5 Relationships within Triangles

- **5.1** Midsegment Theorem and Coordinate Proof
- **5.2** Use Perpendicular Bisectors
- **5.3** Use Angle Bisectors of Triangles
- **5.4** Use Medians and Altitudes
- **5.5** Use Inequalities in a Triangle
- **5.6** Inequalities in Two Triangles and Indirect Proof

Before

In previous courses and in Chapters 1–4, you learned the following skills, which you'll use in Chapter 5: simplifying expressions, finding distances and slopes, using properties of triangles, and solving equations and inequalities.

Prerequisite Skills

VOCABULARY CHECK

1. Is the *distance from point P to line AB* equal to the length of $\overline{PQ}$? *Explain* why or why not.
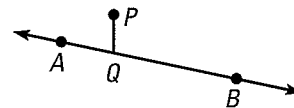
SKILLS AND ALGEBRA CHECK

Simplify the expression. All variables are positive. *(Review pp. 139, 870 for 5.1.)*

2. $\sqrt{(0-h)^2}$ 3. $\dfrac{2m+2n}{2}$ 4. $|(x+a)-a|$ 5. $\sqrt{r^2+r^2}$

$\triangle PQR$ has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle. *(Review p. 217 for 5.1, 5.4.)*

6. $P(2, 0)$, $Q(6, 6)$, and $R(12, 2)$ 7. $P(2, 3)$, $Q(4, 7)$, and $R(11, 3)$

Ray *AD* bisects $\angle BAC$ and point *E* bisects $\overline{CB}$. Find the measurement. *(Review pp. 15, 24, 217 for 5.2, 5.3, 5.5.)*

8. *CE*   9. $m\angle BAC$ 10. $m\angle ACB$
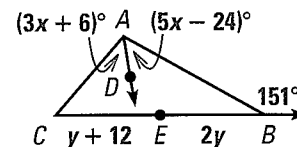
Solve. *(Review pp. 287, 882 for 5.3, 5.5.)*

11. $x^2 + 24^2 = 26^2$ 12. $48 + x^2 = 60$ 13. $43 > x + 35$

@HomeTutor Prerequisite skills practice at classzone.com

In Chapter 5, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 343. You will also use the key vocabulary listed below.

Big Ideas

1 Using properties of special segments in triangles
2 Using triangle inequalities to determine what triangles are possible
3 Extending methods for justifying and proving relationships

KEY VOCABULARY

- midsegment of a triangle, *p. 295*
- coordinate proof, *p. 296*
- perpendicular bisector, *p. 303*
- equidistant, *p. 303*
- point of concurrency, *p. 305*
- circumcenter, *p. 306*

- incenter, *p. 312*
- median of a triangle, *p. 319*
- centroid, *p. 319*
- altitude of a triangle, *p. 320*
- orthocenter, *p. 321*
- indirect proof, *p. 337*

Why?

You can use triangle relationships to find and compare angle measures and distances. For example, if two sides of a triangle represent travel along two roads, then the third side represents the distance back to the starting point.

Animated Geometry

The animation illustrated below for Example 2 on page 336 helps you answer this question: After taking different routes, which group of bikers is farther from the camp?

Two groups of bikers head out from the same point and use different routes.

A diagram of the bikers' travel is shown below. The distances biked and the distances back to start form two triangles, each with a 2 mile side and a 1.2 mile side.

Enter values for *x* and *y*. Predict which bikers are farther from the start.

Animated **Geometry** at classzone.com

Other animations for Chapter 5: pages 296, 304, 312, 321, and 330

5.1 Investigate Segments in Triangles
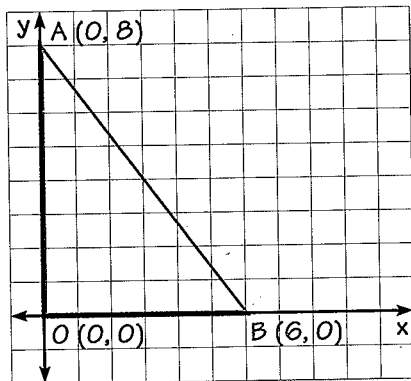
MATERIALS · graph paper · ruler · pencil

QUESTION How are the midsegments of a triangle related to the sides of the triangle?

A *midsegment* of a triangle connects the midpoints of two sides of a triangle.
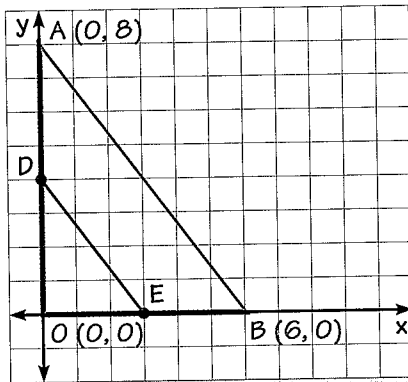
EXPLORE Draw and find a midsegment

STEP 1 *Draw a right triangle*
Draw a right triangle with legs on the *x*-axis and the *y*-axis. Use vertices $A(0, 8)$, $B(6, 0)$, and $O(0, 0)$ as Case 1.

STEP 2 *Draw the midsegment*
Find the midpoints of $\overline{OA}$ and $\overline{OB}$. Plot the midpoints and label them *D* and *E*. Connect them to create the midsegment $\overline{DE}$.

STEP 3 *Make a table*
Draw the Case 2 triangle below. Copy and complete the table.

	Case 1	Case 2
O	(0, 0)	(0, 0)
A	(0, 8)	(0, 11)
B	(6, 0)	(5, 0)
D	?	?
E	?	?
Slope of $\overline{AB}$	?	?
Slope of $\overline{DE}$	?	?
Length of $\overline{AB}$	?	?
Length of $\overline{DE}$	?	?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Choose two other right triangles with legs on the axes. Add these triangles as Cases 3 and 4 to your table.

2. Expand your table in Step 3 for Case 5 with $A(0, n)$, $B(k, 0)$, and $O(0, 0)$.

3. Expand your table in Step 3 for Case 6 with $A(0, 2n)$, $B(2k, 0)$, and $O(0, 0)$.

4. What do you notice about the slopes of $\overline{AB}$ and $\overline{DE}$? What do you notice about the lengths of $\overline{AB}$ and $\overline{DE}$?

5. In each case, is the midsegment $\overline{DE}$ parallel to $\overline{AB}$? *Explain.*

6. Are your observations true for the midsegment created by connecting the midpoints of $\overline{OA}$ and $\overline{AB}$? What about the midsegment connecting the midpoints of $\overline{AB}$ and $\overline{OB}$?

7. Make a conjecture about the relationship between a midsegment and a side of the triangle. Test your conjecture using an acute triangle.
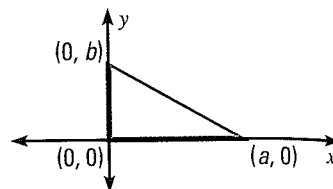
5.1 EXERCISES

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In $\triangle ABC$, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$. $\overline{DE}$ is a __?__ of $\triangle ABC$.

2. ★ **WRITING** *Explain* why it is convenient to place a right triangle on the grid as shown when writing a coordinate proof. How might you want to relabel the coordinates of the vertices if the proof involves midpoints?
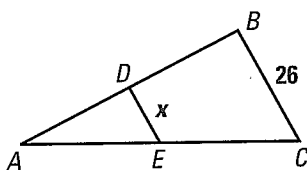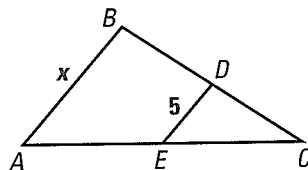
EXAMPLES 1 and 2
on pp. 295–296 for Exs. 3–11

FINDING LENGTHS $\overline{DE}$ is a midsegment of $\triangle ABC$. Find the value of *x*.

3.

4.

5.
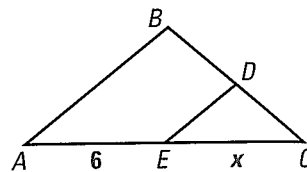
USING THE MIDSEGMENT THEOREM In $\triangle XYZ$, $\overline{XJ} \cong \overline{JY}$, $\overline{YL} \cong \overline{LZ}$, and $\overline{XK} \cong \overline{KZ}$. Copy and complete the statement.

6. $\overline{JK} \parallel$ __?__

7. $\overline{JL} \parallel$ __?__

8. $\overline{XY} \parallel$ __?__

9. $\overline{YJ} \cong$ __?__ $\cong$ __?__

10. $\overline{JL} \cong$ __?__ $\cong$ __?__

11. $\overline{JK} \cong$ __?__ $\cong$ __?__
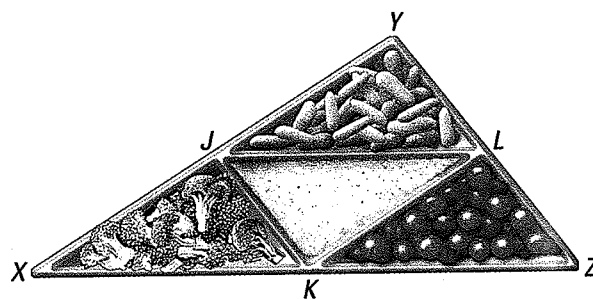
EXAMPLE 3
on p. 296 for Exs. 12–19

PLACING FIGURES Place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex.

12. Right triangle: leg lengths are 3 units and 2 units

13. Isosceles right triangle: leg length is 7 units

14. Square: side length is 3 units

15. Scalene triangle: one side length is 2*m*

16. Rectangle: length is *a* and width is *b*

17. Square: side length is *s*

18. Isosceles right triangle: leg length is *p*

19. Right triangle: leg lengths are *r* and *s*

EXAMPLES 4 and 5
on p. 297 for Exs. 20–23

20. **COMPARING METHODS** Find the length of the hypotenuse in Exercise 19. Then place the triangle another way and use the new coordinates to find the length of the hypotenuse. Do you get the same result?
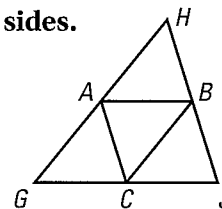
APPLYING VARIABLE COORDINATES Sketch $\triangle ABC$. Find the length and the slope of each side. Then find the coordinates of each midpoint. Is $\triangle ABC$ a right triangle? Is it isosceles? *Explain.* (Assume all variables are positive, $p \neq q$, and $m \neq n$.)

21. $A(0, 0)$, $B(p, q)$, $C(2p, 0)$
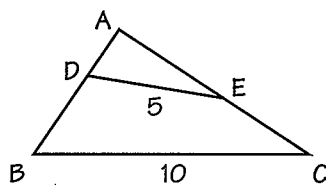
22. $A(0, 0)$, $B(h, h)$, $C(2h, 0)$

23. $A(0, n)$, $B(m, n)$, $C(m, 0)$

XV ALGEBRA Use $\triangle GHJ$, where A, B, and C are midpoints of the sides.

24. If $AB = 3x + 8$ and $GJ = 2x + 24$, what is AB?

25. If $AC = 3y - 5$ and $HJ = 4y + 2$, what is HB?

26. If $GH = 7z - 1$ and $BC = 4z - 3$, what is GH?

**27. ERROR ANALYSIS** *Explain* why the conclusion is incorrect.

$DE = \frac{1}{2}BC$, so by the Midsegment Theorem $\overline{AD} \cong \overline{DB}$ and $\overline{AE} \cong \overline{EC}$.
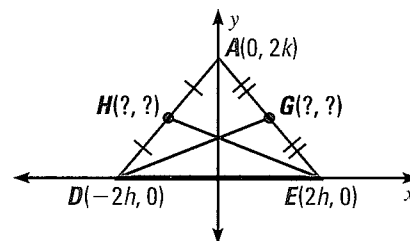
28. FINDING PERIMETER The midpoints of the three sides of a triangle are $P(2, 0)$, $Q(7, 12)$, and $R(16, 0)$. Find the length of each midsegment and the perimeter of $\triangle PQR$. Then find the perimeter of the original triangle.

APPLYING VARIABLE COORDINATES Find the coordinates of the red point(s) in the figure. Then show that the given statement is true.

29. $\triangle OPQ \cong \triangle RSQ$
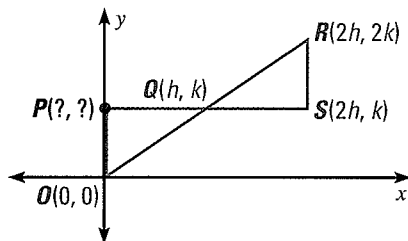
30. slope of $\overline{HE} = -$(slope of $\overline{DG}$)

31. ★ MULTIPLE CHOICE A rectangle with side lengths $3h$ and k has a vertex at $(-h, k)$. Which point *cannot* be a vertex of the rectangle?

A (h, k) **B** $(-h, 0)$ **C** $(2h, 0)$ **D** $(2h, k)$

32. RECONSTRUCTING A TRIANGLE The points $T(2, 1)$, $U(4, 5)$, and $V(7, 4)$ are the midpoints of the sides of a triangle. Graph the three midsegments. Then show how to use your graph and the properties of midsegments to draw the original triangle. Give the coordinates of each vertex.

33. 3-D FIGURES Points A, B, C, and D are the vertices of a *tetrahedron* (a solid bounded by four triangles). $\overline{EF}$ is a midsegment of $\triangle ABC$, $\overline{GE}$ is a midsegment of $\triangle ABD$, and $\overline{FG}$ is a midsegment of $\triangle ACD$.

Show that Area of $\triangle EFG = \frac{1}{4} \cdot$ Area of $\triangle BCD$.
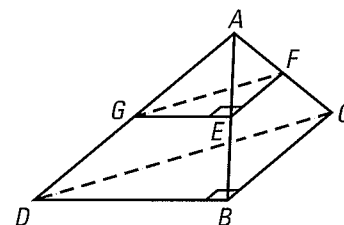
34. CHALLENGE In $\triangle PQR$, the midpoint of $\overline{PQ}$ is $K(4, 12)$, the midpoint of $\overline{QR}$ is $L(5, 15)$, and the midpoint of $\overline{PR}$ is $M(6.4, 10.8)$. Show how to find the vertices of $\triangle PQR$. *Compare* your work for this exercise with your work for Exercise 32. How were your methods different?

35. FLOODLIGHTS A floodlight on the edge of the stage shines upward onto the backdrop as shown. Constance is 5 feet tall. She stands halfway between the light and the backdrop, and the top of her head is at the midpoint of $\overline{AC}$. The edge of the light just reaches the top of her head. How tall is her shadow?
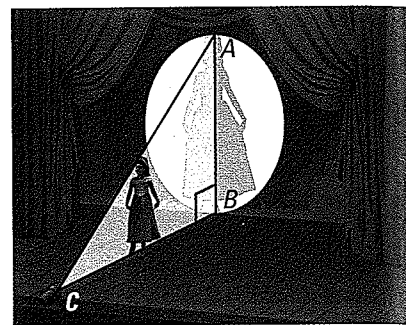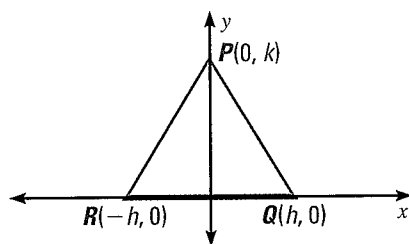
@HomeTutor for problem solving help at classzone.com

EXAMPLE 5

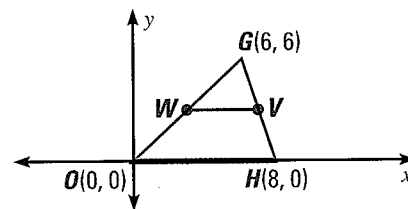on p. 297
for Exs. 36–37

COORDINATE PROOF Write a coordinate proof.

36. GIVEN ▶ $P(0, k)$, $Q(h, 0)$, $R(-h, 0)$

PROVE ▶ $\triangle PQR$ is isosceles.

37. GIVEN ▶ $O(0, 0)$, $G(6, 6)$, $H(8, 0)$, $\overline{WV}$ is a midsegment.

PROVE ▶ $\overline{WV} \parallel \overline{OH}$ and $WV = \frac{1}{2}OH$

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38. CARPENTRY In the set of shelves shown, the third shelf, labeled $\overline{CD}$, is closer to the bottom shelf, $\overline{EF}$, than midsegment $\overline{AB}$ is. If $\overline{EF}$ is 8 feet long, is it possible for $\overline{CD}$ to be 3 feet long? 4 feet long? 6 feet long? 8 feet long? *Explain.*
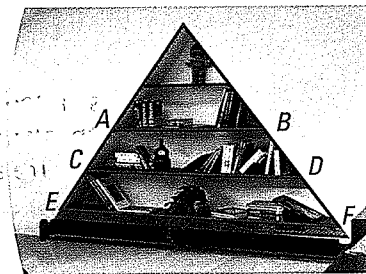
39. ★ SHORT RESPONSE Use the information in the diagram at the right. What is the length of side $\overline{AC}$ of $\triangle ABC$? *Explain* your reasoning.
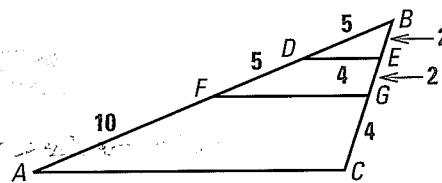
40. PLANNING FOR PROOF Copy and complete the plan for proof.

GIVEN ▶ $\overline{ST}$, $\overline{TU}$, and $\overline{SU}$ are midsegments of $\triangle PQR$.

PROVE ▶ $\triangle PST \cong \triangle SQU$

Use _?_ to show that $\overline{PS} \cong \overline{SQ}$. Use _?_ to show that $\angle QSU \cong \angle SPT$. Use _?_ to show that $\angle\,\underline{\,?\,} \cong \angle\,\underline{\,?\,}$. Use _?_ to show that $\triangle PST \cong \triangle SQU$.
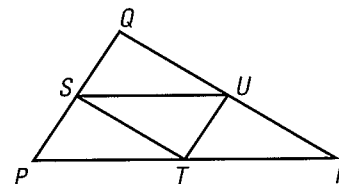
41. PROVING THEOREM 5.1 Use the figure in Example 5. Draw the midpoint F of $\overline{OC}$. Prove that $\overline{DF}$ is parallel to $\overline{BC}$ and $DF = \frac{1}{2}BC$.

○ **= WORKED-OUT SOLUTIONS**
on p. WS1

★ **= STANDARDIZED TEST PRACTICE**

42. COORDINATE PROOF Write a coordinate proof.

GIVEN ▶ $\triangle ABD$ is a right triangle, with the right angle at vertex A.
Point C is the midpoint of hypotenuse BD.

**PROVE** ▶ Point C is the same distance from each vertex of $\triangle ABD$.

43. MULTI-STEP PROBLEM To create the design below, shade the triangle formed by the three midsegments of a triangle. Then repeat the process for each unshaded triangle. Let the perimeter of the original triangle be 1.

Stage 0   Stage 1   Stage 2   Stage 3
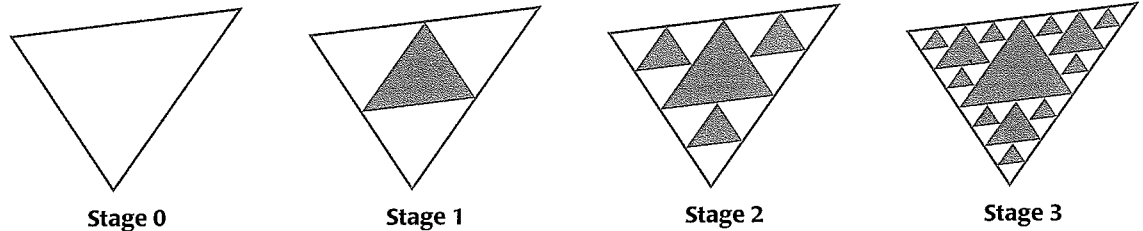
a. What is the perimeter of the triangle that is shaded in Stage 1?

b. What is the total perimeter of all the shaded triangles in Stage 2?

c. What is the total perimeter of all the shaded triangles in Stage 3?

RIGHT ISOSCELES TRIANGLES In Exercises 44 and 45, write a coordinate proof.

44. Any right isosceles triangle can be subdivided into a pair of congruent right isosceles triangles. (*Hint:* Draw the segment from the right angle to the midpoint of the hypotenuse.)

45. Any two congruent right isosceles triangles can be combined to form a single right isosceles triangle.

46. CHALLENGE XY is a midsegment of $\triangle LMN$. Suppose $\overline{DE}$ is called a "quarter-segment" of $\triangle LMN$. What do you think an "eighth-segment" would be? Make a conjecture about the properties of a quarter-segment and of an eighth-segment. Use variable coordinates to verify your conjectures.
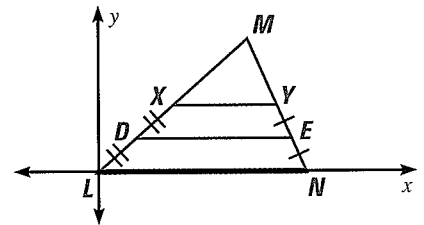
MIXED REVIEW

Line ℓ bisects the segment. Find LN. *(p. 15)*

47.

$3x - 1$ $2x + 9$

L N P

48.

$6x - 10$ $4x$

L M N

49.

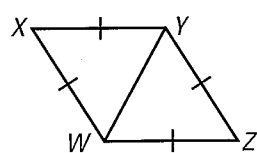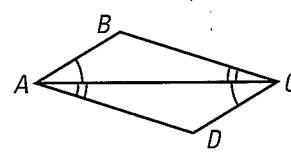$4x + 22$ $7x + 1$

K L N
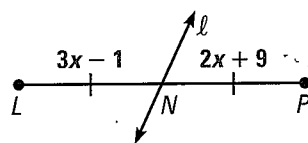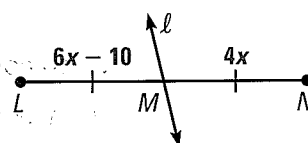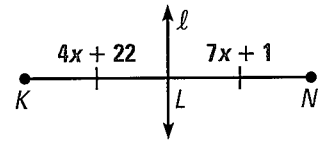
State which postulate or theorem you can use to prove that the triangles are congruent. Then write a congruence statement. *(pp. 225, 249)*
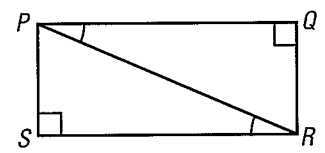
50. X —— Y
W —— Z

51. B
A C
D

52. P —— Q
S —— R

EXTRA PRACTICE for Lesson 5.1, p. 904   🔷 **ONLINE QUIZ** at classzone.com   **301**
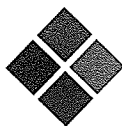
Using ALTERNATIVE METHODS

## Another Way to Solve Example 4, page 297

MULTIPLE REPRESENTATIONS When you write a coordinate proof, you often have several options for how to place the figure in the coordinate plane and how to assign variables.

PROBLEM

Place an isosceles right triangle in a coordinate plane. Then find the length of the hypotenuse and the coordinates of its midpoint M.

METHOD

Placing Hypotenuse on an Axis Place the triangle with point C at $(0, h)$ on the y-axis and the hypotenuse $\overline{AB}$ on the x-axis. To make $\angle ACB$ be a right angle, position A and B so that legs $\overline{CA}$ and $\overline{CB}$ have slopes of 1 and -1.

Slope is 1. Slope is -1.

$C(0, h)$

$A(-h, 0)$ $B(h, 0)$
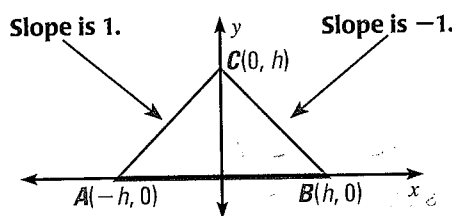
Length of hypotenuse $= 2h$

$$M = \left(\frac{-h + h}{2}, \frac{0 + 0}{2}\right) = (0, 0)$$

PRACTICE

1. **VERIFYING TRIANGLE PROPERTIES** Verify that $\angle C$ above is a right angle. Verify that $\triangle ABC$ is isosceles by showing $AC = BC$.

2. **MULTIPLES OF 2** Find the midpoint and length of each side using the placement below. What is the advantage of using $2h$ instead of h for the leg lengths?

$D(0, 2h)$

$O(0, 0)$ $E(2h, 0)$ x
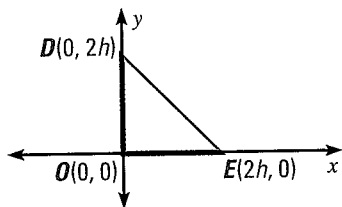
3. **OTHER ALTERNATIVES** Graph $\triangle JKL$ and verify that it is an isosceles right triangle. Then find the length and midpoint of $\overline{JK}$.

   a. $J(0, 0)$, $K(h, h)$, $L(h, 0)$

   b. $J(-2h, 0)$, $K(2h, 0)$, $L(0, 2h)$

4. **CHOOSE** Suppose you need to place a right isosceles triangle on a coordinate grid and assign variable coordinates. You know you will need to find all three side lengths and all three midpoints. How would you place the triangle? *Explain* your reasoning.

5. **RECTANGLES** Place rectangle $PQRS$ with length m and width n in the coordinate plane. Draw $\overline{PR}$ and $\overline{QS}$ connecting opposite corners of the rectangle. Then use coordinates to show that $\overline{PR} \cong \overline{QS}$.

6. **PARK** A square park has paths as shown. Use coordinates to determine whether a snack cart at point N is the same distance from each corner.

N

$2s$
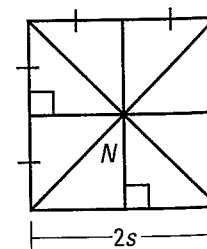
READ VOCABULARY

The prefix *circum-* means "around" or "about" as in *circumference* (distance around a circle).
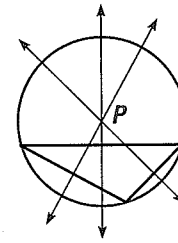
CIRCUMCENTER The point of concurrency of the three perpendicular bisectors of a triangle is called the **circumcenter** of the triangle. The circumcenter *P* is equidistant from the three vertices, so *P* is the center of a circle that passes through all three vertices.
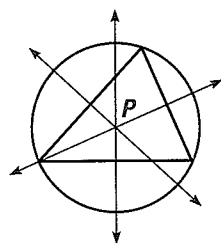
Acute triangle
P is inside triangle.
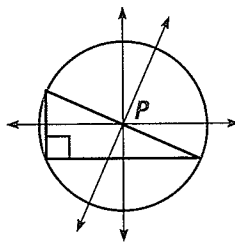
Right triangle
P is on triangle.

Obtuse triangle
P is outside triangle.

As shown above, the location of *P* depends on the type of triangle. The circle with the center *P* is said to be *circumscribed* about the triangle.

5.2 EXERCISES

SKILL PRACTICE

1. **VOCABULARY** Suppose you draw a circle with a compass. You choose three points on the circle to use as the vertices of a triangle. Copy and complete: The center of the circle is also the __?__ of the triangle.

2. ★ **WRITING** Consider $\overline{AB}$. How can you *describe* the set of all points in a plane that are equidistant from *A* and *B*?
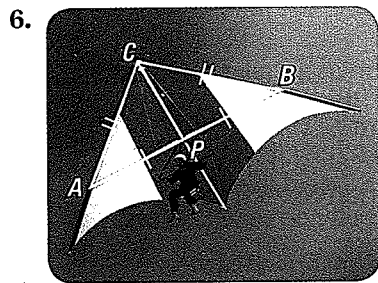
EXAMPLES 1 and 2
on pp. 303–304
for Exs. 3–15

ⓧⒶ ALGEBRA Find the length of $\overline{AB}$.

3.

$5x$ $4x + 3$

4.

$3x - 6$

$x + 18$

5.

$9x + 1$ $7x + 13$
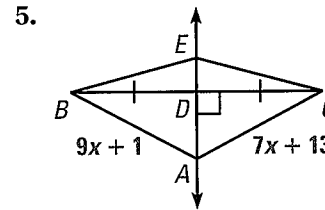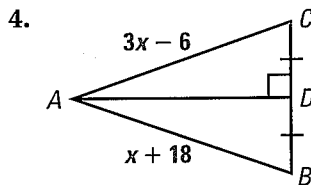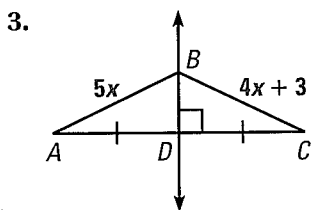
REASONING Tell whether the information in the diagram allows you to conclude that *C* is on the perpendicular bisector of $\overline{AB}$.

6.

7.

8.
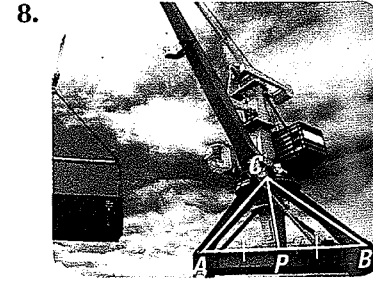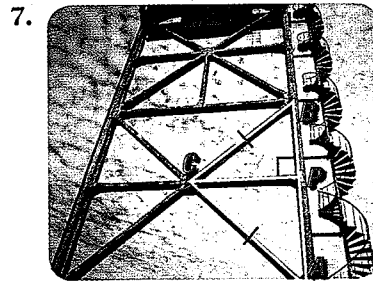
9. ★ **MULTIPLE CHOICE** Point P is inside $\triangle ABC$ and is equidistant from points A and B. On which of the following segments must P be located?
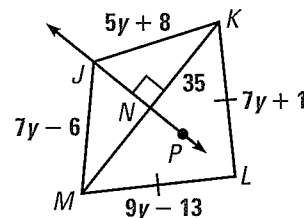
    (A) $\overline{AB}$ (B) The perpendicular bisector of $\overline{AB}$

    (C) The midsegment opposite $\overline{AB}$ (D) The perpendicular bisector of $\overline{AC}$

10. **ERROR ANALYSIS** *Explain* why the conclusion is not correct given the information in the diagram.

$\overleftrightarrow{AB}$ will pass through C.
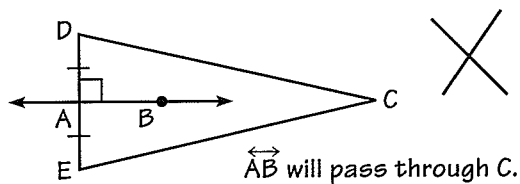
PERPENDICULAR BISECTORS **In Exercises 11–15, use the diagram.** $\overleftrightarrow{JN}$ **is the perpendicular bisector of** $\overline{MK}$.

11. Find NM.   12. Find JK.

13. Find KL.   14. Find ML.

(15.) Is L on $\overleftrightarrow{JP}$? *Explain* your reasoning.

USING CONCURRENCY **In the diagram, the perpendicular bisectors of** $\triangle ABC$ **meet at point** G **and are shown in blue. Find the indicated measure.**

16. Find BG.
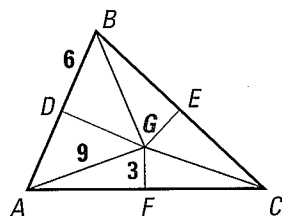
(17.) Find GA.
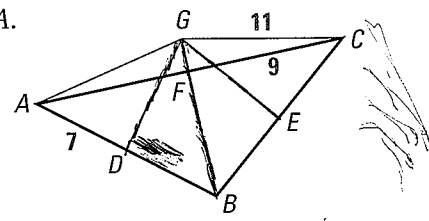
18. **CONSTRUCTING PERPENDICULAR BISECTORS** Use the construction shown on page 33 to construct the bisector of a segment. *Explain* why the bisector you constructed is actually the perpendicular bisector.

19. **CONSTRUCTION** Draw a right triangle. Use a compass and straightedge to find its circumcenter. Use a compass to draw the circumscribed circle.

ANALYZING STATEMENTS **Copy and complete the statement with** *always,* *sometimes,* **or** *never.* *Justify* **your answer.**

20. The circumcenter of a scalene triangle is __?__ inside the triangle.

21. If the perpendicular bisector of one side of a triangle goes through the opposite vertex, then the triangle is __?__ isosceles.

22. The perpendicular bisectors of a triangle intersect at a point that is __?__ equidistant from the midpoints of the sides of the triangle.
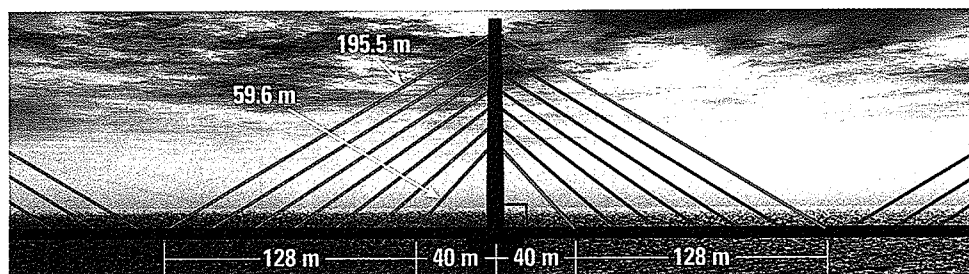
23. **CHALLENGE** Prove the statements in parts (a) – (c).

    GIVEN ▶ Plane P is a perpendicular bisector of $\overline{XZ}$ at Y.

    PROVE ▶ a. $\overline{XW} \cong \overline{ZW}$

             b. $\overline{XV} \cong \overline{ZV}$

             c. $\angle VXW \cong \angle VZW$
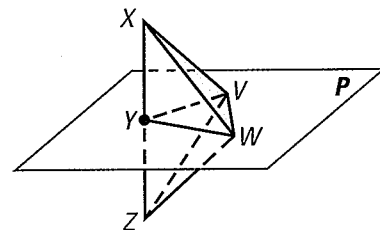
24. BRIDGE A cable-stayed bridge is shown below. Two cable lengths are given. Find the lengths of the blue cables. *Justify* your answer.

195.5 m
59.6 m
128 m — 40 m — 40 m — 128 m

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25. ★ **SHORT RESPONSE** You and two friends plan to walk your dogs together. You want your meeting place to be the same distance from each person's house. *Explain* how you can use the diagram to locate the meeting place.
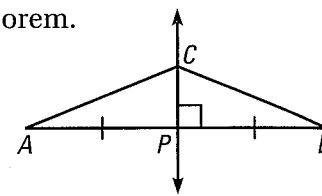
your house   •   Mike's house   •

• Ken's house
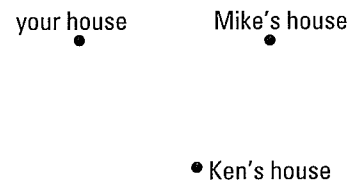
@HomeTutor for problem solving help at classzone.com

26. PROVING THEOREM 5.2 Prove the Perpendicular Bisector Theorem.

GIVEN ▶ $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$.

PROVE ▶ $CA = CB$

Plan for Proof Show that right triangles $\triangle APC$ and $\triangle BPC$ are congruent. Then show that $\overline{CA} \cong \overline{CB}$.

27. PROVING THEOREM 5.3 Prove the converse of Theorem 5.2. (*Hint:* Construct a line through C perpendicular to $\overline{AB}$.)

GIVEN ▶ $CA = CB$

PROVE ▶ C is on the perpendicular bisector of $\overline{AB}$.
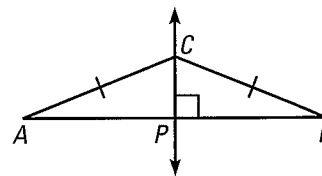
28. ★ **EXTENDED RESPONSE** Archaeologists find three stones. They believe that the stones were once part of a circle of stones with a community firepit at its center. They mark the locations of Stones A, B, and C on a graph where distances are measured in feet.

a. *Explain* how the archaeologists can use a sketch to estimate the center of the circle of stones.

b. Copy the diagram and find the approximate coordinates of the point at which the archaeologists should look for the firepit.
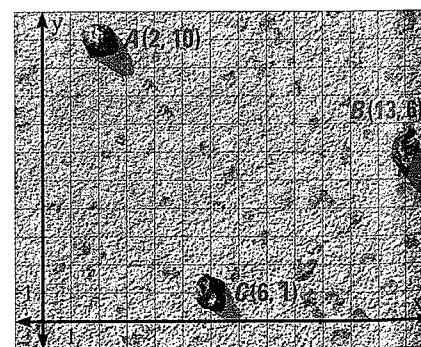

29. TECHNOLOGY Use geometry drawing software to construct $\overline{AB}$. Find the midpoint C. Draw the perpendicular bisector of $\overline{AB}$ through C. Construct a point D along the perpendicular bisector and measure $\overline{DA}$ and $\overline{DB}$. Move D along the perpendicular bisector. What theorem does this construction demonstrate?

○ = **WORKED-OUT SOLUTIONS**
on p. WS1
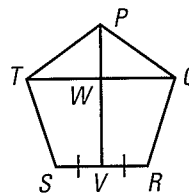
★ = **STANDARDIZED TEST PRACTICE**

30. COORDINATE PROOF Where is the circumcenter located in any right triangle? Write a coordinate proof of this result.

PROOF Use the information in the diagram to prove the given statement.

31. $\overline{AB} \cong \overline{BC}$ if and only if D, E, and B are collinear.
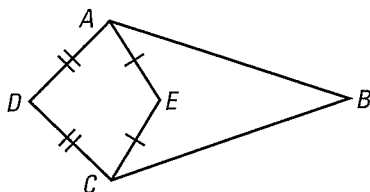
32. $\overleftrightarrow{PV}$ is the perpendicular bisector of $\overline{TQ}$ for regular polygon $PQRST$.

33. CHALLENGE The four towns on the map are building a common high school. They have agreed that the school should be an equal distance from each of the four towns. Is there a single point where they could agree to build the school? If so, find it. If not, *explain* why not. Use a diagram to *explain* your answer.

MIXED REVIEW

Solve the equation. Write your answer in simplest radical form. *(p. 882)*

34. $5^2 + x^2 = 13^2$

35. $x^2 + 15^2 = 17^2$

36. $x^2 + 10 = 38$

Note

PREVIEW

Prepare for Lesson 5.3 in Exs. 37–38.

Ray $\overrightarrow{BD}$ bisects $\angle ABC$. Find the value of x. Then find $m\angle ABC$. *(p. 24)*

37.
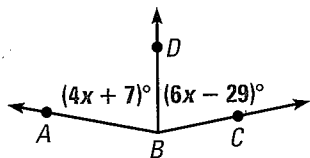
38.
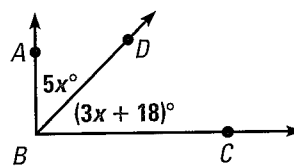
Describe the pattern in the numbers. Write the next number. *(p. 72)*

39. 21, 16, 11, 6, . . .

40. 2, 6, 18, 54, . . .

41. 3, 3, 4, 6, . . .

QUIZ *for Lessons 5.1–5.2*

Find the value of x. Identify the theorem used to find the answer. *(pp. 295, 303)*
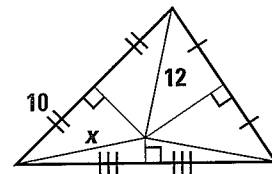
1.
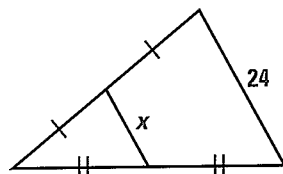
2.
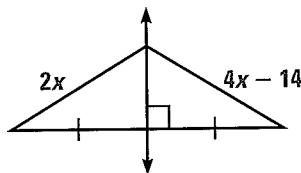
3.

4. Graph the triangle with vertices $R(2a, 0)$, $S(0, 2b)$, and $T(2a, 2b)$, where a and b are positive. Find RT and ST. Then find the slope of $\overline{SR}$ and the coordinates of the midpoint of $\overline{SR}$. *(p. 295)*

EXTRA PRACTICE for Lesson 5.2, p. 904   ⟡ **ONLINE QUIZ** at classzone.com   **309**

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: Point C is in the interior of ∠ABD. If ∠ABC and ∠DBC are congruent, then $\overrightarrow{BC}$ is the __?__ of ∠ABD.
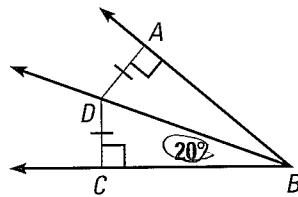
2. ★ **WRITING** How are perpendicular bisectors and angle bisectors of a triangle different? How are they alike?
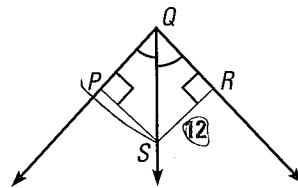
EXAMPLE 1
on p. 310
for Exs. 3–5

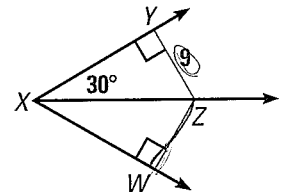**FINDING MEASURES** Use the information in the diagram to find the measure.
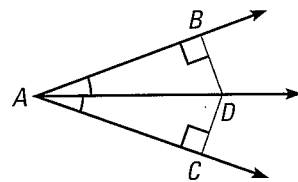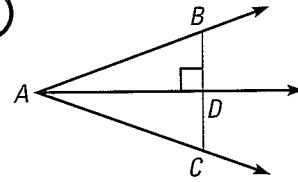
3. Find $m\angle ABD$.

4. Find PS.

5. $m\angle YXW = 60°$. Find WZ.

EXAMPLE 2
on p. 311
for Exs. 6–11

ANGLE BISECTOR THEOREM Is $DB = DC$? *Explain.*
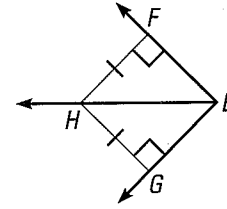
6.

7.

8.
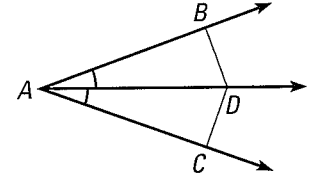
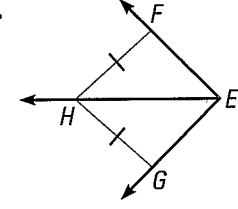**REASONING** Can you conclude that $\overrightarrow{EH}$ bisects ∠FEG? *Explain.*
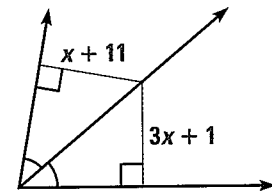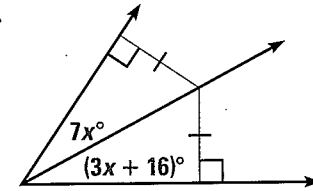
9.
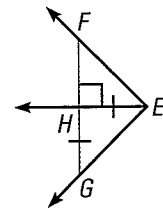
10.

11.

EXAMPLE 3
on p. 311
for Exs. 12–18

ALGEBRA Find the value of x.

12.

13.

14.

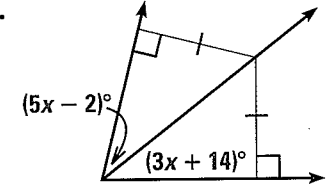$x + 11$ $3x + 1$
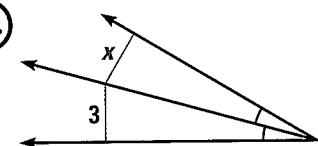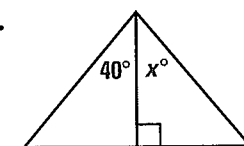
$7x°$ $(3x + 16)°$

$(5x - 2)°$ $(3x + 14)°$

RECOGNIZING MISSING INFORMATION Can you find the value of x? *Explain.*

15.

16.

17.

x 3

$40°$ $x°$

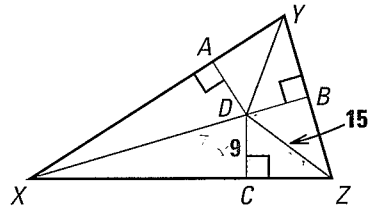7 x
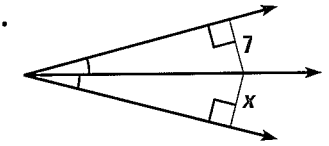
18. ★ **MULTIPLE CHOICE** What is the value of x in the diagram?

(A) 13   (B) 18

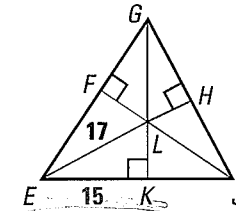(C) 33   (D) Not enough information

$(3x - 9)°$
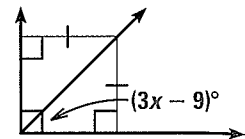
USING INCENTERS Find the indicated measure.

19. Point D is the incenter of $\triangle XYZ$. Find DB.
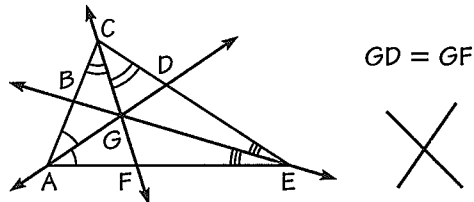
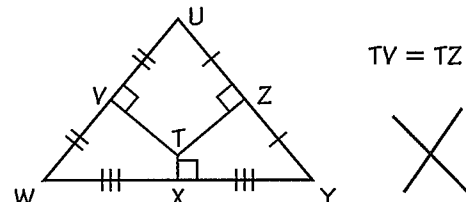**20.** Point L is the incenter of $\triangle EGJ$. Find HL.

ERROR ANALYSIS *Describe* the error in reasoning. Then state a correct conclusion about distances that can be deduced from the diagram.
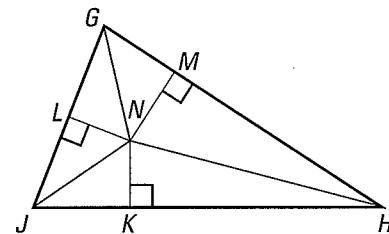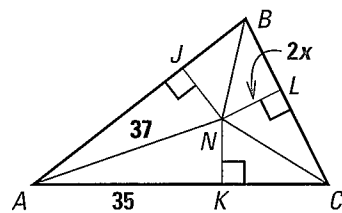
21.

$GD = GF$

22.

$TV = TZ$

23. ★ **MULTIPLE CHOICE** In the diagram, N is the incenter of $\triangle GHJ$. Which statement cannot be deduced from the given information?

(A) $\overline{NM} \cong \overline{NK}$ (B) $\overline{NL} \cong \overline{NM}$

(C) $\overline{NG} \cong \overline{NJ}$ (D) $\overline{HK} \cong \overline{HM}$

(XV) ALGEBRA Find the value of x that makes N the incenter of the triangle.

24.

$2x$

37

35

25.

$14x$

25

24
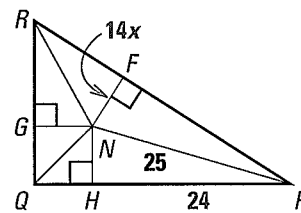
26. **CONSTRUCTION** Use a compass and a straightedge to draw $\triangle ABC$ with incenter D. Label the angle bisectors and the perpendicular segments from D to each of the sides of $\triangle ABC$. Measure each segment. What do you notice? What theorem have you verified for your $\triangle ABC$?

27. **CHALLENGE** Point D is the incenter of $\triangle ABC$. Write an expression for the length x in terms of the three side lengths AB, AC, and BC.

○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**
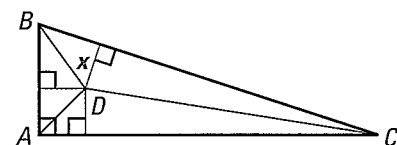
28. FIELD HOCKEY In a field hockey game, the goalkeeper is at point G and a player from the opposing team hits the ball from point B. The goal extends from left goalpost L to right goalpost R. Will the goalkeeper have to move farther to keep the ball from hitting L or R? *Explain.*

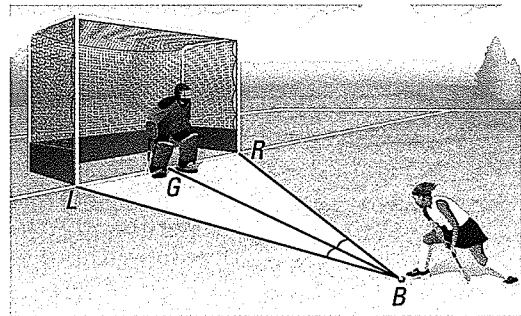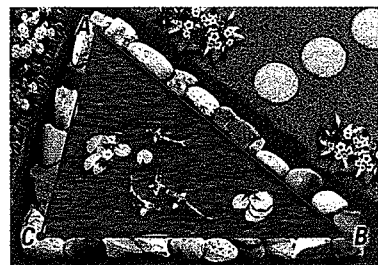*@HomeTutor* for problem solving help at classzone.com

29. KOI POND You are constructing a fountain in a triangular koi pond. You want the fountain to be the same distance from each edge of the pond. Where should you build the fountain? *Explain* your reasoning. Use a sketch to support your answer.

@HomeTutor for problem solving help at classzone.com

30. ★ SHORT RESPONSE What congruence postulate or theorem would you use to prove the Angle Bisector Theorem? to prove the Converse of the Angle Bisector Theorem? Use diagrams to show your reasoning.

31. ★ EXTENDED RESPONSE Suppose you are given a triangle and are asked to draw all of its perpendicular bisectors and angle bisectors.

  a. For what type of triangle would you need the fewest segments? What is the minimum number of segments you would need? *Explain.*

  b. For what type of triangle would you need the most segments? What is the maximum number of segments you would need? *Explain.*

CHOOSING A METHOD In Exercises 32 and 33, tell whether you would use *perpendicular bisectors* or *angle bisectors*. Then solve the problem.

32. BANNER To make a banner, you will cut a triangle from an $8\frac{1}{2}$ inch by 11 inch sheet of white paper and paste a red circle onto it as shown. The circle should just touch each side of the triangle. Use a model to decide whether the circle's radius should be *more* or *less* than $2\frac{1}{2}$ inches. Can you cut the circle from a 5 inch by 5 inch red square? *Explain.*

$8\frac{1}{2}$ in. $4\frac{1}{4}$ in. $4\frac{1}{4}$ in.   11 in.
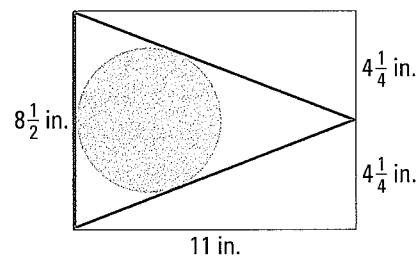
33. CAMP A map of a camp shows a pool at (10, 20), a nature center at (16, 2), and a tennis court at (2, 4). A new circular walking path will connect the three locations. Graph the points and find the approximate center of the circle. Estimate the radius of the circle if each unit on the grid represents 10 yards. Then use the formula $C = 2\pi r$ to estimate the length of the path.

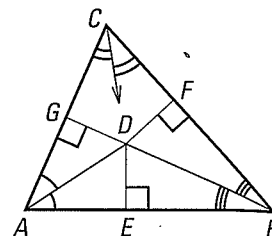**PROVING THEOREMS 5.5 AND 5.6** Use Exercise 30 to prove the theorem.

34. Angle Bisector Theorem   **35.** Converse of the Angle Bisector Theorem

36. PROVING THEOREM 5.7 Write a proof of the Concurrency of Angle Bisectors of a Triangle Theorem.

GIVEN ▶ $\triangle ABC$, $\overline{AD}$ bisects $\angle CAB$, $\overline{BD}$ bisects $\angle CBA$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$, $\overline{DG} \perp \overline{CA}$

PROVE ▶ The angle bisectors intersect at D, which is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

37. CELEBRATION You are planning a graduation party in the triangular courtyard shown. You want to fit as large a circular tent as possible on the site without extending into the walkway.

a. Copy the triangle and show how to place the tent so that it just touches each edge. Then *explain* how you can be sure that there is no place you could fit a larger tent on the site. Use sketches to support your answer.

b. Suppose you want to fit as large a tent as possible while leaving at least one foot of space around the tent. Would you put the center of the tent in the same place as you did in part (a)? *Justify* your answer.
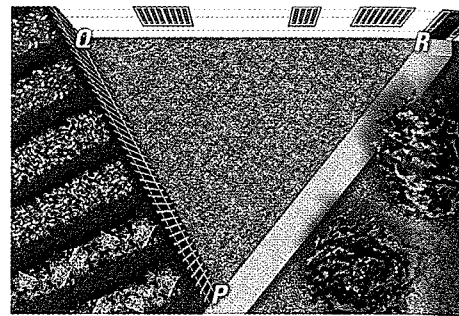
38. CHALLENGE You have seen that there is a point inside any triangle that is equidistant from the three sides of the triangle. Prove that if you extend the sides of the triangle to form lines, you can find three points outside the triangle, each of which is equidistant from those three lines.
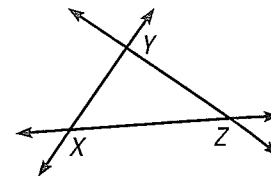
MIXED REVIEW

Find the length of $\overline{AB}$ and the coordinates of the midpoint of $\overline{AB}$. *(p. 15)*

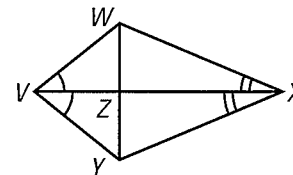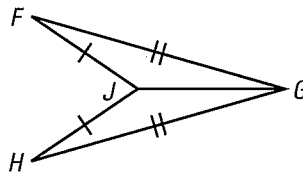**39.** $A(-2, 2)$, $B(-10, 2)$ **40.** $A(0, 6)$, $B(5, 8)$ **41.** $A(-1, -3)$, $B(7, -5)$
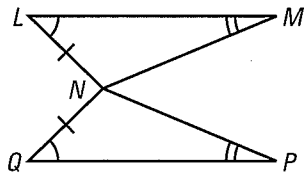
***Explain* how to prove the given statement.** *(p. 256)*

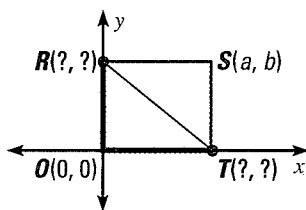**42.** $\angle QNP \cong \angle LNM$ **43.** $\overline{JG}$ bisects $\angle FGH$.   **44.** $\triangle ZWX \cong \triangle ZYX$
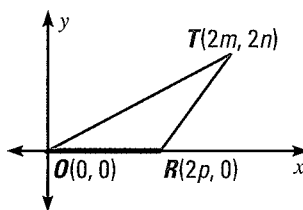
Find the coordinates of the red points in the figure if necessary. Then find OR and the coordinates of the midpoint M of $\overline{RT}$. *(p. 295)*
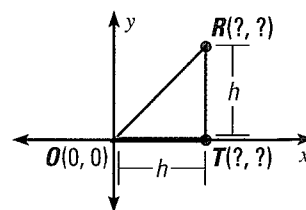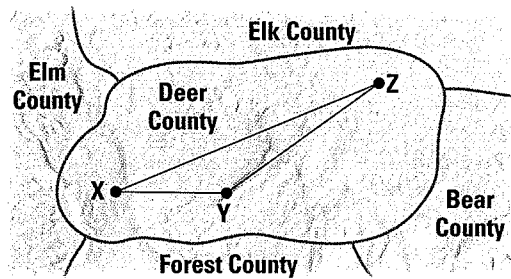
45.   **46.**   **47.**

Lessons 5.1–5.3

1. SHORT RESPONSE A committee has decided to build a park in Deer County. The committee agreed that the park should be equidistant from the three largest cities in the county, which are labeled X, Y, and Z in the diagram. *Explain* why this may not be the best place to build the park. Use a sketch to support your answer.


2. EXTENDED RESPONSE A woodworker is trying to cut as large a wheel as possible from a triangular scrap of wood. The wheel just touches each side of the triangle as shown below.
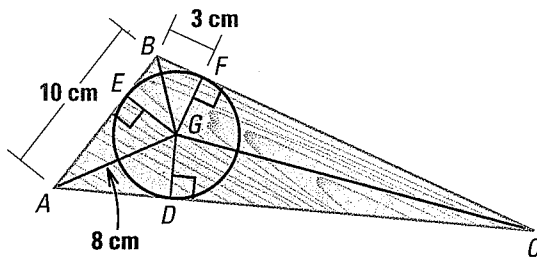

a. Which point of concurrency is the woodworker using for the center of the circle? What type of special segment are $\overline{BG}$, $\overline{CG}$, and $\overline{AG}$?

b. Which postulate or theorem can you use to prove that $\triangle BGF \cong \triangle BGE$?

c. Find the radius of the wheel to the nearest tenth of a centimeter. *Explain* your reasoning.

3. SHORT RESPONSE Graph $\triangle GHJ$ with vertices $G(2, 2)$, $H(6, 8)$, and $J(10, 4)$ and draw its midsegments. Each midsegment is contained in a line. Which of those lines has the greatest y-intercept? Write the equation of that line. *Justify* your answer.

4. GRIDDED ANSWER Three friends are practicing disc golf, in which a flying disk is thrown into a set of targets. Each player is 15 feet from the target. Two players are 24 feet from each other along one edge of the nearby football field. How far is the target from that edge of the football field?
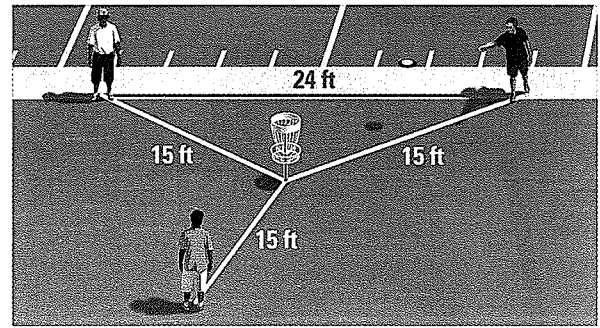
5. MULTI-STEP PROBLEM An artist created a large floor mosaic consisting of eight triangular sections. The grey segments are the midsegments of the two black triangles.
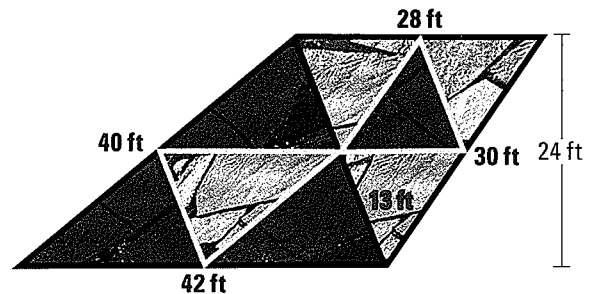
a. The gray and black edging was created using special narrow tiles. What is the total length of all the edging used?

b. What is the total area of the mosaic?

6. OPEN-ENDED If possible, draw a triangle whose incenter and circumcenter are the same point. *Describe* this triangle as specifically as possible.

7. SHORT RESPONSE Points S, T, and U are the midpoints of the sides of $\triangle PQR$. Which angles are congruent to $\angle QST$? *Justify* your answer.
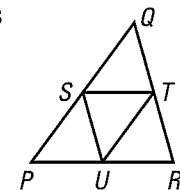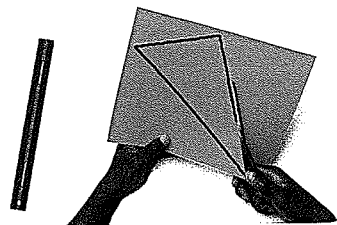
5.4 Intersecting Medians

MATERIALS • cardboard • straightedge • scissors • metric ruler

QUESTION What is the relationship between segments formed by the medians of a triangle?

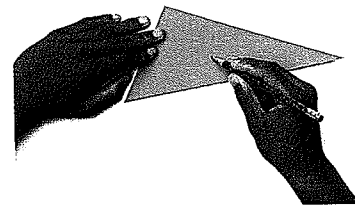**EXPLORE 1** Find the balance point of a triangle

| STEP 1 | STEP 2 | STEP 3 |

Cut out triangle Draw a triangle on a piece of cardboard. Then cut it out.

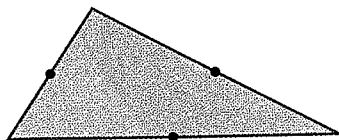***Balance the triangle*** Balance the triangle on the eraser end of a pencil.

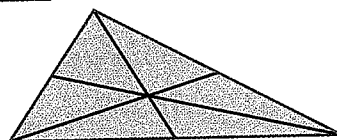***Mark the balance point*** Mark the point on the triangle where it balanced on the pencil.

EXPLORE 2 Construct the medians of a triangle
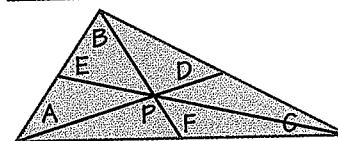
| STEP 1 | STEP 2 | STEP 3 |

Find the midpoint Use a ruler to find the midpoint of each side of the triangle.

Draw medians Draw a segment, or *median*, from each midpoint to the vertex of the opposite angle.

Label points Label your triangle as shown. What do you notice about point P and the balance point in Explore 1?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Copy and complete the table. Measure in millimeters.

Length of segment from vertex to midpoint of opposite side	$AD = ?$	$BF = ?$	$CE = ?$
Length of segment from vertex to *P*	$AP = ?$	$BP = ?$	$CP = ?$
Length of segment from *P* to midpoint	$PD = ?$	$PF = ?$	$PE = ?$

2. How does the length of the segment from a vertex to P compare with the length of the segment from P to the midpoint of the opposite side?

3. How does the length of the segment from a vertex to P compare with the length of the segment from the vertex to the midpoint of the opposite side?

5.4 EXERCISES

SKILL PRACTICE

1. **VOCABULARY** Name the four types of points of concurrency introduced in Lessons 5.2–5.4. When is each type inside the triangle? on the triangle? outside the triangle?

2. ★ **WRITING** *Compare* a perpendicular bisector and an altitude of a triangle. *Compare* a perpendicular bisector and a median of a triangle.
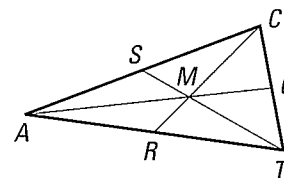
EXAMPLE 1
on p. 319
for Exs. 3–7

FINDING LENGTHS G is the centroid of $\triangle ABC$, $BG = 6$, $AF = 12$, and $AE = 15$. Find the length of the segment.

3. $\overline{FC}$

4. $\overline{BF}$

5. $\overline{AG}$

6. $\overline{GE}$
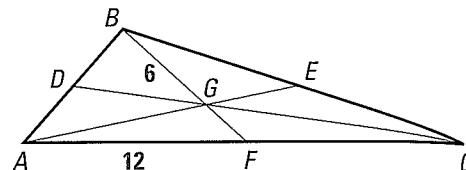
7. ★ **MULTIPLE CHOICE** In the diagram, M is the centroid of $\triangle ACT$, $CM = 36$, $MQ = 30$, and $TS = 56$. What is AM?
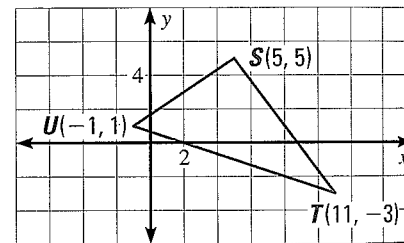
(A) 15

(B) 30

(C) 36

(D) 60

EXAMPLE 2
on p. 320
for Exs. 8–11

8. **FINDING A CENTROID** Use the graph shown.

a. Find the coordinates of P, the midpoint of $\overline{ST}$. Use the median $\overline{UP}$ to find the coordinates of the centroid Q.

b. Find the coordinates of R, the midpoint of $\overline{TU}$. Verify that $SQ = \frac{2}{3}SR$.

GRAPHING CENTROIDS Find the coordinates of the centroid P of $\triangle ABC$.

9. $A(-1, 2)$, $B(5, 6)$, $C(5, -2)$

10. $A(0, 4)$, $B(3, 10)$, $C(6, -2)$

11. ★ **OPEN-ENDED MATH** Draw a large right triangle and find its centroid.

EXAMPLE 3
on p. 321
for Exs. 12–16

12. ★ **OPEN-ENDED MATH** Draw a large obtuse, scalene triangle and find its orthocenter.

IDENTIFYING SEGMENTS Is $\overline{BD}$ a *perpendicular bisector* of $\triangle ABC$? Is $\overline{BD}$ a *median*? an *altitude*?

13.

14.

15.
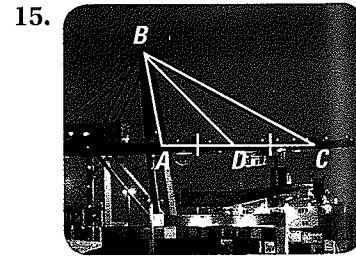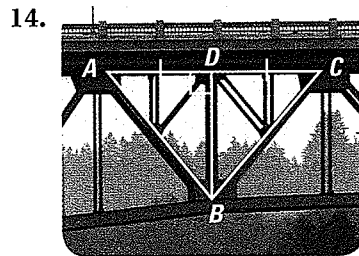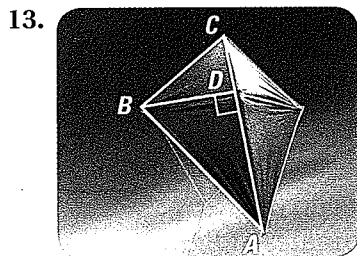
16. ERROR ANALYSIS A student uses the fact that T is a point of concurrency to conclude that $NT = \frac{2}{3}NQ$. *Explain* what is wrong with this reasoning.

$$NT = \frac{2}{3}NQ$$

REASONING Use the diagram shown and the given information to decide whether $\overline{YW}$ is a *perpendicular bisector*, an *angle bisector*, a *median*, or an *altitude* of $\triangle XYZ$. There may be more than one right answer.

17. $\overline{YW} \perp \overline{XZ}$
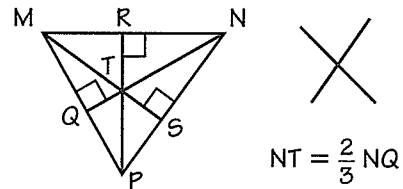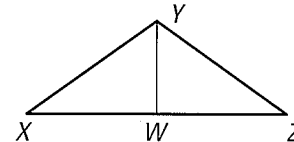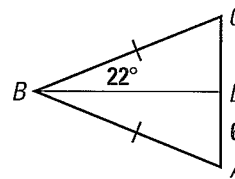
18. $\angle XYW \cong \angle ZYW$

19. $\overline{XW} \cong \overline{ZW}$

20. $\overline{YW} \perp \overline{XZ}$ and $\overline{XW} \cong \overline{ZW}$

21. $\triangle XYW \cong \triangle ZYW$

22. $\overline{YW} \perp \overline{XZ}$ and $\overline{XY} \cong \overline{ZY}$

ISOSCELES TRIANGLES Find the measurements. *Explain* your reasoning.

23. Given that $\overline{DB} \perp \overline{AC}$, find DC and $m\angle ABD$.

24. Given that $AD = DC$, find $m\angle ADB$ and $m\angle ABD$.

RELATING LENGTHS Copy and complete the statement for $\triangle DEF$ with medians $\overline{DH}$, $\overline{EJ}$, and $\overline{FG}$, and centroid K.

25. $EJ = \underline{\ ?\ }\ KJ$

26. $DK = \underline{\ ?\ }\ KH$

27. $FG = \underline{\ ?\ }\ KF$

28. ★ **SHORT RESPONSE** Any isosceles triangle can be placed in the coordinate plane with its base on the x-axis and the opposite vertex on the y-axis as in Guided Practice Exercise 6 on page 321. *Explain* why.

CONSTRUCTION Verify the Concurrency of Altitudes of a Triangle by drawing a triangle of the given type and constructing its altitudes. (*Hint:* To construct an altitude, use the construction in Exercise 25 on page 195.)

29. Equilateral triangle

30. Right scalene triangle

31. Obtuse isosceles triangle

32. **VERIFYING THEOREM 5.8** Use Example 2 on page 320. Verify that Theorem 5.8, the Concurrency of Medians of a Triangle, holds for the median from vertex F and for the median from vertex H.

(XY) ALGEBRA Point D is the centroid of $\triangle ABC$. Use the given information to find the value of x.

33. $BD = 4x + 5$ and $BF = 9x$

34. $GD = 2x - 8$ and $GC = 3x + 3$

35. $AD = 5x$ and $DE = 3x - 2$
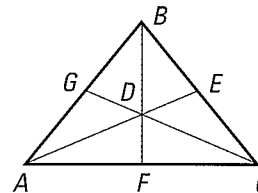
36. **CHALLENGE** $\overline{KM}$ is a median of $\triangle JKL$. Find the areas of $\triangle JKM$ and $\triangle LKM$. Compare the areas. Do you think that the two areas will always compare in this way, regardless of the shape of the triangle? *Explain*.
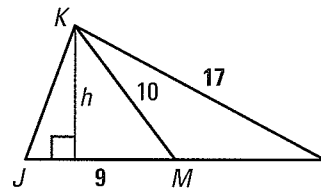
37. MOBILES To complete the mobile, you need to balance the red triangle on the tip of a metal rod. Copy the triangle and decide if you should place the rod at *A* or *B*. *Explain.*
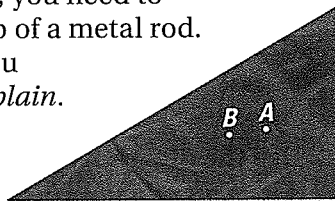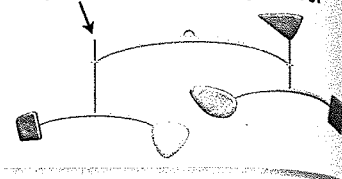
MOBILE INSTRUCTIONS
Step 5: Attach red triangle here.

@HomeTutor for problem solving help at classzone.com

38. DEVELOPING PROOF Show two different ways that you can place an isosceles triangle with base $2n$ and height h on the coordinate plane. Label the coordinates for each vertex.

@HomeTutor for problem solving help at classzone.com

(39.) PAPER AIRPLANE Find the area of the triangular part of the paper airplane wing that is outlined in red. Which special segment of the triangle did you use?

3 in.   9 in.
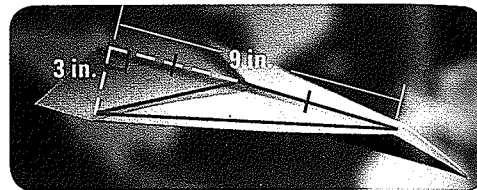
40. ★ SHORT RESPONSE In what type(s) of triangle can a vertex of the triangle be one of the points of concurrency of the triangle? *Explain.*

41. COORDINATE GEOMETRY Graph the lines on the same coordinate plane and find the centroid of the triangle formed by their intersections.

$$y_1 = 3x - 4 \qquad\qquad y_2 = \frac{3}{4}x + 5 \qquad\qquad y_3 = -\frac{3}{2}x - 4$$
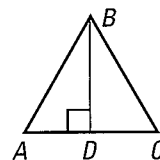
42. PROOF Write proofs using different methods.

GIVEN ▶ $\triangle ABC$ is equilateral.
$\overline{BD}$ is an altitude of $\triangle ABC$.

PROVE ▶ $\overline{BD}$ is also a perpendicular bisector of $\overline{AC}$.

a. Write a proof using congruent triangles.

b. Write a proof using the Perpendicular Postulate on page 148.

43. TECHNOLOGY Use geometry drawing software.

a. Construct a triangle and its medians. Measure the areas of the blue, green, and red triangles.

b. What do you notice about the triangles?

c. If a triangle is of uniform thickness, what can you conclude about the weight of the three interior triangles? How does this support the idea that a triangle will balance on its centroid?
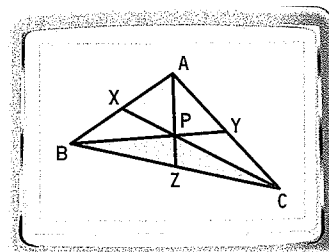
44. ★ EXTENDED RESPONSE Use $P(0, 0)$, $Q(8, 12)$, and $R(14, 0)$.

a. What is the slope of the altitude from R to $\overline{PQ}$?

b. Write an equation for each altitude of $\triangle PQR$. Find the orthocenter by finding the ordered pair that is a solution of the three equations.

c. How would your steps change if you were finding the circumcenter?

◯ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

45. CHALLENGE Prove the results in parts (a) – (c).

GIVEN ▶ $\overrightarrow{LP}$ and $\overline{MQ}$ are medians of scalene $\triangle LMN$. Point R is on $\overrightarrow{LP}$ such that $\overline{LP} \cong \overline{PR}$. Point S is on $\overrightarrow{MQ}$ such that $\overline{MQ} \cong \overline{QS}$.

PROVE ▶ **a.** $\overline{NS} \cong \overline{NR}$

b. $\overline{NS}$ and $\overline{NR}$ are both parallel to $\overline{LM}$.

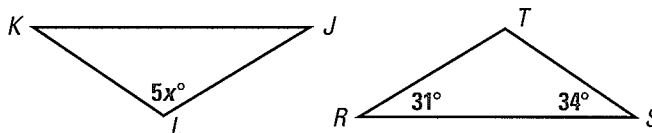**c.** R, N, and S are collinear.

MIXED REVIEW

In Exercises 46–48, write an equation of the line that passes through points A and B. *(p. 180)*

46. $A(0, 7), B(1, 10)$ **47.** $A(4, -8), B(-2, -5)$ **48.** $A(5, -21), B(0, 4)$

49. In the diagram, $\triangle JKL \cong \triangle RST$. Find the value of x. *(p. 225)*
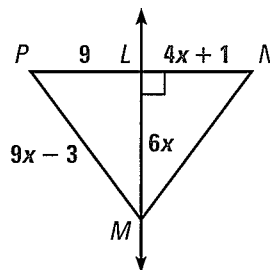
Solve the inequality. *(p. 287)*

50. $2x + 13 < 35$ **51.** $12 > -3x - 6$ **52.** $6x < x + 20$

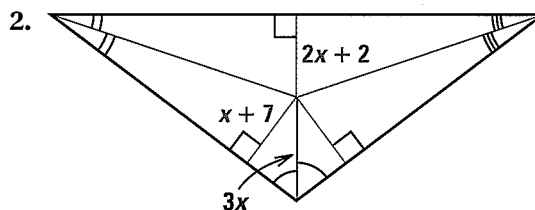**In the diagram, $\overline{LM}$ is the perpendicular bisector of $\overline{PN}$.** *(p. 303)*

53. What segment lengths are equal?

54. What is the value of x?

55. Find MN.

QUIZ for Lessons 5.3–5.4

Find the value of x. Identify the theorem used to find the answer. *(p. 310)*

1.

2.
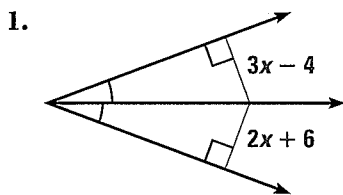
In the figure, P is the centroid of $\triangle XYZ$, $YP = 12$, $LX = 15$, and $LZ = 18$. *(p. 319)*

3. Find the length of $\overline{LY}$.

4. Find the length of $\overline{YN}$.

5. Find the length of $\overline{LP}$.
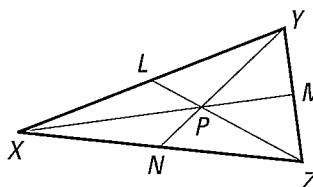
5.4 Investigate Points of Concurrency
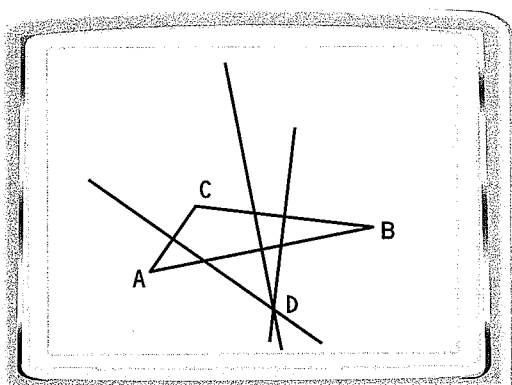
MATERIALS · graphing calculator or computer

QUESTION How are the points of concurrency in a triangle related?

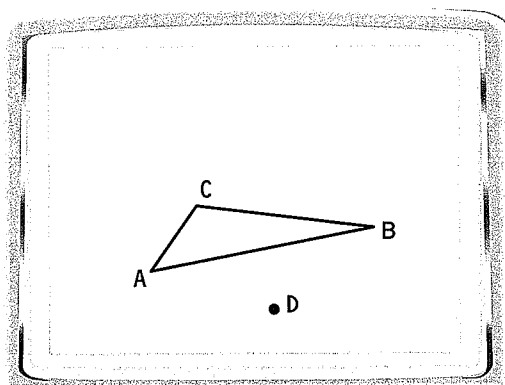You can use geometry drawing software to investigate concurrency.

EXAMPLE 1 Draw the perpendicular bisectors of a triangle

| STEP 1 |

| STEP 2 |

Draw perpendicular bisectors Draw a line perpendicular to each side of a $\triangle ABC$ at the midpoint. Label the point of concurrency D.

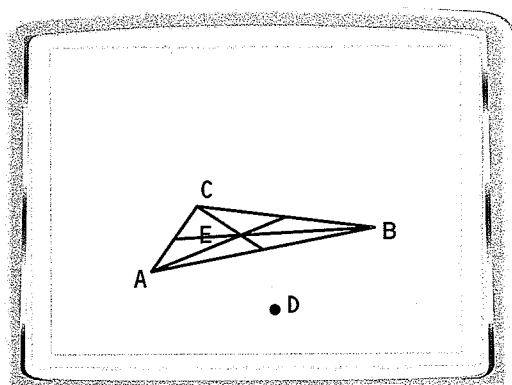***Hide the lines*** Use the *HIDE* feature to hide the perpendicular bisectors. Save as "EXAMPLE1."
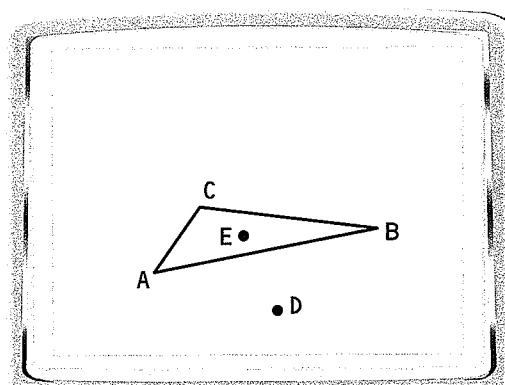
EXAMPLE 2 Draw the medians of the triangle

| STEP 1 |

| STEP 2 |

Draw medians Start with the figure you saved as "EXAMPLE1." Draw the medians of $\triangle ABC$. Label the point of concurrency E.
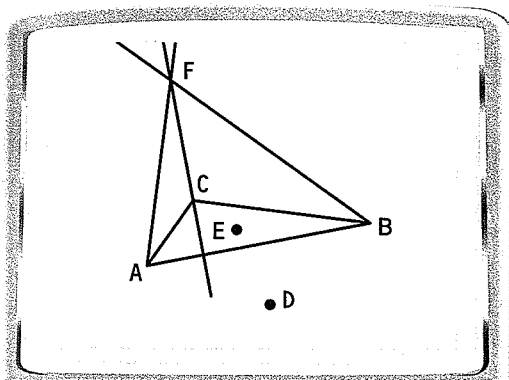
Hide the lines Use the *HIDE* feature to hide the medians. Save as "EXAMPLE2."
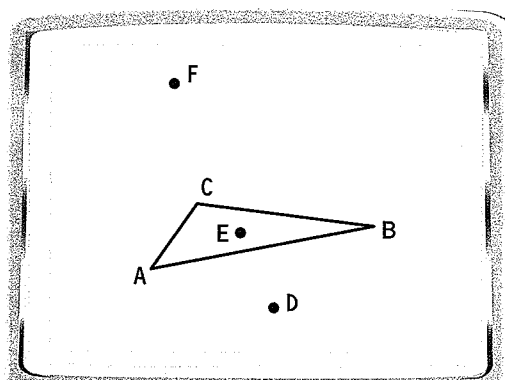
EXAMPLE 3 Draw the altitudes of the triangle

STEP 1

STEP 2

Draw altitudes Start with the figure you saved as "EXAMPLE2." Draw the altitudes of △*ABC*. Label the point of concurrency *F*.

Hide the lines Use the *HIDE* feature to hide the altitudes. Save as "EXAMPLE3."

PRACTICE

1. Try to draw a line through points *D*, *E*, and *F*. Are the points collinear?

2. Try dragging point *A*. Do points *D*, *E*, and *F* remain collinear?

In Exercises 3–5, use the triangle you saved as "EXAMPLE3."

3. Draw the angle bisectors. Label the point of concurrency as point *G*.

4. How does point *G* relate to points *D*, *E*, and *F*?

5. Try dragging point *A*. What do you notice about points *D*, *E*, *F*, and *G*?

DRAW CONCLUSIONS

In 1765, Leonhard Euler (pronounced "oi′-ler") proved that the circumcenter, the centroid, and the orthocenter are all collinear. The line containing these three points is called *Euler's line*. Save the triangle from Exercise 5 as "EULER" and use that for Exercises 6–8.

6. Try moving the triangle's vertices. Can you verify that the same three points lie on Euler's line whatever the shape of the triangle? *Explain.*

7. Notice that some of the four points can be outside of the triangle. Which points lie outside the triangle? Why? What happens when you change the shape of the triangle? Are there any points that never lie outside the triangle? Why?

8. Draw the three midsegments of the triangle. Which, if any, of the points seem contained in the triangle formed by the midsegments? Do those points stay there when the shape of the large triangle is changed?
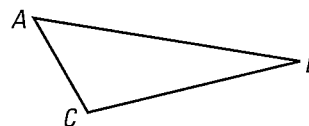
5.5 EXERCISES

HOMEWORK
KEY

◯ = **WORKED-OUT SOLUTIONS**
on p. WS6 for Exs. 9, 17, and 39

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 12, 20, 30, 39, and 45

SKILL PRACTICE

1. **VOCABULARY** Use the diagram at the right. For each angle, name the side that is *opposite* that angle.

2. ★ **WRITING** How can you tell from the angle measures of a triangle which side of the triangle is the longest? the shortest?

EXAMPLE 1
on p. 328
for Exs. 3–5

MEASURING Use a ruler and protractor to draw the given type of triangle. Mark the largest angle and longest side in red and the smallest angle and shortest side in blue. What do you notice?

3. Acute scalene

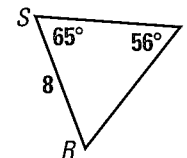4. Right scalene

5. Obtuse isosceles

EXAMPLE 2
on p. 329
for Exs. 6–15

WRITING MEASUREMENTS IN ORDER List the sides and the angles in order from smallest to largest.

6.

7.

8.

9.

10.

11.
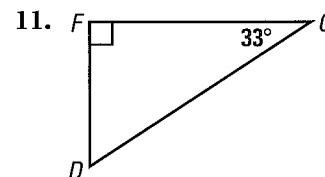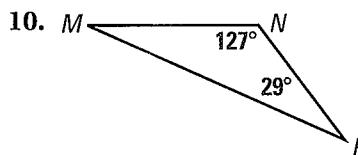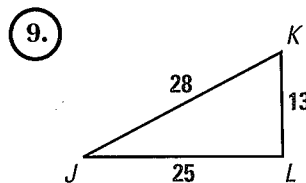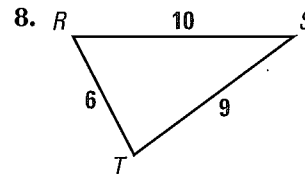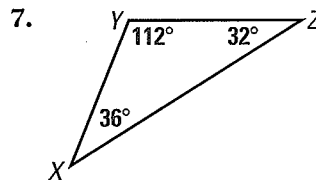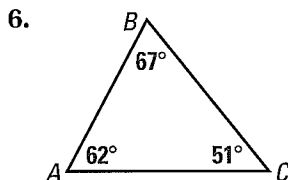
12. ★ **MULTIPLE CHOICE** In △RST, which is a possible side length for ST?

Ⓐ 7   Ⓑ 8

Ⓒ 9   Ⓓ Cannot be determined

DRAWING TRIANGLES Sketch and label the triangle described.

13. Side lengths: about 3 m, 7 m, and 9 m, with longest side on the bottom
    Angle measures: 16°, 41°, and 123°, with smallest angle at the left

14. Side lengths: 37 ft, 35 ft, and 12 ft, with shortest side at the right
    Angle measures: about 71°, about 19°, and 90°, with right angle at the top

15. Side lengths: 11 in., 13 in., and 14 in., with middle-length side at the left
    Two angle measures: about 48° and 71°, with largest angle at the top

EXAMPLE 3
on p. 330
for Exs. 16–26

IDENTIFYING POSSIBLE TRIANGLES Is it possible to construct a triangle with the given side lengths? If not, *explain* why not.

16. 6, 7, 11   17. 3, 6, 9   18. 28, 34, 39   19. 35, 120, 125

20. ★ **MULTIPLE CHOICE** Which group of side lengths can be used to construct a triangle?

    Ⓐ 3 yd, 4 ft, 5 yd   Ⓑ 3 yd, 5 ft, 8 ft

    Ⓒ 11 in., 16 in., 27 in.   Ⓓ 2 ft, 11 in., 12 in.

POSSIBLE SIDE LENGTHS *Describe* the possible lengths of the third side of the triangle given the lengths of the other two sides.

21. 5 inches, 12 inches   **22.** 3 meters, 4 meters   **23.** 12 feet, 18 feet

**24.** 10 yards, 23 yards   **25.** 2 feet, 40 inches   **26.** 25 meters, 25 meters

27. **EXTERIOR ANGLE INEQUALITY** Another triangle inequality relationship is given by the Exterior Angle Inequality Theorem. It states:
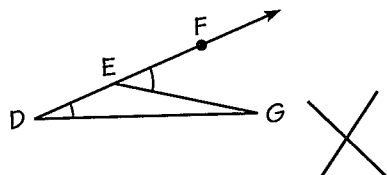
The measure of an exterior angle of a triangle is greater than the measure of either of the nonadjacent interior angles.

Use a relationship from Chapter 4 to *explain* how you know that $m\angle 1 > m\angle A$ and $m\angle 1 > m\angle B$ in $\triangle ABC$ with exterior angle $\angle 1$.
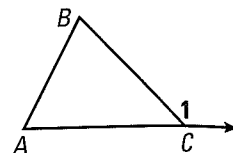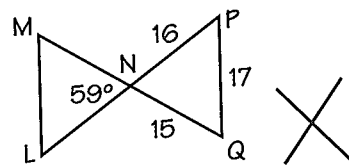

ERROR ANALYSIS Use Theorems 5.10–5.12 and the theorem in Exercise 27 to *explain* why the diagram must be incorrect.

28.

29.

30. ★ **SHORT RESPONSE** *Explain* why the hypotenuse of a right triangle must always be longer than either leg.
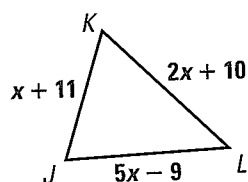
ORDERING MEASURES Is it possible to build a triangle using the given side lengths? If so, list the angles of the triangle in order from least to greatest measure.

31. $PQ = \sqrt{58}$, $QR = 2\sqrt{13}$, $PR = 5\sqrt{2}$ **32.** $ST = \sqrt{29}$, $TU = 2\sqrt{17}$, $SU = 13.9$

⊗ ALGEBRA *Describe* the possible values of *x*.

33.

34.
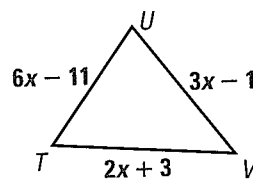
35. **USING SIDE LENGTHS** Use the diagram at the right. Suppose $\overline{XY}$ bisects $\angle WYZ$. List all six angles of $\triangle XYZ$ and $\triangle WXY$ in order from smallest to largest. *Explain* your reasoning.

36. **CHALLENGE** The perimeter of $\triangle HGF$ must be between what two integers? *Explain* your reasoning.

37. TRAY TABLE In the tray table shown, $\overline{PQ} \cong \overline{PR}$ and $QR < PQ$. Write two inequalities about the angles in $\triangle PQR$. What other angle relationship do you know?

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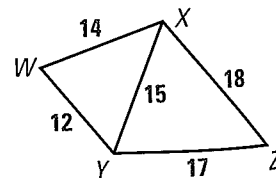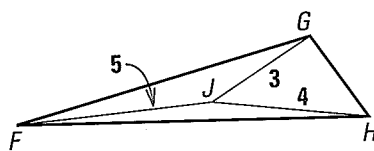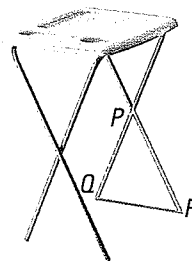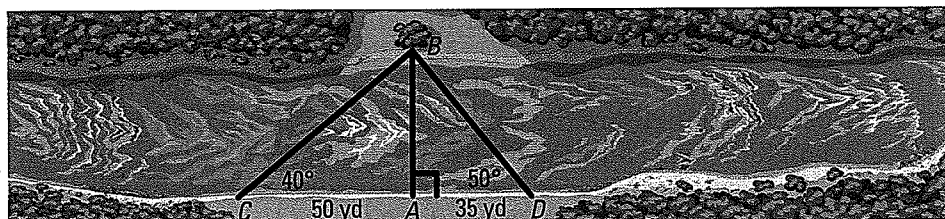
38. INDIRECT MEASUREMENT You can estimate the width of the river at point A by taking several sightings to the tree across the river at point B. The diagram shows the results for locations C and D along the riverbank. Using $\triangle BCA$ and $\triangle BDA$, what can you conclude about AB, the width of the river at point A? What could you do if you wanted a closer estimate?


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(39.) ★ EXTENDED RESPONSE You are planning a vacation to Montana. You want to visit the destinations shown in the map.

a. A brochure states that the distance between Granite Peak and Fort Peck Lake is 1080 kilometers. *Explain* how you know that this distance is a misprint.

b. Could the distance from Granite Peak to Fort Peck Lake be 40 kilometers? *Explain.*

c. Write two inequalities to represent the range of possible distances from Granite Peak to Fort Peck Lake.

d. What can you say about the distance between Granite Peak and Fort Peck Lake if you know that $m\angle 2 < m\angle 1$ and $m\angle 2 < m\angle 3$?
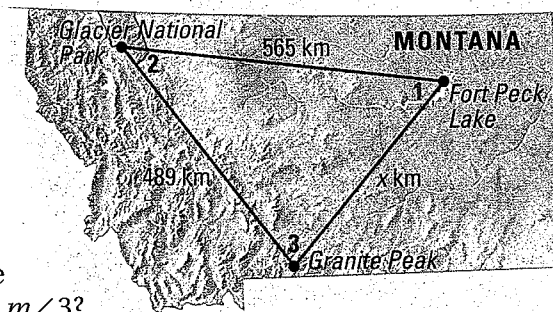
FORMING TRIANGLES In Exercises 40–43, you are given a 24 centimeter piece of string. You want to form a triangle out of the string so that the length of each side is a whole number. Draw figures accurately.

40. Can you decide if three side lengths form a triangle without checking all three inequalities shown for Theorem 5.12? If so, *describe* your shortcut.

41. Draw four possible isosceles triangles and label each side length. Tell whether each of the triangles you formed is *acute, right,* or *obtuse.*

42. Draw three possible scalene triangles and label each side length. Try to form at least one scalene acute triangle and one scalene obtuse triangle.

43. List three combinations of side lengths that will not produce triangles.

44. SIGHTSEEING You get off the Washington, D.C., subway system at the Smithsonian Metro station. First you visit the Museum of Natural History. Then you go to the Air and Space Museum. You record the distances you walk on your map as shown. *Describe* the range of possible distances you might have to walk to get back to the Smithsonian Metro station.
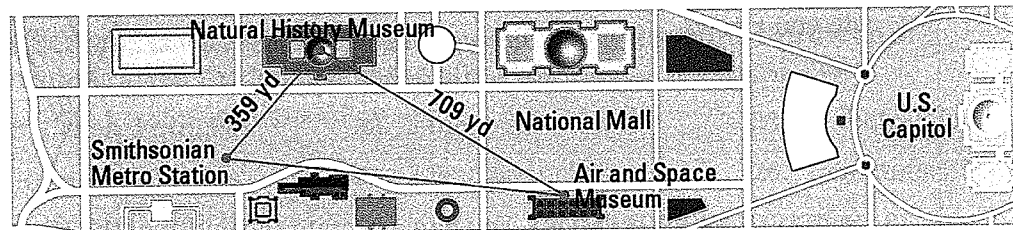
45. ★ SHORT RESPONSE Your house is 2 miles from the library. The library is $\frac{3}{4}$ mile from the grocery store. What do you know about the distance from your house to the grocery store? *Explain*. Include the special case when the three locations are all in a straight line.

46. ISOSCELES TRIANGLES For what combinations of angle measures in an isosceles triangle are the congruent sides shorter than the base of the triangle? longer than the base of the triangle?

47. PROVING THEOREM 5.12 Prove the Triangle Inequality Theorem.

  GIVEN ▶ $\triangle ABC$

  PROVE ▶ (1) $AB + BC > AC$
  (2) $AC + BC > AB$
  (3) $AB + AC > BC$

  Plan for Proof One side, say $\overline{BC}$, is longer than or at least as long as each of the other sides. Then (1) and (2) are true. To prove (3), extend $\overline{AC}$ to D so that $\overline{AB} \cong \overline{AD}$ and use Theorem 5.11 to show that $DC > BC$.

48. CHALLENGE Prove the following statements.

  a. The length of any one median of a triangle is less than half the perimeter of the triangle.

  b. The sum of the lengths of the three medians of a triangle is greater than half the perimeter of the triangle.

MIXED REVIEW

In Exercises 49 and 50, write the if-then form, the converse, the inverse, and the contrapositive of the given statement. *(p. 79)*

49. A redwood is a large tree.   **50.** $5x - 2 = 18$, because $x = 4$.

51. A triangle has vertices $A(22, 21)$, $B(0, 0)$, and $C(22, 2)$. Graph $\triangle ABC$ and classify it by its sides. Then determine if it is a right triangle. *(p. 217)*

Graph figure *LMNP* with vertices $L(-4, 6)$, $M(4, 8)$, $N(2, 2)$, and $P(-4, 0)$. Then draw its image after the transformation. *(p. 272)*

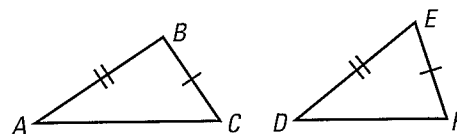**52.** $(x, y) \rightarrow (x + 3, y - 4)$ **53.** $(x, y) \rightarrow (x, -y)$ **54.** $(x, y) \rightarrow (-x, y)$

EXAMPLE 4 **Prove the Converse of the Hinge Theorem**

Write an indirect proof of Theorem 5.14.

GIVEN ▶ $\overline{AB} \cong \overline{DE}$
$\overline{BC} \cong \overline{EF}$
$AC > DF$

PROVE ▶ $m\angle B > m\angle E$

Proof Assume temporarily that $m\angle B \not> m\angle E$. Then, it follows that either $m\angle B = m\angle E$ or $m\angle B < m\angle E$.

Case 1 If $m\angle B = m\angle E$, then $\angle B \cong \angle E$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate and $AC = DF$.

Case 2 If $m\angle B < m\angle E$, then $AC < DF$ by the Hinge Theorem.

Both conclusions contradict the given statement that $AC > DF$. So, the temporary assumption that $m\angle B \not> m\angle E$ cannot be true. This proves that $m\angle B > m\angle E$.

✓ **GUIDED PRACTICE** **for Example 4**

5. Write a temporary assumption you could make to prove the Hinge Theorem indirectly. What two cases does that assumption lead to?

5.6 EXERCISES

HOMEWORK KEY
○ = WORKED-OUT SOLUTIONS on p. WS7 for Exs. 5, 7, and 23
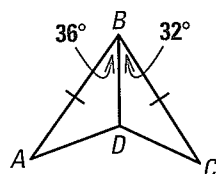★ = STANDARDIZED TEST PRACTICE Exs. 2, 9, 19, and 25

SKILL PRACTICE

1. **VOCABULARY** Why is indirect proof also called *proof by contradiction*?

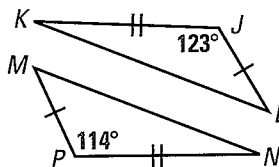2. ★ **WRITING** *Explain* why the name "Hinge Theorem" is used for Theorem 5.13.

EXAMPLE 1 on p. 336 for Exs. 3–10

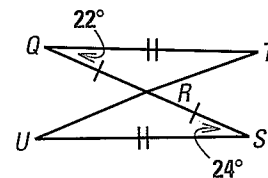**APPLYING THEOREMS** Copy and complete with <, >, or =. *Explain.*
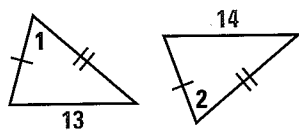
3. AD __?__ CD

4. MN __?__ LK

5. TR __?__ UR

6. $m\angle 1$ __?__ $m\angle 2$

7. $m\angle 1$ __?__ $m\angle 2$

8. $m\angle 1$ __?__ $m\angle 2$

338 Chapter 5 Relationships within Triangles
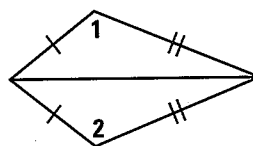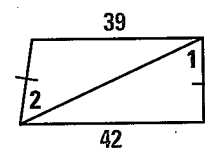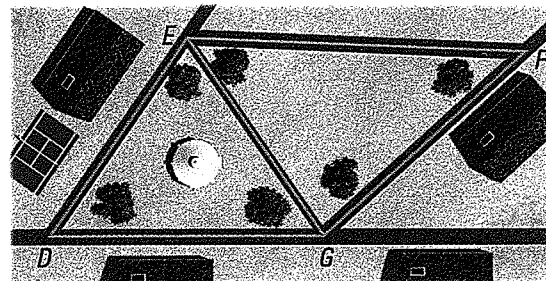
9. ★ MULTIPLE CHOICE Which is a possible measure for $\angle JKM$?

   Ⓐ 20°   Ⓑ 25°

   Ⓒ 30°   Ⓓ Cannot be determined
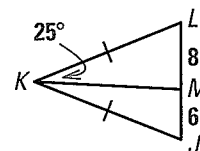
10. USING A DIAGRAM The path from E to F is longer than the path from E to D. The path from G to D is the same length as the path from G to F. What can you conclude about the angles of the paths? *Explain* your reasoning.

STARTING AN INDIRECT PROOF In Exercises 11 and 12, write a temporary assumption you could make to prove the conclusion indirectly.

11. If x and y are odd integers, then xy is odd.

12. In $\triangle ABC$, if $m\angle A = 100°$, then $\angle B$ is not a right angle.

13. REASONING Your study partner is planning to write an indirect proof to show that $\angle A$ is an obtuse angle. She states "Assume temporarily that $\angle A$ is an acute angle." What has your study partner overlooked?
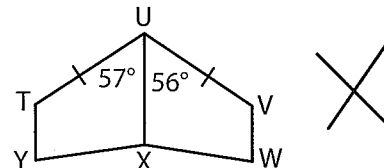
ERROR ANALYSIS *Explain* why the student's reasoning is not correct.

14.
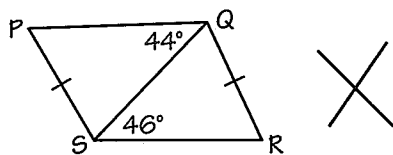
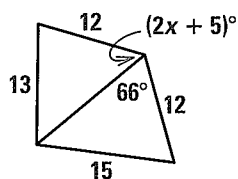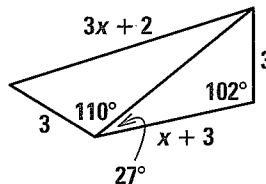By the Hinge Theorem, PQ < SR.

15.

By the Hinge Theorem, XW < XY.

⊗ ALGEBRA Use the Hinge Theorem or its converse and properties of triangles to write and solve an inequality to describe a restriction on the value of x.

16.

17.

18.
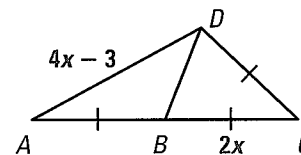
19. ★ SHORT RESPONSE If $\overline{NR}$ is a median of $\triangle NPQ$ and $NQ > NP$, *explain* why $\angle NRQ$ is obtuse.

20. ANGLE BISECTORS In $\triangle EFG$, the bisector of $\angle F$ intersects the bisector of $\angle G$ at point H. *Explain* why $\overline{FG}$ must be longer than $\overline{FH}$ or $\overline{HG}$.

21. CHALLENGE In $\triangle ABC$, the altitudes from B and C meet at D. What is true about $\triangle ABC$ if $m\angle BAC > m\angle BDC$? *Justify* your answer.

22. HIKING Two hikers start at the visitor center. The first hikes 4 miles due west, then turns 40° toward south and hikes 1.8 miles. The second hikes 4 miles due east, then turns 52° toward north and and hikes 1.8 miles. Which hiker is farther from the visitor center? *Explain* how you know.
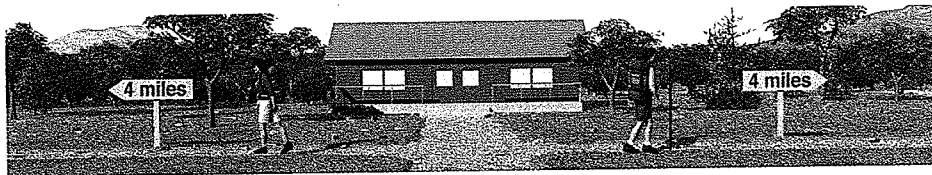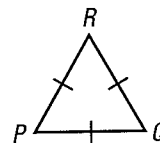
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23. INDIRECT PROOF Arrange statements A–E in order to write an indirect proof of the corollary: If $\triangle PQR$ is *equilateral*, then it is *equiangular*.

GIVEN ▶ $\triangle PQR$ is equilateral.

A. That means that for some pair of vertices, say P and Q, $m\angle P > m\angle Q$.

B. But this contradicts the given statement that $\triangle PQR$ is equilateral.

C. The contradiction shows that the temporary assumption that $\triangle PQR$ is not equiangular is false. This proves that $\triangle PQR$ is equiangular.

D. Then, by Theorem 5.11, you can conclude that $QR > PR$.

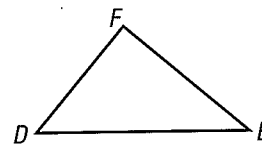**E.** Temporarily assume that $\triangle PQR$ is not equiangular.

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24. PROVING THEOREM 5.11 Write an indirect proof of Theorem 5.11, page 328.

GIVEN ▶ $m\angle D > m\angle E$

PROVE ▶ $EF > DF$

Plan for Proof In Case 1, assume that $EF < DF$. In Case 2, assume that $EF = DF$.

25. ★ EXTENDED RESPONSE A scissors lift can be used to adjust the height of a platform.

   a. Interpret As the mechanism expands, $\overline{KL}$ gets longer. As KL increases, what happens to $m\angle LNK$? to $m\angle KNM$?

   b. Apply Name a distance that decreases as $\overline{KL}$ gets longer.

   c. Writing *Explain* how the adjustable mechanism illustrates the Hinge Theorem.
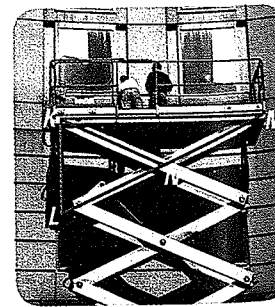
26. PROOF Write a proof that the shortest distance from a point to a line is the length of the perpendicular segment from the point to the line.

   GIVEN ▶ Line k; point A not on k; point B on k such that $\overline{AB} \perp k$

   PROVE ▶ $\overline{AB}$ is the shortest segment from A to k.

   Plan for Proof Assume that there is a shorter segment from A to k and use Theorem 5.10 to show that this leads to a contradiction.

○ **= WORKED-OUT SOLUTIONS** on p. WS1

★ **= STANDARDIZED TEST PRACTICE**
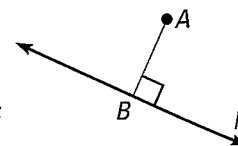
**27. USING A CONTRAPOSITIVE** Because the contrapositive of a conditional is equivalent to the original statement, you can prove the statement by proving its contrapositive. Look back at the conditional in Example 3 on page 337. Write a proof of the contrapositive that uses direct reasoning. How is your proof similar to the indirect proof of the original statement?

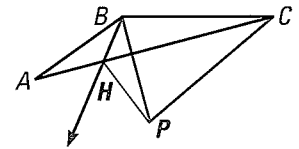**28. CHALLENGE** Write a proof of Theorem 5.13, the Hinge Theorem.

GIVEN ▶ $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $m\angle ABC > m\angle DEF$

PROVE ▶ $AC > DF$
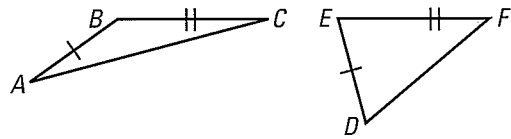
Plan for Proof

1. Because $m\angle ABC > m\angle DEF$, you can locate a point P in the interior of $\angle ABC$ so that $\angle CBP \cong \angle FED$ and $\overline{BP} \cong \overline{ED}$. Draw $\overline{BP}$ and show that $\triangle PBC \cong \triangle DEF$.

2. Locate a point H on $\overline{AC}$ so that $\overrightarrow{BH}$ bisects $\angle PBA$ and show that $\triangle ABH \cong \triangle PBH$.

3. Give reasons for each statement below to show that $AC > DF$.
   $$AC = AH + HC = PH + HC > PC = DF$$

MIXED REVIEW

Write the conversion factor you would multiply by to change units as specified. *(p. 886)*

29. inches to feet

30. liters to kiloliters

31. pounds to ounces

Solve the equation. Write a reason for each step. *(p. 105)*

32. $1.5(x + 4) = 5(2.4)$

**33.** $-3(-2x + 5) = 12$

34. $2(5x) = 3(4x + 6)$

35. Simplify the expression $\dfrac{-6xy^2}{21x^2y}$ if possible. *(p. 139)*


QUIZ for Lessons 5.5–5.6

1. Is it possible to construct a triangle with side lengths 5, 6, and 12? If not, *explain* why not. *(p. 328)*

2. The lengths of two sides of a triangle are 15 yards and 27 yards. *Describe* the possible lengths of the third side of the triangle. *(p. 328)*

3. In $\triangle PQR$, $m\angle P = 48°$ and $m\angle Q = 79°$. List the sides of $\triangle PQR$ in order from shortest to longest. *(p. 328)*

Copy and complete with <, >, or =. *(p. 335)*

4. $BA \underline{\ ?\ } DA$
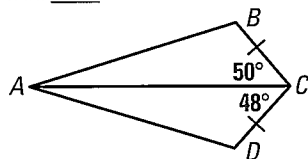
5. $m\angle 1 \underline{\ ?\ } m\angle 2$
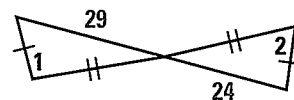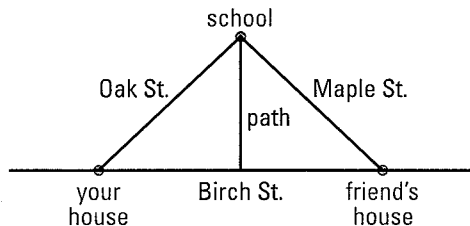
Lessons 5.4–5.6

1. MULTI-STEP PROBLEM In the diagram below, the entrance to the path is halfway between your house and your friend's house.


a. Can you conclude that you and your friend live the same distance from the school if the path bisects the angle formed by Oak and Maple Streets?

b. Can you conclude that you and your friend live the same distance from the school if the path is perpendicular to Birch Street?

c. Your answers to parts (a) and (b) show that a triangle must be isosceles if which two special segments are equal in length?

2. SHORT RESPONSE The map shows your driving route from Allentown to Bakersville and from Allentown to Dawson. Which city, Bakersville or Dawson, is located closer to Allentown? *Explain* your reasoning.
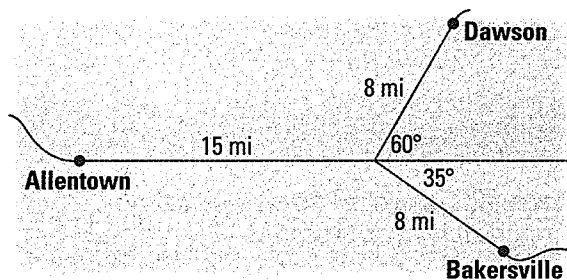

**3. GRIDDED RESPONSE** Find the length of $\overline{AF}$.
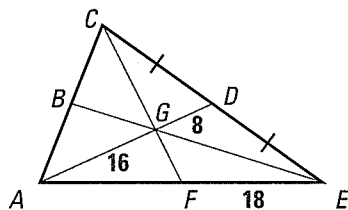
4. SHORT RESPONSE In the instructions for creating the terrarium shown, you are given a pattern for the pieces that form the roof. Does the diagram for the red triangle appear to be correct? *Explain* why or why not.
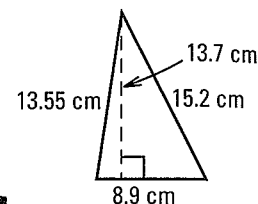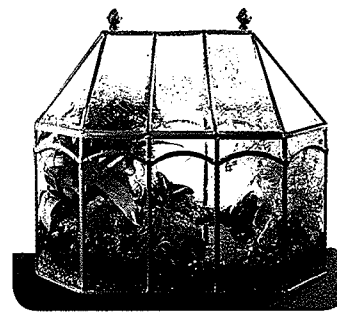
5. EXTENDED RESPONSE You want to create a triangular fenced pen for your dog. You have the two pieces of fencing shown, so you plan to move those to create two sides of the pen.
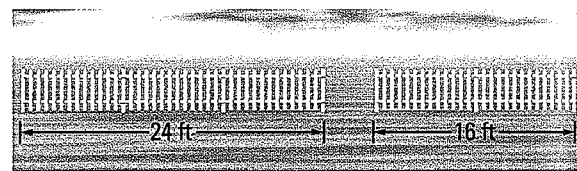
a. *Describe* the possible lengths for the third side of the pen.

b. The fencing is sold in 8 foot sections. If you use whole sections, what lengths of fencing are possible for the third side?

c. You want your dog to have a run within the pen that is at least 25 feet long. Which pen(s) could you use? *Explain*.

6. OPEN-ENDED In the gem shown, give a possible side length of $\overline{DE}$ if $m\angle EFD > 90°$, $DF = 0.4$ mm, and $EF = 0.63$ mm.
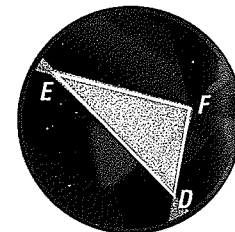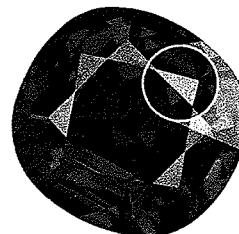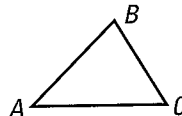
BIG IDEAS
For Your Notebook

Big Idea 1

Using Properties of Special Segments in Triangles

Special segment	*Properties to remember*
Midsegment	Parallel to side opposite it and half the length of side opposite it
Perpendicular bisector	Concurrent at the circumcenter, which is: • equidistant from 3 vertices of △ • center of *circumscribed* circle that passes through 3 vertices of △
Angle bisector	Concurrent at the incenter, which is: • equidistant from 3 sides of △ • center of *inscribed* circle that just touches each side of △
Median (connects vertex to midpoint of opposite side)	Concurrent at the centroid, which is: • located two thirds of the way from vertex to midpoint of opposite side • balancing point of △
Altitude (perpendicular to side of △ through opposite vertex)	Concurrent at the orthocenter Used in finding area: If b is length of any side and h is length of altitude to that side, then $A = \frac{1}{2}bh$.

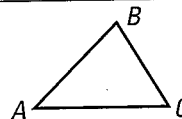**Big Idea 2**

Using Triangle Inequalities to Determine What Triangles are Possible

Sum of lengths of any two sides of a △ is greater than length of third side.		$AB + BC > AC$ $AB + AC > BC$ $BC + AC > AB$
In a △, longest side is opposite largest angle and shortest side is opposite smallest angle.		If $AC > AB > BC$, then $m\angle B > m\angle C > m\angle A$. If $m\angle B > m\angle C > m\angle A$, then $AC > AB > BC$.
If two sides of a △ are ≅ to two sides of another △, then the △ with longer third side also has larger included angle.		If $BC > EF$, then $m\angle A > m\angle D$. If $m\angle A > m\angle D$, then $BC > EF$.
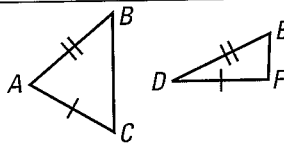
Big Idea 3

Extending Methods for Justifying and Proving Relationships

Coordinate proof uses the coordinate plane and variable coordinates. *Indirect proof* involves assuming the conclusion is false and then showing that the assumption leads to a contradiction.

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

• midsegment of a triangle, *p. 295*
• coordinate proof, *p. 296*
• perpendicular bisector, *p. 303*
• equidistant, *p. 303*
• concurrent, *p. 305*
• point of concurrency, *p. 305*
• circumcenter, *p. 306*

• incenter, *p. 312*
• median of a triangle, *p. 319*
• centroid, *p. 319*
• altitude of a triangle, *p. 320*
• orthocenter, *p. 321*
• indirect proof, *p. 337*

VOCABULARY EXERCISES

1. Copy and complete: A __?__ is a segment, ray, line, or plane that is perpendicular to a segment at its midpoint.

2. **WRITING** *Explain* how to draw a circle that is circumscribed about a triangle. What is the center of the circle called? *Describe* its radius.

In Exercises 3–5, match the term with the correct definition.

3. Incenter   **A.** The point of concurrency of the medians of a triangle

4. Centroid   **B.** The point of concurrency of the angle bisectors of a triangle

5. Orthocenter   **C.** The point of concurrency of the altitudes of a triangle

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 5.
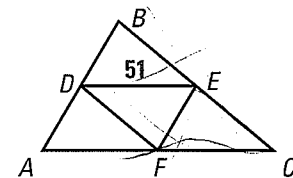
5.1 Midsegment Theorem and Coordinate Proof   *pp. 295–301*

EXAMPLE

In the diagram, $\overline{DE}$ is a midsegment of $\triangle ABC$. Find AC.

By the Midsegment Theorem, $DE = \frac{1}{2}AC$.

So, $AC = 2DE = 2(51) = 102$.

EXERCISES

EXAMPLES 1, 4, and 5 on pp. 295, 297 for Exs. 6–8

Use the diagram above where $\overline{DF}$ and $\overline{EF}$ are midsegments of $\triangle ABC$.

6. If $AB = 72$, find EF.

7. If $DF = 45$, find EC.

8. Graph $\triangle PQR$, with vertices $P(2a, 2b)$, $Q(2a, 0)$, and $O(0, 0)$. Find the coordinates of midpoint S of $\overline{PQ}$ and midpoint T of $\overline{QO}$. Show $\overline{ST} \parallel \overline{PO}$.

5.2 Use Perpendicular Bisectors

EXAMPLE

Use the diagram at the right to find XZ.

$\overleftrightarrow{WZ}$ is the perpendicular bisector of $\overline{XY}$.

$5x - 5 = 3x + 3$	**By the Perpendicular Bisector Theorem, $ZX = ZY$.**
$x = 4$	**Solve for x.**

▶ So, $XZ = 5x - 5 = 5(4) - 5 = 15$.

EXERCISES

EXAMPLES 1 and 2
on pp. 303–304
for Exs. 9–11
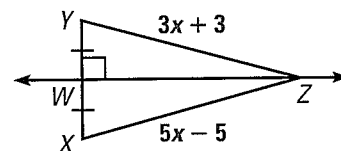
In the diagram, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.

9. What segment lengths are equal?

10. What is the value of x?

11. Find AB.
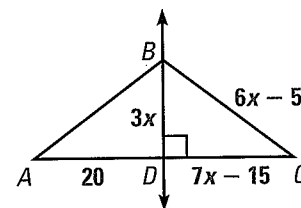
5.3 Use Angle Bisectors of Triangles

EXAMPLE

In the diagram, N is the incenter of $\triangle XYZ$. Find NL.

Use the Pythagorean Theorem to find NM in $\triangle NMY$.

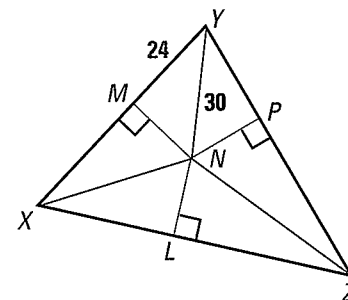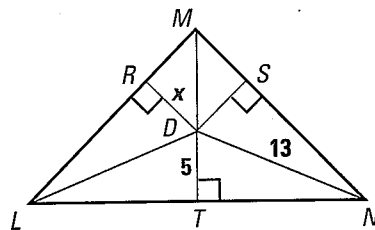|	
$c^2 = a^2 + b^2$	**Pythagorean Theorem**
$30^2 = NM^2 + 24^2$	**Substitute known values.**
$900 = NM^2 + 576$	**Multiply.**
$324 = NM^2$	**Subtract 576 from each side.**
$18 = NM$	**Take positive square root of each side.**

▶ By the Concurrency of Angle Bisectors of a Triangle, the incenter N of $\triangle XYZ$ is equidistant from all three sides of $\triangle XYZ$. So, because $NM = NL$, $NL = 18$.
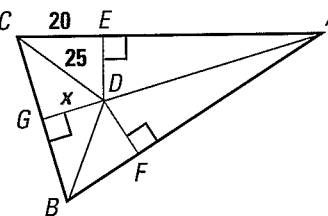
EXERCISES

EXAMPLE 4
on p. 312
for Exs. 12–13

Point D is the incenter of the triangle. Find the value of x.

12.

13.

5.4 Use Medians and Altitudes

EXAMPLE

The vertices of $\triangle ABC$ are $A(-6, 8)$, $B(0, -4)$, and $C(-12, 2)$. Find the coordinates of its centroid P.

Sketch $\triangle ABC$. Then find the midpoint M of $\overline{BC}$ and sketch median $\overline{AM}$.
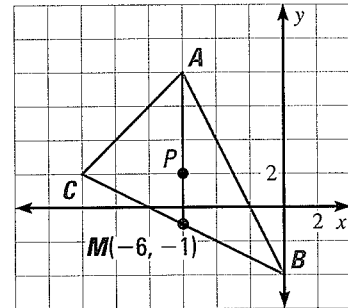
$$M\left(\frac{-12 + 0}{2}, \frac{2 + (-4)}{2}\right) = M(-6, -1)$$

The centroid is two thirds of the distance from a vertex to the midpoint of the opposite side.

The distance from vertex $A(-6, 8)$ to midpoint $M(-6, -1)$ is $8 - (-1) = 9$ units.

So, the centroid P is $\frac{2}{3}(9) = 6$ units down from A on $\overline{AM}$.

▶ The coordinates of the centroid P are $(-6, 8 - 6)$, or $(-6, 2)$.

EXERCISES

Find the coordinates of the centroid D of $\triangle RST$.

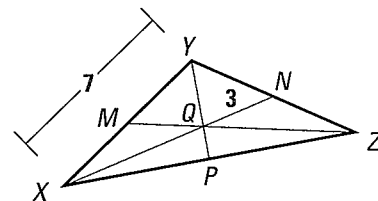**14.** $R(-4, 0)$, $S(2, 2)$, $T(2, -2)$ **15.** $R(-6, 2)$, $S(-2, 6)$, $T(2, 4)$

Point Q is the centroid of $\triangle XYZ$.

16. Find XQ.   **17.** Find XM.

**18.** Draw an obtuse $\triangle ABC$. Draw its three altitudes. Then label its orthocenter D.

5.5 Use Inequalities in a Triangle
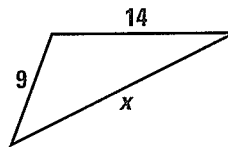
EXAMPLE

A triangle has one side of length 9 and another of length 14. Describe the possible lengths of the third side.

Let x represent the length of the third side. Draw diagrams and use the Triangle Inequality Theorem to write inequalities involving x.

$x + 9 > 14$ $9 + 14 > x$
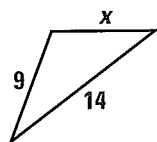
$x > 5$ $23 > x$, or $x < 23$

▶ The length of the third side must be greater than 5 and less than 23.

EXERCISES

Describe the possible lengths of the third side of the triangle given the lengths of the other two sides.
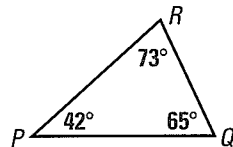
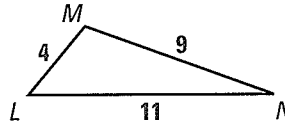**19.** 4 inches, 8 inches   **20.** 6 meters, 9 meters   **21.** 12 feet, 20 feet

List the sides and the angles in order from smallest to largest.

22.   **23.**   **24.**

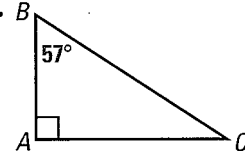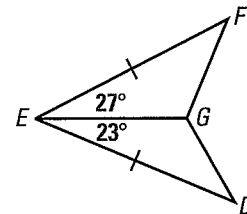## 5.6 Inequalities in Two Triangles and Indirect Proof   *pp. 335–341*

EXAMPLE

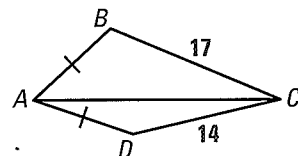How does the length of $\overline{DG}$ compare to the length of $\overline{FG}$?

▶ Because 27° > 23°, $m\angle GEF > m\angle GED$. You are given that $\overline{DE} \cong \overline{FE}$ and you know that $\overline{EG} \cong \overline{EG}$. Two sides of $\triangle GEF$ are congruent to two sides of $\triangle GED$ and the included angle is larger so, by the Hinge Theorem, $FG > DG$.
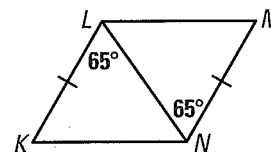

EXERCISES

Copy and complete with <, >, or =.

25. $m\angle BAC \underline{\ ?\ } m\angle DAC$

26. $LM \underline{\ ?\ } KN$

27. Arrange statements A–D in correct order to write an indirect proof of the statement: *If two lines intersect, then their intersection is exactly one point.*

  GIVEN ▶ Intersecting lines *m* and *n*

  PROVE ▶ The intersection of lines *m* and *n* is exactly one point.

  A. But this contradicts Postulate 5, which states that through any two points there is exactly one line.

  B. Then there are two lines (*m* and *n*) through points *P* and *Q*.

  C. Assume that there are two points, *P* and *Q*, where *m* and *n* intersect.

  D. It is false that *m* and *n* can intersect in two points, so they must intersect in exactly one point.

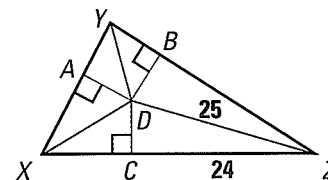**Two midsegments of △ABC are $\overline{DE}$ and $\overline{DF}$.**

1. Find DB.   **2.** Find DF.

3. What can you conclude about $\overline{EF}$?
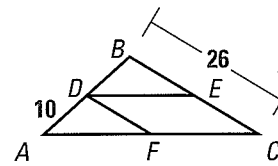
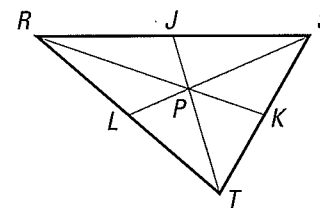**Find the value of x. Explain your reasoning.**

4.

5.

6.
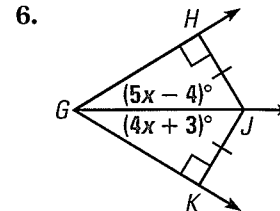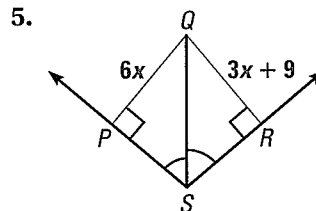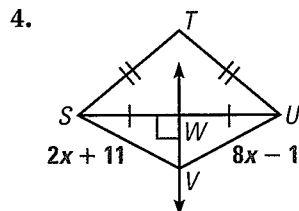
7. In Exercise 4, is point T on the perpendicular bisector of $\overline{SU}$? Explain.

8. In the diagram at the right, the angle bisectors of △XYZ meet at point D. Find DB.

In the diagram at the right, P is the centroid of △RST.

9. If LS = 36, find PL and PS.

10. If TP = 20, find TJ and PJ.

11. If JR = 25, find JS and RS.

12. Is it possible to construct a triangle with side lengths 9, 12, and 22? If not, *explain* why not.

13. In △ABC, AB = 36, BC = 18, and AC = 22. Sketch and label the triangle. List the angles in order from smallest to largest.

In the diagram for Exercises 14 and 15, JL = MK.

14. If m∠JKM > m∠LJK, which is longer, $\overline{LK}$ or $\overline{MJ}$? *Explain.*

15. If MJ < LK, which is larger, ∠LJK or ∠JKM? *Explain.*

16. Write a temporary assumption you could make to prove the conclusion indirectly: *If RS + ST ≠ 12 and ST = 5, then RS ≠ 7.*
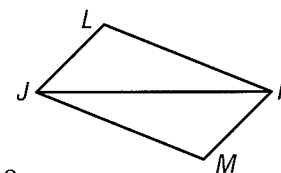
Use the diagram in Exercises 17 and 18.

17. *Describe* the range of possible distances from the beach to the movie theater.

18. A market is the same distance from your house, the movie theater, and the beach. Copy the diagram and locate the market.
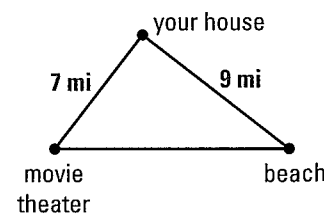
USE RATIOS AND PERCENT OF CHANGE

ⓧⓨ **EXAMPLE 1** *Write a ratio in simplest form*

A team won 18 of its 30 games and lost the rest. Find its win-loss ratio.

The ratio of a to b, $b \neq 0$, can be written as a to b, $a:b$, and $\frac{a}{b}$.

$$\frac{\text{wins}}{\text{losses}} = \frac{18}{30-18}$$ **To find losses, subtract wins from total.**

$$= \frac{18}{12} = \frac{3}{2}$$ **Simplify.**

▶ The team's win-loss ratio is $3:2$.

ⓧⓨ **EXAMPLE 2** *Find and interpret a percent of change*

A $50 sweater went on sale for $28. What is the percent of change in price? The new price is what percent of the old price?

$$\text{Percent of change} = \frac{\text{Amount of increase or decrease}}{\text{Original amount}} = \frac{50-28}{50} = \frac{22}{50} = 0.44$$

▶ The price went down, so the change is a decrease. The percent of decrease is 44%. So, the new price is $100\% - 44\% = 56\%$ of the original price.

EXERCISES

EXAMPLE 1
for Exs. 1–3

1. A team won 12 games and lost 4 games. Write each ratio in simplest form.

   a. wins to losses   **b.** losses out of total games

2. A scale drawing that is 2.5 feet long by 1 foot high was used to plan a mural that is 15 feet long by 6 feet high. Write each ratio in simplest form.

   a. length to height of mural   **b.** length of scale drawing to length of mural

3. There are 8 males out of 18 members in the school choir. Write the ratio of females to males in simplest form.

EXAMPLE 2
for Exs. 4–13

Find the percent of change.

4. From 75 campsites to 120 campsites   **5.** From 150 pounds to 136.5 pounds

6. From $480 to $408   **7.** From 16 employees to 18 employees

8. From 24 houses to 60 houses   **9.** From 4000 ft^2 to 3990 ft^2

Write the percent comparing the new amount to the original amount. Then find the new amount.

10. 75 feet increased by 4%   **11.** 45 hours decreased by 16%

12. $16,500 decreased by 85%   **13.** 80 people increased by 7.5%