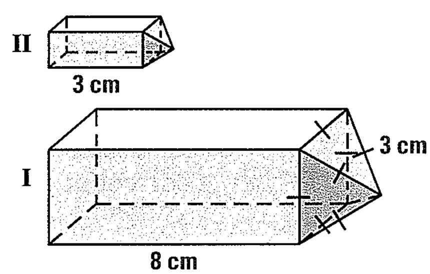
Ⓐ $\frac{8}{27}$ Ⓑ $\frac{1}{3}$ Ⓒ $\frac{4}{9}$ Ⓓ $\frac{2}{3}$

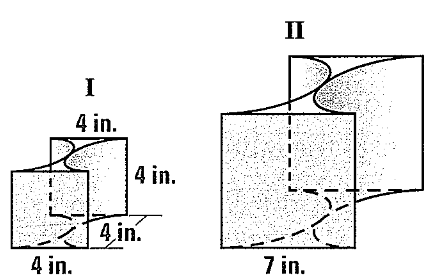
17. **FINDING A RATIO** Two spheres have volumes of 2π cubic feet and 16π cubic feet. What is the ratio of the surface area of the smaller sphere to the surface area of the larger sphere?

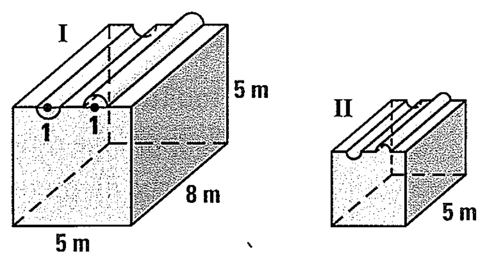
18. **FINDING SURFACE AREA** Two similar cylinders have a scale factor of 2 : 3. The smaller cylinder has a surface area of 78π square meters. Find the surface area of the larger cylinder.

COMPOSITE SOLIDS In Exercises 19–22, Solid I is similar to Solid II. Find the surface area and volume of Solid II.

19.


20.


21.


22.


23. **ALGEBRA** Two similar cylinders have surface areas of 54π square feet and 384π square feet. The height of each cylinder is equal to its diameter. Find the radius and height of both cylinders.

24. **CHALLENGE** A plane parallel to the base of a cone divides the cone into two pieces with the dimensions shown. Find each ratio described.

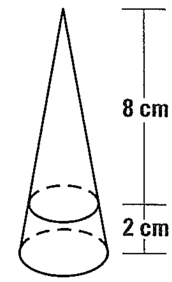
a. The area of the top shaded circle to the area of the bottom shaded circle

b. The slant height of the top part of the cone to the slant height of the whole cone

c. The lateral area of the top part of the cone to the lateral area of the whole cone

d. The volume of the top part of the cone to the volume of the whole cone

e. The volume of the top part of the cone to the volume of the bottom part



PROBLEM SOLVING

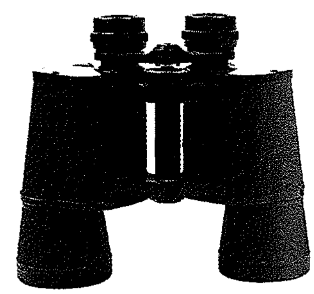
EXAMPLE 4 on p. 849 for Exs. 25–27

25. **COFFEE MUGS** The heights of two similar coffee mugs are 3.5 inches and 4 inches. The larger mug holds 12 fluid ounces. What is the capacity of the smaller mug?

@HomeTutor for problem solving help at classzone.com

26. **ARCHITECTURE** You have a pair of binoculars that is similar in shape to the structure on page 847. Your binoculars are 6 inches high, and the height of the structure is 45 feet. Find the ratio of the volume of your binoculars to the volume of the structure.

@HomeTutor for problem solving help at classzone.com



27. **PARTY PLANNING** Two similar punch bowls have a scale factor of 3 : 4. The amount of lemonade to be added is proportional to the volume. How much lemonade does the smaller bowl require if the larger bowl requires 64 fluid ounces?

28. ★ **OPEN-ENDED MATH** Using the scale factor 2 : 5, sketch a pair of solids in the correct proportions. Label the dimensions of the solids.

29. **MULTI-STEP PROBLEM** Two oranges are both spheres with diameters 3.2 inches and 4 inches. The skin on both oranges has an average thickness of $\frac{1}{8}$ inch.

a. Find the volume of each unpeeled orange.

b. *Compare* the ratio of the diameters to the ratio of the volumes.

c. Find the diameter of each orange after being peeled.

d. *Compare* the ratio of surface areas of the peeled oranges to the ratio of the volumes of the peeled oranges.

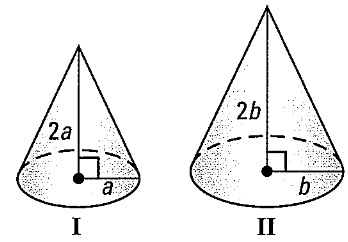
Animated Geometry at classzone.com

30. **ALGEBRA** Use the two similar cones shown.

   a. What is the scale factor of Cone I to Cone II? What should the ratio of the volume of Cone I to the volume of Cone II be?

   b. Write an expression for the volume of each solid.

   c. Write and simplify an expression for the ratio of the volume of Cone I to the volume of Cone II. Does your answer agree with your answer to part (a)? *Explain.*

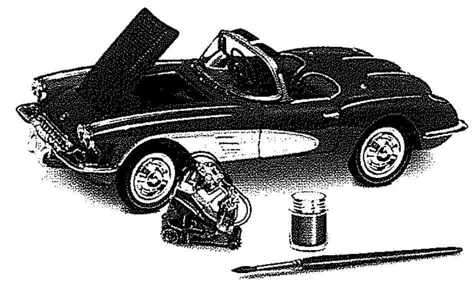


31. ★ **EXTENDED RESPONSE** The scale factor of the model car at the right to the actual car is 1 : 18.

   a. The model has length 8 inches. What is the length of the actual car?

   b. Each tire of the model has a surface area of 12.1 square inches. What is the surface area of each tire of the actual car?

   c. The actual car's engine has volume 8748 cubic inches. Find the volume of the model car's engine.



32. **USING VOLUMES** Two similar cylinders have volumes 16π and 432π. The larger cylinder has lateral area 72π. Find the lateral area of the smaller cylinder.

33. ★ **SHORT RESPONSE** A snow figure is made using three balls of snow with diameters 25 centimeters, 35 centimeters, and 45 centimeters. The smallest weighs about 1.2 kilograms. Find the total weight of the snow used to make the snow figure. *Explain* your reasoning.

34. **MULTIPLE REPRESENTATIONS** A gas is enclosed in a cubical container with side length s in centimeters. Its temperature remains constant while the side length varies. By the *Ideal Gas Law,* the pressure P in atmospheres (atm) of the gas varies inversely with its volume.

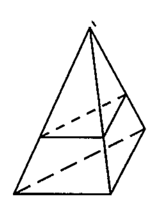
   a. **Writing an Equation** Write an equation relating P and s. You will need to introduce a constant of variation k.

   b. **Making a Table** Copy and complete the table below for various side lengths. Express the pressure P in terms of the constant k.

Side length s (cm)	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
Pressure P (atm)	?	$8k$	k	?	?

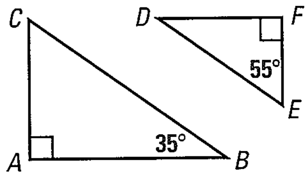
   c. **Drawing a Graph** For this particular gas, $k = 1$. Use your table to sketch a graph of P versus s. Place P on the vertical axis and s on the horizontal axis. Does the graph show a linear relationship? *Explain.*

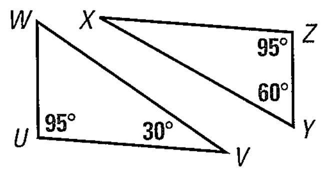
35. **CHALLENGE** A plane parallel to the base of a pyramid separates the pyramid into two pieces with equal volumes. The height of the pyramid is 12 feet. Find the height of the top piece.

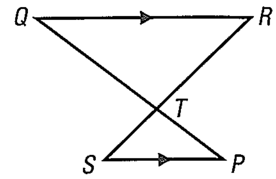


MIXED REVIEW

Determine whether the triangles are similar. If they are, write a similarity statement. *(p. 381)*

36. 

37. 

38. 

The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides. *(p. 507)*

39. 900° 40. 180° 41. 540° 42. 1080°

Write a standard equation of the circle with the given center and radius. *(p. 699)*

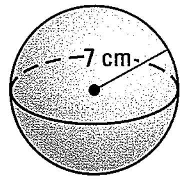
43. Center (2, 5), radius 4 44. Center (−3, 2), radius 6

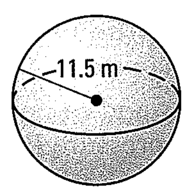
Sketch the described solid and find its surface area. Round your answer to two decimal places, if necessary. *(p. 803)*

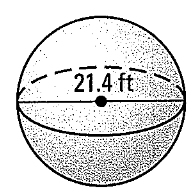
45. Right rectangular prism with length 8 feet, width 6 feet, and height 3 feet
46. Right regular pentagonal prism with all edges measuring 12 millimeters
47. Right cylinder with radius 4 inches and height 4 inches
48. Right cylinder with diameter 9 centimeters and height 7 centimeters

QUIZ *for Lessons 12.6–12.7*

Find the surface area and volume of the sphere. Round your answers to two decimal places. *(p. 838)*

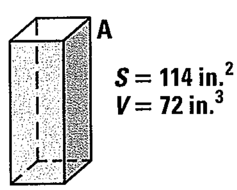
1. 

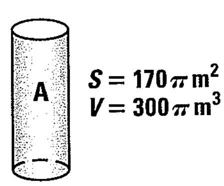
2. 

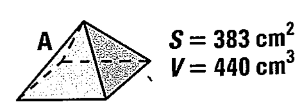
3. 

Solid A (shown) is similar to Solid B (not shown) with the given scale factor of A to B. Find the surface area S and volume V of Solid B. *(p. 847)*

4. Scale factor of 1 : 3 5. Scale factor of 2 : 3 6. Scale factor of 5 : 4







7. Two similar cones have volumes 729π cubic feet and 343π cubic feet. What is the scale factor of the larger cone to the smaller cone? *(p. 847)*

MIXED REVIEW of Problem Solving

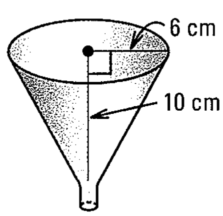
STATE TEST PRACTICE
classzone.com

Lessons 12.4–12.7

1. **MULTI-STEP PROBLEM** You have a container in the shape of a right rectangular prism with inside dimensions of length 24 inches, width 16 inches, and height 20 inches.
   - **a.** Find the volume of the inside of the container.
   - **b.** You are going to fill the container with boxes of cookies that are congruent right rectangular prisms. Each box has length 8 inches, width 2 inches, and height 3 inches. Find the volume of one box of cookies.
   - **c.** How many boxes of cookies will fit inside the cardboard container?

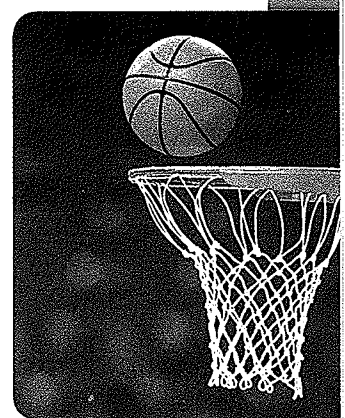
2. **SHORT RESPONSE** You have a cup in the shape of a cylinder with inside dimensions of diameter 2.5 inches and height 7 inches.
   - **a.** Find the volume of the inside of the cup.
   - **b.** You have an 18 ounce bottle of orange juice that you want to pour into the cup. Will all of the juice fit? *Explain* your reasoning. ($1 \text{ in.}^3 \approx 0.554$ fluid ounces)

3. **EXTENDED RESPONSE** You have a funnel with the dimensions shown.

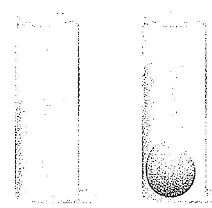


   - **a.** Find the approximate volume of the funnel.
   - **b.** You are going to use the funnel to put oil in a car. Oil flows out of the funnel at a rate of 45 milliliters per second. How long will it take to empty the funnel when it is full of oil? ($1 \text{ mL} = 1 \text{ cm}^3$)
   - **c.** How long would it take to empty a funnel with radius 10 cm and height 6 cm?
   - **d.** *Explain* why you can claim that the time calculated in part (c) is greater than the time calculated in part (b) without doing any calculations.

4. **EXTENDED RESPONSE** An official men's basketball has circumference 29.5 inches. An official women's basketball has circumference 28.5 inches.
   - **a.** Find the surface area and volume of the men's basketball.
   - **b.** Find the surface area and volume of the women's basketball using the formulas for surface area and volume of a sphere.
   - **c.** Use your answers in part (a) and the Similar Solids Theorem to find the surface area and volume of the women's basketball. Do your results match your answers in part (b)?



5. **GRIDDED ANSWER** To accurately measure the radius of a spherical rock, you place the rock into a cylindrical glass containing water. When you do so, the water level rises $\frac{9}{64}$ inch. The radius of the glass is 2 inches. What is the radius of the rock?



6. **SHORT RESPONSE** Sketch a rectangular prism and label its dimensions. Change the dimensions of the prism so that its surface area increases and its volume decreases.

7. **SHORT RESPONSE** A hemisphere and a right cone have the same radius and the height of the cone is equal to the radius. *Compare* the volumes of the solids.

8. **SHORT RESPONSE** *Explain* why the height of a right cone is always less than its slant height. Include a diagram in your answer.

12 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

Exploring Solids and Their Properties

Euler's Theorem is useful when finding the number of faces, edges, or vertices on a polyhedron, especially when one of those quantities is difficult to count by hand.

For example, suppose you want to find the number of edges on a regular icosahedron, which has 20 faces. You count 12 vertices on the solid. To calculate the number of edges, use Euler's Theorem:

$F + V = E + 2$ **Write Euler's Theorem.**

$20 + 12 = E + 2$ **Substitute known values.**

$30 = E$ **Solve for *E*.**

Big Idea 2

Solving Problems Using Surface Area and Volume

Figure	Surface Area	Volume
Right prism	$S = 2B + Ph$	$V = Bh$
Right cylinder	$S = 2B + Ch$	$V = Bh$
Regular pyramid	$S = B + \frac{1}{2}P\ell$	$V = \frac{1}{3}Bh$
Right cone	$S = B + \frac{1}{2}C\ell$	$V = \frac{1}{3}Bh$
Sphere	$S = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

The volume formulas for prisms, cylinders, pyramids, and cones can be used for oblique solids.

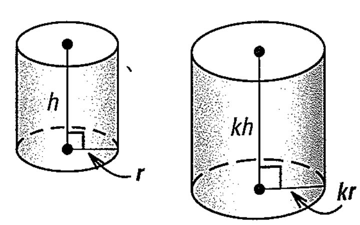
While many of the above formulas can be written in terms of more detailed variables, it is more important to remember the more general formulas for a greater understanding of why they are true.

Big Idea 3

Connecting Similarity to Solids

The similarity concepts learned in Chapter 6 can be extended to 3-dimensional figures as well.

Suppose you have a right cylindrical can whose surface area and volume are known. You are then given a new can whose linear dimensions are k times the dimensions of the original can. If the surface area of the original can is S and the volume of the original can is V, then the surface area and volume of the new can can be expressed as k^2S and k^3V, respectively.



12 CHAPTER REVIEW

@HomeTutor
classzone.com
- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- polyhedron, *p. 794*
  face, edge, vertex, base
- regular polyhedron, *p. 796*
- convex polyhedron, *p. 796*
- Platonic solids, *p. 796*
- tetrahedron, *p. 796*
- cube, *p. 796*
- octahedron, *p. 796*
- dodecahedron, *p. 796*
- icosahedron, *p. 796*
- cross section, *p. 797*
- prism, *p. 803*
  lateral faces, lateral edges
- surface area, *p. 803*
- lateral area, *p. 803*
- net, *p. 803*
- right prism, *p. 804*
- oblique prism, *p. 804*
- cylinder, *p. 805*
- right cylinder, *p. 805*
- pyramid, *p. 810*
- vertex of a pyramid, *p. 810*
- regular pyramid, *p. 810*
- slant height, *p. 810*
- cone, *p. 812*
- vertex of a cone, *p. 812*
- right cone, *p. 812*
- lateral surface, *p. 812*
- volume, *p. 819*
- sphere, *p. 838*
  center, radius, chord, diameter
- great circle, *p. 839*
- hemisphere, *p. 839*
- similar solids, *p. 847*

VOCABULARY EXERCISES

1. Copy and complete: A __?__ is the set of all points in space equidistant from a given point.

2. **WRITING** Sketch a right rectangular prism and an oblique rectangular prism. *Compare* the prisms.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 12.

12.1 Explore Solids

pp. 794–801

EXAMPLE

A polyhedron has 16 vertices and 24 edges. How many faces does the polyhedron have?

$F + V = E + 2$	**Euler's Theorem**
$F + 16 = 24 + 2$	**Substitute known values.**
$F = 10$	**Solve for *F*.**

▶ The polyhedron has 10 faces.

EXAMPLES 2 and 3 on pp. 796–797 for Exs. 3–5

EXERCISES

Use Euler's Theorem to find the value of n.

3. Faces: 20
   Vertices: n
   Edges: 30

4. Faces: n
   Vertices: 6
   Edges: 12

5. Faces: 14
   Vertices: 24
   Edges: n

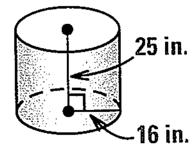
12 CHAPTER REVIEW

12.2 Surface Area of Prisms and Cylinders

pp. 803–809

EXAMPLE

Find the surface area of the right cylinder.



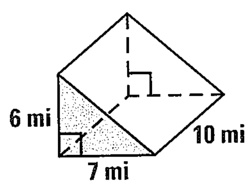
$S = 2\pi r^2 + 2\pi rh$	**Write formula.**
$= 2\pi(16)^2 + 2\pi(16)(25)$	**Substitute for r and h.**
$= 1312\pi$	**Simplify.**
≈ 4121.77	**Use a calculator.**

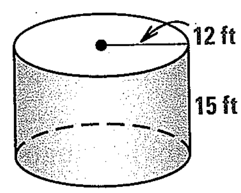
▸ The surface area of the cylinder is about 4121.77 square inches.

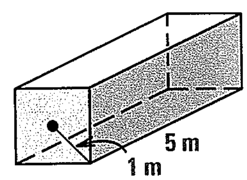
EXERCISES

EXAMPLES 2, 3, and 4 on pp. 804–806 for Exs. 6–9

Find the surface area of the right prism or right cylinder. Round your answer to two decimal places, if necessary.

6. 

7. 

8. 

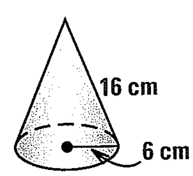
9. A cylinder has a surface area of 44π square meters and a radius of 2 meters. Find the height of the cylinder.

12.3 Surface Area of Pyramids and Cones

pp. 810–817

EXAMPLE

Find the lateral area of the right cone.



Lateral area $= \pi r\ell$	**Write formula.**
$= \pi(6)(16)$	**Substitute for r and ℓ.**
$= 96\pi$	**Simplify.**
≈ 301.59	**Use a calculator.**

▸ The lateral area of the cone is about 301.59 square centimeters.

EXERCISES

EXAMPLES 1, 2, and 4 on pp. 810–813 for Exs. 10–12

10. Find the surface area of a right square pyramid with base edge length 2 feet and height 5 feet.
11. The surface area of a cone with height 15 centimeters is 500π square centimeters. Find the radius of the base of the cone. Round your answer to two decimal places.
12. Find the surface area of a right octagonal pyramid with height 2.5 yards, and its base has apothem length 1.5 yards.

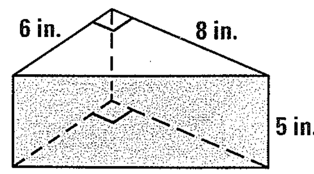
@HomeTutor
classzone.com
Chapter Review Practice

12.4 Volume of Prisms and Cylinders

pp. 819–825

EXAMPLE

Find the volume of the right triangular prism.



The area of the base is $B = \frac{1}{2}(6)(8) = 24$ square inches. Use $h = 5$ to find the volume.

$V = Bh$	**Write formula.**
$= 24(5)$	**Substitute for *B* and *h*.**
$= 120$	**Simplify.**

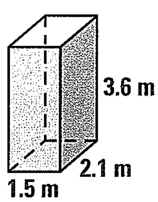
▶ The volume of the prism is 120 cubic inches.

EXERCISES

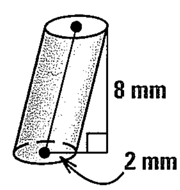
EXAMPLES 2 and 4 on pp. 820–821 for Exs. 13–15

Find the volume of the right prism or oblique cylinder. Round your answer to two decimal places.

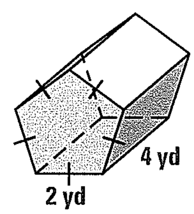
13.



14.



15.

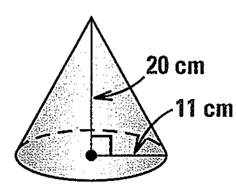


12.5 Volume of Pyramids and Cones

pp. 829–836

EXAMPLE

Find the volume of the right cone.



The area of the base is $B = \pi r^2 = \pi(11)^2 \approx 380.13 \text{ cm}^2$. Use $h = 20$ to find the volume.

$V = \frac{1}{3}Bh$	**Write formula.**
$\approx \frac{1}{3}(380.13)(20)$	**Substitute for *B* and *h*.**
≈ 2534.2	**Simplify.**

▶ The volume of the cone is about 2534.2 cubic centimeters.

EXERCISES

EXAMPLES 1 and 2 on pp. 829–830 for Exs. 16–17

16. A cone with diameter 16 centimeters has height 15 centimeters. Find the volume of the cone. Round your answer to two decimal places.

17. The volume of a pyramid is 60 cubic inches and the height is 15 inches. Find the area of the base.

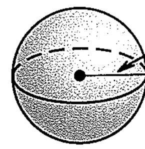
12 CHAPTER REVIEW

12.6 Surface Area and Volume of Spheres

pp. 838–845

EXAMPLE

Find the surface area of the sphere.



$S = 4\pi r^2$ **Write formula.**

$= 4\pi(7)^2$ **Substitute 7 for *r*.**

$= 196\pi$ **Simplify.**

▶ The surface area of the sphere is 196π, or about 615.75 square meters.

EXERCISES

EXAMPLES 1, 4, and 5 on pp. 839, 841 for Exs. 18–19

18. **ASTRONOMY** The shape of Pluto can be approximated as a sphere of diameter 2390 kilometers. Find the surface area and volume of Pluto.

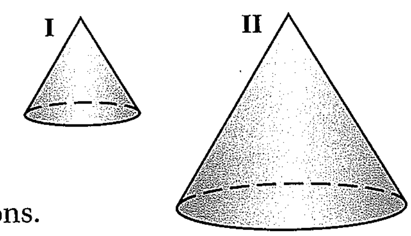
19. A solid is composed of a cube with side length 6 meters and a hemisphere with diameter 6 meters. Find the volume of the solid. Round your answer to two decimal places.

12.7 Explore Similar Solids

pp. 847–854

EXAMPLE

The cones are similar with a scale factor of 1 : 2. Find the surface area and volume of Cone II given that the surface area of Cone I is 384π square inches and the volume of Cone I is 768π cubic inches.



Use Theorem 12.13 to write and solve two proportions.

$$\frac{\text{Surface area of I}}{\text{Surface area of II}} = \frac{a^2}{b^2} \qquad \frac{\text{Volume of I}}{\text{Volume of II}} = \frac{a^3}{b^3}$$

$$\frac{384\pi}{\text{Surface area of II}} = \frac{1^2}{2^2} \qquad \frac{768\pi}{\text{Volume of II}} = \frac{1^3}{2^3}$$

$$\text{Surface area of II} = 1536\pi \text{ in.}^2 \qquad \text{Volume of II} = 6144\pi \text{ in.}^3$$

▶ The surface area of Cone II is 1536π, or about 4825.49 square inches, and the volume of Cone II is 6144π, or about 19,301.95 cubic inches.

EXERCISES

EXAMPLE 2 on p. 848 for Exs. 20–22

Solid A is similar to Solid B with the given scale factor of A to B. The surface area and volume of Solid A are given. Find the surface area and volume of Solid B.

20. Scale factor of 1 : 4
 $S = 62 \text{ cm}^2$
 $V = 30 \text{ cm}^3$

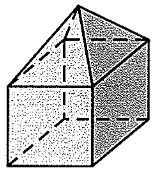
21. Scale factor of 1 : 3
 $S = 112\pi \text{ m}^2$
 $V = 160\pi \text{ m}^3$

22. Scale factor of 2 : 5
 $S = 144\pi \text{ yd}^2$
 $V = 288\pi \text{ yd}^3$

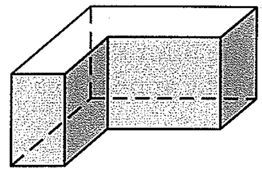
12 CHAPTER TEST

Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.

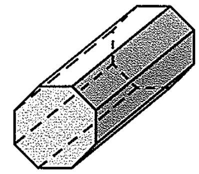
1.



2.

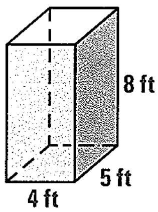


3.

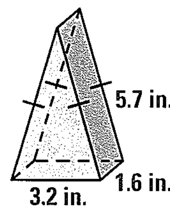


Find the surface area of the solid. The prisms, pyramids, cylinders, and cones are right. Round your answer to two decimal places, if necessary.

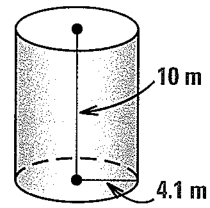
4.



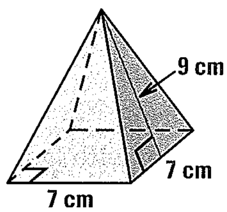
5.



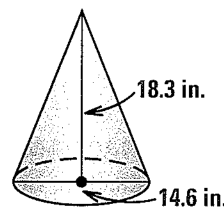
6.



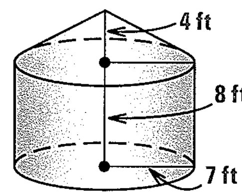
7.



8.

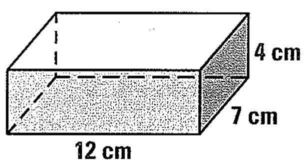


9.

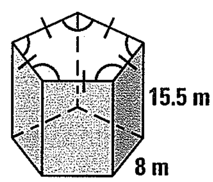


Find the volume of the right prism or right cylinder. Round your answer to two decimal places, if necessary.

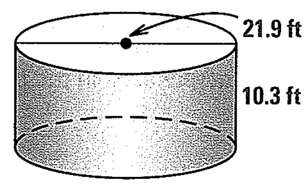
10.



11.

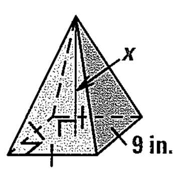


12.

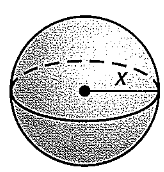


In Exercises 13–15, solve for *x*.

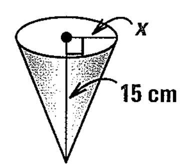
13. Volume = 324 in.^{3}



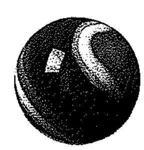
14. Volume = $\frac{32\pi}{3}$ ft^3



15. Volume = 180π cm^3



16. MARBLES The diameter of the marble shown is 35 millimeters. Find the surface area and volume of the marble.



17. PACKAGING Two similar cylindrical cans have a scale factor of 2 : 3. The smaller can has surface area 308π square inches and volume 735π cubic inches. Find the surface area and volume of the larger can.