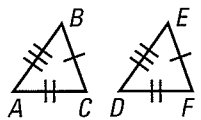
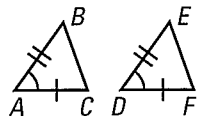
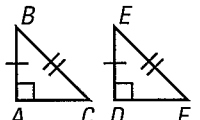
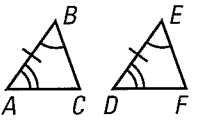
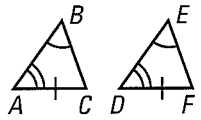


Triangle Congruence Postulates and Theorems

You have learned five methods for proving that triangles are congruent.

SSS	SAS	HL (right \triangle only)	ASA	AAS
				
All three sides are congruent.	Two sides and the included angle are congruent.	The hypotenuse and one of the legs are congruent.	Two angles and the included side are congruent.	Two angles and a (non-included) side are congruent.

In the Exercises, you will prove three additional theorems about the congruence of right triangles: **Angle-Leg**, **Leg-Leg**, and **Hypotenuse-Angle**.

4.5 EXERCISES

HOMWORK KEY

- \bigcirc = WORKED-OUT SOLUTIONS on p. WS5 for Exs. 5, 9, and 27
- \star = STANDARDIZED TEST PRACTICE Exs. 2, 7, 21, and 26

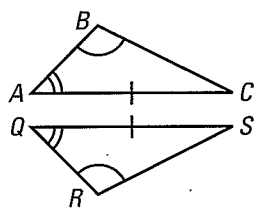
SKILL PRACTICE

- VOCABULARY** Name one advantage of using a flow proof rather than a two-column proof.
- \star **WRITING** You know that a pair of triangles has two pairs of congruent corresponding angles. What other information do you need to show that the triangles are congruent?

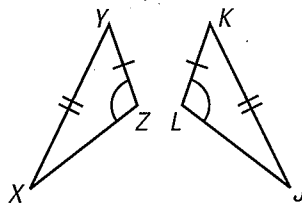
EXAMPLE 1
on p. 250
for Exs. 3–7

IDENTIFY CONGRUENT TRIANGLES Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use.

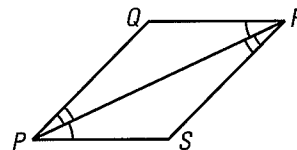
3. $\triangle ABC, \triangle QRS$



4. $\triangle XYZ, \triangle JKL$

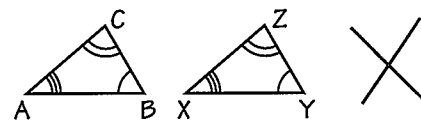


5. $\triangle PQR, \triangle RSP$



6. **ERROR ANALYSIS** Describe the error in concluding that $\triangle ABC \cong \triangle XYZ$.

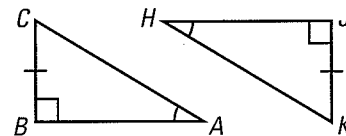
By AAA,
 $\triangle ABC \cong \triangle XYZ$.



EXAMPLE 2
on p. 250
for Exs. 8–13

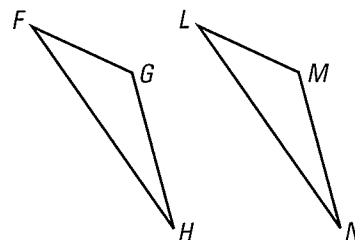
7. ★ **MULTIPLE CHOICE** Which postulate or theorem can you use to prove that $\triangle ABC \cong \triangle HJK$?

- (A) HL (B) AAS
(C) SAS (D) Not enough information



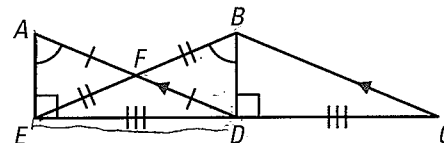
DEVELOPING PROOF State the third congruence that is needed to prove that $\triangle FGH \cong \triangle LMN$ using the given postulate or theorem.

8. **GIVEN** $\overline{GH} \cong \overline{MN}$, $\angle G \cong \angle M$, $\underline{\quad} \cong \underline{\quad}$
Use the AAS Congruence Theorem.
9. **GIVEN** $\overline{FG} \cong \overline{LM}$, $\angle G \cong \angle M$, $\underline{\quad} \cong \underline{\quad}$
Use the ASA Congruence Postulate.
10. **GIVEN** $\overline{FH} \cong \overline{LN}$, $\angle H \cong \angle N$, $\underline{\quad} \cong \underline{\quad}$
Use the SAS Congruence Postulate.



OVERLAPPING TRIANGLES Explain how you can prove that the indicated triangles are congruent using the given postulate or theorem.

11. $\triangle AFE \cong \triangle DFB$ by SAS
12. $\triangle AED \cong \triangle BDE$ by AAS
13. $\triangle AED \cong \triangle BDC$ by ASA

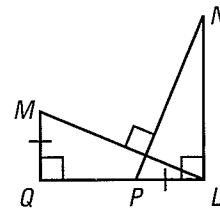
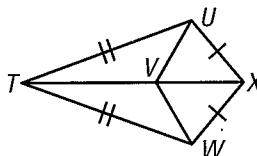
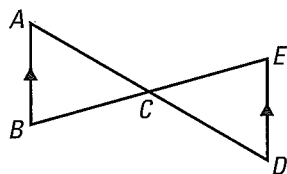


DETERMINING CONGRUENCE Tell whether you can use the given information to determine whether $\triangle ABC \cong \triangle DEF$. Explain your reasoning.

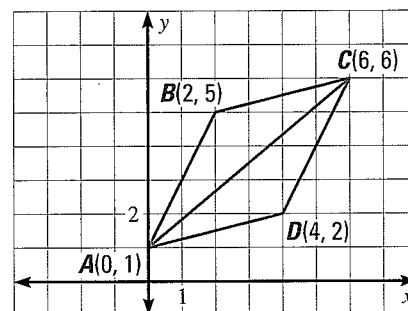
14. $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$ 15. $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$
16. $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AC} \cong \overline{DE}$ 17. $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{FD}$, $\overline{AC} \cong \overline{DE}$

IDENTIFY CONGRUENT TRIANGLES Is it possible to prove that the triangles are congruent? If so, state the postulate(s) or theorem(s) you would use.

18. $\triangle ABC$, $\triangle DEC$ 19. $\triangle TUV$, $\triangle TWV$ 20. $\triangle QML$, $\triangle LPN$



21. ★ **EXTENDED RESPONSE** Use the graph at the right.
- a. Show that $\angle CAD \cong \angle ACB$. Explain your reasoning.
b. Show that $\angle ACD \cong \angle CAB$. Explain your reasoning.
c. Show that $\triangle ABC \cong \triangle CDA$. Explain your reasoning.

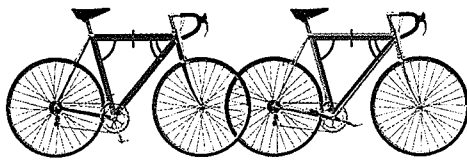


22. **CHALLENGE** Use a coordinate plane.
- a. Graph the lines $y = 2x + 5$, $y = 2x - 3$, and $x = 0$ in the same coordinate plane.
b. Consider the equation $y = mx + 1$. For what values of m will the graph of the equation form two triangles if added to your graph? For what values of m will those triangles be congruent? Explain.

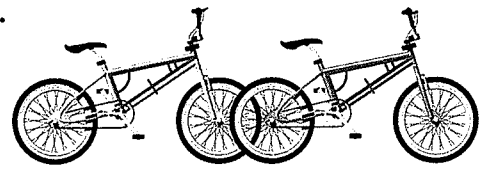
PROBLEM SOLVING

CONGRUENCE IN BICYCLES Explain why the triangles are congruent.

23.



24.



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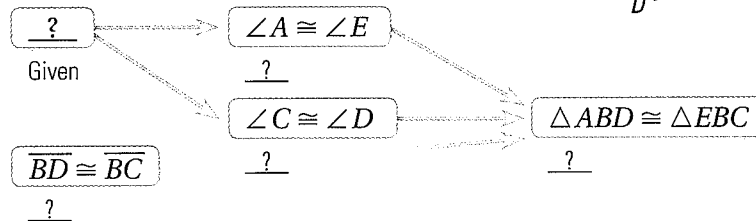
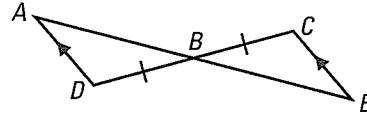
EXAMPLE 3

on p. 251
for Ex. 25

25. **FLOW PROOF** Copy and complete the flow proof.

GIVEN $\triangleright \overline{AD} \parallel \overline{CE}, \overline{BD} \cong \overline{BC}$

PROVE $\triangleright \triangle ABD \cong \triangle EBC$



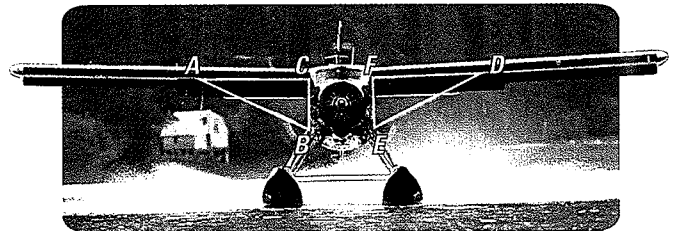
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EXAMPLE 4

on p. 251
for Ex. 26

26. ★ **SHORT RESPONSE** You are making a map for an orienteering race. Participants start at a large oak tree, find a boulder 250 yards due east of the oak tree, and then find a maple tree that is 50° west of north of the boulder and 35° east of north of the oak tree. Sketch a map. Can you locate the maple tree? Explain.

27. **AIRPLANE** In the airplane at the right, $\angle C$ and $\angle F$ are right angles, $\overline{BC} \cong \overline{EF}$, and $\angle A \cong \angle D$. What postulate or theorem allows you to conclude that $\triangle ABC \cong \triangle DEF$?



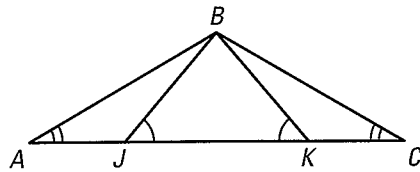
RIGHT TRIANGLES In Lesson 4.4, you learned the Hypotenuse-Leg Theorem for right triangles. In Exercises 28–30, write a paragraph proof for these other theorems about right triangles.

- 28. **Leg-Leg (LL) Theorem** If the legs of two right triangles are congruent, then the triangles are congruent.
- 29. **Angle-Leg (AL) Theorem** If an angle and a leg of a right triangle are congruent to an angle and a leg of a second right triangle, then the triangles are congruent.
- 30. **Hypotenuse-Angle (HA) Theorem** If an angle and the hypotenuse of a right triangle are congruent to an angle and the hypotenuse of a second right triangle, then the triangles are congruent.

31. **PROOF** Write a two-column proof.

GIVEN ▶ $\overline{AK} \cong \overline{CJ}$, $\angle BJK \cong \angle BKJ$,
 $\angle A \cong \angle C$

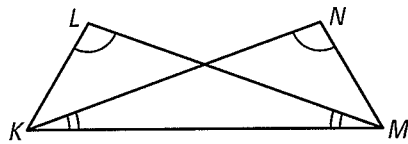
PROVE ▶ $\triangle ABK \cong \triangle CBJ$



33. **PROOF** Write a proof.

GIVEN ▶ $\angle NKM \cong \angle LMK$, $\angle L \cong \angle N$

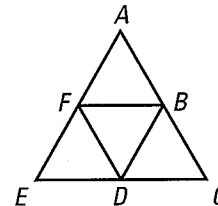
PROVE ▶ $\triangle NMK \cong \triangle LKM$



35. **CHALLENGE** Write a proof.

GIVEN ▶ $\triangle ABF \cong \triangle DFB$, F is the midpoint of \overline{AE} ,
 B is the midpoint of \overline{AC} .

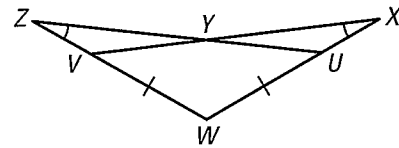
PROVE ▶ $\triangle FDE \cong \triangle BCD \cong \triangle ABF$



32. **PROOF** Write a flow proof.

GIVEN ▶ $\overline{VW} \cong \overline{UW}$, $\angle X \cong \angle Z$

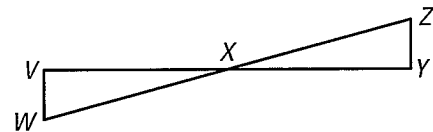
PROVE ▶ $\triangle XWV \cong \triangle ZWU$



34. **PROOF** Write a proof.

GIVEN ▶ X is the midpoint of \overline{VY} and \overline{WZ} .

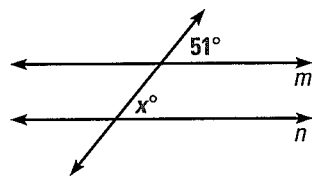
PROVE ▶ $\triangle VWX \cong \triangle YZX$



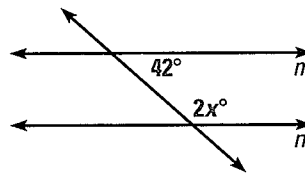
MIXED REVIEW

Find the value of x that makes $m \parallel n$. (p. 161)

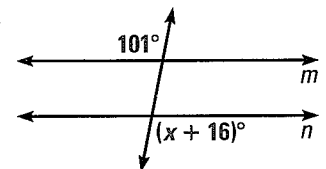
36.



37.



38.



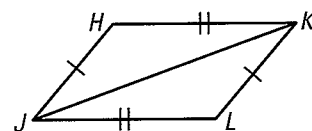
Write an equation of the line that passes through point P and is parallel to the line with the given equation. (p. 180)

39. $P(0, 3)$, $y = x - 8$

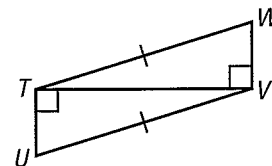
40. $P(-2, 4)$, $y = -2x + 3$

Decide which method, SSS, SAS, or HL, can be used to prove that the triangles are congruent. (pp. 234, 240)

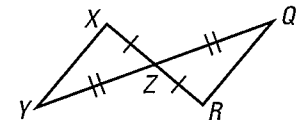
41. $\triangle HJK \cong \triangle LKJ$



42. $\triangle UTV \cong \triangle WVT$



43. $\triangle XYZ \cong \triangle RQZ$



PREVIEW

Prepare for
Lesson 4.6 in
Exs. 41–43.

4.6 EXERCISES

HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS on p. WS5 for Exs. 19, 23, and 31

★ = STANDARDIZED TEST PRACTICE Exs. 2, 14, 31, and 36

SKILL PRACTICE

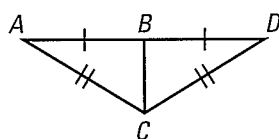
- VOCABULARY** Copy and complete: Corresponding parts of congruent triangles are ?.
- ★ **WRITING** Explain why you might choose to use congruent triangles to measure the distance across a river. Give another example where it may be easier to measure with congruent triangles rather than directly.

EXAMPLES 1 and 2

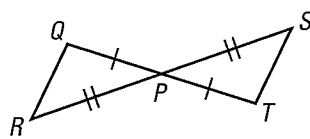
on p. 256–257 for Exs. 3–11

CONGRUENT TRIANGLES Tell which triangles you can show are congruent in order to prove the statement. What postulate or theorem would you use?

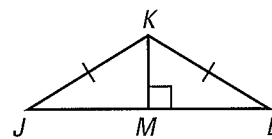
3. $\angle A \cong \angle D$



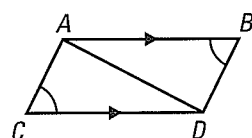
4. $\angle Q \cong \angle T$



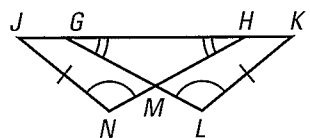
5. $\overline{JM} \cong \overline{LM}$



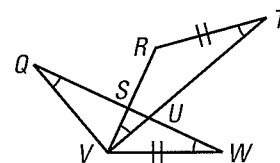
6. $\overline{AC} \cong \overline{BD}$



7. $\overline{GK} \cong \overline{HJ}$

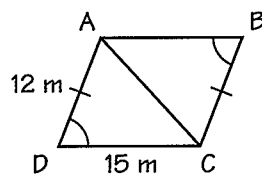


8. $\overline{QW} \cong \overline{TV}$



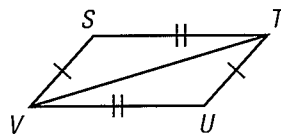
9. **ERROR ANALYSIS** Describe the error in the statement.

$\triangle ABC \cong \triangle CDA$ by SAS.
So, $AB = 15$ meters.

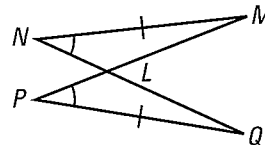


PLANNING FOR PROOF Use the diagram to write a plan for proof.

10. **PROVE** $\angle S \cong \angle U$



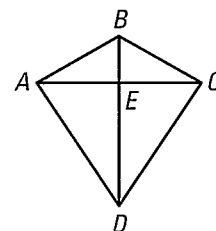
11. **PROVE** $\overline{LM} \cong \overline{LQ}$



12. **PENTAGONS** Explain why segments connecting any pair of corresponding vertices of congruent pentagons are congruent. Make a sketch to support your answer.

13. **ALGEBRA** Given that $\triangle ABC \cong \triangle DEF$, $m\angle A = 70^\circ$, $m\angle B = 60^\circ$, $m\angle C = 50^\circ$, $m\angle D = (3x + 10)^\circ$, $m\angle E = \left(\frac{y}{3} + 20\right)^\circ$, and $m\angle F = (z^2 + 14)^\circ$, find the values of x , y , and z .

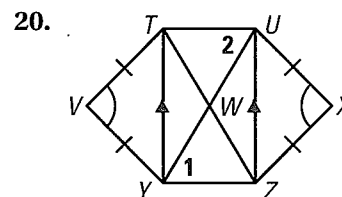
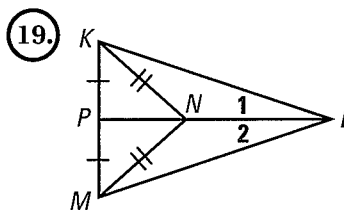
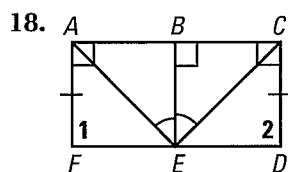
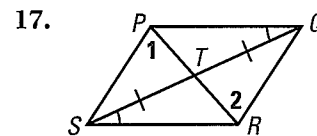
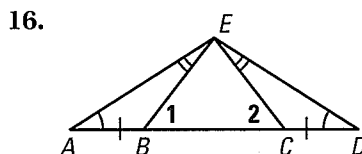
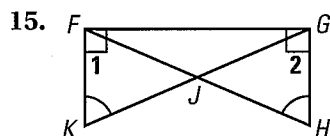
14. ★ **MULTIPLE CHOICE** Which set of given information does *not* allow you to conclude that $\overline{AD} \cong \overline{CD}$?



- (A) $\overline{AE} \cong \overline{CE}, m\angle BEA = 90^\circ$
 (B) $\overline{BA} \cong \overline{BC}, \angle BDC \cong \angle BDA$
 (C) $\overline{AB} \cong \overline{CB}, \angle ABE \cong \angle CBE$
 (D) $\overline{AE} \cong \overline{CE}, \overline{AB} \cong \overline{CB}$

EXAMPLE 3
 on p. 257
 for Exs. 15–20

PLANNING FOR PROOF Use the information given in the diagram to write a plan for proving that $\angle 1 \cong \angle 2$.

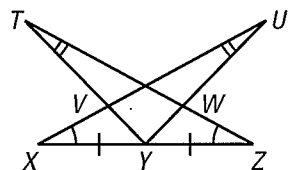


USING COORDINATES Use the vertices of $\triangle ABC$ and $\triangle DEF$ to show that $\angle A \cong \angle D$. Explain your reasoning.

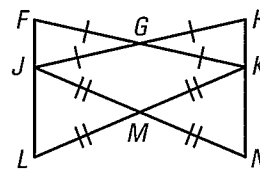
21. $A(3, 7), B(6, 11), C(11, 13), D(2, -4), E(5, -8), F(10, -10)$
 22. $A(3, 8), B(3, 2), C(11, 2), D(-1, 5), E(5, 5), F(5, 13)$

PROOF Use the information given in the diagram to write a proof.

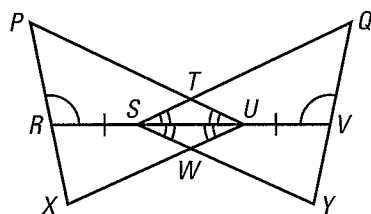
23. **PROVE** $\angle VYX \cong \angle WYZ$



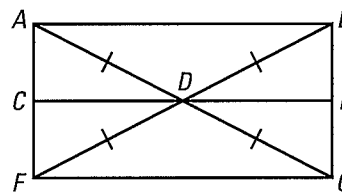
24. **PROVE** $\overline{FL} \cong \overline{HN}$



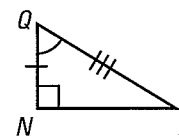
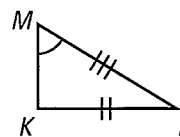
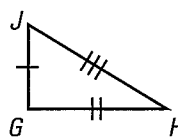
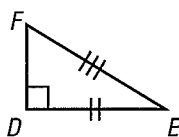
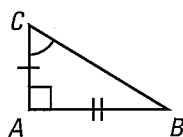
25. **PROVE** $\triangle PUX \cong \triangle QSY$



26. **PROVE** $\overline{AC} \cong \overline{GE}$



27. **CHALLENGE** Which of the triangles below are congruent?

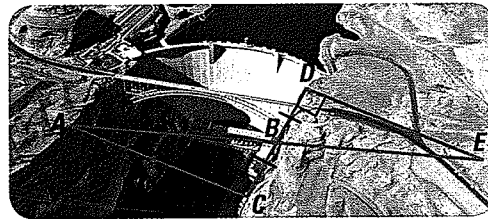


PROBLEM SOLVING

EXAMPLE 2
on p. 257
for Ex. 28

28. **CANYON** Explain how you can find the distance across the canyon.

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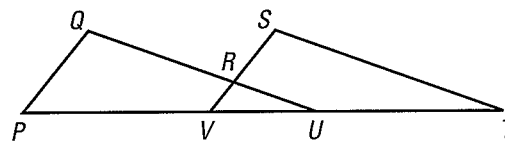


29. **PROOF** Use the given information and the diagram to write a two-column proof.

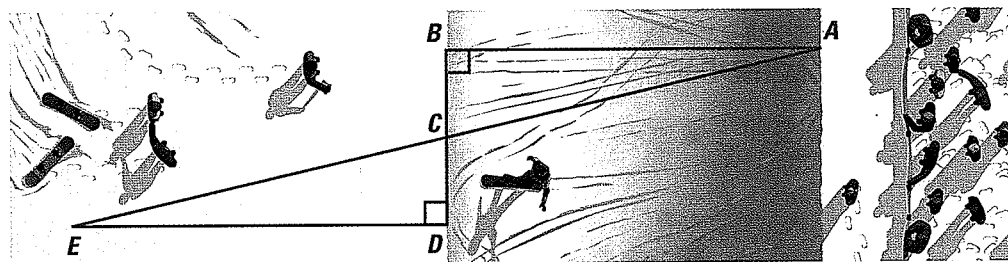
GIVEN $\triangleright \overline{PQ} \parallel \overline{VS}, \overline{QU} \parallel \overline{ST}, \overline{PQ} \cong \overline{VS}$

PROVE $\triangleright \angle Q \cong \angle S$

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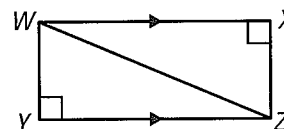


30. **SNOWBOARDING** In the diagram of the half pipe below, C is the midpoint of \overline{BD} . If $EC \approx 11.5$ m, and $CD \approx 2.5$ m, find the approximate distance across the half pipe. Explain your reasoning.



31. **★ MULTIPLE CHOICE** Using the information in the diagram, you can prove that $\overline{WY} \cong \overline{ZX}$. Which reason would *not* appear in the proof?

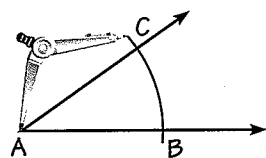
- (A) SAS Congruence Postulate
- (B) AAS Congruence Theorem
- (C) Alternate Interior Angles Theorem
- (D) Right Angles Congruence Theorem



EXAMPLE 4
on p. 258
for Ex. 32

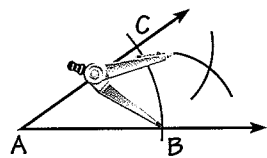
32. **PROVING A CONSTRUCTION** The diagrams below show the construction on page 34 used to bisect $\angle A$. By construction, you can assume that $\overline{AB} \cong \overline{AC}$ and $\overline{BG} \cong \overline{CG}$. Write a proof to verify that \overrightarrow{AG} bisects $\angle A$.

STEP 1



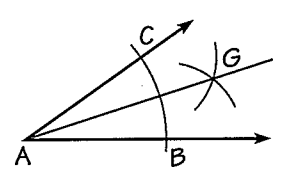
First draw an arc with center A . Label the points where the arc intersects the sides of the angle points B and C .

STEP 2



Draw an arc with center C . Using the same radius, draw an arc with center B . Label the intersection point G .

STEP 3



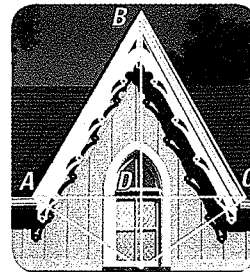
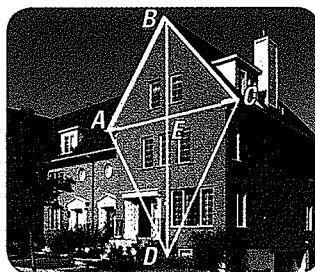
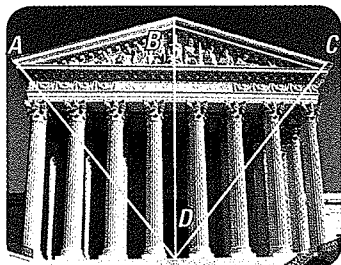
Draw \overrightarrow{AG} . It follows that $\angle BAG \cong \angle CAG$.

ARCHITECTURE Can you use the given information to determine that $\overline{AB} \cong \overline{BC}$? Justify your answer.

33. $\angle ABD \cong \angle CBD$,
 $AD = CD$

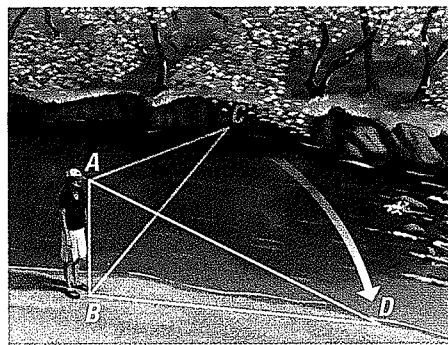
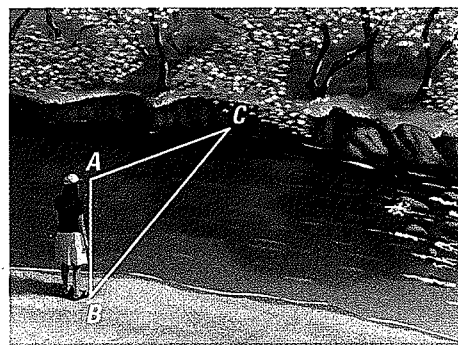
34. $\overline{AC} \perp \overline{BD}$,
 $\triangle ADE \cong \triangle CDE$

35. \overline{BD} bisects \overline{AC} ,
 $\overline{AD} \perp \overline{BD}$



36. ★ **EXTENDED RESPONSE** You can use the method described below to find the distance across a river. You will need a cap with a visor.

- Stand on one side of the river and look straight across to a point on the other side. Align the visor of your cap with that point.
- Without changing the inclination of your neck and head, turn sideways until the visor is in line with a point on your side of the stream.
- Measure the distance BD between your feet and that point.

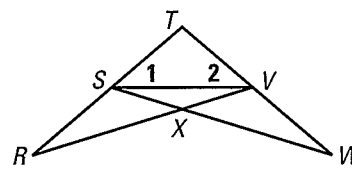
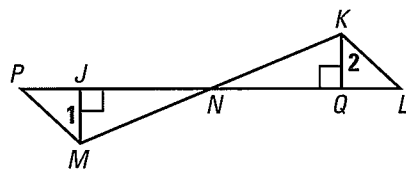


- What corresponding parts of the two triangles can you assume are congruent? What postulate or theorem can you use to show that the two triangles are congruent?
- Explain why BD is also the distance across the stream.

PROOF Use the given information and the diagram to prove that $\angle 1 \cong \angle 2$.

37. **GIVEN** $\overline{MN} \cong \overline{KN}$, $\angle PMN \cong \angle NKL$

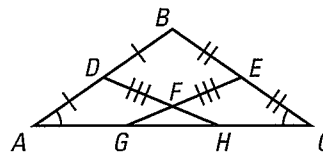
38. **GIVEN** $\overline{TS} \cong \overline{TV}$, $\overline{SR} \cong \overline{VW}$



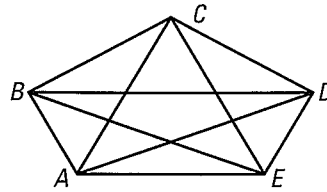
39. **PROOF** Write a proof.

GIVEN $\overline{BA} \cong \overline{BC}$, D and E are midpoints,
 $\angle A \cong \angle C$, $\overline{DF} \cong \overline{EF}$

PROVE $\overline{FG} \cong \overline{FH}$



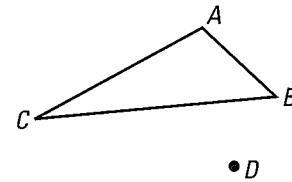
40. **CHALLENGE** In the diagram of pentagon $ABCDE$, $\overline{AB} \parallel \overline{EC}$, $\overline{AC} \parallel \overline{ED}$, $\overline{AB} \cong \overline{ED}$, and $\overline{AC} \cong \overline{EC}$. Write a proof that shows $\overline{AD} \cong \overline{EB}$.



MIXED REVIEW

How many lines can be drawn that fit each description?
Copy the diagram and sketch all the lines. (p. 147)

41. Line(s) through B and parallel to \overleftrightarrow{AC}
 42. Line(s) through A and perpendicular to \overleftrightarrow{BC}
 43. Line(s) through D and C



PREVIEW

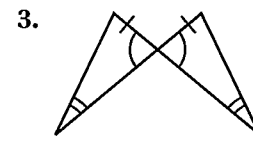
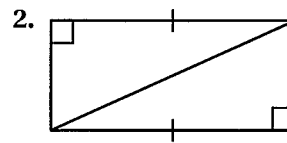
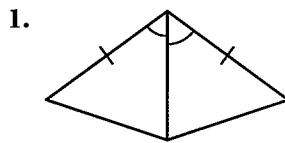
Prepare for
Lesson 4.7 in
Exs. 44–46.

The variable expressions represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles. (p. 217)

- | | | |
|---|---|---|
| 44. $m\angle A = x^\circ$
$m\angle B = (4x)^\circ$
$m\angle C = (5x)^\circ$ | 45. $m\angle A = x^\circ$
$m\angle B = (5x)^\circ$
$m\angle C = (x + 19)^\circ$ | 46. $m\angle A = (x - 22)^\circ$
$m\angle B = (x + 16)^\circ$
$m\angle C = (2x - 14)^\circ$ |
|---|---|---|

QUIZ for Lessons 4.4–4.6

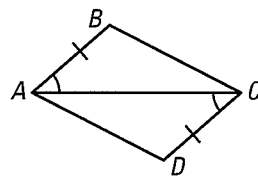
Decide which method, SAS, ASA, AAS, or HL, can be used to prove that the triangles are congruent. (pp. 240, 249)



Use the given information to write a proof.

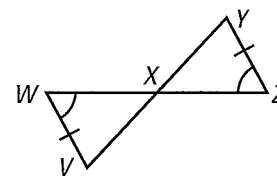
4. **GIVEN** $\angle BAC \cong \angle DCA$, $\overline{AB} \cong \overline{CD}$

PROVE $\triangle ABC \cong \triangle CDA$ (p. 240)



5. **GIVEN** $\angle W \cong \angle Z$, $\overline{VW} \cong \overline{YZ}$

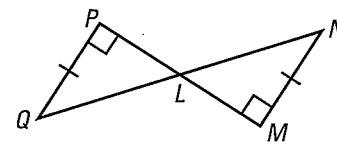
PROVE $\triangle VWX \cong \triangle YZX$ (p. 249)



6. Write a plan for a proof. (p. 256)

GIVEN $\overline{PQ} \cong \overline{MN}$, $m\angle P = m\angle M = 90^\circ$

PROVE $\overline{QL} \cong \overline{NL}$



4.7 EXERCISES

HOMWORK KEY

○ = WORKED-OUT SOLUTIONS
on p. WS5 for Exs. 5, 17, and 41

★ = STANDARDIZED TEST PRACTICE
Exs. 2, 18, 19, 30, 31, and 46

SKILL PRACTICE

1. **VOCABULARY** Define the *vertex angle* of an isosceles triangle.

2. ★ **WRITING** What is the relationship between the base angles of an isosceles triangle? *Explain.*

EXAMPLE 1

on p. 264
for Exs. 3–6

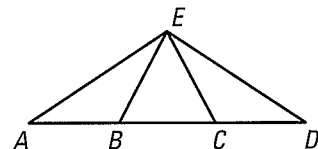
USING DIAGRAMS In Exercises 3–6, use the diagram. Copy and complete the statement. Tell what theorem you used.

3. If $\overline{AE} \cong \overline{DE}$, then $\angle _? \cong \angle _?$.

4. If $\overline{AB} \cong \overline{EB}$, then $\angle _? \cong \angle _?$.

5. If $\angle D \cong \angle CED$, then $_? \cong _?$.

6. If $\angle EBC \cong \angle ECB$, then $_? \cong _?$.

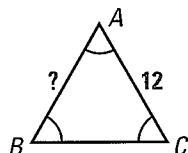


EXAMPLE 2

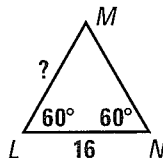
on p. 265
for Exs. 7–14

REASONING Find the unknown measure.

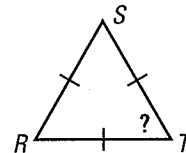
7.



8.

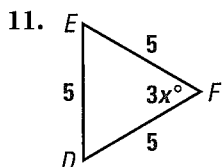


9.

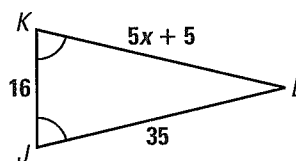


10. **DRAWING DIAGRAMS** A base angle in an isosceles triangle measures 37° . Draw and label the triangle. What is the measure of the vertex angle?

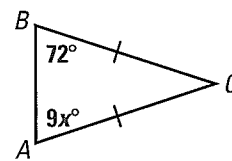
11. **ALGEBRA** Find the value of x .



12.

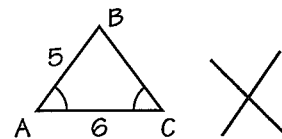


13.



14. **ERROR ANALYSIS** Describe and correct the error made in finding BC in the diagram shown.

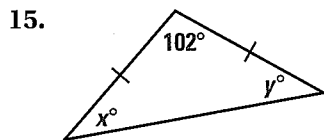
$\angle A \cong \angle C$, therefore
 $\overline{AC} \cong \overline{BC}$. So,
 $BC = 6$



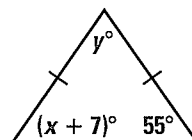
EXAMPLE 3

on p. 266
for Exs. 15–17

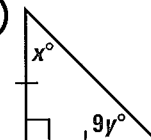
15. **ALGEBRA** Find the values of x and y .



16.



17.



18. ★ **SHORT RESPONSE** Are isosceles triangles always acute triangles? *Explain* your reasoning.

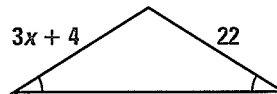
19. ★ **MULTIPLE CHOICE** What is the value of x in the diagram?

(A) 5

(B) 6

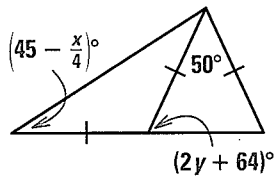
(C) 7

(D) 9

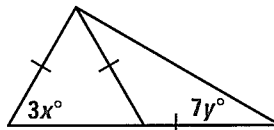


20. **ALGEBRA** Find the values of x and y , if possible. *Explain your reasoning.*

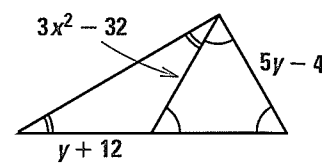
20.



21.

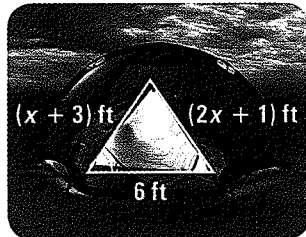


22.

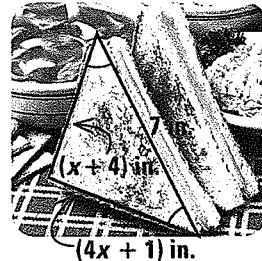


23. **ALGEBRA** Find the perimeter of the triangle.

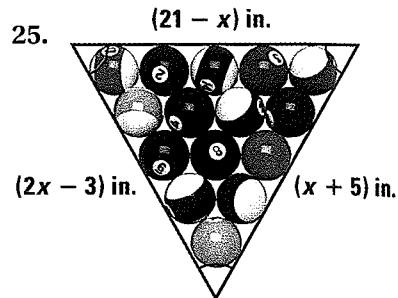
23.



24.



25.



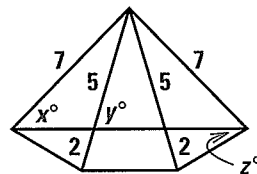
REASONING In Exercises 26–29, use the diagram. State whether the given values for x , y , and z are possible or not. If not, *explain*.

26. $x = 90$, $y = 68$, $z = 42$

27. $x = 40$, $y = 72$, $z = 36$

28. $x = 25$, $y = 25$, $z = 15$

29. $x = 42$, $y = 72$, $z = 33$

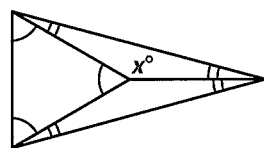


30. ★ **SHORT RESPONSE** In $\triangle DEF$, $m\angle D = (4x + 2)^\circ$, $m\angle E = (6x - 30)^\circ$, and $m\angle F = 3x^\circ$. What type of triangle is $\triangle DEF$? *Explain your reasoning.*

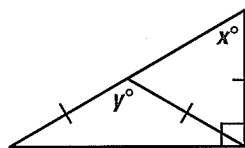
31. ★ **SHORT RESPONSE** In $\triangle ABC$, D is the midpoint of \overline{AC} , and \overline{BD} is perpendicular to \overline{AC} . *Explain why $\triangle ABC$ is isosceles.*

32. **ALGEBRA** Find the value(s) of the variable(s). *Explain your reasoning.*

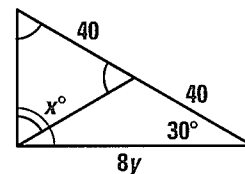
32.



33.



34.



35. **REASONING** The measure of an exterior angle of an isosceles triangle is 130° . What are the possible angle measures of the triangle? *Explain.*

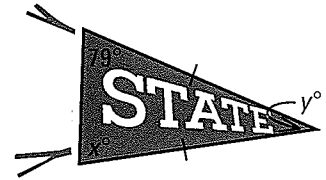
36. **PROOF** Let $\triangle ABC$ be isosceles with vertex angle $\angle A$. Suppose $\angle A$, $\angle B$, and $\angle C$ have integer measures. Prove that $m\angle A$ must be even.

37. **CHALLENGE** The measure of an exterior angle of an isosceles triangle is x° . What are the possible angle measures of the triangle in terms of x ? *Describe all the possible values of x .*

PROBLEM SOLVING

38. **SPORTS** The dimensions of a sports pennant are given in the diagram. Find the values of x and y .

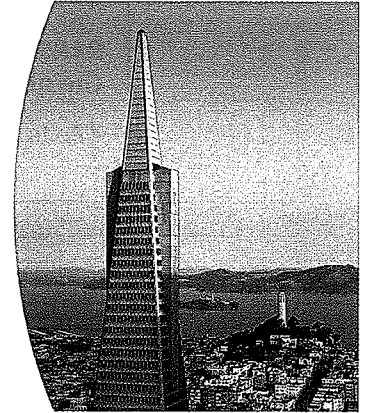
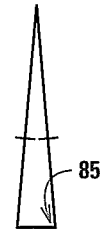
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39. **ADVERTISING** A logo in an advertisement is an equilateral triangle with a side length of 5 centimeters. Sketch the logo and give the measure of each side and angle.

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40. **ARCHITECTURE** The Transamerica Pyramid building shown in the photograph has four faces shaped like isosceles triangles. The measure of a base angle of one of these triangles is about 85° . What is the approximate measure of the vertex angle of the triangle?

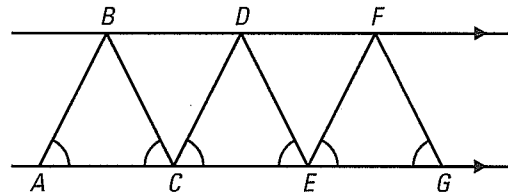


EXAMPLE 4

on p. 266
for Exs. 41–42

41. **MULTI-STEP PROBLEM** To make a zig-zag pattern, a graphic designer sketches two parallel line segments. Then the designer draws blue and green triangles as shown below.

- Prove that $\triangle ABC \cong \triangle BCD$.
- Name all the isosceles triangles in the diagram.
- Name four angles that are congruent to $\angle ABC$.

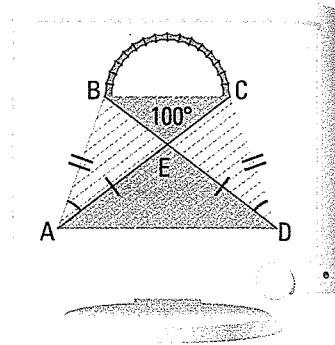


42. **★ VISUAL REASONING** In the pattern below, each small triangle is an equilateral triangle with an area of 1 square unit.

Triangle				
Area	1 square unit	?	?	?

- Reasoning** Explain how you know that any triangle made out of equilateral triangles will be an equilateral triangle.
 - Area** Find the areas of the first four triangles in the pattern.
 - Make a Conjecture** Describe any patterns in the areas. Predict the area of the seventh triangle in the pattern. Explain your reasoning.
43. **REASONING** Let $\triangle PQR$ be an isosceles right triangle with hypotenuse \overline{QR} . Find $m\angle P$, $m\angle Q$, and $m\angle R$.
44. **REASONING** Explain how the Corollary to the Base Angles Theorem follows from the Base Angles Theorem.
45. **PROVING THEOREM 4.8** Write a proof of the Converse of the Base Angles Theorem.

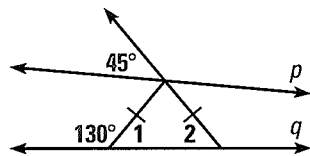
46. ★ **EXTENDED RESPONSE** Sue is designing fabric purses that she will sell at the school fair. Use the diagram of one of her purses.



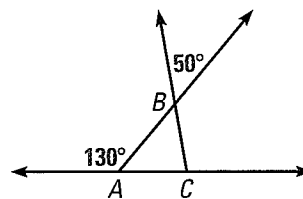
- Prove that $\triangle ABE \cong \triangle DCE$.
- Name the isosceles triangles in the purse.
- Name three angles that are congruent to $\angle EAD$.
- What If?** If the measure of $\angle BEC$ changes, does your answer to part (c) change? *Explain.*

REASONING FROM DIAGRAMS Use the information in the diagram to answer the question. *Explain* your reasoning.

47. Is $p \parallel q$?



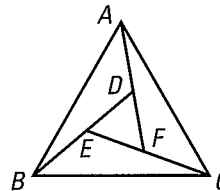
48. Is $\triangle ABC$ isosceles?



49. **PROOF** Write a proof.

GIVEN ▶ $\triangle ABC$ is equilateral,
 $\angle CAD \cong \angle ABE \cong \angle BCF$.

PROVE ▶ $\triangle DEF$ is equilateral.



50. **COORDINATE GEOMETRY** The coordinates of two vertices of $\triangle TUV$ are $T(0, 4)$ and $U(4, 0)$. *Explain* why the triangle will always be an isosceles triangle if V is any point on the line $y = x$ except $(2, 2)$.

51. **CHALLENGE** The lengths of the sides of a triangle are $3t$, $5t - 12$, and $t + 20$. Find the values of t that make the triangle isosceles. *Explain.*

MIXED REVIEW

What quadrant contains the point? (p. 878)

52. $(-1, -3)$

53. $(-2, 4)$

54. $(5, -2)$

Copy and complete the given function table. (p. 884)

55.

x	-7	0	5
$y = x - 4$?	?	?

56.

?	-2	0	1
?	-6	0	3

PREVIEW
Prepare for
Lesson 4.8 in
Exs. 57–60.

Use the Distance Formula to decide whether $\overline{AB} \cong \overline{AC}$. (p. 15)

57. $A(0, 0)$, $B(-5, -6)$, $C(6, 5)$

58. $A(3, -3)$, $B(0, 1)$, $C(-1, 0)$

59. $A(0, 1)$, $B(4, 7)$, $C(-6, 3)$

60. $A(-3, 0)$, $B(2, 2)$, $C(2, -2)$

4.8 Investigate Slides and Flips

MATERIALS • graph paper • pencil

QUESTION What happens when you slide or flip a triangle?

EXPLORE 1 Slide a triangle

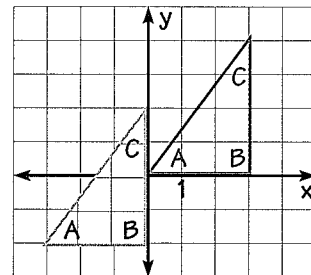
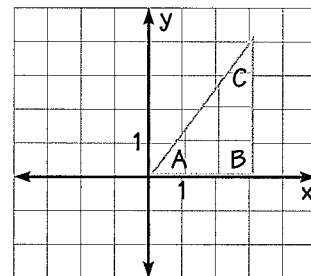
STEP 1 Draw a triangle Draw a scalene right triangle with legs of length 3 units and 4 units on a piece of graph paper. Cut out the triangle.

STEP 2 Draw coordinate plane Draw axes on the graph paper. Place the cut-out triangle so that the coordinates of the vertices are integers. Trace around the triangle and label the vertices.

STEP 3 Slide triangle Slide the cut-out triangle so it moves left and down. Write a description of the *transformation* and record ordered pairs in a table like the one shown. Repeat this step three times, sliding the triangle left or right *and* up or down to various places in the coordinate plane.

Slide 3 units left and 2 units down.

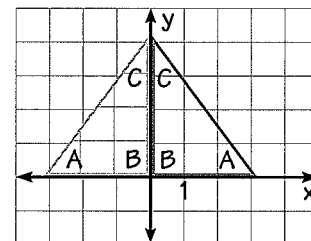
Vertex	Original position	New position
A	(0, 0)	(-3, -2)
B	(3, 0)	(0, -2)
C	(3, 4)	(0, 2)



EXPLORE 2 Flip a triangle

STEP 1 Draw a coordinate plane Draw and label a second coordinate plane. Place the cut-out triangle so that one vertex is at the origin and one side is along the y -axis, as shown.

STEP 2 Flip triangle Flip the cut-out triangle over the y -axis. Record a description of the *transformation* and record the ordered pairs in a table. Repeat this step, flipping the triangle over the x -axis.



DRAW CONCLUSIONS Use your observations to complete these exercises

- How are the coordinates of the original position of the triangle related to the new position in a slide? in a flip?
- Is the original triangle congruent to the new triangle in a slide? in a flip? Explain your reasoning.

4.8 EXERCISES

HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS
on p. WS5 for Exs. 11, 23, and 39

★ = STANDARDIZED TEST PRACTICE
Exs. 2, 25, 40, 41, and 43

SKILL PRACTICE

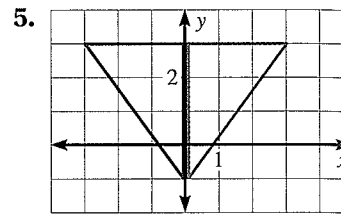
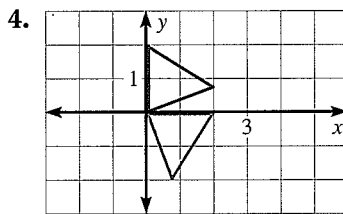
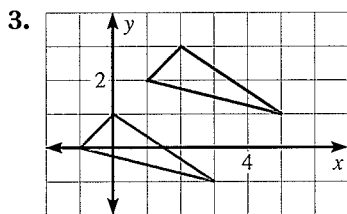
1. **VOCABULARY** Describe the translation $(x, y) \rightarrow (x - 1, y + 4)$ in words.

2. ★ **WRITING** Explain why the term *congruence transformation* is used in describing translations, reflections, and rotations.

EXAMPLE 1

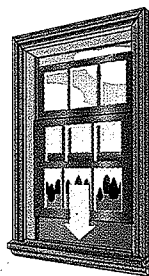
on p. 272
for Exs. 3–8

IDENTIFYING TRANSFORMATIONS Name the type of transformation shown.

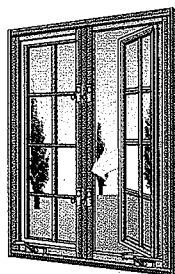


WINDOWS Decide whether the moving part of the window is a translation.

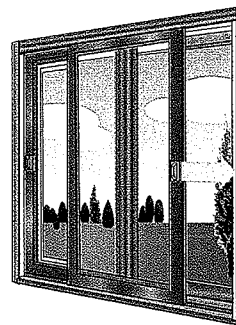
6. Double hung



7. Casement



8. Sliding



EXAMPLE 2

on p. 273
for Exs. 9–16

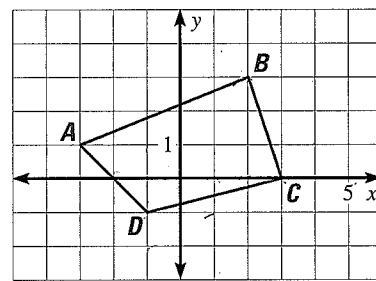
DRAWING A TRANSLATION Copy figure $ABCD$ and draw its image after the translation.

9. $(x, y) \rightarrow (x + 2, y - 3)$

10. $(x, y) \rightarrow (x - 1, y - 5)$

11. $(x, y) \rightarrow (x + 4, y + 1)$

12. $(x, y) \rightarrow (x - 2, y + 3)$



COORDINATE NOTATION Use coordinate notation to describe the translation.

13. 4 units to the left, 2 units down

14. 6 units to the right, 3 units up

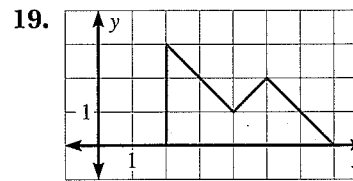
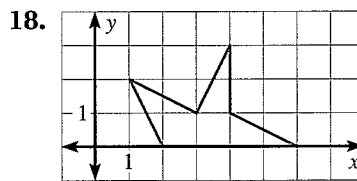
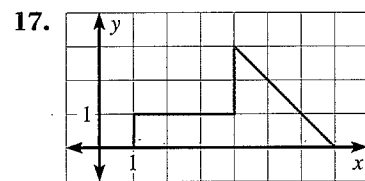
15. 2 units to the right, 1 unit down

16. 7 units to the left, 9 units up

EXAMPLE 3

on p. 274
for Exs. 17–19

DRAWING Use a reflection in the x -axis to draw the other half of the figure.



EXAMPLE 4
on p. 275
for Exs. 20–23

ROTATIONS Use the coordinates to graph \overline{AB} and \overline{CD} . Tell whether \overline{CD} is a rotation of \overline{AB} about the origin. If so, give the angle and direction of rotation.

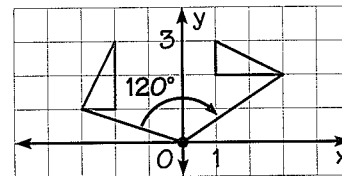
20. $A(1, 2), B(3, 4), C(2, -1), D(4, -3)$

21. $A(-2, -4), B(-1, -2), C(4, 3), D(2, 1)$

22. $A(-4, 0), B(-4, -4), C(4, 4), D(0, 4)$

23. $A(1, 2), B(3, 0), C(2, -1), D(2, -3)$

24. **ERROR ANALYSIS** A student says that the red triangle is a 120° clockwise rotation of the blue triangle about the origin. Describe and correct the error.



25. **★ WRITING** Can a point or a line segment be its own image under a transformation? Explain and illustrate your answer.

APPLYING TRANSLATIONS Complete the statement using the description of the translation. In the description, points $(0, 3)$ and $(2, 5)$ are two vertices of a hexagon.

26. If $(0, 3)$ translates to $(0, 0)$, then $(2, 5)$ translates to $?$.
27. If $(0, 3)$ translates to $(1, 2)$, then $(2, 5)$ translates to $?$.
28. If $(0, 3)$ translates to $(-3, -2)$, then $(2, 5)$ translates to $?$.

29. **ALGEBRA** A point on an image and the translation are given. Find the corresponding point on the original figure.

29. Point on image: $(4, 0)$; translation: $(x, y) \rightarrow (x + 2, y - 3)$

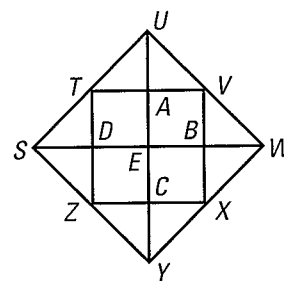
30. Point on image: $(-3, 5)$; translation: $(x, y) \rightarrow (-x, y)$

31. Point on image: $(6, -9)$; translation: $(x, y) \rightarrow (x - 7, y - 4)$

32. **CONGRUENCE** Show that the transformation in Exercise 3 is a congruence transformation.

DESCRIBING AN IMAGE State the segment or triangle that represents the image. You can use tracing paper to help you see the rotation.

33. 90° clockwise rotation of \overline{ST} about E
34. 90° counterclockwise rotation of \overline{BX} about E
35. 180° rotation of $\triangle BWX$ about E
36. 180° rotation of $\triangle TUA$ about E



37. **CHALLENGE** Solve for the variables in the transformation of \overline{AB} to \overline{CD} and then to \overline{EF} .

$A(2, 3),$
 $B(4, 2a)$

Translation:
 $(x, y) \rightarrow (x - 2, y + 1)$

$C(m - 3, 4),$
 $D(n - 9, 5)$

Reflection:
in x -axis

$E(0, g - 6),$
 $F(8h, -5)$

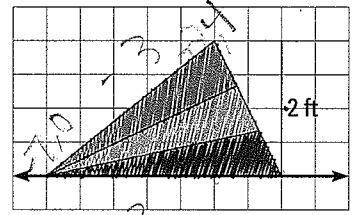
PROBLEM SOLVING

EXAMPLE 3

on p. 274
for Ex. 38

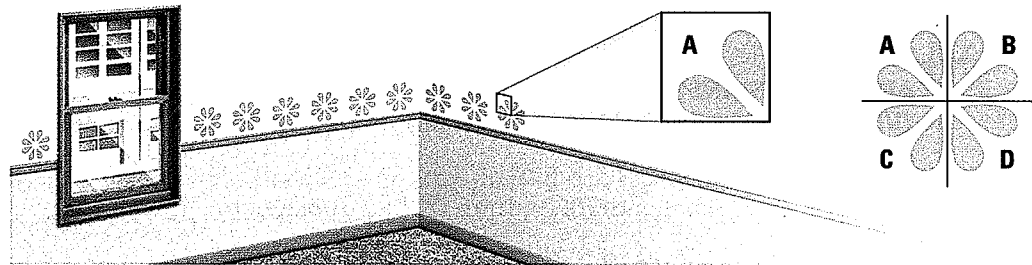
38. KITES The design for a kite shows the layout and dimensions for only half of the kite.

- a. What type of transformation can a designer use to create plans for the entire kite?
- b. What is the maximum width of the entire kite?



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39. STENCILING You are stenciling a room in your home. You want to use the stencil pattern below on the left to create the design shown. Give the angles and directions of rotation you will use to move the stencil from A to B and from A to C.



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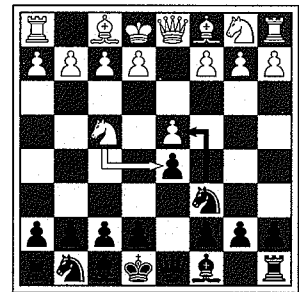
40. ★ OPEN-ENDED MATH Some words reflect onto themselves through a vertical line of reflection. An example is shown.

- a. Find two other words with vertical lines of reflection. Draw the line of reflection for each word.
- b. Find two words with horizontal lines of reflection. Draw the line of reflection for each word.



41. ★ SHORT RESPONSE In chess, six different kinds of pieces are moved according to individual rules. The Knight (shaped like a horse) moves in an "L" shape. It moves two squares horizontally or vertically and then one additional square perpendicular to its original direction. When a knight lands on a square with another piece, it *captures* that piece.

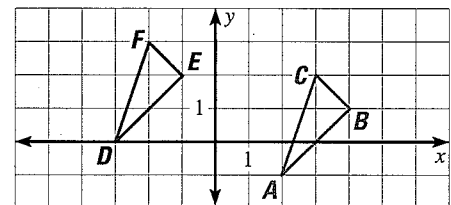
- a. Describe the translation used by the Black Knight to capture the White Pawn.
- b. Describe the translation used by the White Knight to capture the Black Pawn.
- c. After both pawns are captured, can the Black Knight capture the White Knight? *Explain.*



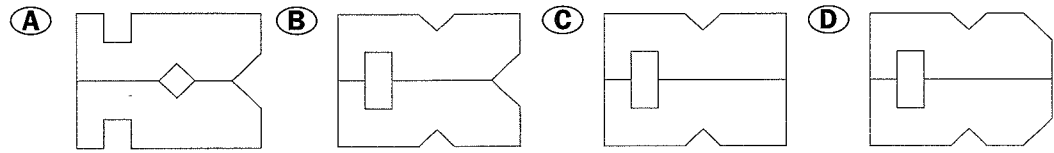
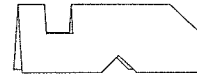
EXAMPLE 5

on p. 275
for Ex. 42

42. VERIFYING CONGRUENCE Show that $\triangle ABC$ and $\triangle DEF$ are right triangles and use the HL Congruence Theorem to verify that $\triangle DEF$ is a congruence transformation of $\triangle ABC$.



43. ★ **MULTIPLE CHOICE** A piece of paper is folded in half and some cuts are made, as shown. Which figure represents the unfolded piece of paper?



44. **CHALLENGE** A triangle is rotated 90° counterclockwise and then translated three units up. The vertices of the final image are $A(-4, 4)$, $B(-1, 6)$, and $C(-1, 4)$. Find the vertices of the original triangle. Would the final image be the same if the original triangle was translated 3 units up and then rotated 90° counterclockwise? *Explain* your reasoning.

MIXED REVIEW

PREVIEW

Prepare for
Lesson 5.1 in
Exs. 45–50.

Simplify the expression. Variables a and b are positive.

45. $\frac{-a-0}{0-(-b)}$ (p. 870)

46. $|(a+b)-a|$ (p. 870)

47. $\frac{2a+2b}{2}$ (p. 139)

Simplify the expression. Variables a and b are positive. (p. 139)

48. $\sqrt{(-b)^2}$

49. $\sqrt{(2a)^2}$

50. $\sqrt{(2a-a)^2 + (0-b)^2}$

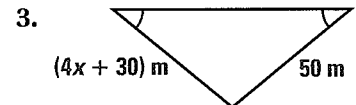
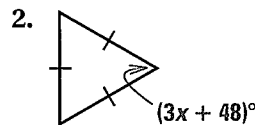
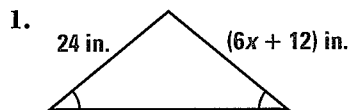
51. Use the SSS Congruence Postulate to show $\triangle RST \cong \triangle UVW$. (p. 234)

$R(1, -4), S(1, -1), T(6, -1)$

$U(1, 4), V(1, 1), W(6, 1)$

QUIZ for Lessons 4.7–4.8

Find the value of x . (p. 264)



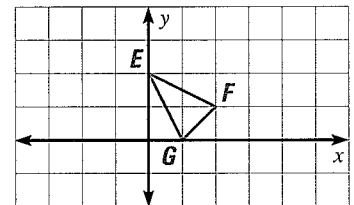
Copy $\triangle EFG$ and draw its image after the transformation. Identify the type of transformation. (p. 272)

4. $(x, y) \rightarrow (x + 4, y - 1)$

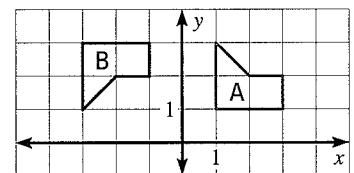
5. $(x, y) \rightarrow (-x, y)$

6. $(x, y) \rightarrow (x, -y)$

7. $(x, y) \rightarrow (x - 3, y + 2)$



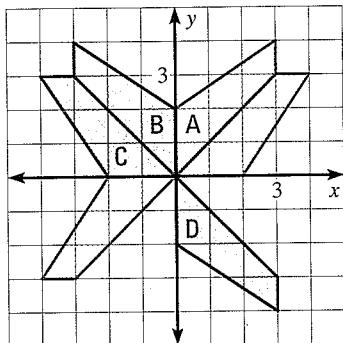
8. Is Figure B a rotation of Figure A about the origin? If so, give the angle and direction of rotation. (p. 272)



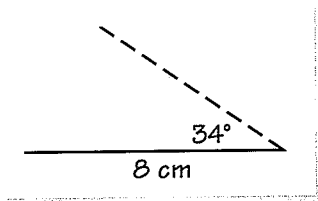


Lessons 4.5–4.8

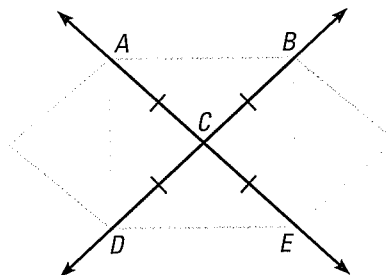
1. **MULTI-STEP PROBLEM** Use the quilt pattern shown below.



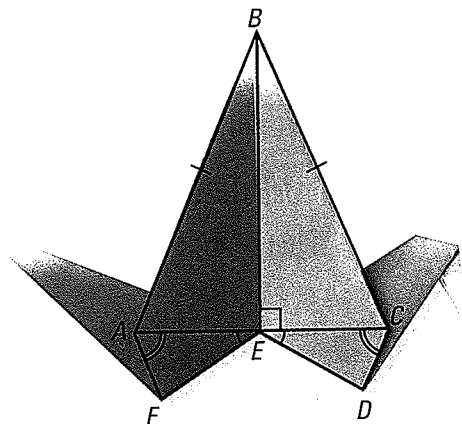
- Figure B is the image of Figure A. Name and describe the transformation.
 - Figure C is the image of Figure A. Name and describe the transformation.
 - Figure D is the image of Figure A. Name and describe the transformation.
 - Explain how you could complete the quilt pattern using transformations of Figure A.
2. **SHORT RESPONSE** You are told that a triangle has sides that are 5 centimeters and 3 centimeters long. You are also told that the side that is 5 centimeters long forms an angle with the third side that measures 28° . Is there only one triangle that has these given dimensions? Explain why or why not.
3. **OPEN-ENDED** A friend has drawn a triangle on a piece of paper and she is describing the triangle so that you can draw one that is congruent to hers. So far, she has told you that the length of one side is 8 centimeters and one of the angles formed with this side is 34° . Describe three pieces of additional information you could use to construct the triangle.



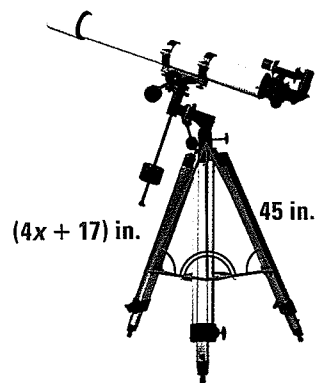
4. **SHORT RESPONSE** Can the triangles ACD and BCE be proven congruent using the information given in the diagram? Can you show that $\overline{AD} \cong \overline{BE}$? Explain.



5. **EXTENDED RESPONSE** Use the information given in the diagram to prove the statements below.



- Prove that $\angle BCE \cong \angle BAE$.
 - Prove that $\overline{AF} \cong \overline{CD}$.
6. **GRIDDED ANSWER** Find the value of x in the diagram.

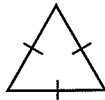


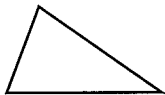

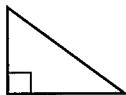
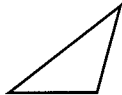


BIG IDEAS

For Your Notebook

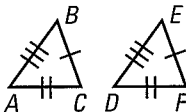
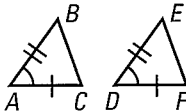
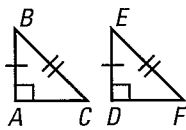
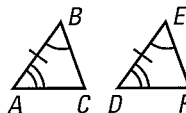
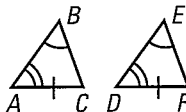
Big Idea 1

Classifying Triangles by Sides and Angles

	Equilateral	Isosceles	Scalene
Sides	 3 congruent sides	 2 or 3 congruent sides	 No congruent sides
Angles	 3 angles $< 90^\circ$	 3 angles $= 60^\circ$	 1 angle $= 90^\circ$
	 1 angle $> 90^\circ$		

Big Idea 2

Proving That Triangles Are Congruent

SSS	All three sides are congruent.	$\triangle ABC \cong \triangle DEF$	
SAS	Two sides and the included angle are congruent.	$\triangle ABC \cong \triangle DEF$	
HL	The hypotenuse and one of the legs are congruent. (Right triangles only)	$\triangle ABC \cong \triangle DEF$	
ASA	Two angles and the included side are congruent.	$\triangle ABC \cong \triangle DEF$	
AAS	Two angles and a (non-included) side are congruent.	$\triangle ABC \cong \triangle DEF$	

Big Idea 3

Using Coordinate Geometry to Investigate Triangle Relationships

You can use the Distance and Midpoint Formulas to apply postulates and theorems to triangles in the coordinate plane.

4

CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- triangle, p. 217
scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- interior angles, p. 218
- exterior angles, p. 218
- corollary to a theorem, p. 220
- congruent figures, p. 225
- corresponding parts, p. 225
- right triangle, p. 241
legs, hypotenuse
- flow proof, p. 250
- isosceles triangle, p. 264
legs, vertex angle, base, base angles
- transformation, p. 272
- image, p. 272
- congruence transformation, p. 272
translation, reflection, rotation

VOCABULARY EXERCISES

1. Copy and complete: A triangle with three congruent angles is called ? .
2. **WRITING** Compare vertex angles and base angles.
3. **WRITING** Describe the difference between isosceles and scalene triangles.
4. Sketch an acute scalene triangle. Label its interior angles 1, 2, and 3. Then draw and shade its exterior angles.
5. If $\triangle PQR \cong \triangle LMN$, which angles are corresponding angles? Which sides are corresponding sides?

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 4.

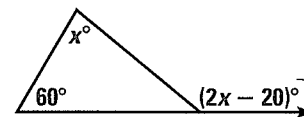
4.1 Apply Triangle Sum Properties

pp. 217–224

EXAMPLE

Find the measure of the exterior angle shown.

Use the Exterior Angle Theorem to write and solve an equation to find the value of x .



$$(2x - 20)^\circ = 60^\circ + x^\circ \quad \text{Apply the Exterior Angle Theorem.}$$

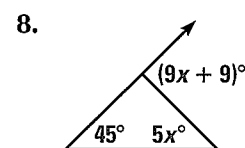
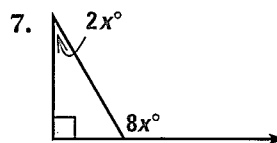
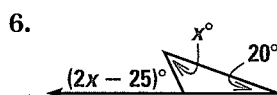
$$x = 80 \quad \text{Solve for } x.$$

The measure of the exterior angle is $(2 \cdot 80 - 20)^\circ$, or 140° .

EXERCISES

Find the measure of the exterior angle shown.

EXAMPLE 3
on p. 219
for Exs. 6–8



4.2 Apply Congruence and Triangles

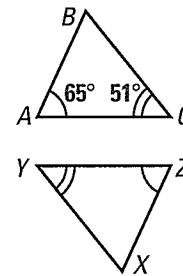
pp. 225–231

EXAMPLE

Use the Third Angles Theorem to find $m\angle X$.

In the diagram, $\angle A \cong \angle Z$ and $\angle C \cong \angle Y$. By the Third Angles Theorem, $\angle B \cong \angle X$. Then by the Triangle Sum Theorem, $m\angle B = 180^\circ - 65^\circ - 51^\circ = 64^\circ$.

So, $m\angle X = m\angle B = 64^\circ$ by the definition of congruent angles.

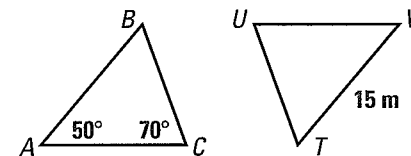


EXERCISES

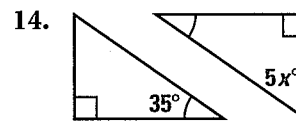
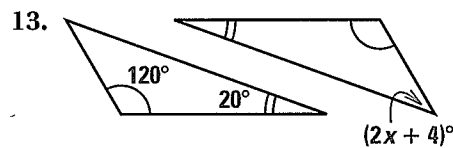
In the diagram, $\triangle ABC \cong \triangle VTU$.

Find the indicated measure.

9. $m\angle B$ 10. AB
11. $m\angle T$ 12. $m\angle V$



Find the value of x .



EXAMPLES 2 and 4

on pp. 226–227
for Exs. 9–14

EXAMPLE 1

on p. 234
for Exs. 15–16

4.3 Prove Triangles Congruent by SSS

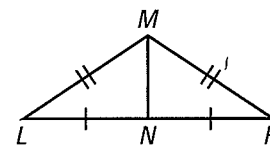
pp. 234–239

EXAMPLE

Prove that $\triangle LMN \cong \triangle PMN$.

The marks on the diagram show that $\overline{LM} \cong \overline{PM}$ and $\overline{LN} \cong \overline{PN}$. By the Reflexive Property, $\overline{MN} \cong \overline{MN}$.

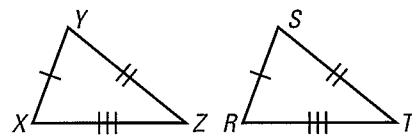
So, by the SSS Congruence Postulate, $\triangle LMN \cong \triangle PMN$.



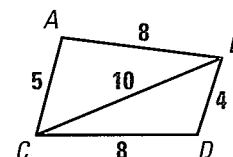
EXERCISES

Decide whether the congruence statement is true. *Explain your reasoning.*

15. $\triangle XYZ \cong \triangle RST$



16. $\triangle ABC \cong \triangle DCB$



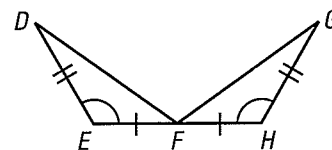
4.4 Prove Triangles Congruent by SAS and HL

pp. 240–246

EXAMPLE

Prove that $\triangle DEF \cong \triangle GHF$.

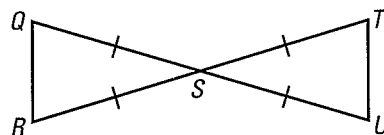
From the diagram, $\overline{DE} \cong \overline{GH}$, $\angle E \cong \angle H$, and $\overline{EF} \cong \overline{HF}$.
By the SAS Congruence Postulate, $\triangle DEF \cong \triangle GHF$.



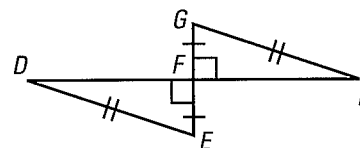
EXERCISES

Decide whether the congruence statement is true. Explain your reasoning.

17. $\triangle QRS \cong \triangle TUS$



18. $\triangle DEF \cong \triangle GHF$



EXAMPLES
1 and 3
on pp. 240, 242
for Exs. 17–18

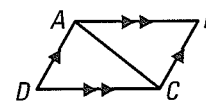
4.5 Prove Triangles Congruent by ASA and AAS

pp. 249–255

EXAMPLE

Prove that $\triangle DAC \cong \triangle BCA$.

By the Reflexive Property, $\overline{AC} \cong \overline{AC}$. Because $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$, $\angle DAC \cong \angle BCA$ and $\angle DCA \cong \angle BAC$ by the Alternate Interior Angles Theorem. So, by the ASA Congruence Postulate, $\triangle ADC \cong \triangle CBA$.



EXERCISES

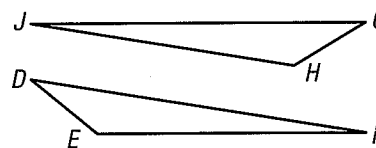
State the third congruence that is needed to prove that $\triangle DEF \cong \triangle GHJ$ using the given postulate or theorem.

19. **GIVEN** $\triangleright \overline{DE} \cong \overline{GH}$, $\angle D \cong \angle G$, $\underline{\quad} \cong \underline{\quad}$

Use the AAS Congruence Theorem.

20. **GIVEN** $\triangleright \overline{DF} \cong \overline{GJ}$, $\angle F \cong \angle J$, $\underline{\quad} \cong \underline{\quad}$

Use the ASA Congruence Postulate.



EXAMPLES
1 and 2
on p. 250
for Exs. 19–20

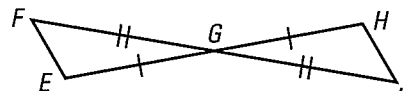
4.6 Use Congruent Triangles

pp. 256–263

EXAMPLE

GIVEN $\triangleright \overline{FG} \cong \overline{JG}$, $\overline{EG} \cong \overline{HG}$

PROVE $\triangleright \overline{EF} \cong \overline{HJ}$

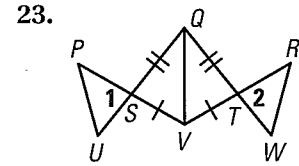
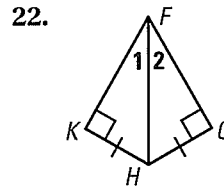
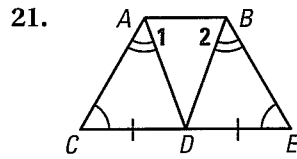


You are given that $\overline{FG} \cong \overline{JG}$ and $\overline{EG} \cong \overline{HG}$. By the Vertical Angles Congruence Theorem, $\angle FGE \cong \angle JGH$. So, $\triangle FGE \cong \triangle JGH$ by the SAS Congruence Postulate. Corresponding parts of $\cong \triangle$ are \cong , so $\overline{EF} \cong \overline{HJ}$.

EXAMPLE 3
on p. 257
for Exs. 21–23

EXERCISES

Write a plan for proving that $\angle 1 \cong \angle 2$.



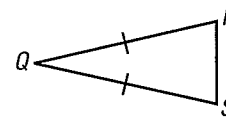
4.7 Use Isosceles and Equilateral Triangles

pp. 264–270

EXAMPLE

$\triangle QRS$ is isosceles. Name two congruent angles.

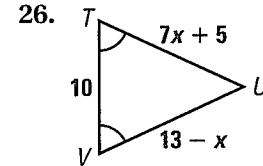
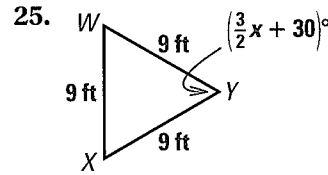
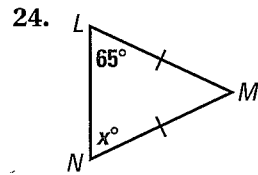
$\overline{QR} \cong \overline{QS}$, so by the Base Angles Theorem, $\angle R \cong \angle S$.



EXAMPLE 3
on p. 266
for Exs. 24–26

EXERCISES

Find the value of x .



4.8 Perform Congruence Transformations

pp. 272–279

EXAMPLE

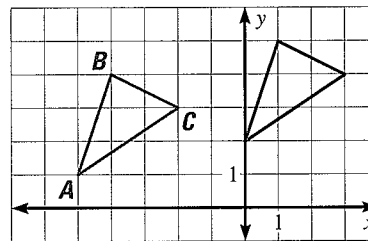
Triangle ABC has vertices $A(-5, 1)$, $B(-4, 4)$, and $C(-2, 3)$. Sketch $\triangle ABC$ and its image after the translation $(x, y) \rightarrow (x + 5, y + 1)$.

$(x, y) \rightarrow (x + 5, y + 1)$

$A(-5, 1) \rightarrow (0, 2)$

$B(-4, 4) \rightarrow (1, 5)$

$C(-2, 3) \rightarrow (3, 4)$



EXAMPLES 2 and 3
on pp. 273–274
for Exs. 27–29

EXERCISES

Triangle QRS has vertices $Q(2, -1)$, $R(5, -2)$, and $S(2, -3)$. Sketch $\triangle QRS$ and its image after the transformation.

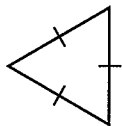
27. $(x, y) \rightarrow (x - 1, y + 5)$

28. $(x, y) \rightarrow (x, -y)$

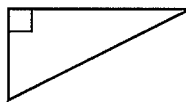
29. $(x, y) \rightarrow (-x, -y)$

Classify the triangle by its sides and by its angles.

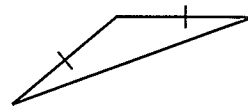
1.



2.

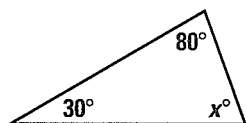


3.

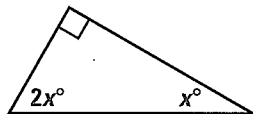


In Exercises 4–6, find the value of x .

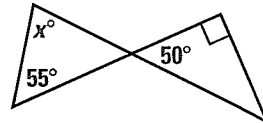
4.



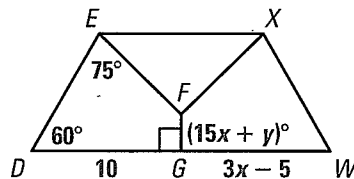
5.



6.



7. In the diagram, $DEFG \cong WXFG$. Find the values of x and y .

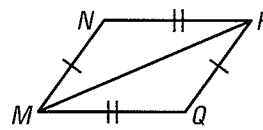
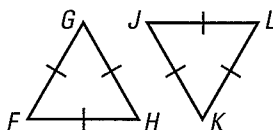
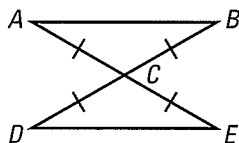


In Exercises 8–10, decide whether the triangles can be proven congruent by the given postulate.

8. $\triangle ABC \cong \triangle EDC$ by SAS

9. $\triangle FGH \cong \triangle JKL$ by ASA

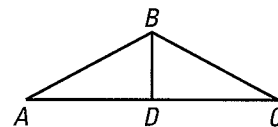
10. $\triangle MNP \cong \triangle PQM$ by SSS



11. Write a proof.

GIVEN $\triangle ABC$ is isosceles with base \overline{AC} , \overline{BD} bisects $\angle B$.

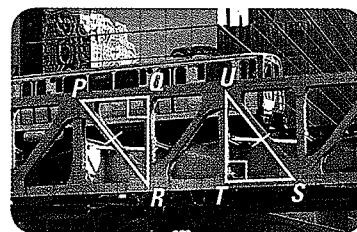
PROVE $\triangle ABD \cong \triangle CBD$



12. What is the third congruence needed to prove that $\triangle PQR \cong \triangle STU$ using the indicated theorem?

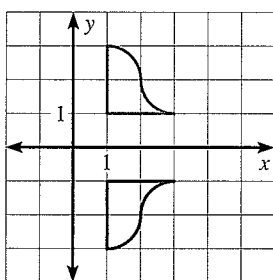
a. HL

b. AAS

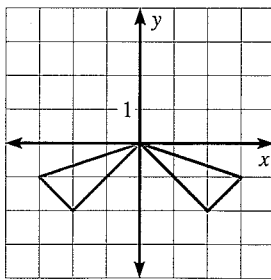


Decide whether the transformation is a *translation*, *reflection*, or *rotation*.

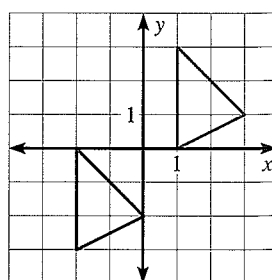
13.



14.



15.



SOLVE INEQUALITIES AND ABSOLUTE VALUE EQUATIONS

xy

EXAMPLE 1 Solve inequalities

Solve $-3x + 7 \leq 28$. Then graph the solution.

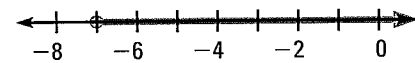
When you multiply or divide each side of an inequality by a *negative* number, you must reverse the inequality symbol to obtain an equivalent inequality.

$$-3x + 7 \leq 28 \quad \text{Write original inequality.}$$

$$-3x \leq 21 \quad \text{Subtract 7 from both sides.}$$

$$x \geq -7 \quad \text{Divide each side by } -3. \text{ Reverse the inequality symbol.}$$

► The solutions are all real numbers greater than or equal to -7 . The graph is shown at the right.



xy

EXAMPLE 2 Solve absolute value equations

Solve $|2x + 1| = 5$.

The expression inside the absolute value bars can represent 5 or -5 .

STEP 1 Assume $2x + 1$ represents 5.

$$2x + 1 = 5$$

$$2x = 4$$

$$x = 2$$

STEP 2 Assume $2x + 1$ represents -5 .

$$2x + 1 = -5$$

$$2x = -6$$

$$x = -3$$

► The solutions are 2 and -3 .

EXERCISES

EXAMPLE 1
for Exs. 1–12

Solve the inequality. Then graph the solution.

1. $x - 6 > -4$

2. $7 - c \leq -1$

3. $-54 \geq 6x$

4. $\frac{5}{2}t + 8 \leq 33$

5. $3(y + 2) < 3$

6. $\frac{1}{4}z < 2$

7. $5k + 1 \geq -11$

8. $13.6 > -0.8 - 7.2r$

9. $6x + 7 < 2x - 3$

10. $-v + 12 \leq 9 - 2v$

11. $4(n + 5) \geq 5 - n$

12. $5y + 3 \geq 2(y - 9)$

EXAMPLE 2
for Exs. 13–27

Solve the equation.

13. $|x - 5| = 3$

14. $|x + 6| = 2$

15. $|4 - x| = 4$

16. $|2 - x| = 0.5$

17. $|3x - 1| = 8$

18. $|4x + 5| = 7$

19. $|x - 1.3| = 2.1$

20. $|3x - 15| = 0$

21. $|6x - 2| = 4$

22. $|8x + 1| = 17$

23. $|9 - 2x| = 19$

24. $|0.5x - 4| = 2$

25. $|5x - 2| = 8$

26. $|7x + 4| = 11$

27. $|3x - 11| = 4$