

Geo

4 Congruent Triangles

- 4.1 Apply Triangle Sum Properties
- 4.2 Apply Congruence and Triangles
- 4.3 Prove Triangles Congruent by SSS
- 4.4 Prove Triangles Congruent by SAS and HL
- 4.5 Prove Triangles Congruent by ASA and AAS
- 4.6 Use Congruent Triangles
- 4.7 Use Isosceles and Equilateral Triangles
- 4.8 Perform Congruence Transformations

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 4: classifying angles, solving linear equations, finding midpoints, and using angle relationships.

Prerequisite Skills

VOCABULARY CHECK

Classify the angle as *acute*, *obtuse*, *right*, or *straight*.

1. $m\angle A = 115^\circ$ 2. $m\angle B = 90^\circ$ 3. $m\angle C = 35^\circ$ 4. $m\angle D = 95^\circ$

SKILLS AND ALGEBRA CHECK

Solve the equation. (Review p. 65 for 4.1, 4.2.)

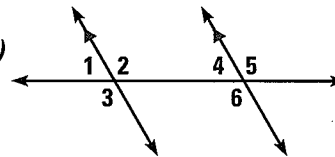
5. $70 + 2y = 180$ 6. $2x = 5x - 54$ 7. $40 + x + 65 = 180$

Find the coordinates of the midpoint of \overline{PQ} . (Review p. 15 for 4.3.)

8. $P(2, -5), Q(-1, -2)$ 9. $P(-4, 7), Q(1, -5)$ 10. $P(h, k), Q(h, 0)$

Name the theorem or postulate that justifies the statement about the diagram. (Review p. 154 for 4.3–4.5.)

11. $\angle 2 \cong \angle 3$ 12. $\angle 1 \cong \angle 4$
13. $\angle 2 \cong \angle 6$ 14. $\angle 3 \cong \angle 5$



@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 4, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 281. You will also use the key vocabulary listed below.

Big Ideas

1. **Classifying triangles by sides and angles**
2. **Proving that triangles are congruent**
3. **Using coordinate geometry to investigate triangle relationships**

KEY VOCABULARY

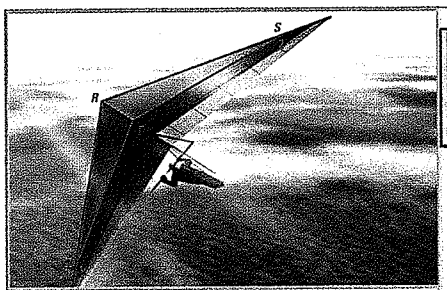
- triangle, p. 217
- corollary, p. 220
- isosceles triangle, p. 264
- scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- congruent figures, p. 225
- legs, vertex angle, base, base angles
- interior angles, p. 218
- corresponding parts, p. 225
- transformation, p. 272
- exterior angles, p. 218
- right triangle, p. 241
- translation, reflection, rotation
- legs, hypotenuse
- flow proof, p. 250

Why?

Triangles are used to add strength to structures in real-world situations. For example, the frame of a hang glider involves several triangles.

Animated Geometry

The animation illustrated below for Example 1 on page 256 helps you answer this question: What must be true about \overline{QT} and \overline{ST} for the hang glider to fly straight?



You will use congruent segments and angles in the hang glider to write a proof.

Given:	Statement	Reasons
$\angle 1 \cong \angle 2$		
$\angle RTQ \cong \angle RTS$	1.	
Statements:		
$\angle RQT$ is supplementary to $\angle 1$, and $\angle RST$ is supplementary to $\angle 2$.	2.	
$\angle RQT \cong \angle RST$	3.	
$RT = RT$	4.	
$\triangle RQT \cong \triangle RST$	5.	
$QT \cong ST$		
Reasons:		
Given	6.	
Given		
Reflexive Property of Segment Congruence	7.	
AAS Congruence Theorem		

Scroll down to see the information needed to prove that $\overline{QT} \cong \overline{ST}$.

Animated Geometry at classzone.com

Other animations for Chapter 4: pages 234, 242, 250, 257, and 274

4.1 Angle Sums in Triangles

MATERIALS • paper • pencil • scissors • ruler

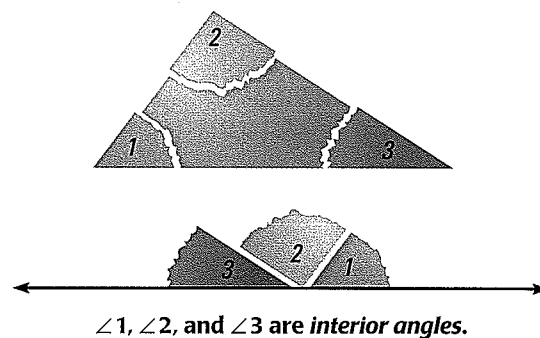
QUESTION What are some relationships among the *interior angles* of a triangle and *exterior angles* of a triangle?

EXPLORE 1 Find the sum of the measures of interior angles

STEP 1 *Draw triangles* Draw and cut out several different triangles.

STEP 2 *Tear off corners* For each triangle, tear off the three corners and place them next to each other, as shown in the diagram.

STEP 3 *Make a conjecture* Make a conjecture about the sum of the measures of the interior angles of a triangle.

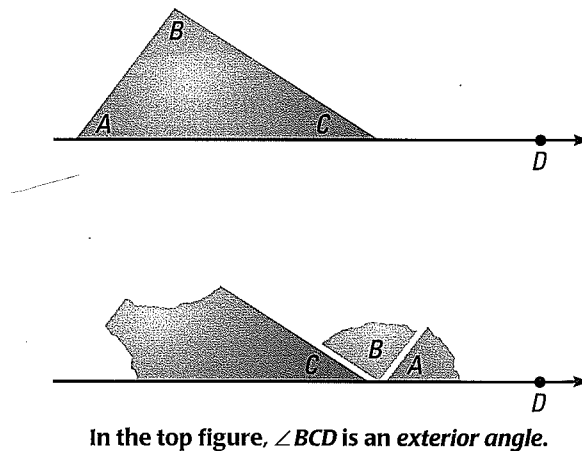


EXPLORE 2 Find the measure of an exterior angle of a triangle

STEP 1 *Draw exterior angle* Draw and cut out several different triangles. Place each triangle on a piece of paper and extend one side to form an *exterior angle*, as shown in the diagram.

STEP 2 *Tear off corners* For each triangle, tear off the corners that are not next to the exterior angle. Use them to fill the exterior angle, as shown.

STEP 3 *Make a conjecture* Make a conjecture about the relationship between the measure of an exterior angle of a triangle and the measures of the nonadjacent interior angles.



DRAW CONCLUSIONS Use your observations to complete these exercises

- Given the measures of two interior angles of a triangle, how can you find the measure of the third angle?
- Draw several different triangles that each have one right angle. Show that the two acute angles of a right triangle are complementary.

4.1 EXERCISES

HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS on p. WS4 for Exs. 9, 15, and 41

★ = STANDARDIZED TEST PRACTICE Exs. 7, 20, 31, 43, and 51

SKILL PRACTICE

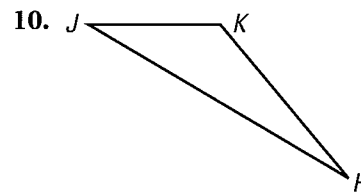
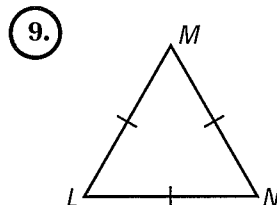
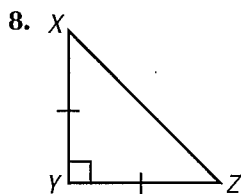
VOCABULARY Match the triangle description with the most specific name.

- | | |
|----------------------------------------------------|----------------|
| 1. Angle measures: $30^\circ, 60^\circ, 90^\circ$ | A. Isosceles |
| 2. Side lengths: 2 cm, 2 cm, 2 cm | B. Scalene |
| 3. Angle measures: $60^\circ, 60^\circ, 60^\circ$ | C. Right |
| 4. Side lengths: 6 m, 3 m, 6 m | D. Obtuse |
| 5. Side lengths: 5 ft, 7 ft, 9 ft | E. Equilateral |
| 6. Angle measures: $20^\circ, 125^\circ, 35^\circ$ | F. Equiangular |

7. ★ **WRITING** Can a right triangle also be obtuse? *Explain* why or why not.

EXAMPLE 1
on p. 217
for Exs. 8–10

CLASSIFYING TRIANGLES Copy the triangle and measure its angles. Classify the triangle by its sides and by its angles.



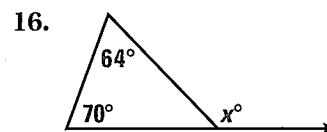
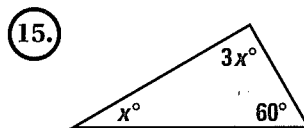
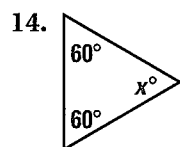
EXAMPLE 2
on p. 218
for Exs. 11–13

COORDINATE PLANE A triangle has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle.

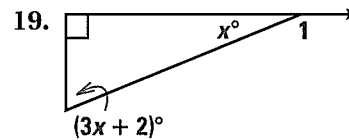
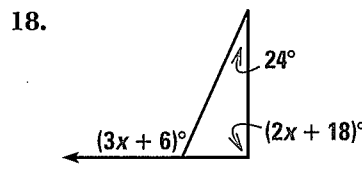
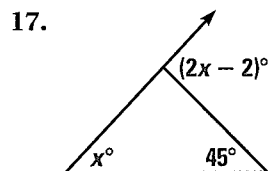
11. $A(2, 3), B(6, 3), C(2, 7)$ 12. $A(3, 3), B(6, 9), C(6, -3)$ 13. $A(1, 9), B(4, 8), C(2, 5)$

EXAMPLE 3
on p. 219
for Exs. 14–19

FINDING ANGLE MEASURES Find the value of x . Then classify the triangle by its angles.

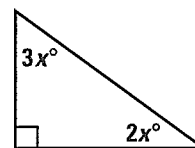


17. **ALGEBRA** Find the measure of the exterior angle shown.



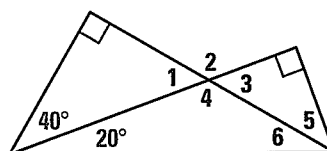
EXAMPLE 4
on p. 220
for Ex. 20

20. ★ **SHORT RESPONSE** *Explain* how to use the Corollary to the Triangle Sum Theorem to find the measure of each angle.



ANGLE RELATIONSHIPS Find the measure of the numbered angle.

21. $\angle 1$ 22. $\angle 2$
 23. $\angle 3$ 24. $\angle 4$
 25. $\angle 5$ 26. $\angle 6$



27. **ALGEBRA** In $\triangle PQR$, $\angle P \cong \angle R$ and the measure of $\angle Q$ is twice the measure of $\angle R$. Find the measure of each angle.
28. **ALGEBRA** In $\triangle EFG$, $m\angle F = 3(m\angle G)$, and $m\angle E = m\angle F - 30^\circ$. Find the measure of each angle.

ERROR ANALYSIS In Exercises 29 and 30, describe and correct the error.

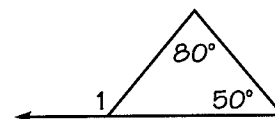
29.

All equilateral triangles are also isosceles. So, if $\triangle ABC$ is isosceles, then it is equilateral as well.



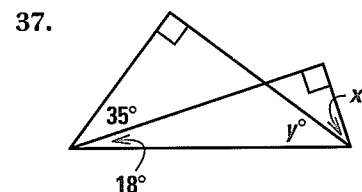
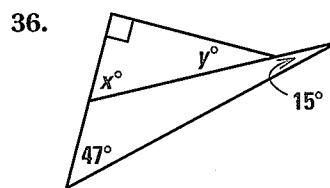
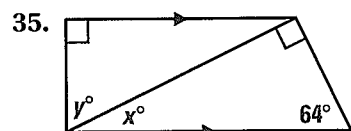
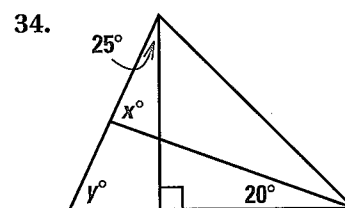
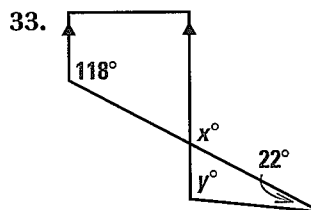
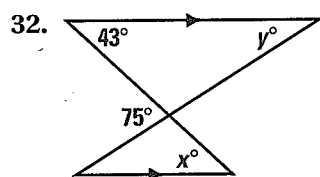
30.

$$m\angle 1 + 80^\circ + 50^\circ = 180^\circ$$



31. **★ MULTIPLE CHOICE** Which of the following is not possible?
- (A) An acute scalene triangle (B) A triangle with two acute exterior angles
 (C) An obtuse isosceles triangle (D) An equiangular acute triangle

ALGEBRA In Exercises 32–37, find the values of x and y .



38. **VISUALIZATION** Is there an angle measure that is so small that any triangle with that angle measure will be an obtuse triangle? Explain.
39. **CHALLENGE** Suppose you have the equations $y = ax + b$, $y = cx + d$, and $y = ex + f$.
- When will these three lines form a triangle?
 - Let $c = 1$, $d = 2$, $e = 4$, and $f = -7$. Find values of a and b so that no triangle is formed by the three equations.
 - Draw the triangle formed when $a = \frac{4}{3}$, $b = \frac{1}{3}$, $c = -\frac{4}{3}$, $d = \frac{41}{3}$, $e = 0$, and $f = -1$. Then classify the triangle by its sides.

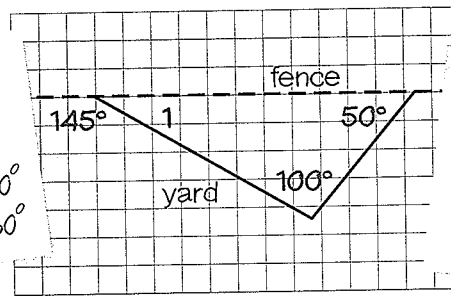
51. ★ **EXTENDED RESPONSE** The figure below shows an initial plan for a triangular flower bed that Mary and Tom plan to build along a fence. They are discussing what the measure of $\angle 1$ should be.

Mary's conclusion:

Use the Triangle Sum Theorem.

$$50^\circ + 100^\circ + m\angle 1 = 180^\circ$$

$$m\angle 1 = 30^\circ$$



Tom's conclusion:

Use the definition of a linear pair.

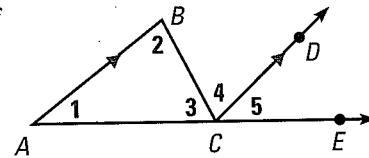
$$145^\circ + m\angle 1 = 180^\circ$$

$$m\angle 1 = 35^\circ$$

Did Mary and Tom both reason correctly? If not, who made a mistake and what mistake was made? If they did both reason correctly, what can you conclude about their initial plan? *Explain.*

52. **ALGEBRA** $\triangle ABC$ is isosceles. $AB = x$ and $BC = 2x - 4$.
- Find two possible values for x if the perimeter of $\triangle ABC$ is 32.
 - How many possible values are there for x if the perimeter of $\triangle ABC$ is 12?

53. **CHALLENGE** Use the diagram to write a proof of the Triangle Sum Theorem. Your proof should be different than the proof of the Triangle Sum Theorem on page 219.



MIXED REVIEW

$\angle A$ and $\angle B$ are complementary. Find $m\angle A$ and $m\angle B$. (p. 35)

54. $m\angle A = (3x + 16)^\circ$
 $m\angle B = (4x - 3)^\circ$

55. $m\angle A = (4x - 2)^\circ$
 $m\angle B = (7x + 4)^\circ$

56. $m\angle A = (3x + 4)^\circ$
 $m\angle B = (2x + 6)^\circ$

PREVIEW

Prepare for
Lesson 4.2
in Exs. 57–59.

Each figure is a regular polygon. Find the value of x . (p. 42)

57. $12x - 10$ and $4x + 6$

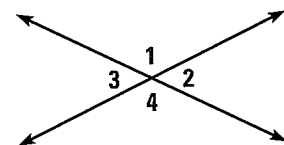
58. $6x + 1$ and $3x + 7$

59. $2x - 5$ and $x + 2$

60. Use the Symmetric Property of Congruence to complete the statement:
 If $\underline{\quad} \cong \underline{\quad}$, then $\angle DEF \cong \angle PQR$. (p. 112)

Use the diagram at the right. (p. 124)

- If $m\angle 1 = 127^\circ$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.
- If $m\angle 4 = 170^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.
- If $m\angle 3 = 54^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 4$.



PROPERTIES OF CONGRUENT TRIANGLES The properties of congruence that are true for segments and angles are also true for triangles.

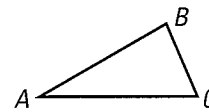
THEOREM

For Your Notebook

THEOREM 4.4 Properties of Congruent Triangles

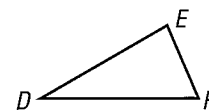
Reflexive Property of Congruent Triangles

For any triangle ABC , $\triangle ABC \cong \triangle ABC$.



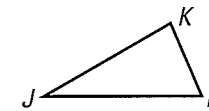
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



4.2 EXERCISES

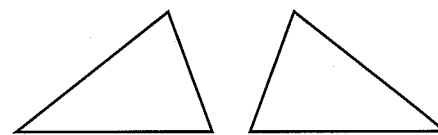
HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS on p. WS4 for Exs. 9, 15, and 25

★ = STANDARDIZED TEST PRACTICE Exs. 2, 18, 21, 24, 27, and 30

SKILL PRACTICE

1. **VOCABULARY** Copy the congruent triangles shown. Then label the vertices of the triangles so that $\triangle JKL \cong \triangle RST$. Identify all pairs of congruent corresponding angles and corresponding sides.

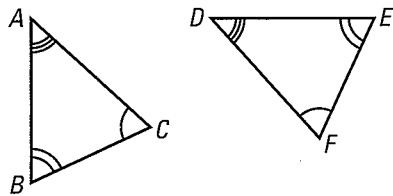


2. ★ **WRITING** Based on this lesson, what information do you need to prove that two triangles are congruent? *Explain.*

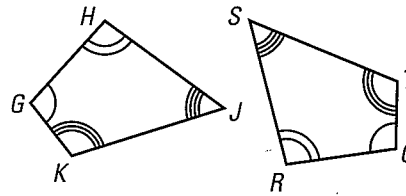
EXAMPLE 1
on p. 225
for Exs. 3–4

USING CONGRUENCE Identify all pairs of congruent corresponding parts. Then write another congruence statement for the figures.

3. $\triangle ABC \cong \triangle DEF$



4. $GHIK \cong QRST$



EXAMPLE 2
on p. 226
for Exs. 5–10

READING A DIAGRAM In the diagram, $\triangle XYZ \cong \triangle MNL$. Copy and complete the statement.

5. $m\angle Y = \underline{\quad?}$

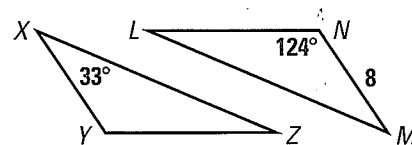
6. $m\angle M = \underline{\quad?}$

7. $YX = \underline{\quad?}$

8. $\overline{YZ} \cong \underline{\quad?}$

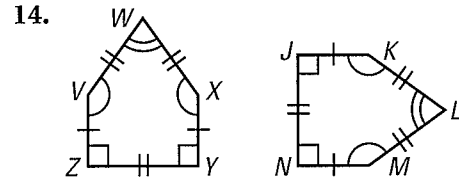
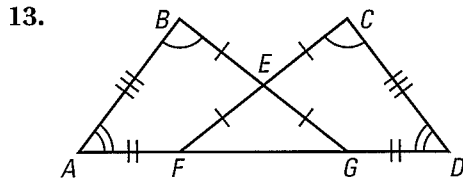
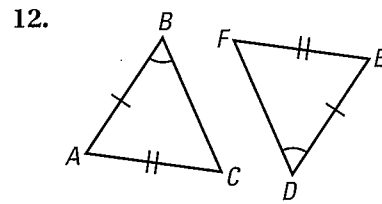
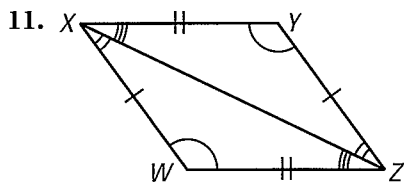
9. $\triangle LNM \cong \underline{\quad?}$

10. $\triangle YXZ \cong \underline{\quad?}$



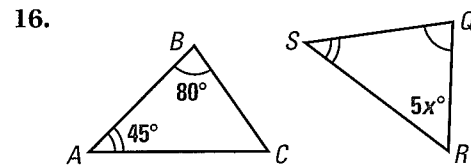
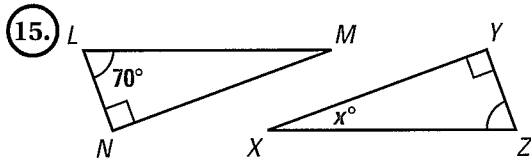
EXAMPLE 3
on p. 226
for Exs. 11–14

NAMING CONGRUENT FIGURES Write a congruence statement for any figures that can be proved congruent. *Explain* your reasoning.

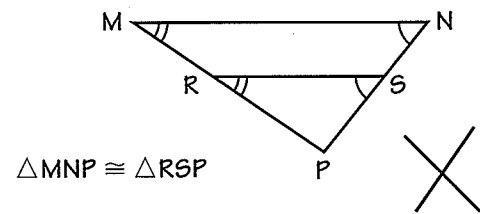


EXAMPLE 4
on p. 227
for Exs. 15–16

THIRD ANGLES THEOREM Find the value of x .

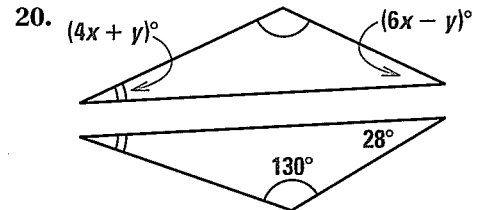
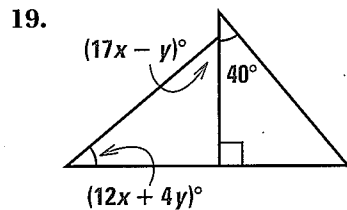


17. **ERROR ANALYSIS** A student says that $\triangle MNP \cong \triangle RSP$ because the corresponding angles of the triangles are congruent. *Describe* the error in this statement.



18. **★ OPEN-ENDED MATH** Graph the triangle with vertices $L(3, 1)$, $M(8, 1)$, and $N(8, 8)$. Then graph a triangle congruent to $\triangle LMN$.

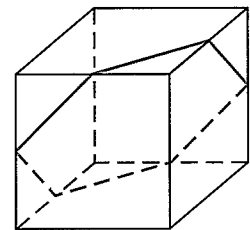
ALGEBRA Find the values of x and y .



21. **★ MULTIPLE CHOICE** Suppose $\triangle ABC \cong \triangle EFD$, $\triangle EFD \cong \triangle GIH$, $m\angle A = 90^\circ$, and $m\angle F = 20^\circ$. What is $m\angle H$?

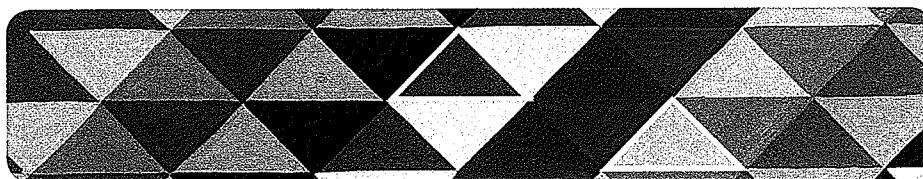
- (A) 20° (B) 70° (C) 90° (D) Cannot be determined

22. **CHALLENGE** A hexagon is contained in a cube, as shown. Each vertex of the hexagon lies on the midpoint of an edge of the cube. This hexagon is equiangular. *Explain* why it is also regular.



PROBLEM SOLVING

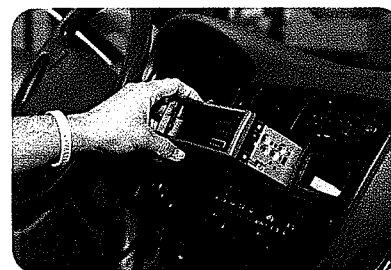
23. **RUG DESIGNS** The rug design is made of congruent triangles. One triangular shape is used to make all of the triangles in the design. Which property guarantees that all the triangles are congruent?



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24. ★ **OPEN-ENDED MATH** Create a design for a rug made with congruent triangles that is different from the one in the photo above.

25. **CAR STEREO** A car stereo fits into a space in your dashboard. You want to buy a new car stereo, and it must fit in the existing space. What measurements need to be the same in order for the new stereo to be congruent to the old one?



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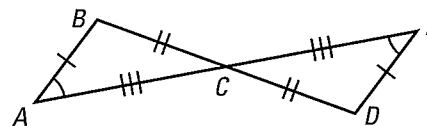
EXAMPLE 5

on p. 227
for Ex. 26

26. **PROOF** Copy and complete the proof.

GIVEN ▶ $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$,
 $\angle BAC \cong \angle DEC$

PROVE ▶ $\triangle ABC \cong \triangle EDC$



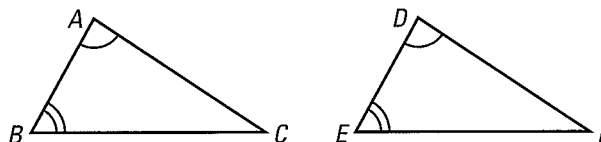
STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$, $\angle BAC \cong \angle DEC$	1. Given
2. $\angle BCA \cong \angle DCE$	2. ?
3. ?	3. Third Angles Theorem
4. $\triangle ABC \cong \triangle EDC$	4. ?

27. ★ **SHORT RESPONSE** Suppose $\triangle ABC \cong \triangle DCB$, and the triangles share vertices at points B and C . Draw a figure that illustrates this situation. Is $\overline{AC} \parallel \overline{BD}$? Explain.

28. **PROVING THEOREM 4.3** Use the plan to prove the Third Angles Theorem.

GIVEN ▶ $\angle A \cong \angle D$, $\angle B \cong \angle E$

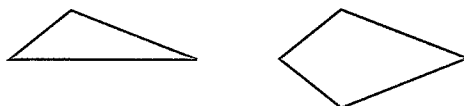
PROVE ▶ $\angle C \cong \angle F$



Plan for Proof Use the Triangle Sum Theorem to show that the sums of the angle measures are equal. Then use substitution to show $\angle C \cong \angle F$.

29. **REASONING** Given that $\triangle AFC \cong \triangle DFE$, must F be the midpoint of \overline{AD} and \overline{EC} ? Include a drawing with your answer.

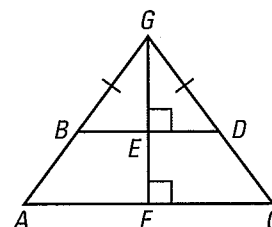
30. **★ SHORT RESPONSE** You have a set of tiles that come in two different shapes, as shown. You can put two of the triangular tiles together to make a quadrilateral that is the same size and shape as the quadrilateral tile.



Explain how you can find all of the angle measures of each tile by measuring only two angles.

31. **MULTI-STEP PROBLEM** In the diagram, quadrilateral $ABEF \cong$ quadrilateral $CDEF$.

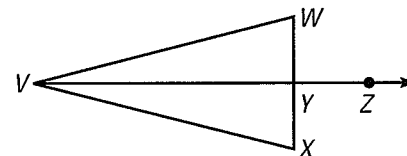
- Explain how you know that $\overline{BE} \cong \overline{DE}$ and $\angle ABE \cong \angle CDE$.
- Explain how you know that $\angle GBE \cong \angle GDE$.
- Explain how you know that $\angle GEB \cong \angle GED$.
- Do you have enough information to prove that $\triangle BEG \cong \triangle DEG$? Explain.



32. **CHALLENGE** Use the diagram to write a proof.

GIVEN $\overline{WX} \perp \overline{VZ}$ at Y , Y is the midpoint of \overline{WX} , $\overline{VW} \cong \overline{VX}$, and \overline{VZ} bisects $\angle WVX$.

PROVE $\triangle VWY \cong \triangle VXY$

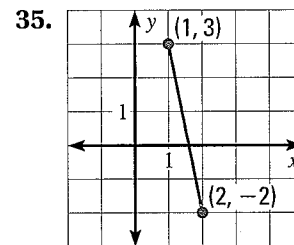
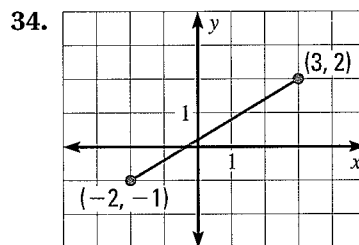
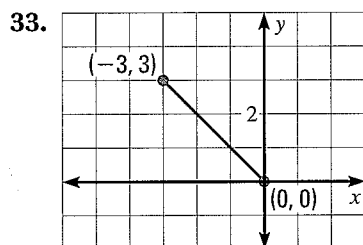


MIXED REVIEW

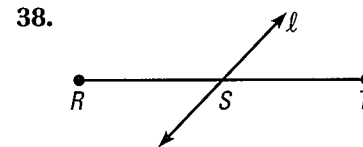
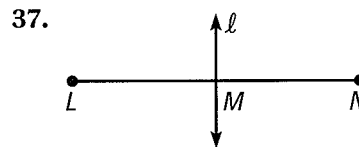
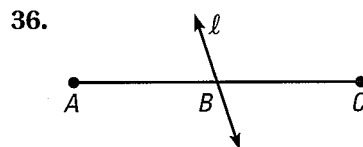
PREVIEW

Prepare for Lesson 4.3 in Exs. 33–35.

Use the Distance Formula to find the length of the segment. Round your answer to the nearest tenth of a unit. (p. 15)



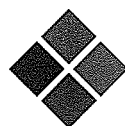
Line l bisects the segment. Write a congruence statement. (p. 15)



Write the converse of the statement. (p. 79)

- If three points are coplanar, then they lie in the same plane.
- If the sky is cloudy, then it is raining outside.

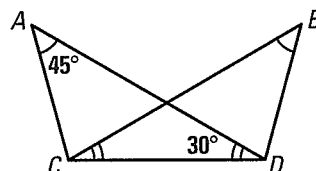
Another Way to Solve Example 4, page 227



MULTIPLE REPRESENTATIONS In Example 4 on page 227, you used congruencies in triangles that overlapped. When you solve problems like this, it may be helpful to redraw the art so that the triangles do not overlap.

PROBLEM

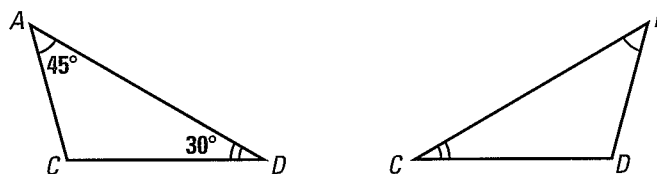
Find $m\angle BDC$.



METHOD

Drawing A Diagram

STEP 1 Identify the triangles that overlap. Then redraw them so that they are separate. Copy all labels and markings.

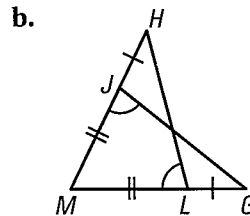
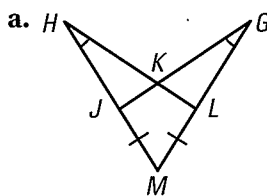


STEP 2 Analyze the situation. By the Triangle Sum Theorem, $m\angle ACD = 180^\circ - 45^\circ - 30^\circ = 105^\circ$.

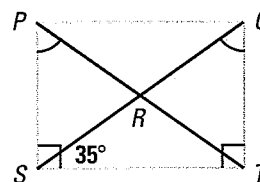
Also, because $\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, by the Third Angles Theorem, $\angle ACD \cong \angle BDC$, and $m\angle ACD = m\angle BDC = 105^\circ$.

PRACTICE

1. DRAWING FIGURES Draw $\triangle HLM$ and $\triangle GJM$ so they do not overlap. Copy all labels and mark any known congruencies.



2. ENVELOPE Draw $\triangle PQS$ and $\triangle QPT$ so that they do not overlap. Find $m\angle PTS$.



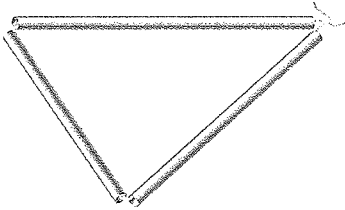
4.3 Investigate Congruent Figures

MATERIALS • straws • string • ruler • protractor

QUESTION How much information is needed to tell whether two figures are congruent?

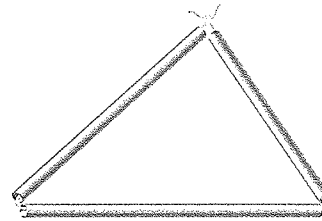
EXPLORE 1 Compare triangles with congruent sides

STEP 1



Make a triangle Cut straws to make side lengths of 8 cm, 10 cm, and 12 cm. Thread the string through the straws. Make a triangle by connecting the ends of the string.

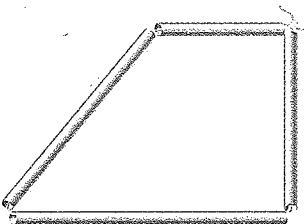
STEP 2



Make another triangle Use the same length straws to make another triangle. If possible, make it different from the first. Compare the triangles. What do you notice?

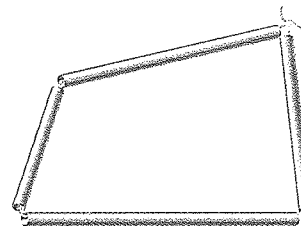
EXPLORE 2 Compare quadrilaterals with congruent sides

STEP 1



Make a quadrilateral Cut straws to make side lengths of 5 cm, 7 cm, 9 cm, and 11 cm. Thread the string through the straws. Make a quadrilateral by connecting the string.

STEP 2



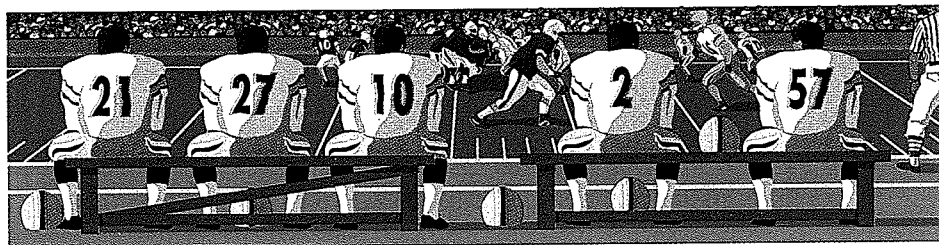
Make another quadrilateral Make a second quadrilateral using the same length straws. If possible, make it different from the first. Compare the quadrilaterals. What do you notice?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Can you make two triangles with the same side lengths that are different shapes? *Justify* your answer.
2. If you know that three sides of a triangle are congruent to three sides of another triangle, can you say the triangles are congruent? *Explain*.
3. Can you make two quadrilaterals with the same side lengths that are different shapes? *Justify* your answer.
4. If four sides of a quadrilateral are congruent to four sides of another quadrilateral, can you say the quadrilaterals are congruent? *Explain*.

EXAMPLE 3 Solve a real-world problem

STRUCTURAL SUPPORT Explain why the bench with the diagonal support is stable, while the one without the support can collapse.

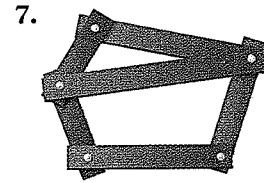
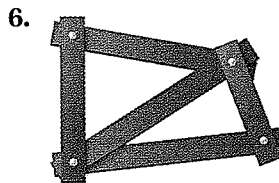
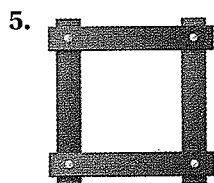


Solution

The bench with a diagonal support forms triangles with fixed side lengths. By the SSS Congruence Postulate, these triangles cannot change shape, so the bench is stable. The bench without a diagonal support is not stable because there are many possible quadrilaterals with the given side lengths.

GUIDED PRACTICE for Example 3

Determine whether the figure is stable. *Explain your reasoning.*



4.3 EXERCISES

HOMEWORK KEY

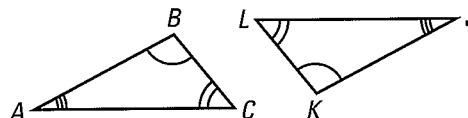
○ = WORKED-OUT SOLUTIONS on p. WS4 for Exs. 7, 9, and 25

★ = STANDARDIZED TEST PRACTICE Exs. 16, 17, and 28

SKILL PRACTICE

VOCABULARY Tell whether the angles or sides are *corresponding angles*, *corresponding sides*, or *neither*.

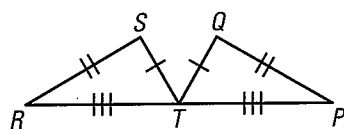
- $\angle C$ and $\angle L$
- \overline{AC} and \overline{JK}
- \overline{BC} and \overline{KL}
- $\angle B$ and $\angle L$



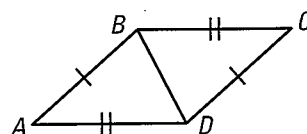
EXAMPLE 1
on p. 234
for Exs. 5–7

DETERMINING CONGRUENCE Decide whether the congruence statement is true. *Explain your reasoning.*

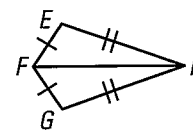
5. $\triangle RST \cong \triangle TQP$



6. $\triangle ABD \cong \triangle CDB$



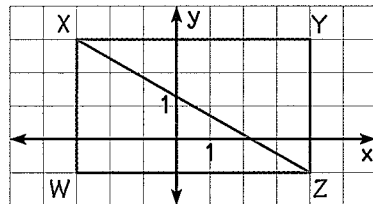
7. $\triangle DEF \cong \triangle DGF$



EXAMPLE 2

on p. 235
for Exs. 8–12

8. **ERROR ANALYSIS** Describe and correct the error in writing a congruence statement for the triangles in the coordinate plane.



$$\triangle WXZ \cong \triangle ZYX$$



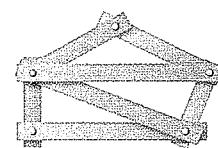
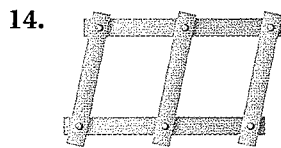
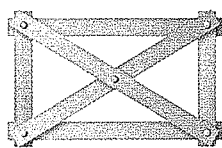
9. **ALGEBRA** Use the given coordinates to determine if $\triangle ABC \cong \triangle DEF$.

9. $A(-2, -2), B(4, -2), C(4, 6), D(5, 7), E(5, 1), F(13, 1)$
 10. $A(-2, 1), B(3, -3), C(7, 5), D(3, 6), E(8, 2), F(10, 11)$
 11. $A(0, 0), B(6, 5), C(9, 0), D(0, -1), E(6, -6), F(9, -1)$
 12. $A(-5, 7), B(-5, 2), C(0, 2), D(0, 6), E(0, 1), F(4, 1)$

EXAMPLE 3

on p. 236
for Exs. 13–15

13. 14. 15. **USING DIAGRAMS** Decide whether the figure is stable. Explain.



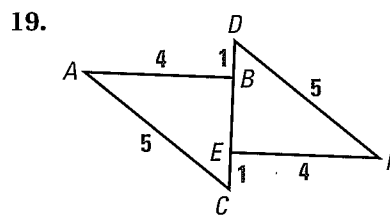
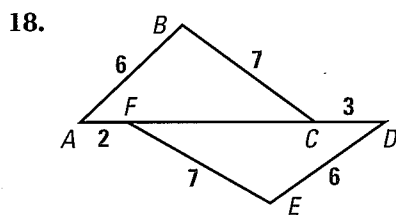
16. **★ MULTIPLE CHOICE** Let $\triangle FGH$ be an equilateral triangle with point J as the midpoint of \overline{FG} . Which of the statements below is *not* true?

- (A) $\overline{FH} \cong \overline{GH}$ (B) $\overline{FJ} \cong \overline{FH}$ (C) $\overline{FJ} \cong \overline{GJ}$ (D) $\triangle FJH \cong \triangle GJH$

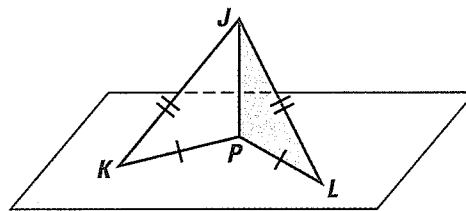
17. **★ MULTIPLE CHOICE** Let $ABCD$ be a rectangle separated into two triangles by \overline{DB} . Which of the statements below is *not* true?

- (A) $\overline{AD} \cong \overline{CB}$ (B) $\overline{AB} \cong \overline{AD}$ (C) $\overline{AB} \cong \overline{CD}$ (D) $\triangle DAB \cong \triangle BCD$

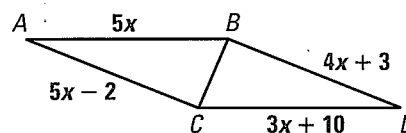
- APPLYING SEGMENT ADDITION** Determine whether $\triangle ABC \cong \triangle DEF$. If they are congruent, write a congruence statement. Explain your reasoning.



20. **3-D FIGURES** In the diagram, $\overline{PK} \cong \overline{PL}$ and $\overline{JK} \cong \overline{JL}$. Show that $\triangle JPK \cong \triangle JPL$.



21. **CHALLENGE** Find all values of x that make the triangles congruent. Explain.



PROBLEM SOLVING

EXAMPLE 1

on p. 234
for Ex. 22

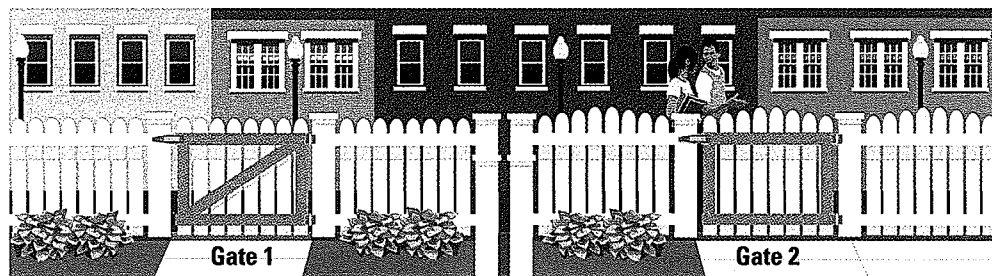
EXAMPLE 3

on p. 236
for Ex. 23

- 22. TILE FLOORS** You notice two triangles in the tile floor of a hotel lobby. You want to determine if the triangles are congruent, but you only have a piece of string. Can you determine if the triangles are congruent? *Explain.*

@HomeTutor for problem solving help at classzone.com

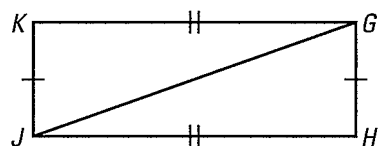
- 23. GATES** Which gate is stable? *Explain* your reasoning.



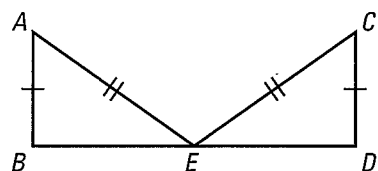
@HomeTutor for problem solving help at classzone.com

PROOF Write a proof.

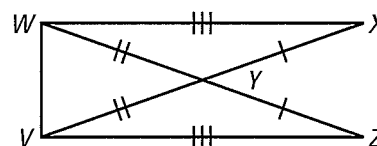
- 24. GIVEN** $\overline{GH} \cong \overline{JK}$, $\overline{HJ} \cong \overline{KG}$
PROVE $\triangle GHJ \cong \triangle JKG$



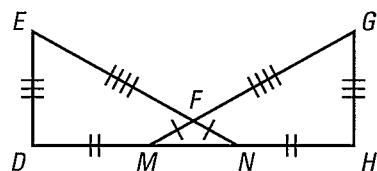
- 26. GIVEN** $\overline{AE} \cong \overline{CE}$, $\overline{AB} \cong \overline{CD}$,
 E is the midpoint of \overline{BD} .
PROVE $\triangle EAB \cong \triangle ECD$



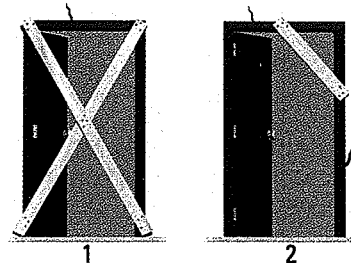
- 25. GIVEN** $\overline{WX} \cong \overline{VZ}$, $\overline{WY} \cong \overline{VY}$, $\overline{YZ} \cong \overline{YX}$
PROVE $\triangle VWX \cong \triangle WVZ$



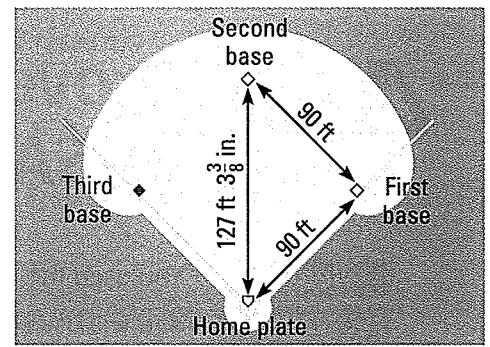
- 27. GIVEN** $\overline{FM} \cong \overline{FN}$, $\overline{DM} \cong \overline{HN}$,
 $\overline{EF} \cong \overline{GF}$, $\overline{DE} \cong \overline{HG}$
PROVE $\triangle DEN \cong \triangle HGM$



- 28. ★ EXTENDED RESPONSE** When rescuers enter a partially collapsed building they often have to reinforce damaged doors for safety.
- Diagonal braces are added to Door 1 as shown below. *Explain* why the door is more stable with the braces.
 - Would these braces be a good choice for rescuers needing to enter and exit the building through this doorway?
 - In the diagram, Door 2 has only a corner brace. Does this solve the problem from part (b)?
 - Explain* why the corner brace makes the door more stable.



29. **BASEBALL FIELD** To create a baseball field, start by placing home plate. Then, place second base 127 feet $3\frac{3}{8}$ inches from home plate. Then, you can find first base using two tape measures. Stretch one from second base toward first base and the other from home plate toward first base. The point where the two tape measures cross at the 90 foot mark is first base. You can find third base in a similar manner. *Explain* how and why this process will always work.



30. **CHALLENGE** Draw and label the figure described below. Then, identify what is given and write a two-column proof.
In an isosceles triangle, if a segment is added from the vertex between the congruent sides to the midpoint of the third side, then two congruent triangles are formed.

MIXED REVIEW

PREVIEW

Prepare for Lesson 4.4 in Exs. 31–33.

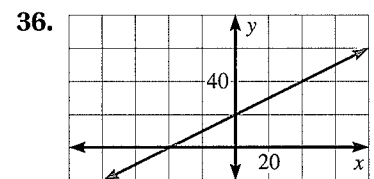
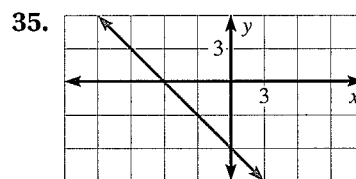
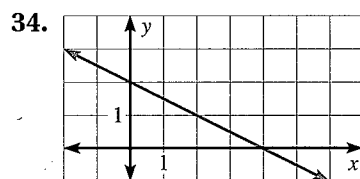
Find the slope of the line that passes through the points. (p. 171)

31. $A(3, 0), B(7, 4)$

32. $F(1, 8), G(-9, 2)$

33. $M(-4, -10), N(6, 2)$

Use the x - and y -intercepts to write an equation of the line. (p. 180)



37. Write an equation of a line that passes through $(-3, -1)$ and is parallel to $y = 3x + 2$. (p. 180)

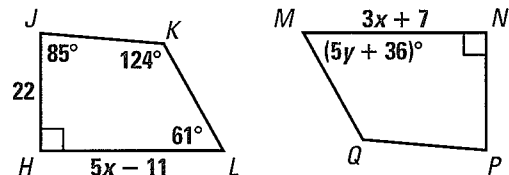
QUIZ for Lessons 4.1–4.3

A triangle has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle. (p. 217)

1. $A(-3, 0), B(0, 4), C(3, 0)$ 2. $A(2, -4), B(5, -1), C(2, -1)$ 3. $A(-7, 0), B(1, 6), C(-3, 4)$

In the diagram, $HJKL \cong NPQM$. (p. 225)

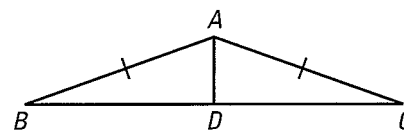
4. Find the value of x .
5. Find the value of y .



6. Write a proof. (p. 234)

GIVEN $\triangleright \overline{AB} \cong \overline{AC}, \overline{AD}$ bisects \overline{BC} .

PROVE $\triangleright \triangle ABD \cong \triangle ACD$



4.4 EXERCISES

HOMWORK KEY

○ = WORKED-OUT SOLUTIONS on p. WS4 for Exs. 13, 19, and 31

★ = STANDARDIZED TEST PRACTICE Exs. 2, 15, 23, and 39

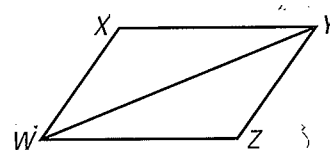
SKILL PRACTICE

- VOCABULARY** Copy and complete: The angle between two sides of a triangle is called the ? angle.
- ★ **WRITING** Explain the difference between proving triangles congruent using the SAS and SSS Congruence Postulates.

EXAMPLE 1
on p. 240
for Exs. 3–15

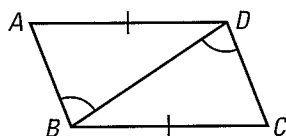
NAMING INCLUDED ANGLES Use the diagram to name the included angle between the given pair of sides.

- \overline{XY} and \overline{YW}
- \overline{ZW} and \overline{YW}
- \overline{XY} and \overline{YZ}
- \overline{WZ} and \overline{ZY}
- \overline{WX} and \overline{YX}
- \overline{WX} and \overline{WZ}

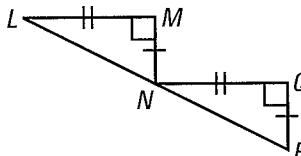


REASONING Decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Postulate.

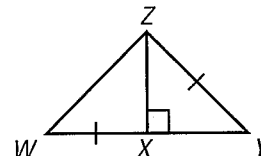
9. $\triangle ABD, \triangle CDB$



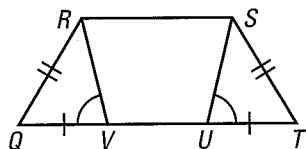
10. $\triangle LMN, \triangle NQP$



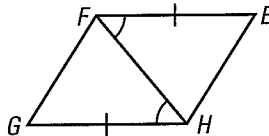
11. $\triangle YXZ, \triangle WXZ$



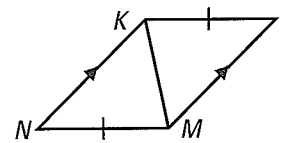
12. $\triangle QRV, \triangle TSU$



13. $\triangle EFH, \triangle GHF$



14. $\triangle KLM, \triangle MNK$



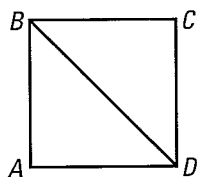
15. ★ **MULTIPLE CHOICE** Which of the following sets of information does not allow you to conclude that $\triangle ABC \cong \triangle DEF$?

- (A) $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, \angle B \cong \angle E$ (B) $\overline{AB} \cong \overline{DF}, \overline{AC} \cong \overline{DE}, \angle C \cong \angle E$
 (C) $\overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}, \overline{BA} \cong \overline{DE}$ (D) $\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \angle A \cong \angle D$

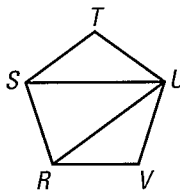
EXAMPLE 2
on p. 241
for Exs. 16–18

APPLYING SAS In Exercises 16–18, use the given information to name two triangles that are congruent. Explain your reasoning.

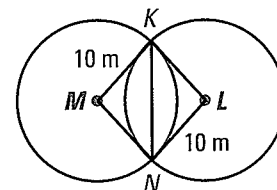
16. $ABCD$ is a square with four congruent sides and four congruent angles.



17. $RSTUV$ is a regular pentagon.

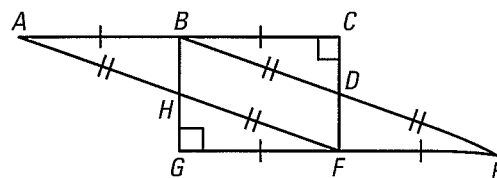


18. $\overline{MK} \perp \overline{MN}$ and $\overline{KL} \perp \overline{NL}$.



EXAMPLE 3
 on p. 242
 for Ex. 19

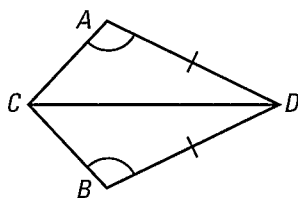
- 19. OVERLAPPING TRIANGLES** Redraw $\triangle ACF$ and $\triangle EGB$ so they are side by side with corresponding parts in the same position. Explain how you know that $\triangle ACF \cong \triangle EGB$.



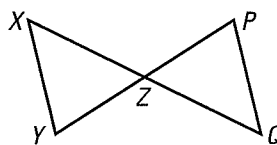
EXAMPLE 4
 on p. 242
 for Exs. 20–22

- REASONING** Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.

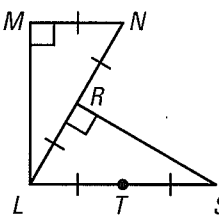
20.



21. Z is the midpoint of \overline{PY} and \overline{XQ} .

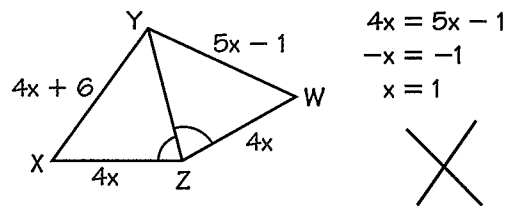


22.



23. **★ WRITING** Suppose both pairs of corresponding legs of two right triangles are congruent. Are the triangles congruent? Explain.

24. **ERROR ANALYSIS** Describe and correct the error in finding the value of x .

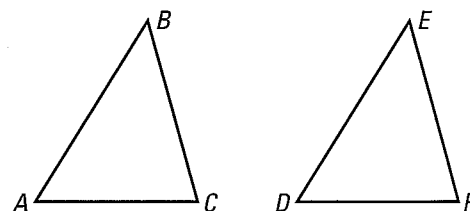


USING DIAGRAMS In Exercises 25–27, state the third congruence that must be given to prove that $\triangle ABC \cong \triangle DEF$ using the indicated postulate.

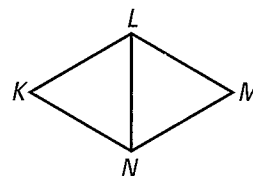
25. **GIVEN** $\overline{AB} \cong \overline{DE}$, $\overline{CB} \cong \overline{FE}$, $\underline{\quad} \cong \underline{\quad}$
 Use the SSS Congruence Postulate.

26. **GIVEN** $\angle A \cong \angle D$, $\overline{CA} \cong \overline{FD}$, $\underline{\quad} \cong \underline{\quad}$
 Use the SAS Congruence Postulate.

27. **GIVEN** $\angle B \cong \angle E$, $\overline{AB} \cong \overline{DE}$, $\underline{\quad} \cong \underline{\quad}$
 Use the SAS Congruence Postulate.

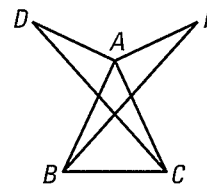


28. **USING ISOSCELES TRIANGLES** Suppose $\triangle KLN$ and $\triangle MLN$ are isosceles triangles with $\overline{KL} \cong \overline{LN}$ and $\overline{ML} \cong \overline{LN}$, and \overline{NL} bisects $\angle KLM$. Is there enough information to prove that $\triangle KLN \cong \triangle MLN$? Explain.



29. **REASONING** Suppose M is the midpoint of \overline{PQ} in $\triangle PQR$. If $\overline{RM} \perp \overline{PQ}$, explain why $\triangle RMP \cong \triangle RMQ$.

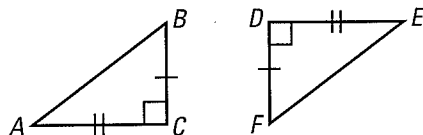
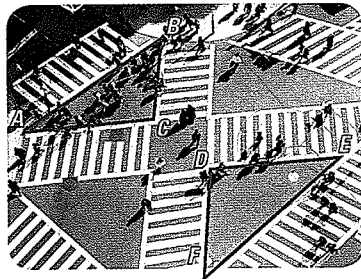
30. **CHALLENGE** Suppose $\overline{AB} \cong \overline{AC}$, $\overline{AD} \cong \overline{AF}$, $\overline{AD} \perp \overline{AB}$, and $\overline{AF} \perp \overline{AC}$. Explain why you can conclude that $\triangle ACD \cong \triangle ABF$.



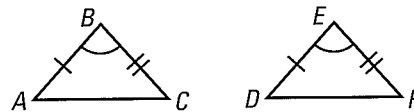
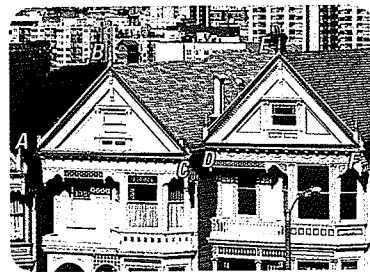
PROBLEM SOLVING

CONGRUENT TRIANGLES In Exercises 31 and 32, identify the theorem or postulate you would use to prove the triangles congruent.

31.



32.



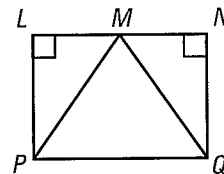
33. **SAILBOATS** Suppose you have two sailboats. What information do you need to know to prove that the triangular sails are congruent using SAS? using HL?

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34. **DEVELOPING PROOF** Copy and complete the proof.

GIVEN ▶ Point M is the midpoint of \overline{LN} .
 $\triangle PMQ$ is an isosceles triangle with $\overline{MP} \cong \overline{MQ}$.
 $\angle L$ and $\angle N$ are right angles.

PROVE ▶ $\triangle LMP \cong \triangle NMQ$



STATEMENTS

1. $\angle L$ and $\angle N$ are right angles.
2. $\triangle LMP$ and $\triangle NMQ$ are right triangles.
3. Point M is the midpoint of \overline{LN} .
4. $\underline{\hspace{1cm}}$
5. $\overline{MP} \cong \overline{MQ}$
6. $\triangle LMP \cong \triangle NMQ$

REASONS

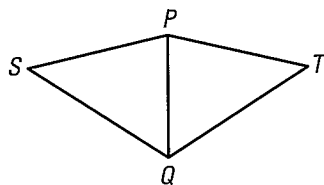
1. Given
2. $\underline{\hspace{1cm}}$
3. $\underline{\hspace{1cm}}$
4. Definition of midpoint
5. Given
6. $\underline{\hspace{1cm}}$

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PROOF In Exercises 35 and 36, write a proof.

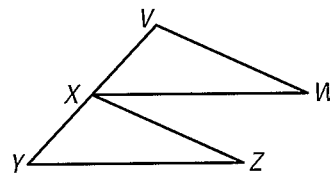
35. **GIVEN** ▶ \overline{PQ} bisects $\angle SPT$, $\overline{SP} \cong \overline{TP}$

PROVE ▶ $\triangle SPQ \cong \triangle TPQ$



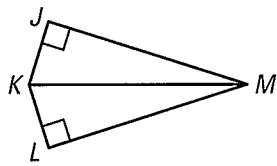
36. **GIVEN** ▶ $\overline{VX} \cong \overline{XY}$, $\overline{XW} \cong \overline{YZ}$, $\overline{XW} \parallel \overline{YZ}$

PROVE ▶ $\triangle VXW \cong \triangle XYZ$

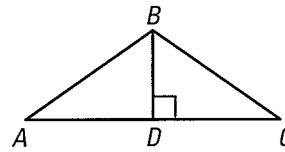


PROOF In Exercises 37 and 38, write a proof.

37. **GIVEN** $\overline{JM} \cong \overline{LM}$
PROVE $\triangle JKM \cong \triangle LKM$

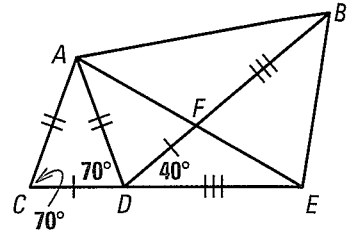


38. **GIVEN** D is the midpoint of \overline{AC} .
PROVE $\triangle ABD \cong \triangle CBD$



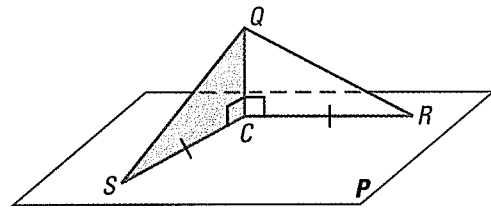
39. **★ MULTIPLE CHOICE** Which triangle congruence can you prove, then use to prove that $\angle FED \cong \angle ABF$?

- (A) $\triangle ABE \cong \triangle ABF$ (B) $\triangle AED \cong \triangle ABD$
 (C) $\triangle ACD \cong \triangle ADF$ (D) $\triangle AEC \cong \triangle ABD$

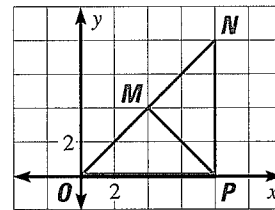


40. **PROOF** Write a two-column proof.

- GIVEN** $\overline{CR} \cong \overline{CS}$, $\overline{QC} \perp \overline{CR}$, $\overline{QC} \perp \overline{CS}$
PROVE $\triangle QCR \cong \triangle QCS$



41. **CHALLENGE** Describe how to show that $\triangle PMO \cong \triangle PMN$ using the SSS Congruence Postulate. Then show that the triangles are congruent using the SAS Congruence Postulate without measuring any angles. Compare the two methods.



MIXED REVIEW

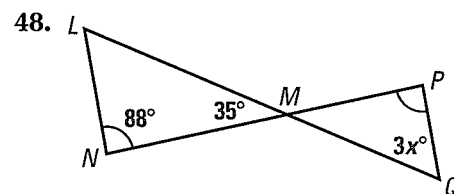
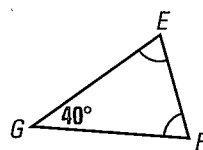
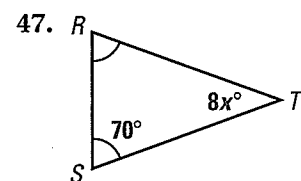
Draw a figure that fits the description. (p. 42)

42. A pentagon that is not regular.
 43. A quadrilateral that is equilateral but not equiangular.

Write an equation of the line that passes through point P and is perpendicular to the line with the given equation. (p. 180)

44. $P(3, -1)$, $y = -x + 2$ 45. $P(3, 3)$, $y = \frac{1}{3}x + 2$ 46. $P(-4, -7)$, $y = -5$

Find the value of x . (p. 225)



PREVIEW
 Prepare for
 Lesson 4.5 in
 Exs. 47–48.

4.4 Investigate Triangles and Congruence

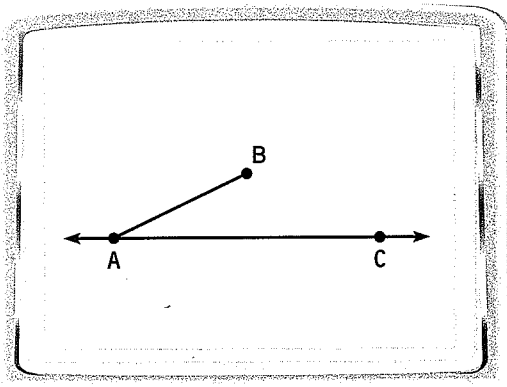
MATERIALS • graphing calculator or computer

QUESTION Can you prove triangles are congruent by SSA?

You can use geometry drawing software to show that if two sides and a nonincluded angle of one triangle are congruent to two sides and a nonincluded angle of another triangle, the triangles are not necessarily congruent.

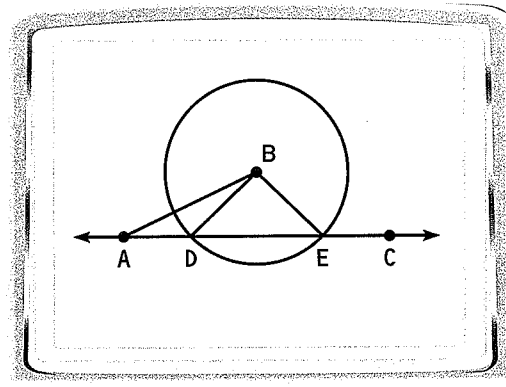
EXAMPLE Draw two triangles

STEP 1



Draw a line Draw points A and C. Draw line \overleftrightarrow{AC} . Then choose point B so that $\angle BAC$ is acute. Draw \overline{AB} .

STEP 2



Draw a circle Draw a circle with center at B so that the circle intersects \overleftrightarrow{AC} at two points. Label the points D and E. Draw \overline{BD} and \overline{BE} . Save as "EXAMPLE".

STEP 3 Use your drawing

Explain why $\overline{BD} \cong \overline{BE}$. In $\triangle ABD$ and $\triangle ABE$, what other sides are congruent? What angles are congruent?

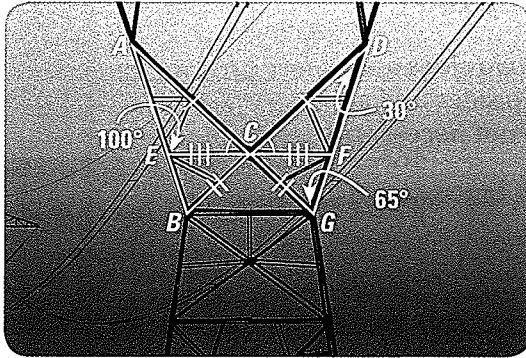
PRACTICE

1. Explain how your drawing shows that $\triangle ABD \not\cong \triangle ABE$.
2. Change the diameter of your circle so that it intersects \overleftrightarrow{AC} in only one point. Measure $\angle BDA$. Explain why there is exactly one triangle you can draw with the measures AB, BD, and a 90° angle at $\angle BDA$.
3. Explain why your results show that SSA cannot be used to show that two triangles are congruent but that HL can.



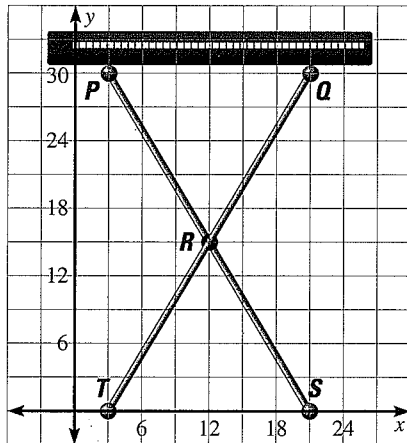
Lessons 4.1–4.4

1. **MULTI-STEP PROBLEM** In the diagram, $\overline{AC} \cong \overline{CD}$, $\overline{BC} \cong \overline{CG}$, $\overline{EC} \cong \overline{CF}$, and $\angle ACE \cong \angle DCF$.

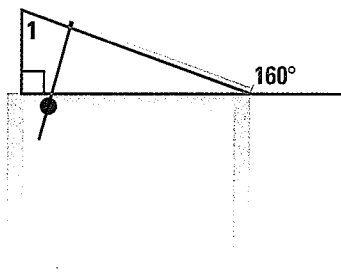


- Classify each triangle in the figure by angles.
- Classify each triangle in the figure by sides.

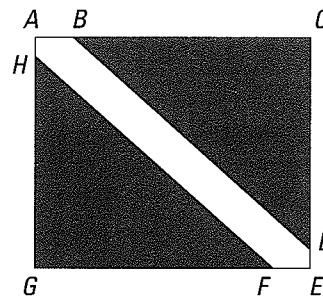
2. **OPEN-ENDED** Explain how you know that $\triangle PQR \cong \triangle STR$ in the keyboard stand shown.



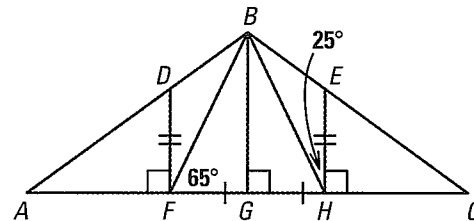
3. **GRIDDED ANSWER** In the diagram below, find the measure of $\angle 1$ in degrees.



4. **SHORT RESPONSE** A rectangular “diver down” flag is used to indicate that scuba divers are in the water. On the flag, $\overline{AB} \cong \overline{FE}$, $\overline{AH} \cong \overline{DE}$, $\overline{CE} \cong \overline{AG}$, and $\overline{EG} \cong \overline{AC}$. Also, $\angle A$, $\angle C$, $\angle E$, and $\angle G$ are right angles. Is $\triangle BCD \cong \triangle FGH$? Explain.



5. **EXTENDED RESPONSE** A roof truss is a network of pieces of wood that forms a stable structure to support a roof, as shown below.



- Prove that $\triangle FGB \cong \triangle HGB$.
 - Is $\triangle BDF \cong \triangle BEH$? If so, prove it.
6. **GRIDDED ANSWER** In the diagram below, $\triangle AFC \cong \triangle DEC$. Find the value of x .

