

Geo

# 6

# Similarity

- 6.1 Ratios, Proportions, and the Geometric Mean
- 6.2 Use Proportions to Solve Geometry Problems
- 6.3 Use Similar Polygons
- 6.4 Prove Triangles Similar by AA
- 6.5 Prove Triangles Similar by SSS and SAS
- 6.6 Use Proportionality Theorems
- 6.7 Perform Similarity Transformations

## Before

In previous courses and in Chapters 1–5, you learned the following skills, which you'll use in Chapter 6: using properties of parallel lines, using properties of triangles, simplifying expressions, and finding perimeter.

## Prerequisite Skills

### VOCABULARY CHECK

1. The alternate interior angles formed when a transversal intersects two ? lines are congruent.
2. Two triangles are congruent if and only if their corresponding parts are ?.

### SKILLS AND ALGEBRA CHECK

Simplify the expression. (Review pp. 870, 874 for 6.1.)

3.  $\frac{9 \cdot 20}{15}$

4.  $\frac{15}{25}$

5.  $\frac{3 + 4 + 5}{6 + 8 + 10}$

6.  $\sqrt{5(5 \cdot 7)}$

Find the perimeter of the rectangle with the given dimensions.  
(Review p. 49 for 6.1, 6.2.)

7.  $l = 5$  in.,  $w = 12$  in.

8.  $l = 30$  ft,  $w = 10$  ft

9.  $A = 56$  m<sup>2</sup>,  $l = 8$  m

10. Find the slope of a line parallel to the line whose equation is  $y - 4 = 7(x + 2)$ . (Review p. 171 for 6.5.)

@HomeTutor Prerequisite skills practice at [classzone.com](http://classzone.com)

## Now

In Chapter 6, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 417. You will also use the key vocabulary listed below.

## Big Ideas

- 1 Using ratios and proportions to solve geometry problems
- 2 Showing that triangles are similar
- 3 Using indirect measurement and similarity

### KEY VOCABULARY

- ratio, p. 356
- proportion, p. 358
- geometric mean, p. 359
- scale drawing, p. 365
- scale, p. 365
- similar polygons, p. 372
- scale factor of two similar polygons, p. 373
- dilation, p. 409
- center of dilation, p. 409
- scale factor of a dilation, p. 409
- reduction, p. 409
- enlargement, p. 409

## Why?

You can use similarity to measure lengths indirectly. For example, you can use similar triangles to find the height of a tree.

## Animated Geometry

The animation illustrated below for Exercise 33 on page 394 helps you answer this question: What is the height of the tree?

If a person who is 5.5 ft tall casts a shadow of 7 ft, how tall is a tree with a shadow of 102 ft?

$$\frac{5.5}{7} = \frac{x}{102}$$

$x =$  ft

Round your answer to two decimal places.

Use similar triangles to write a proportion. Then find the value of  $x$ .

**Animated Geometry** at [classzone.com](http://classzone.com)

Other animations for Chapter 6: pages 365, 375, 391, 407, and 414

# 6.1 EXERCISES

## HOMEWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS7 for Exs. 5, 27, and 59
- ★ = STANDARDIZED TEST PRACTICE Exs. 2, 47, 48, 52, and 63
- ◆ = MULTIPLE REPRESENTATIONS Ex. 66

### SKILL PRACTICE

- VOCABULARY** Copy the proportion  $\frac{m}{n} = \frac{p}{q}$ . Identify the means of the proportion and the extremes of the proportion.
- ★ WRITING** Write three ratios that are equivalent to the ratio 3 : 4. Explain how you found the ratios.

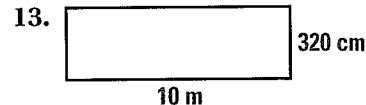
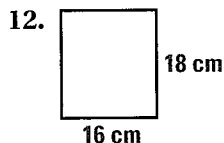
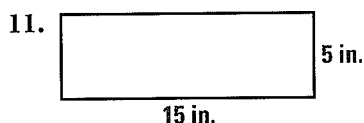
#### EXAMPLE 1

on p. 356  
for Exs. 3–17

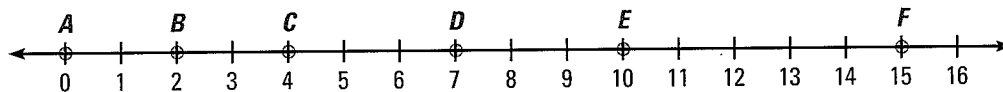
#### SIMPLIFYING RATIOS Simplify the ratio.

- \$20 : \$5
- $\frac{15 \text{ cm}^2}{12 \text{ cm}^2}$
- 6 L : 10 mL
- $\frac{1 \text{ mi}}{20 \text{ ft}}$
- $\frac{7 \text{ ft}}{12 \text{ in.}}$
- $\frac{80 \text{ cm}}{2 \text{ m}}$
- $\frac{3 \text{ lb}}{10 \text{ oz}}$
- $\frac{2 \text{ gallons}}{18 \text{ quarts}}$

#### WRITING RATIOS Find the ratio of the width to the length of the rectangle. Then simplify the ratio.



#### FINDING RATIOS Use the number line to find the ratio of the distances.



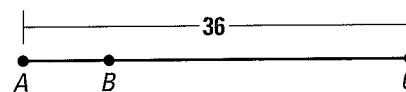
- $\frac{AD}{CF}$
- $\frac{BD}{AB}$
- $\frac{CE}{EF}$
- $\frac{BE}{CE}$

#### EXAMPLE 2

on p. 357  
for Exs. 18–19

- PERIMETER** The perimeter of a rectangle is 154 feet. The ratio of the length to the width is 10 : 1. Find the length and the width.

- SEGMENT LENGTHS** In the diagram,  $AB : BC$  is 2 : 7 and  $AC = 36$ . Find  $AB$  and  $BC$ .



#### EXAMPLE 3

on p. 357  
for Exs. 20–22

#### USING EXTENDED RATIOS The measures of the angles of a triangle are in the extended ratio given. Find the measures of the angles of the triangle.

- 3 : 5 : 10
- 2 : 7 : 9
- 11 : 12 : 13

#### EXAMPLE 4

on p. 358  
for Exs. 23–30

#### ALGEBRA Solve the proportion.

- $\frac{6}{x} = \frac{3}{2}$
- $\frac{y}{20} = \frac{3}{10}$
- $\frac{2}{7} = \frac{12}{z}$
- $\frac{j+1}{5} = \frac{4}{10}$
- $\frac{1}{c+5} = \frac{3}{24}$
- $\frac{4}{a-3} = \frac{2}{5}$
- $\frac{1+3b}{4} = \frac{5}{2}$
- $\frac{3}{2p+5} = \frac{1}{9p}$

**EXAMPLE 6**

on p. 359  
for Exs. 31–36

**GEOMETRIC MEAN** Find the geometric mean of the two numbers.

31. 2 and 18

32. 4 and 25

33. 32 and 8

34. 4 and 16

35. 2 and 25

36. 6 and 20

37. **ERROR ANALYSIS** A student incorrectly simplified the ratio. *Describe* and correct the student's error.

$$\frac{8 \text{ in.}}{3 \text{ ft}} = \frac{8 \text{ in.}}{3 \text{ ft}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = \frac{96 \text{ in.}}{3 \text{ ft}} = \frac{32 \text{ in.}}{1 \text{ ft}}$$

**WRITING RATIOS** Let  $x = 10$ ,  $y = 3$ , and  $z = 8$ . Write the ratio in simplest form.

38.  $x : z$

39.  $\frac{8y}{x}$

40.  $\frac{4}{2x + 2z}$

41.  $\frac{2x - z}{3y}$

**ALGEBRA** Solve the proportion.

42.  $\frac{2x + 5}{3} = \frac{x - 5}{4}$

43.  $\frac{2 - s}{3} = \frac{2s + 1}{5}$

44.  $\frac{15}{m} = \frac{m}{5}$

45.  $\frac{7}{q + 1} = \frac{q - 1}{5}$

46. **ANGLE MEASURES** The ratio of the measures of two supplementary angles is 5 : 3. Find the measures of the angles.

47. **★ SHORT RESPONSE** The ratio of the measure of an exterior angle of a triangle to the measure of the adjacent interior angle is 1 : 4. Is the triangle *acute* or *obtuse*? *Explain* how you found your answer.

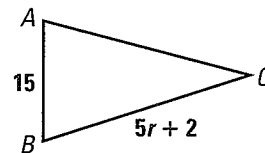
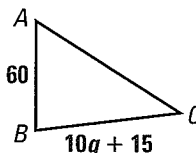
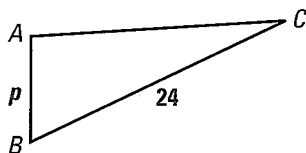
48. **★ SHORT RESPONSE** Without knowing its side lengths, can you determine the ratio of the perimeter of a square to the length of one of its sides? *Explain*.

**ALGEBRA** In Exercises 49–51, the ratio of two side lengths for the triangle is given. Solve for the variable.

49.  $AB : BC$  is 3 : 8.

50.  $AB : BC$  is 3 : 4.

51.  $AB : BC$  is 5 : 9.



52. **★ MULTIPLE CHOICE** What is a value of  $x$  that makes  $\frac{x}{3} = \frac{4x}{x + 3}$  true?

**(A)** 3

**(B)** 4

**(C)** 9

**(D)** 12

53. **AREA** The area of a rectangle is 4320 square inches. The ratio of the width to the length is 5 : 6. Find the length and the width.

54. **COORDINATE GEOMETRY** The points  $(-3, 2)$ ,  $(1, 1)$ , and  $(x, 0)$  are collinear. Use slopes to write a proportion to find the value of  $x$ .

55. **ALGEBRA** Use the proportions  $\frac{a + b}{2a - b} = \frac{5}{4}$  and  $\frac{b}{a + 9} = \frac{5}{9}$  to find  $a$  and  $b$ .

56. **CHALLENGE** Find the ratio of  $x$  to  $y$  given that  $\frac{5}{y} + \frac{7}{x} = 24$  and  $\frac{12}{y} + \frac{2}{x} = 24$ .

## PROBLEM SOLVING

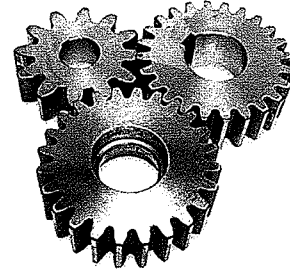
**EXAMPLE 2**

on p. 357  
for Ex. 57

- 57. TILING** The perimeter of a room is 66 feet. The ratio of its length to its width is 6 : 5. You want to tile the floor with 12 inch square tiles. Find the length and width of the room, and the area of the floor. How many tiles will you need? The tiles cost \$1.98 each. What is the total cost to tile the floor?

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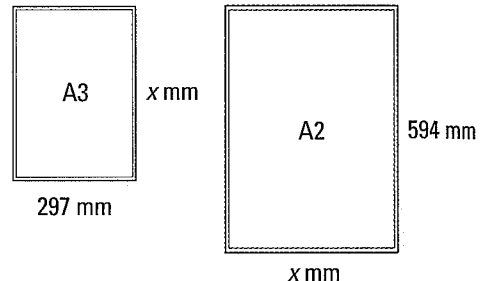
- 58. GEARS** The *gear ratio* of two gears is the ratio of the number of teeth of the larger gear to the number of teeth of the smaller gear. In a set of three gears, the ratio of Gear A to Gear B is equal to the ratio of Gear B to Gear C. Gear A has 36 teeth and Gear C has 16 teeth. How many teeth does Gear B have?



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- 59. TRAIL MIX** You need to make 36 one-half cup bags of trail mix for a class trip. The recipe calls for peanuts, chocolate chips, and raisins in the extended ratio 5 : 1 : 4. How many cups of each item do you need?

- 60. PAPER SIZES** International standard paper sizes are commonly used all over the world. The various sizes all have the same width-to-length ratios. Two sizes of paper are shown, called A3 and A2. The distance labeled  $x$  is the geometric mean of 297 mm and 594 mm. Find the value of  $x$ .



- 61. BATTING AVERAGE** The batting average of a baseball player is the ratio of the number of hits to the number of official at-bats. In 2004, Johnny Damon of the Boston Red Sox had 621 official at-bats and a batting average of .304. Use the proportion to find the number of hits made by Johnny Damon.

$$\frac{\text{Number of hits}}{\text{Number of at-bats}} = \frac{\text{Batting average}}{1.000}$$

**EXAMPLE 5**

on p. 359  
for Ex. 62

- 62. MULTI-STEP PROBLEM** The population of Red-tailed hawks is increasing in many areas of the United States. One long-term survey of bird populations suggests that the Red-tailed hawk population is increasing nationally by 2.7% each year.
- Write the ratio of hawks in year  $n$  to hawks in year  $(n - 1)$ .
  - In 2004, observers in Corpus Christi, TX, spotted 180 migrating Red-tailed hawks. Assuming this population follows the national trend, about how many Red-tailed hawks can they expect to see in 2005?
  - Observers in Lipan Point, AZ, spotted 951 migrating Red-tailed hawks in 2004. Assuming this population follows the national trend, about how many Red-tailed hawks can they expect to see in 2006?

63. ★ **SHORT RESPONSE** Some common computer screen resolutions are 1024 : 768, 800 : 600, and 640 : 480. *Explain* why these ratios are equivalent.

64. **BIOLOGY** The larvae of the Mother-of-Pearl moth is the fastest moving caterpillar. It can run at a speed of 15 inches per second. When threatened, it can curl itself up and roll away 40 times faster than it can run. How fast can it run in miles per hour? How fast can it roll?



65. **CURRENCY EXCHANGE** Emily took 500 U.S. dollars to the bank to exchange for Canadian dollars. The exchange rate on that day was 1.2 Canadian dollars per U.S. dollar. How many Canadian dollars did she get in exchange for the 500 U.S. dollars?

66. ◆ **MULTIPLE REPRESENTATIONS** Let  $x$  and  $y$  be two positive numbers whose geometric mean is 6.

a. **Making a Table** Make a table of ordered pairs  $(x, y)$  such that  $\sqrt{xy} = 6$ .

b. **Drawing a Graph** Use the ordered pairs to make a scatter plot. Connect the points with a smooth curve.

c. **Analyzing Data** Is the data linear? Why or why not?

67. ⓧ **ALGEBRA** Use algebra to verify Property 1, the Cross Products Property.

68. ⓧ **ALGEBRA** Show that the geometric mean of two numbers is equal to the arithmetic mean (or average) of the two numbers only when the numbers are equal. (*Hint*: Solve  $\sqrt{xy} = \frac{x+y}{2}$  with  $x, y \geq 0$ .)

**CHALLENGE** In Exercises 69–71, use the given information to find the value(s) of  $x$ . Assume that the given quantities are nonnegative.

69. The geometric mean of the quantities  $(\sqrt{x})$  and  $(3\sqrt{x})$  is  $(x - 6)$ .

70. The geometric mean of the quantities  $(x + 1)$  and  $(2x + 3)$  is  $(x + 3)$ .

71. The geometric mean of the quantities  $(2x + 1)$  and  $(6x + 1)$  is  $(4x - 1)$ .

## MIXED REVIEW

### PREVIEW

Prepare for  
Lesson 6.2  
in Exs. 72–75.

Find the reciprocal. (p. 869)

72.  $-6$

73.  $\frac{1}{13}$

74.  $\frac{-36}{3}$

75.  $-0.2$

Solve the quadratic equation. (p. 882)

76.  $5x^2 = 35$

77.  $x^2 - 20 = 29$

78.  $(x - 3)(x + 3) = 27$

Write the equation of the line with the given description. (p. 180)

79. Parallel to  $y = 3x - 7$ , passing through  $(1, 2)$

80. Perpendicular to  $y = \frac{1}{4}x + 5$ , passing through  $(0, 24)$

# 6.2 EXERCISES

## HOMEWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS7 for Exs. 11, 13, and 25  
 ★ = STANDARDIZED TEST PRACTICE Exs. 2, 18, and 24

### SKILL PRACTICE

- VOCABULARY** Copy and complete: A ? is a drawing that has the same shape as the object it represents.
- ★ **WRITING** Suppose the scale of a model of the Eiffel Tower is 1 inch : 20 feet. *Explain* how to determine how many times taller the actual tower is than the model.

#### EXAMPLE 1

on p. 364  
for Exs. 3–10

**REASONING** Copy and complete the statement.

3. If  $\frac{8}{x} = \frac{3}{y}$ , then  $\frac{8}{3} = \frac{?}{?}$ .

4. If  $\frac{x}{9} = \frac{y}{20}$ , then  $\frac{x}{y} = \frac{?}{?}$ .

5. If  $\frac{x}{6} = \frac{y}{15}$ , then  $\frac{x+6}{6} = \frac{?}{?}$ .

6. If  $\frac{14}{3} = \frac{x}{y}$ , then  $\frac{17}{3} = \frac{?}{?}$ .

**REASONING** Decide whether the statement is *true* or *false*.

7. If  $\frac{8}{m} = \frac{n}{9}$ , then  $\frac{8+m}{m} = \frac{n+9}{9}$ .

8. If  $\frac{5}{7} = \frac{a}{b}$ , then  $\frac{7}{5} = \frac{a}{b}$ .

9. If  $\frac{d}{2} = \frac{g+10}{11}$ , then  $\frac{d}{g+10} = \frac{2}{11}$ .

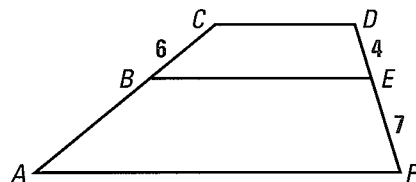
10. If  $\frac{4+x}{4} = \frac{3+y}{y}$ , then  $\frac{x}{4} = \frac{3}{y}$ .

#### EXAMPLE 2

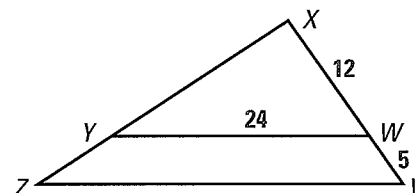
on p. 365  
for Exs. 11–12

**PROPERTIES OF PROPORTIONS** Use the diagram and the given information to find the unknown length.

11. Given  $\frac{CB}{BA} = \frac{DE}{EF}$ , find BA.



12. Given  $\frac{XW}{XV} = \frac{YW}{ZV}$ , find ZV.

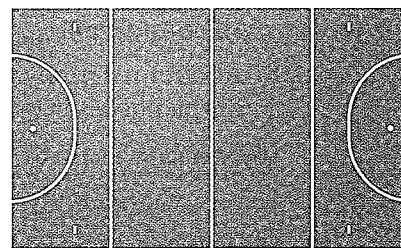


#### EXAMPLES 3 and 4

on pp. 365–366  
for Exs. 13–14

**SCALE DIAGRAMS** In Exercises 13 and 14, use the diagram of the field hockey field in which 1 inch = 50 yards. Use a ruler to approximate the dimension.

- Find the actual length of the field.
- Find the actual width of the field.



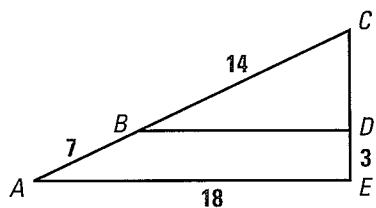
15. **ERROR ANALYSIS** Describe and correct the error made in the reasoning.

If  $\frac{a}{3} = \frac{c}{4}$ , then  $\frac{a+3}{3} = \frac{c+3}{4}$ .

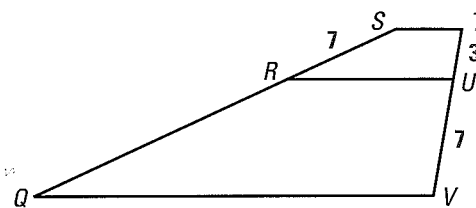


**PROPERTIES OF PROPORTIONS** Use the diagram and the given information to find the unknown length.

16. Given  $\frac{CA}{CB} = \frac{AE}{BD}$ , find  $BD$ .



17. Given  $\frac{SQ}{SR} = \frac{TV}{TU}$ , find  $RQ$ .



18. ★ **MULTIPLE CHOICE** If  $x$ ,  $y$ ,  $z$ , and  $q$  are four different numbers, and the proportion  $\frac{x}{y} = \frac{z}{q}$  is true, which of the following is false?

(A)  $\frac{y}{x} = \frac{q}{z}$

(B)  $\frac{x}{z} = \frac{y}{q}$

(C)  $\frac{y}{x} = \frac{z}{q}$

(D)  $\frac{x+y}{y} = \frac{z+q}{q}$

**CHALLENGE** Two number patterns are *proportional* if there is a nonzero number  $k$  such that  $(a_1, b_1, c_1, \dots) = k(a_2, b_2, c_2, \dots) = ka_2, kb_2, kc_2, \dots$

19. Given the relationship  $(8, 16, 20) = k(2, 4, 5)$ , find  $k$ .

20. Given that  $a_1 = ka_2$ ,  $b_1 = kb_2$ , and  $c_1 = kc_2$ , show that  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ .

21. Given that  $a_1 = ka_2$ ,  $b_1 = kb_2$ , and  $c_1 = kc_2$ , show that  $\frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} = k$ .

## PROBLEM SOLVING

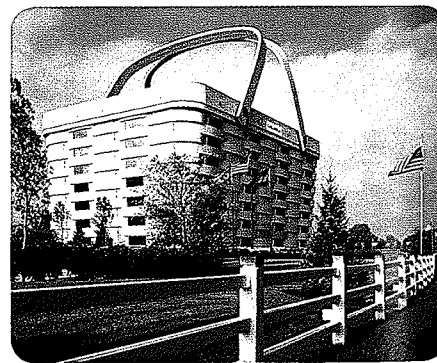
**EXAMPLE 5**  
on p. 366  
for Ex. 22

22. **ARCHITECTURE** A basket manufacturer has headquarters in an office building that has the same shape as a basket they sell.

a. The bottom of the basket is a rectangle with length 15 inches and width 10 inches. The base of the building is a rectangle with length 192 feet. What is the width of the base of the building?

b. About how many times as long as the bottom of the basket is the base of the building?

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Longaberger Company Home Office  
Newark, Ohio

23. **MAP SCALE** A street on a map is 3 inches long. The actual street is 1 mile long. Find the scale of the map.

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24. ★ **MULTIPLE CHOICE** A model train engine is 12 centimeters long. The actual engine is 18 meters long. What is the scale of the model?

(A) 3 cm : 2 m

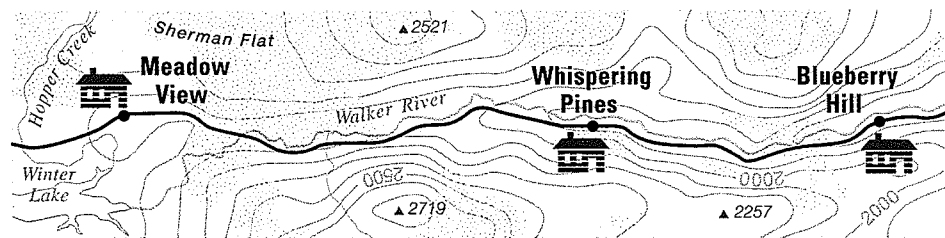
(B) 1 cm : 1.5 m

(C) 1 cm : 3 m

(D) 200 cm : 3 m

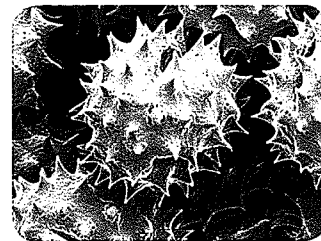


**MAP READING** The map of a hiking trail has a scale of 1 inch : 3.2 miles. Use a ruler to approximate the actual distance between the two shelters.

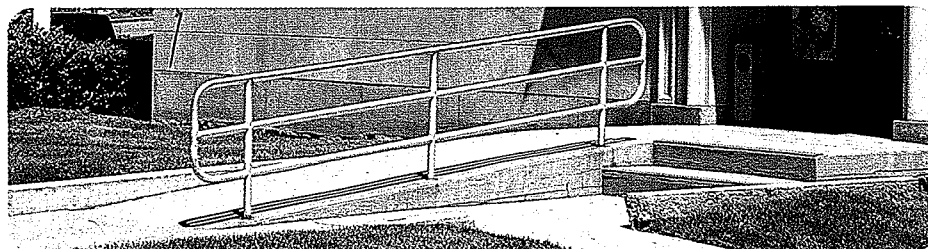


25. Meadow View and Whispering Pines      26. Whispering Pines and Blueberry Hill

27. **POLLEN** The photograph shows a particle of goldenrod pollen that has been magnified under a microscope. The scale of the photograph is 900 : 1. Use a ruler to estimate the width in millimeters of the particle.



**RAMP DESIGN** Assume that the wheelchair ramps described each have a slope of  $\frac{1}{12}$ , which is the maximum slope recommended for a wheelchair ramp.



28. A wheelchair ramp has a 21 foot run. What is its rise?  
 29. A wheelchair ramp rises 4 feet. What is its run?  
 30. **STATISTICS** Researchers asked 4887 people to pick a number between 1 and 10. The results are shown in the table below.

<b>Answer</b>	1	2	3	4	5
<b>Percent</b>	4.2%	5.1%	11.4%	10.5%	10.7%
<b>Answer</b>	6	7	8	9	10
<b>Percent</b>	10.0%	27.2%	8.8%	6.0%	6.1%

- a. Estimate the number of people who picked the number 3.  
 b. You ask a participant what number she picked. Is the participant more likely to answer 6 or 7? *Explain.*  
 c. Conduct this experiment with your classmates. Make a table in which you compare the new percentages with the ones given in the original survey. Why might they be different?

**XV ALGEBRA** Use algebra to verify the property of proportions.

31. Property 2      32. Property 3      33. Property 4

**REASONING** Use algebra to *explain* why the property of proportions is true.

34. If  $\frac{a-b}{a+b} = \frac{c-d}{c+d}$ , then  $\frac{a}{b} = \frac{c}{d}$ .

35. If  $\frac{a+c}{b+d} = \frac{a-c}{b-d}$ , then  $\frac{a}{b} = \frac{c}{d}$ .

36. If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ , then  $\frac{a+c+e}{b+d+f} = \frac{a}{b}$ . (Hint: Let  $\frac{a}{b} = r$ .)

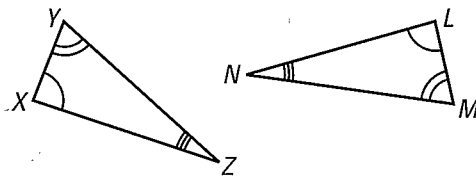
37. **CHALLENGE** When fruit is dehydrated, water is removed from the fruit. The water content in fresh apricots is about 86%. In dehydrated apricots, the water content is about 75%. Suppose 5 kilograms of raw apricots are dehydrated. How many kilograms of water are removed from the fruit? What is the approximate weight of the dehydrated apricots?

### MIXED REVIEW

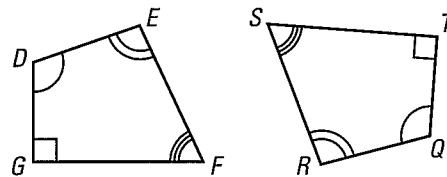
38. Over the weekend, Claudia drove a total of 405 miles, driving twice as far on Saturday as on Sunday. How far did Claudia travel each day? (p. 65)

Identify all pairs of congruent corresponding parts. Then write another congruence statement for the figures. (p. 225)

39.  $\triangle XYZ \cong \triangle LMN$



40.  $DEFG \cong QRST$



**PREVIEW**

Prepare for  
Lesson 6.3  
in Exs. 39–40.

### QUIZ for Lessons 6.1–6.2

Solve the proportion. (p. 356)

1.  $\frac{10}{y} = \frac{5}{2}$

2.  $\frac{x}{6} = \frac{9}{3}$

3.  $\frac{1}{a+3} = \frac{4}{16}$

4.  $\frac{6}{d-6} = \frac{4}{8}$

Copy and complete the statement. (p. 364)

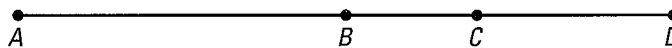
5. If  $\frac{9}{x} = \frac{5}{2}$ , then  $\frac{9}{5} = \frac{?}{?}$ .

6. If  $\frac{x}{15} = \frac{y}{21}$ , then  $\frac{x}{y} = \frac{?}{?}$ .

7. If  $\frac{x}{8} = \frac{y}{12}$ , then  $\frac{x+8}{8} = \frac{?}{?}$ .

8. If  $\frac{32}{5} = \frac{x}{y}$ , then  $\frac{37}{5} = \frac{?}{?}$ .

9. In the diagram,  $AD = 10$ ,  $B$  is the midpoint of  $\overline{AD}$ , and  $AC$  is the geometric mean of  $AB$  and  $AD$ . Find  $AC$ . (p. 364)



## 6.3 Similar Polygons

**MATERIALS** • metric ruler • protractor

**QUESTION** When a figure is reduced, how are the corresponding angles related? How are the corresponding lengths related?

**EXPLORE** Compare measures of lengths and angles in two photos

**STEP 1 Measure segments** Photo 2 is a reduction of Photo 1. In each photo, find  $\overline{AB}$  to the nearest millimeter. Write the ratio of the length of  $\overline{AB}$  in Photo 1 to the length of  $\overline{AB}$  in Photo 2.

**STEP 2 Measure angles** Use a protractor to find the measure of  $\angle 1$  in each photo. Write the ratio of  $m\angle 1$  in Photo 1 to  $m\angle 1$  in Photo 2.

**STEP 3 Find measurements** Copy and complete the table. Use the same units for each measurement. Record your results in a table.

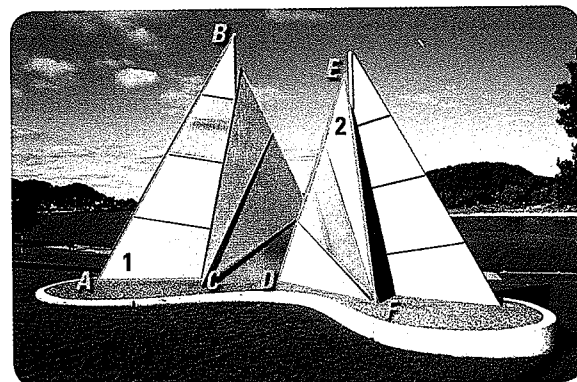


Photo 1

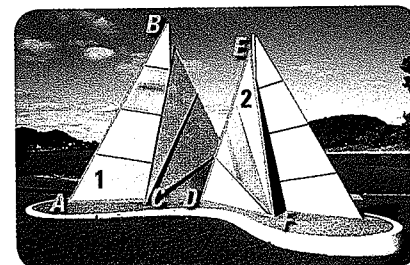


Photo 2

Measurement	Photo 1	Photo 2	$\frac{\text{Photo 1}}{\text{Photo 2}}$
$\overline{AB}$	?	?	?
$\overline{AC}$	?	?	?
$\overline{DE}$	?	?	?
$m\angle 1$	?	?	?
$m\angle 2$	?	?	?

**DRAW CONCLUSIONS** Use your observations to complete these exercises

1. Make a conjecture about the relationship between corresponding lengths when a figure is reduced.
2. Make a conjecture about the relationship between corresponding angles when a figure is reduced.
3. Suppose the measure of an angle in Photo 2 is  $35^\circ$ . What is the measure of the corresponding angle in Photo 1?
4. Suppose a segment in Photo 2 is 1 centimeter long. What is the measure of the corresponding segment in Photo 1?
5. Suppose a segment in Photo 1 is 5 centimeters long. What is the measure of the corresponding segment in Photo 2?

# 6.3 EXERCISES

## HOMWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS7 for Exs. 3, 7, and 31
- ★ = STANDARDIZED TEST PRACTICE Exs. 2, 6, 18, 27, 28, 35, 36, and 37
- ◆ = MULTIPLE REPRESENTATIONS Ex. 33

### SKILL PRACTICE

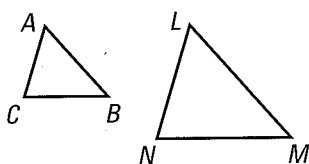
- VOCABULARY** Copy and complete: Two polygons are similar if corresponding angles are ? and corresponding side lengths are ?.
- ★ **WRITING** If two polygons are congruent, must they be similar? If two polygons are similar, must they be congruent? *Explain.*

#### EXAMPLE 1

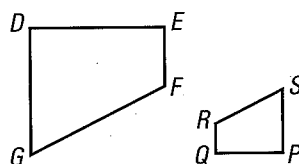
on p. 372  
for Exs. 3–6

**USING SIMILARITY** List all pairs of congruent angles for the figures. Then write the ratios of the corresponding sides in a statement of proportionality.

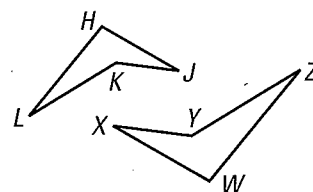
3.  $\triangle ABC \sim \triangle LMN$



4.  $DEFG \sim PQRS$



5.  $HJKL \sim WXYZ$



6. ★ **MULTIPLE CHOICE** Triangles  $ABC$  and  $DEF$  are similar. Which statement is *not* correct?

(A)  $\frac{BC}{EF} = \frac{AC}{DF}$

(B)  $\frac{AB}{DE} = \frac{CA}{FD}$

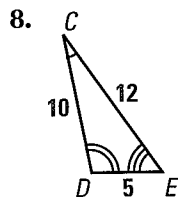
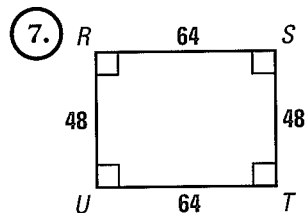
(C)  $\frac{CA}{FD} = \frac{BC}{EF}$

(D)  $\frac{AB}{EF} = \frac{BC}{DE}$

#### EXAMPLES 2 and 3

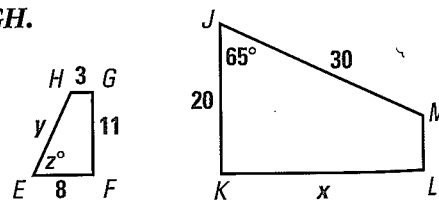
on p. 373  
for Exs. 7–10

**DETERMINING SIMILARITY** Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.



**USING SIMILAR POLYGONS** In the diagram,  $JKLM \sim EFGH$ .

- Find the scale factor of  $JKLM$  to  $EFGH$ .
- Find the values of  $x$ ,  $y$ , and  $z$ .
- Find the perimeter of each polygon.

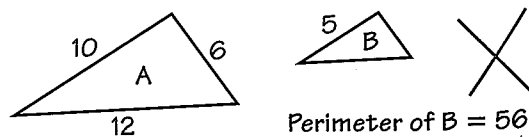


#### EXAMPLE 4

on p. 374  
for Exs. 11–13

12. **PERIMETER** Two similar FOR SALE signs have a scale factor of 5:3. The large sign's perimeter is 60 inches. Find the small sign's perimeter.

13. **ERROR ANALYSIS** The triangles are similar. *Describe* and correct the error in finding the perimeter of Triangle B.

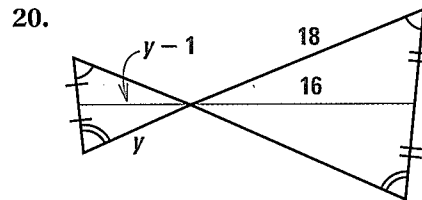
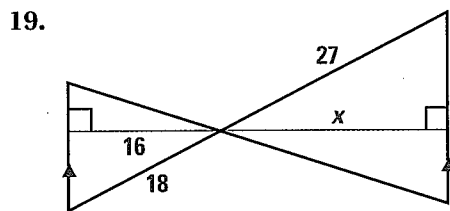


**REASONING** Are the polygons *always*, *sometimes*, or *never* similar?

14. Two isosceles triangles  
 15. Two equilateral triangles  
 16. A right triangle and an isosceles triangle  
 17. A scalene triangle and an isosceles triangle

18. ★ **SHORT RESPONSE** The scale factor of Figure A to Figure B is  $1 : x$ . What is the scale factor of Figure B to Figure A? *Explain* your reasoning.

**SIMILAR TRIANGLES** The black triangles are similar. Identify the type of special segment shown in blue, and find the value of the variable.



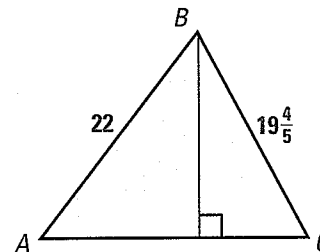
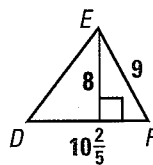
**EXAMPLE 5**  
 on p. 375  
 for Exs. 21–22

**USING SCALE FACTOR** Triangles  $NPQ$  and  $RST$  are similar. The side lengths of  $\triangle NPQ$  are 6 inches, 8 inches, and 10 inches, and the length of an altitude is 4.8 inches. The shortest side of  $\triangle RST$  is 8 inches long.

21. Find the lengths of the other two sides of  $\triangle RST$ .  
 22. Find the length of the corresponding altitude in  $\triangle RST$ .

**USING SIMILAR TRIANGLES** In the diagram,  $\triangle ABC \sim \triangle DEF$ .

23. Find the scale factor of  $\triangle ABC$  to  $\triangle DEF$ .  
 24. Find the unknown side lengths in both triangles.  
 25. Find the length of the altitude shown in  $\triangle ABC$ .  
 26. Find and compare the areas of both triangles.



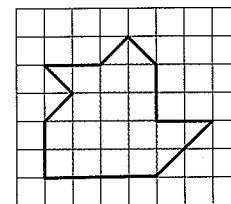
27. ★ **SHORT RESPONSE** Suppose you are told that  $\triangle PQR \sim \triangle XYZ$  and that the extended ratio of the angle measures in  $\triangle PQR$  is  $x : x + 30 : 3x$ . Do you need to know anything about  $\triangle XYZ$  to be able to write its extended ratio of angle measures? *Explain* your reasoning.

28. ★ **MULTIPLE CHOICE** The lengths of the legs of right triangle  $ABC$  are 3 feet and 4 feet. The shortest side of  $\triangle UVW$  is 4.5 feet and  $\triangle UVW \sim \triangle ABC$ . How long is the hypotenuse of  $\triangle UVW$ ?

- (A) 1.5 ft      (B) 5 ft      (C) 6 ft      (D) 7.5 ft

29. **CHALLENGE** Copy the figure at the right and divide it into two similar figures.

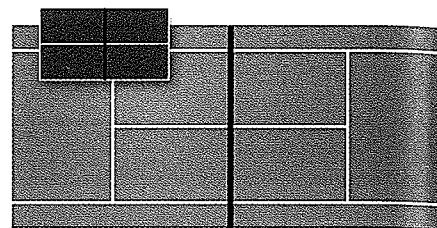
30. **REASONING** Is similarity reflexive? symmetric? transitive? Give examples to support your answers.



## PROBLEM SOLVING

**EXAMPLE 2**  
on p. 373 for  
Exs. 31–32

- 31. TENNIS** In table tennis, the table is a rectangle 9 feet long and 5 feet wide. A tennis court is a rectangle 78 feet long and 36 feet wide. Are the two surfaces similar? *Explain.* If so, find the scale factor of the tennis court to the table.



**@HomeTutor** for problem solving help at classzone.com

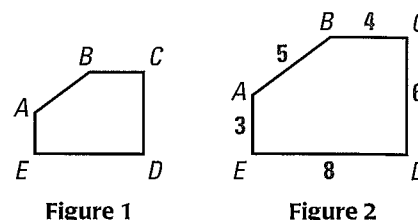
- 32. DIGITAL PROJECTOR** You are preparing a computer presentation to be digitally projected onto the wall of your classroom. Your computer screen is 13.25 inches wide and 10.6 inches high. The projected image on the wall is 53 inches wide and 42.4 inches high. Are the two shapes similar? If so, find the scale factor of the computer screen to the projected image.

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- 33. MULTIPLE REPRESENTATIONS** Use the similar figures shown. The scale factor of Figure 1 to Figure 2 is 7:10.

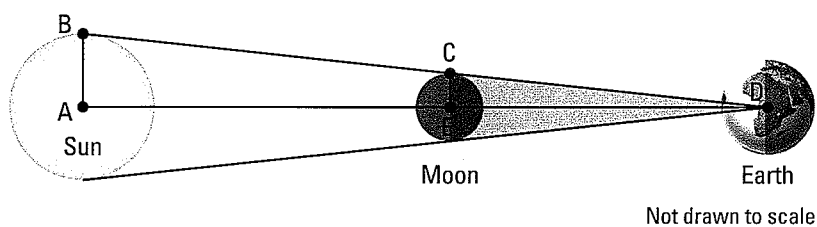
a. **Making a Table** Copy and complete the table.

	<i>AB</i>	<i>BC</i>	<i>CD</i>	<i>DE</i>	<i>EA</i>
<b>Figure 1</b>	3.5	?	?	?	?
<b>Figure 2</b>	5.0	4.0	6.0	8.0	3.0



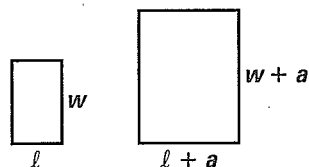
- b. **Drawing a Graph** Graph the data in the table. Let  $x$  represent the length of a side in Figure 1 and let  $y$  represent the length of the corresponding side in Figure 2. Is the relationship linear?
- c. **Writing an Equation** Write an equation that relates  $x$  and  $y$ . What is its slope? How is the slope related to the scale factor?

- 34. MULTI-STEP PROBLEM** During a total eclipse of the sun, the moon is directly in line with the sun and blocks the sun's rays. The distance  $ED$  between Earth and the moon is 240,000 miles, the distance  $DA$  between Earth and the sun is 93,000,000 miles, and the radius  $AB$  of the sun is 432,500 miles.



- a. Copy the diagram and label the known distances.
- b. In the diagram,  $\triangle BDA \sim \triangle CDE$ . Use this fact to explain a total eclipse of the sun.
- c. Estimate the radius  $CE$  of the moon.

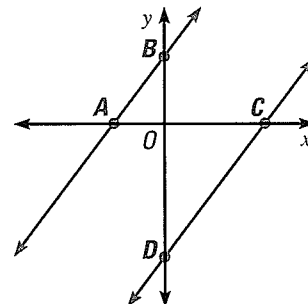
35. ★ **SHORT RESPONSE** A rectangular image is enlarged on each side by the same amount. The angles remain unchanged. Can the larger image be similar to the original? *Explain* your reasoning, and give an example to support your answer.



36. ★ **SHORT RESPONSE** How are the areas of similar rectangles related to the scale factor? Use examples to *justify* your reasoning.

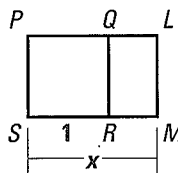
37. ★ **EXTENDED RESPONSE** The equations of two lines in the coordinate plane are  $y = \frac{4}{3}x + 4$  and  $y = \frac{4}{3}x - 8$ .

- Explain why the two lines are parallel.
- Show that  $\angle BOA \cong \angle DOC$ ,  $\angle OBA \cong \angle ODC$ , and  $\angle BAO \cong \angle DCO$ .
- Find the coordinates of points A, B, C, and D. Find the lengths of the sides of  $\triangle AOB$  and  $\triangle COD$ .
- Show that  $\triangle AOB \sim \triangle COD$ .



38. **PROVING THEOREM 6.1** Prove the Perimeters of Similar Polygons Theorem for similar rectangles. Include a diagram in your proof.

39. **CHALLENGE** In the diagram, PQRS is a square, and  $PLMS \sim LMRQ$ . Find the exact value of x. This value is called the *golden ratio*. Golden rectangles have their length and width in this ratio. Show that the similar rectangles in the diagram are golden rectangles.



## MIXED REVIEW

### PREVIEW

Prepare for  
Lesson 6.4  
in Exs. 40–42.

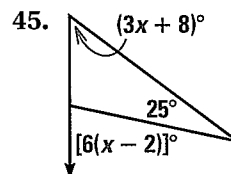
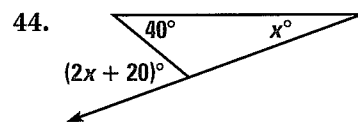
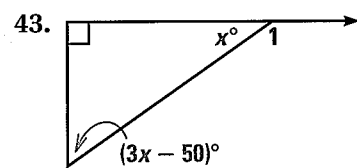
- Given  $A(1, 1)$ ,  $B(3, 2)$ ,  $C(2, 4)$ , and  $D(1, \frac{7}{2})$ , determine whether the following lines are *parallel*, *perpendicular*, or *neither*. (p. 171)

40.  $\vec{AB}$  and  $\vec{BC}$

41.  $\vec{CD}$  and  $\vec{AD}$

42.  $\vec{AB}$  and  $\vec{CD}$

Find the measure of the exterior angle shown. (p. 217)

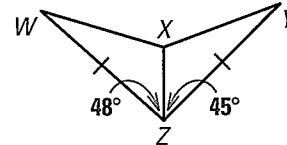
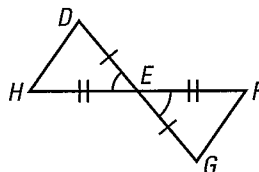
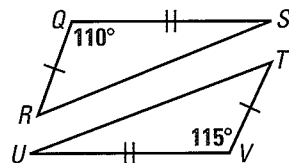


Copy and complete the statement with  $<$ ,  $>$ , or  $=$ . (p. 335)

46.  $RS$   $?$   $TU$

47.  $FG$   $?$   $HD$

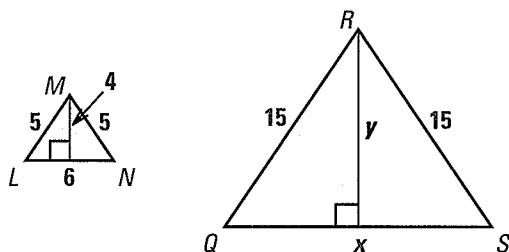
48.  $WX$   $?$   $YX$



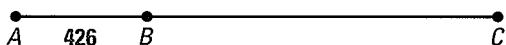


## Lessons 6.1–6.3

1. **MULTI-STEP PROBLEM** In the diagram,  $\triangle LMN \sim \triangle QRS$ .



- Find the scale factor of  $\triangle LMN$  to  $\triangle QRS$ . Then find the values of  $x$  and  $y$ .
  - Find the perimeters of  $\triangle LMN$  and  $\triangle QRS$ .
  - Find the areas of  $\triangle LMN$  and  $\triangle QRS$ .
  - Compare the ratio of the perimeters to the ratio of the areas of  $\triangle LMN$  to  $\triangle QRS$ . What do you notice?
2. **GRIDDED ANSWER** In the diagram,  $AB:BC$  is  $3:8$ . Find  $AC$ .



3. **OPEN-ENDED**  $\triangle UVW$  is a right triangle with side lengths of 3 cm, 4 cm, and 5 cm. Draw and label  $\triangle UVW$ . Then draw a triangle similar to  $\triangle UVW$  and label its side lengths. What scale factor did you use?
4. **MULTI-STEP PROBLEM** Kelly is going on a trip to England. She takes 600 U.S. dollars with her.

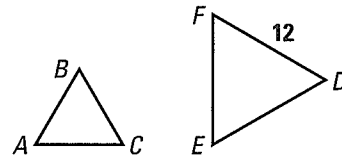
One U.S. Dollar Buys	
	EURO
	GREAT BRITAIN
	CANADA

- In England, she exchanges her U.S. dollars for British pounds. During her stay, Kelly spends 150 pounds. How many British pounds does she have left?
- When she returns home, she exchanges her money back to U.S. dollars. How many U.S. dollars does she have at the end of her trip?

5. **SHORT RESPONSE** Kelly bought a 3-D scale model of the Tower Bridge in London, England. The towers of the model are 9 inches tall. The towers of the actual bridge are 206 feet tall, and there are two walkways that are 140 feet high.



- Approximate the height of the walkways on the model.
  - About how many times as tall as the model is the actual structure?
6. **GRIDDED ANSWER** In the diagram,  $\triangle ABC \sim \triangle DEF$ . The scale factor of  $\triangle ABC$  to  $\triangle DEF$  is  $3:5$ . Find  $AC$ .



7. **EXTENDED RESPONSE** In the United States, 4634 million pounds of apples were consumed in 2002. The population of the United States in that year was 290 million.
- Divide the total number of apples consumed by the population to find the per capita consumption.
  - About how many pounds of apples would a family of four have consumed in one year? in one month?
  - A medium apple weighs about 5 ounces. Estimate how many apples a family of four would have consumed in one month.
  - Is it reasonable to assume that a family of four would have eaten that many apples? What other factors could affect the per capita consumption? *Explain.*



# 6.4 EXERCISES

**HOMEWORK KEY**

○ = WORKED-OUT SOLUTIONS  
on p. WS7 for Exs. 9, 13, and 33

★ = STANDARDIZED TEST PRACTICE  
Exs. 2, 16, 18, 19, 20, 33, and 38

## SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are ?.

2. ★ **WRITING** Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent? Explain.

**EXAMPLE 1**

on p. 382  
for Exs. 3–11

**REASONING** Use the diagram to complete the statement.

3.  $\triangle ABC \sim \underline{\quad?}$

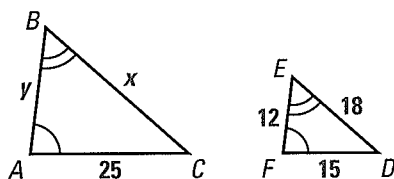
4.  $\frac{BA}{?} = \frac{AC}{?} = \frac{CB}{?}$

5.  $\frac{25}{?} = \frac{?}{12}$

6.  $\frac{?}{25} = \frac{18}{?}$

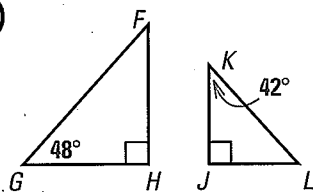
7.  $y = \underline{\quad?}$

8.  $x = \underline{\quad?}$

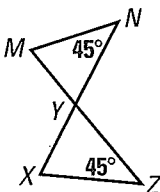


**AA SIMILARITY POSTULATE** In Exercises 9–14, determine whether the triangles are similar. If they are, write a similarity statement.

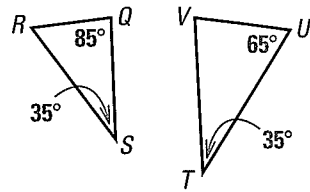
9.



10.



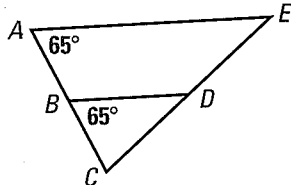
11.



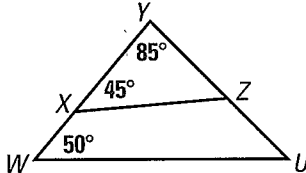
**EXAMPLE 2**

on p. 382  
for Exs. 12–16

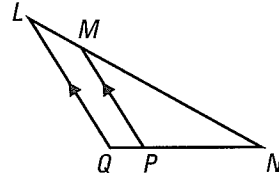
12.



13.

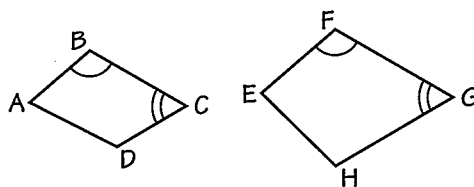


14.



15. **ERROR ANALYSIS** Explain why the student's similarity statement is incorrect.

$ABCD \sim EFGH$   
by AA Similarity  
Postulate



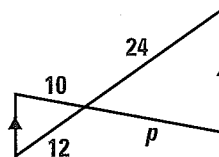
16. ★ **MULTIPLE CHOICE** What is the value of  $p$ ?

(A) 5

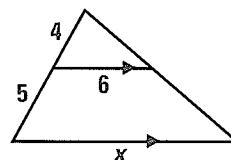
(B) 20

(C) 28.8

(D) Cannot be determined



17. **ERROR ANALYSIS** A student uses the proportion  $\frac{4}{6} = \frac{5}{x}$  to find the value of  $x$  in the figure. *Explain* why this proportion is incorrect and write a correct proportion.

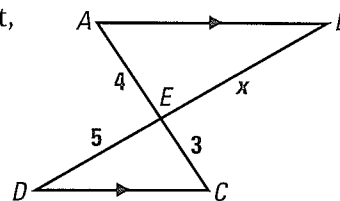


★ **OPEN-ENDED MATH** In Exercises 18 and 19, make a sketch that can be used to show that the statement is false.

18. If two pairs of sides of two triangles are congruent, then the triangles are similar.
19. If the ratios of two pairs of sides of two triangles are proportional, then the triangles are similar.

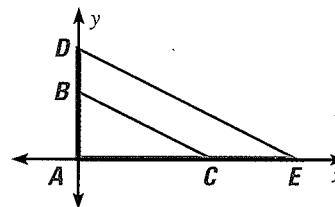
20. ★ **MULTIPLE CHOICE** In the figure at the right, find the length of  $\overline{BD}$ .

- (A)  $\frac{35}{3}$                       (B)  $\frac{37}{5}$   
 (C)  $\frac{20}{3}$                       (D)  $\frac{12}{5}$



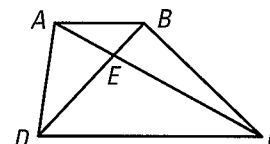
① **ALGEBRA** Find coordinates for point  $E$  so that  $\triangle ABC \sim \triangle ADE$ .

21.  $A(0, 0), B(0, 4), C(8, 0), D(0, 5), E(x, y)$   
 22.  $A(0, 0), B(0, 3), C(4, 0), D(0, 7), E(x, y)$   
 23.  $A(0, 0), B(0, 1), C(6, 0), D(0, 4), E(x, y)$   
 24.  $A(0, 0), B(0, 6), C(3, 0), D(0, 9), E(x, y)$



25. **MULTI-STEP PROBLEM** In the diagram,  $\vec{AB} \parallel \vec{DC}$ ,  $AE = 6$ ,  $AB = 8$ ,  $CE = 15$ , and  $DE = 10$ .

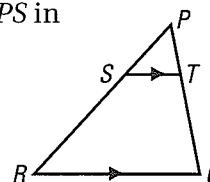
- a. Copy the diagram and mark all given information.  
 b. List two pairs of congruent angles in the diagram.  
 c. Name a pair of similar triangles and write a similarity statement.  
 d. Find  $BE$  and  $DC$ .



**REASONING** In Exercises 26–29, is it possible for  $\triangle JKL$  and  $\triangle XYZ$  to be similar? *Explain why or why not.*

26.  $m\angle J = 71^\circ$ ,  $m\angle K = 52^\circ$ ,  $m\angle X = 71^\circ$ , and  $m\angle Z = 57^\circ$   
 27.  $\triangle JKL$  is a right triangle and  $m\angle X + m\angle Y = 150^\circ$ .  
 28.  $m\angle J = 87^\circ$  and  $m\angle Y = 94^\circ$   
 29.  $m\angle J + m\angle K = 85^\circ$  and  $m\angle Y + m\angle Z = 80^\circ$

30. **CHALLENGE** If  $PT = x$ ,  $PQ = 3x$ , and  $SR = \frac{8}{3}x$ , find  $PS$  in terms of  $x$ . *Explain* your reasoning.

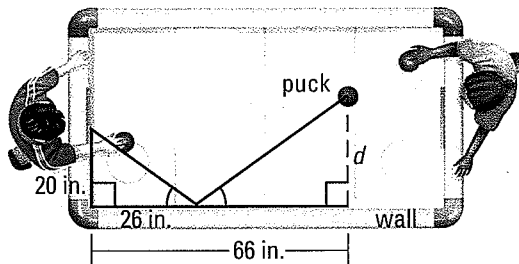


## PROBLEM SOLVING

**EXAMPLE 3**  
on p. 383  
for Exs. 31–32

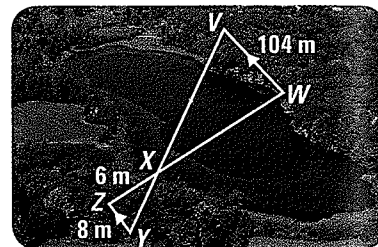
- 31. AIR HOCKEY** An air hockey player returns the puck to his opponent by bouncing the puck off the wall of the table as shown. From physics, the angles that the path of the puck makes with the wall are congruent. What is the distance  $d$  between the puck and the wall when the opponent returns it?

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- 32. LAKES** You can measure the width of the lake using a surveying technique, as shown in the diagram.

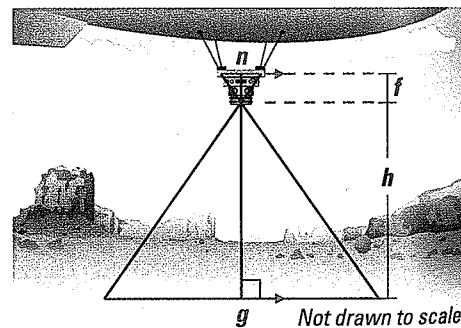
- What postulate or theorem can you use to show that the triangles are similar?
- Find the width of the lake,  $WX$ .
- If  $XY = 10$  meters, find  $VX$ .



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- 33. ★ SHORT RESPONSE** Explain why all equilateral triangles are similar. Include sketches in your answer.

- 34. AERIAL PHOTOGRAPHY** Low-level aerial photos can be taken using a remote-controlled camera suspended from a blimp. You want to take an aerial photo that covers a ground distance  $g$  of 50 meters. Use the proportion  $\frac{f}{h} = \frac{n}{g}$  to estimate the altitude  $h$  that the blimp should fly at to take the photo. In the proportion, use  $f = 8$  centimeters and  $n = 3$  centimeters. These two variables are determined by the type of camera used.



- 35. PROOF** Use the given information to draw a sketch. Then write a proof.

**GIVEN** ▶  $\triangle STU \sim \triangle PQR$

Point  $V$  lies on  $\overline{TU}$  so that  $\overline{SV}$  bisects  $\angle TSU$ .

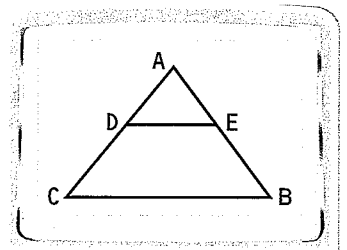
Point  $N$  lies on  $\overline{QR}$  so that  $\overline{PN}$  bisects  $\angle QPR$ .

**PROVE** ▶  $\frac{SV}{PN} = \frac{ST}{PQ}$

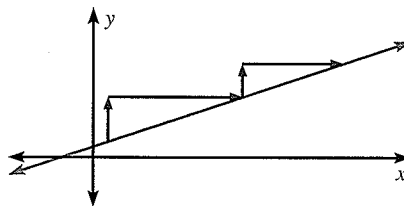
- 36. PROOF** Prove that if an acute angle in one right triangle is congruent to an acute angle in another right triangle, then the triangles are similar.

37. **TECHNOLOGY** Use a graphing calculator or computer.

- Draw  $\triangle ABC$ . Draw  $\overline{DE}$  through two sides of the triangle, parallel to the third side.
- Measure  $\angle ADE$  and  $\angle ACB$ . Measure  $\angle AED$  and  $\angle ABC$ . What do you notice?
- What does a postulate in this lesson tell you about  $\triangle ADE$  and  $\triangle ACB$ ?
- Measure all the sides. Show that corresponding side lengths are proportional.
- Move vertex  $A$  to form new triangles. How do your measurements in parts (b) and (d) change? Are the new triangles still similar? *Explain.*



38. **★ EXTENDED RESPONSE** *Explain* how you could use similar triangles to show that any two points on a line can be used to calculate its slope.



39. **CORRESPONDING LENGTHS** Without using the Corresponding Lengths Property on page 375, prove that the ratio of two corresponding angle bisectors in similar triangles is equal to the scale factor.

40. **CHALLENGE** Prove that if the lengths of two sides of a triangle are  $a$  and  $b$  respectively, then the lengths of the corresponding altitudes to those sides are in the ratio  $\frac{b}{a}$ .

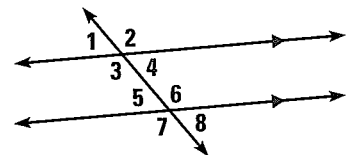
## MIXED REVIEW

### PREVIEW

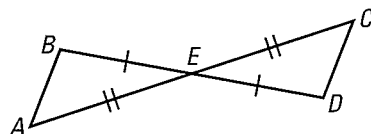
Prepare for  
Lesson 6.5  
in Exs. 41–44.

In Exercises 41–44, use the diagram.

- Name three pairs of corresponding angles. (p. 147)
- Name two pairs of alternate interior angles. (p. 147)
- Name two pairs of alternate exterior angles. (p. 147)
- Find  $m\angle 1 + m\angle 7$ . (p. 154)



45. **CONGRUENCE** Explain why  $\triangle ABE \cong \triangle CDE$ . (p. 240)



Simplify the ratio. (p. 356)

46.  $\frac{4}{20}$

47.  $\frac{36}{18}$

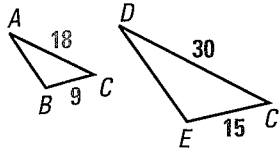
48. 21:63

49. 42:28

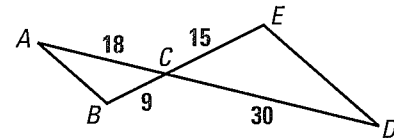
### EXAMPLE 4 Choose a method

#### VISUAL REASONING

To identify corresponding parts, redraw the triangles so that the corresponding parts have the same orientation.



Tell what method you would use to show that the triangles are similar.



#### Solution

Find the ratios of the lengths of the corresponding sides.

Shorter sides  $\frac{BC}{EC} = \frac{9}{15} = \frac{3}{5}$

Longer sides  $\frac{CA}{CD} = \frac{18}{30} = \frac{3}{5}$

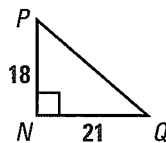
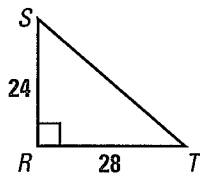
The corresponding side lengths are proportional. The included angles  $\angle ACB$  and  $\angle DCE$  are congruent because they are vertical angles. So,  $\triangle ACB \sim \triangle DCE$  by the SAS Similarity Theorem.

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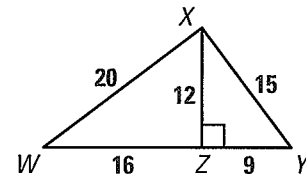
### GUIDED PRACTICE for Examples 3 and 4

Explain how to show that the indicated triangles are similar.

3.  $\triangle SRT \sim \triangle PNQ$



4.  $\triangle XZW \sim \triangle YZX$



## 6.5 EXERCISES

#### HOMEWORK KEY

$\bigcirc$  = WORKED-OUT SOLUTIONS on p. WS7 for Exs. 3, 7, and 31

$\star$  = STANDARDIZED TEST PRACTICE Exs. 2, 14, 32, 34, and 36

### SKILL PRACTICE

1. **VOCABULARY** You plan to prove that  $\triangle ACB$  is similar to  $\triangle PXQ$  by the SSS Similarity Theorem. Copy and complete the proportion that is needed to use this theorem:  $\frac{AC}{?} = \frac{?}{XQ} = \frac{AB}{?}$ .

2.  $\star$  **WRITING** If you know two triangles are similar by the SAS Similarity Theorem, what additional piece(s) of information would you need to know to show that the triangles are congruent?

#### EXAMPLES 1 and 2

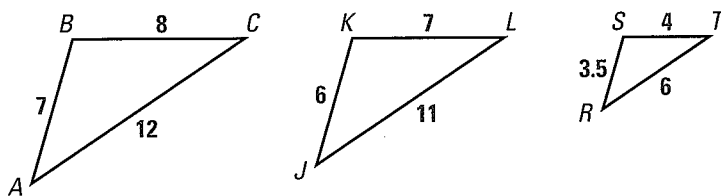
on pp. 388–389 for Exs. 3–6

**SSS SIMILARITY THEOREM** Verify that  $\triangle ABC \sim \triangle DEF$ . Find the scale factor of  $\triangle ABC$  to  $\triangle DEF$ .

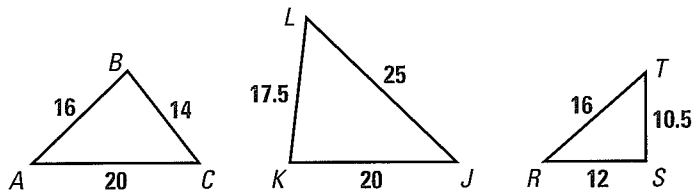
3.  $\triangle ABC$ :  $BC = 18$ ,  $AB = 15$ ,  $AC = 12$   
 $\triangle DEF$ :  $EF = 12$ ,  $DE = 10$ ,  $DF = 8$

4.  $\triangle ABC$ :  $AB = 10$ ,  $BC = 16$ ,  $CA = 20$   
 $\triangle DEF$ :  $DE = 25$ ,  $EF = 40$ ,  $FD = 50$

5. **SSS SIMILARITY THEOREM** Is either  $\triangle JKL$  or  $\triangle RST$  similar to  $\triangle ABC$ ?

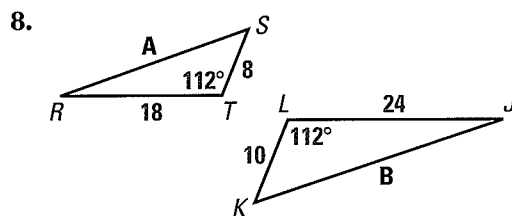
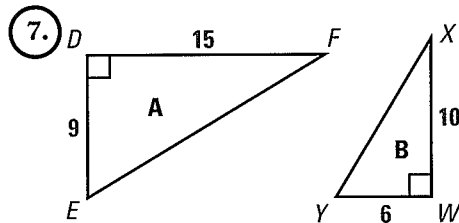


6. **SSS SIMILARITY THEOREM** Is either  $\triangle JKL$  or  $\triangle RST$  similar to  $\triangle ABC$ ?



**EXAMPLE 3**  
on p. 390  
for Exs. 7–9

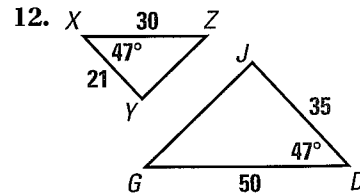
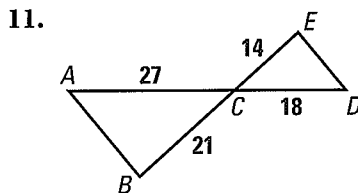
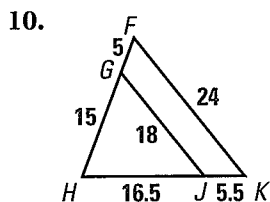
**SAS SIMILARITY THEOREM** Determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A.



9. **ALGEBRA** Find the value of  $n$  that makes  $\triangle PQR \sim \triangle XYZ$  when  $PQ = 4$ ,  $QR = 5$ ,  $XY = 4(n + 1)$ ,  $YZ = 7n - 1$ , and  $\angle Q \cong \angle Y$ . Include a sketch.

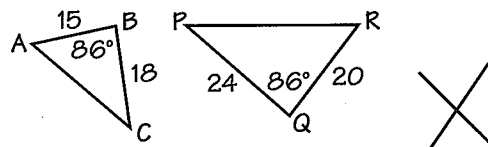
**EXAMPLE 4**  
on p. 391  
for Exs. 10–12

**SHOWING SIMILARITY** Show that the triangles are similar and write a similarity statement. *Explain your reasoning.*



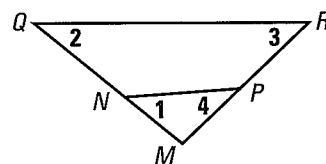
13. **ERROR ANALYSIS** Describe and correct the student's error in writing the similarity statement.

$\triangle ABC \sim \triangle PQR$  by SAS Similarity Theorem



14. **★ MULTIPLE CHOICE** In the diagram,  $\frac{MN}{MR} = \frac{MP}{MQ}$ . Which of the statements must be true?

- (A)  $\angle 1 \cong \angle 2$       (B)  $\overline{QR} \parallel \overline{NP}$   
(C)  $\angle 1 \cong \angle 4$       (D)  $\triangle MNP \sim \triangle MRQ$

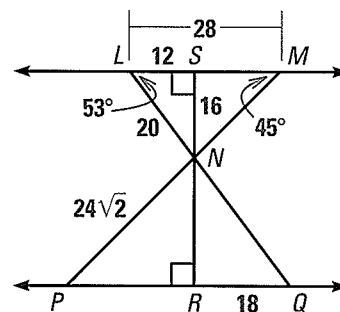


**DRAWING TRIANGLES** Sketch the triangles using the given description. Explain whether the two triangles can be similar.

- In  $\triangle XYZ$ ,  $m\angle X = 66^\circ$  and  $m\angle Y = 34^\circ$ . In  $\triangle LMN$ ,  $m\angle M = 34^\circ$  and  $m\angle N = 80^\circ$ .
- In  $\triangle RST$ ,  $RS = 20$ ,  $ST = 32$ , and  $m\angle S = 16^\circ$ . In  $\triangle FGH$ ,  $GH = 30$ ,  $HF = 48$ , and  $m\angle H = 24^\circ$ .
- The side lengths of  $\triangle ABC$  are 24,  $8x$ , and 54, and the side lengths of  $\triangle DEF$  are 15, 25, and  $7x$ .

**FINDING MEASURES** In Exercises 18–23, use the diagram to copy and complete the statements.

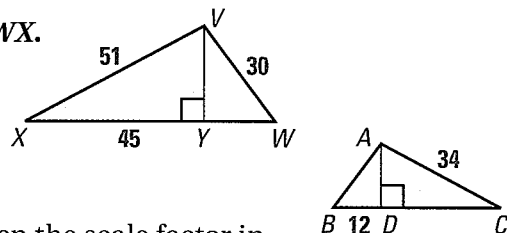
- $m\angle NQP = ?$
- $m\angle PNQ = ?$
- $PQ = ?$
- $m\angle QPN = ?$
- $RN = ?$
- $NM = ?$



- SIMILAR TRIANGLES** In the diagram at the right, name the three pairs of triangles that are similar.

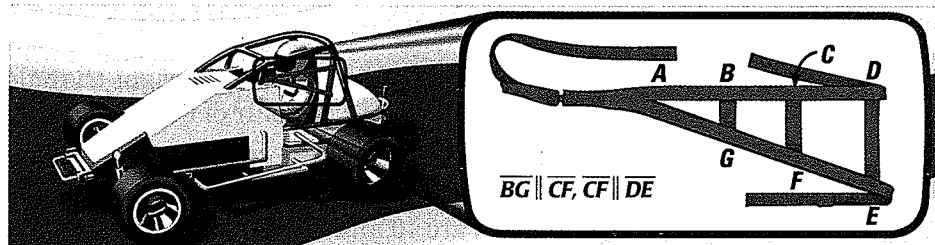
**CHALLENGE** In the figure at the right,  $\triangle ABC \sim \triangle VWX$ .

- Find the scale factor of  $\triangle VWX$  to  $\triangle ABC$ .
- Find the ratio of the area of  $\triangle VWX$  to the area of  $\triangle ABC$ .
- Make a conjecture about the relationship between the scale factor in Exercise 25 and the ratio in Exercise 26. Justify your conjecture.



## PROBLEM SOLVING

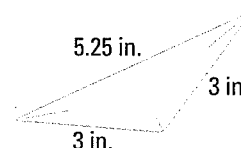
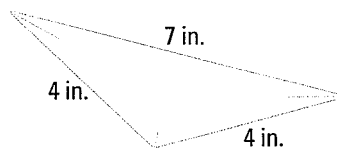
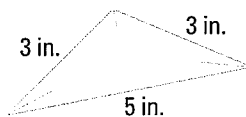
- RACECAR NET** Which postulate or theorem could you use to show that the three triangles that make up the racecar window net are similar? Explain.



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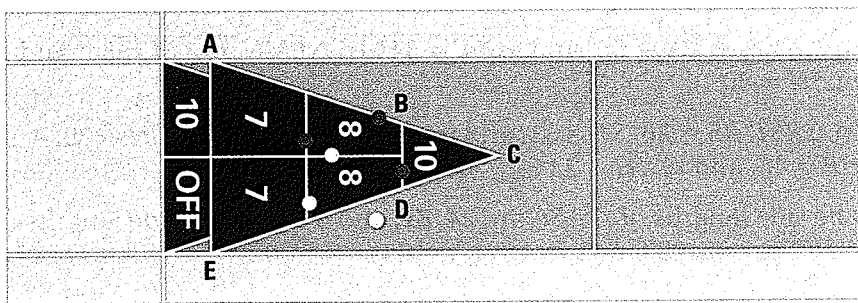
**EXAMPLE 1**  
on p. 388  
for Ex. 29

- STAINED GLASS** Certain sections of stained glass are sold in triangular beveled pieces. Which of the three beveled pieces, if any, are similar?



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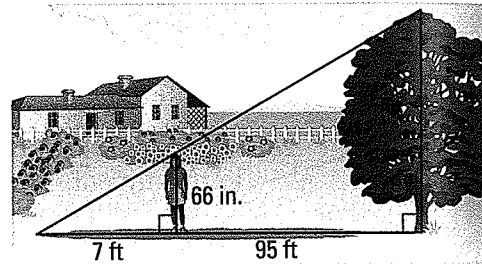
**SHUFFLEBOARD** In the portion of the shuffleboard court shown,  $\frac{BC}{AC} = \frac{BD}{AE}$ .



30. What additional piece of information do you need in order to show that  $\triangle BCD \sim \triangle ACE$  using the SSS Similarity Theorem?
31. What additional piece of information do you need in order to show that  $\triangle BCD \sim \triangle ACE$  using the SAS Similarity Theorem?
32. **★ OPEN-ENDED MATH** Use a diagram to show why there is no Side-Side-Angle Similarity Postulate.

**EXAMPLE 4**  
on p. 391  
for Ex. 33

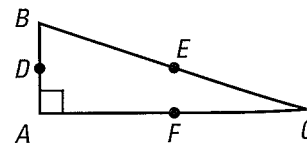
33. **MULTI-STEP PROBLEM** Ruby is standing in her back yard and she decides to estimate the height of a tree. She stands so that the tip of her shadow coincides with the tip of the tree's shadow, as shown. Ruby is 66 inches tall. The distance from the tree to Ruby is 95 feet and the distance between the tip of the shadows and Ruby is 7 feet.



- What postulate or theorem can you use to show that the triangles in the diagram are similar?
- About how tall is the tree, to the nearest foot?
- What If?** Curtis is 75 inches tall. At a different time of day, he stands so that the tip of his shadow and the tip of the tree's shadow coincide, as described above. His shadow is 6 feet long. How far is Curtis from the tree?

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34. **★ EXTENDED RESPONSE** Suppose you are given two right triangles with one pair of corresponding legs and the pair of corresponding hypotenuses having the same length ratios.
- The lengths of the given pair of corresponding legs are 6 and 18, and the lengths of the hypotenuses are 10 and 30. Use the Pythagorean Theorem to solve for the lengths of the other pair of corresponding legs. Draw a diagram.
  - Write the ratio of the lengths of the second pair of corresponding legs.
  - Are these triangles similar? Does this suggest a Hypotenuse-Leg Similarity Theorem for right triangles?
35. **PROOF** Given that  $\triangle ABC$  is a right triangle and  $D$ ,  $E$ , and  $F$  are midpoints, prove that  $m\angle DEF = 90^\circ$ .
36. **★ WRITING** Can two triangles have all pairs of corresponding angles in proportion? *Explain.*

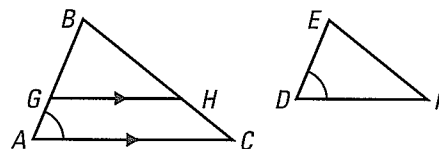




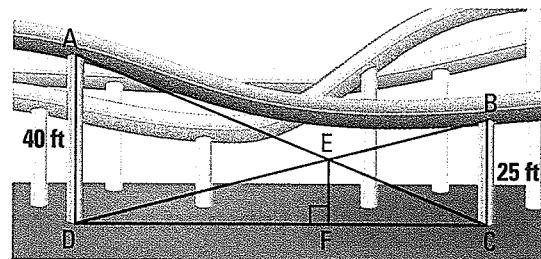
37. **PROVING THEOREM 6.3** Write a paragraph proof of the SAS Similarity Theorem.

**GIVEN**  $\angle A \cong \angle D, \frac{AB}{DE} = \frac{AC}{DF}$

**PROVE**  $\triangle ABC \sim \triangle DEF$



38. **CHALLENGE** A portion of a water slide in an amusement park is shown. Find the length of  $\overline{EF}$ . (Note: The posts form right angles with the ground.)



### MIXED REVIEW

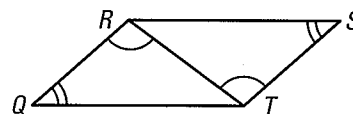
Find the slope of the line that passes through the given points. (p. 171)

39.  $(0, -8), (4, 16)$

40.  $(-2, -9), (1, -3)$

41.  $(-3, 9), (7, 2)$

42. State the postulate or theorem you would use to prove the triangles congruent. Then write a congruence statement. (p. 249)

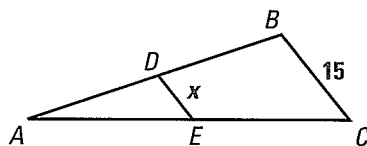


#### PREVIEW

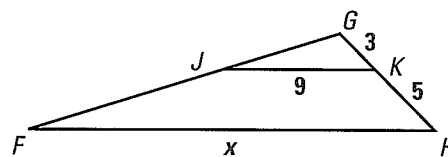
Prepare for Lesson 6.6 in Exs. 43–44.

Find the value of  $x$ .

43.  $\overline{DE}$  is a midsegment of  $\triangle ABC$ . (p. 295)



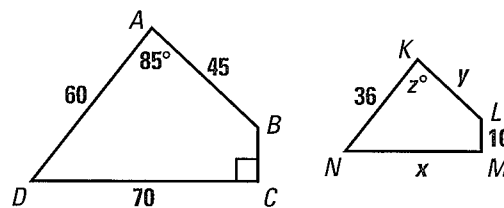
44.  $\frac{GK}{GH} = \frac{JK}{FH}$  (p. 364)



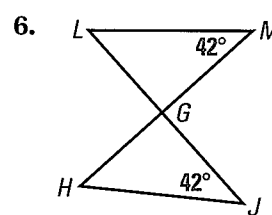
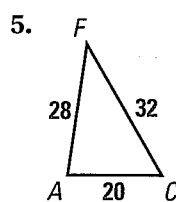
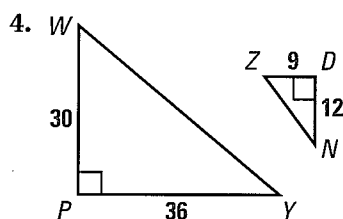
### QUIZ for Lessons 6.3–6.5

In the diagram,  $ABCD \sim KLMN$ . (p. 372)

- Find the scale factor of  $ABCD$  to  $KLMN$ .
- Find the values of  $x, y,$  and  $z$ .
- Find the perimeter of each polygon.



Determine whether the triangles are similar. If they are similar, write a similarity statement. (pp. 381, 388)



## 6.6 Investigate Proportionality

**MATERIALS** • graphing calculator or computer

**QUESTION** How can you use geometry drawing software to compare segment lengths in triangles?

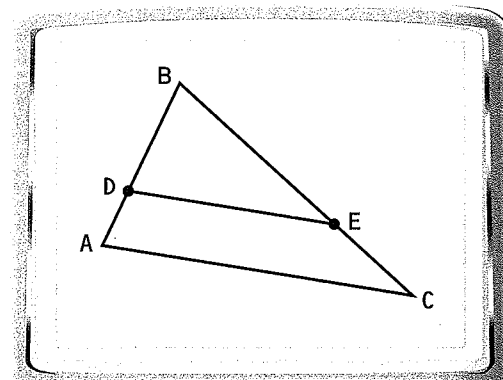
**EXPLORE 1** Construct a line parallel to a triangle's third side

**STEP 1** *Draw a triangle* Draw a triangle. Label the vertices  $A$ ,  $B$ , and  $C$ . Draw a point on  $\overline{AB}$ . Label the point  $D$ .

**STEP 2** *Draw a parallel line* Draw a line through  $D$  that is parallel to  $\overline{AC}$ . Label the intersection of the line and  $\overline{BC}$  as point  $E$ .

**STEP 3** *Measure segments* Measure  $\overline{BD}$ ,  $\overline{DA}$ ,  $\overline{BE}$ , and  $\overline{EC}$ . Calculate the ratios  $\frac{BD}{DA}$  and  $\frac{BE}{EC}$ .

**STEP 4** *Compare ratios* Move one or more of the triangle's vertices to change its shape. Compare the ratios from Step 3 as the shape changes. Save as "EXPLORE1."

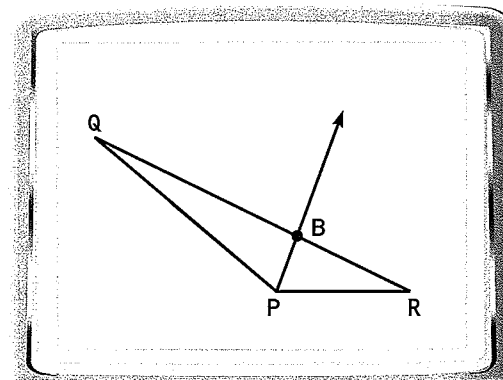


**EXPLORE 2** Construct an angle bisector of a triangle

**STEP 1** *Draw a triangle* Draw a triangle. Label the vertices  $P$ ,  $Q$ , and  $R$ . Draw the angle bisector of  $\angle QPR$ . Label the intersection of the angle bisector and  $\overline{QR}$  as point  $B$ .

**STEP 2** *Measure segments* Measure  $\overline{BR}$ ,  $\overline{RP}$ ,  $\overline{BQ}$ , and  $\overline{QP}$ . Calculate the ratios  $\frac{BR}{BQ}$  and  $\frac{RP}{QP}$ .

**STEP 3** *Compare ratios* Move one or more of the triangle's vertices to change its shape. Compare the ratios from Step 3. Save as "EXPLORE2."



**DRAW CONCLUSIONS** Use your observations to complete these exercises

1. Make a conjecture about the ratios of the lengths of the segments formed when two sides of a triangle are cut by a line parallel to the triangle's third side.
2. Make a conjecture about how the ratio of the lengths of two sides of a triangle is related to the ratio of the lengths of the segments formed when an angle bisector is drawn to the third side.

# 6.6 EXERCISES

## HOMWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS8 for Exs. 5, 9, and 21
- ★ = STANDARDIZED TEST PRACTICE Exs. 2, 8, 13, 25, and 28

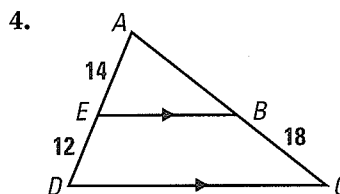
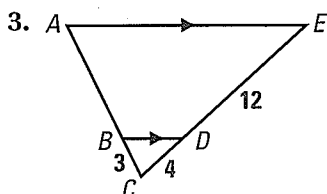
### SKILL PRACTICE

- VOCABULARY** State the Triangle Proportionality Theorem. Draw a diagram.
- ★ WRITING** Compare the Midsegment Theorem (see page 295) and the Triangle Proportionality Theorem. How are they related?

#### EXAMPLE 1

on p. 397  
for Exs. 3–4

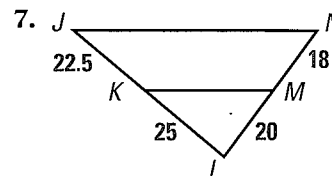
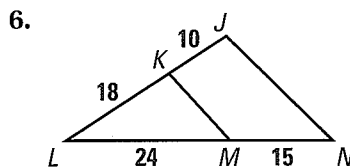
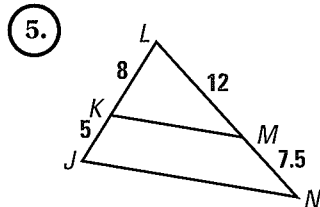
#### FINDING THE LENGTH OF A SEGMENT Find the length of $\overline{AB}$ .



#### EXAMPLE 2

on p. 398  
for Exs. 5–7

#### REASONING Use the given information to determine whether $\overline{KM} \parallel \overline{JN}$ . Explain your reasoning.

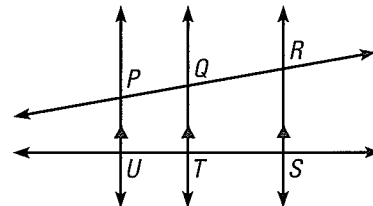


#### EXAMPLE 3

on p. 399  
for Ex. 8

8. **★ MULTIPLE CHOICE** For the figure at the right, which statement is *not* necessarily true?

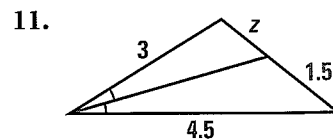
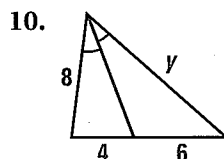
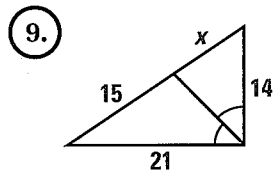
- (A)  $\frac{PQ}{QR} = \frac{UT}{TS}$       (B)  $\frac{TS}{UT} = \frac{QR}{PQ}$   
(C)  $\frac{QR}{RS} = \frac{TS}{RS}$       (D)  $\frac{PQ}{PR} = \frac{UT}{US}$



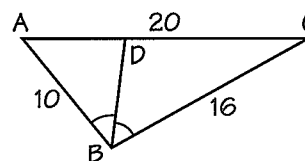
#### EXAMPLE 4

on p. 399  
for Exs. 9–12

#### ALGEBRA Find the value of the variable.



12. **ERROR ANALYSIS** A student begins to solve for the length of  $\overline{AD}$  as shown. Describe and correct the student's error.

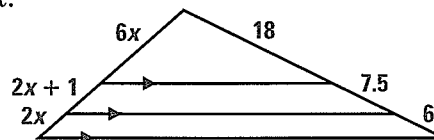


$$\frac{AB}{BC} = \frac{AD}{CD} \quad \frac{10}{16} = \frac{20 - x}{20}$$

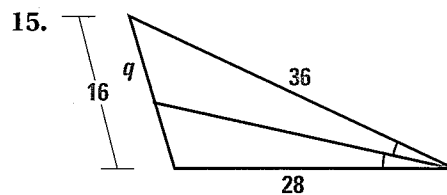
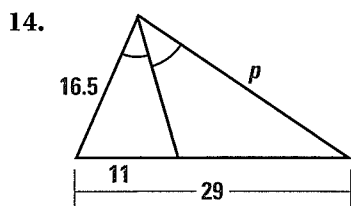


13. ★ **MULTIPLE CHOICE** Find the value of  $x$ .

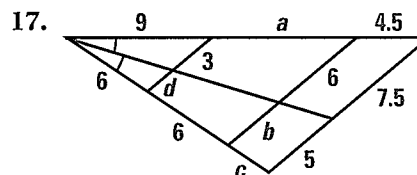
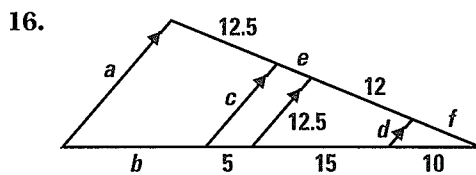
- (A)  $\frac{1}{2}$                       (B) 1  
 (C) 2                              (D) 3



14. **ALGEBRA** Find the value of the variable.

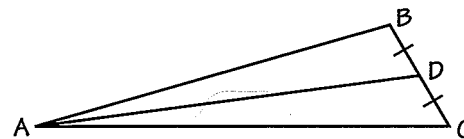


**FINDING SEGMENT LENGTHS** Use the diagram to find the value of each variable.



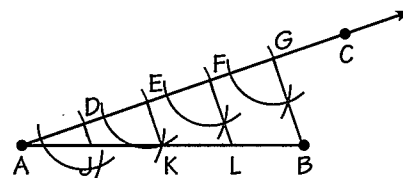
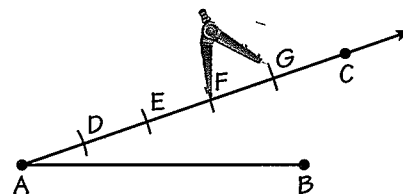
18. **ERROR ANALYSIS** A student claims that  $AB = AC$  using the method shown. Describe and correct the student's error.

By Theorem 6.7,  $\frac{BD}{CD} = \frac{AB}{AC}$ . Because  $BD = CD$ , it follows that  $AB = AC$ .

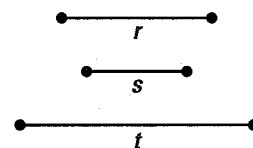


19. **CONSTRUCTION** Follow the instructions for constructing a line segment that is divided into four equal parts.

- Draw a line segment that is about 3 inches long, and label its endpoints  $A$  and  $B$ . Choose any point  $C$  not on  $\overline{AB}$ . Draw  $\overline{AC}$ .
- Using any length, place the compass point at  $A$  and make an arc intersecting  $\overline{AC}$  at  $D$ . Using the same compass setting, make additional arcs on  $\overline{AC}$ . Label the points  $E$ ,  $F$ , and  $G$  so that  $AD = DE = EF = FG$ .
- Draw  $\overline{GB}$ . Construct a line parallel to  $\overline{GB}$  through  $D$ . Continue constructing parallel lines and label the points as shown. Explain why  $AJ = JK = KL = LB$ .

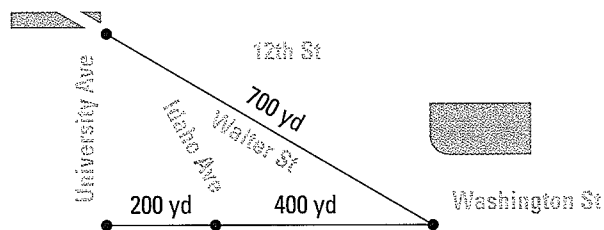


20. **CHALLENGE** Given segments with lengths  $r$ ,  $s$ , and  $t$ , construct a segment of length  $x$ , such that  $\frac{r}{s} = \frac{t}{x}$ .



## PROBLEM SOLVING

21. **CITY MAP** On the map below, Idaho Avenue bisects the angle between University Avenue and Walter Street. To the nearest yard, what is the distance along University Avenue from 12th Street to Washington Street?

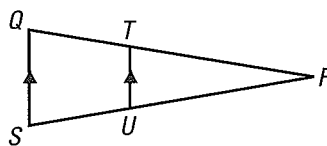


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22. **PROVING THEOREM 6.4** Prove the Triangle Proportionality Theorem.

**GIVEN** ▶  $\overline{QS} \parallel \overline{TU}$

**PROVE** ▶  $\frac{QT}{TR} = \frac{SU}{UR}$

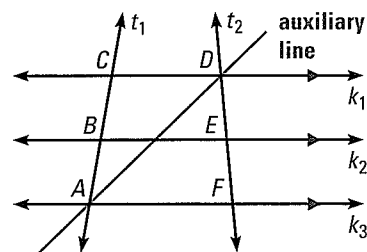


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23. **PROVING THEOREM 6.6** Use the diagram with the auxiliary line drawn to write a paragraph proof of Theorem 6.6.

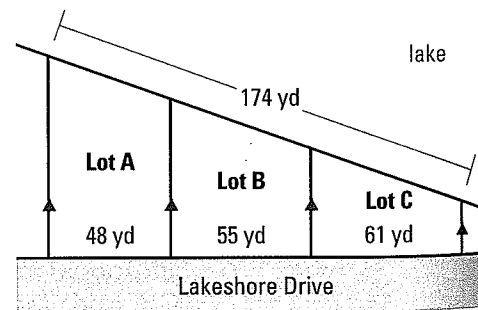
**GIVEN** ▶  $k_1 \parallel k_2, k_2 \parallel k_3$

**PROVE** ▶  $\frac{CB}{BA} = \frac{DE}{EF}$



24. **MULTI-STEP PROBLEM** The real estate term *lake frontage* refers to the distance along the edge of a piece of property that touches a lake.

- Find the lake frontage (to the nearest tenth of a yard) for each lot shown.
- In general, the more lake frontage a lot has, the higher its selling price. Which of the lots should be listed for the highest price?
- Suppose that lot prices are in the same ratio as lake frontages. If the least expensive lot is \$100,000, what are the prices of the other lots? *Explain* your reasoning.

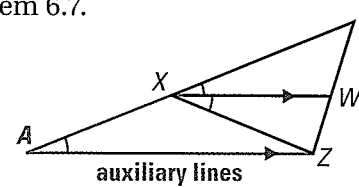


25. **★ SHORT RESPONSE** Sketch an isosceles triangle. Draw a ray that bisects the angle opposite the base. This ray divides the base into two segments. By Theorem 6.7, the ratio of the legs is proportional to the ratio of these two segments. *Explain* why this ratio is 1 : 1 for an isosceles triangle.
26. **PLAN FOR PROOF** Use the diagram given for the proof of Theorem 6.4 in Exercise 22 to write a plan for proving Theorem 6.5, the Triangle Proportionality Converse.

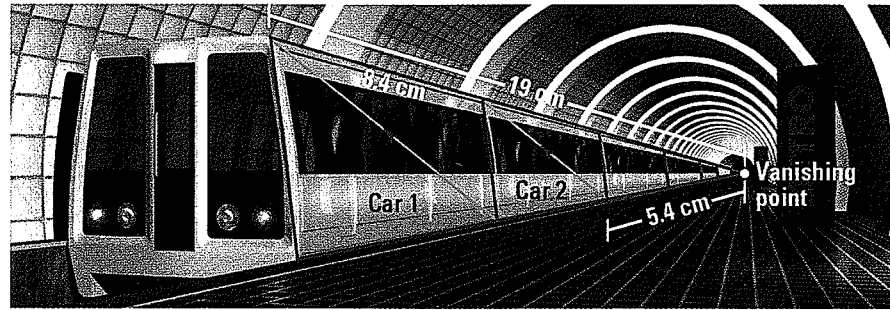
27. **PROVING THEOREM 6.7** Use the diagram with the auxiliary lines drawn to write a paragraph proof of Theorem 6.7.

**GIVEN**  $\angle YXW \cong \angle WXZ$

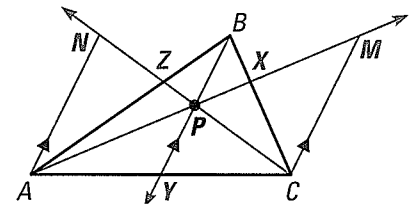
**PROVE**  $\frac{YW}{WZ} = \frac{XY}{XZ}$



28. **★ EXTENDED RESPONSE** In *perspective drawing*, lines that are parallel in real life must meet at a vanishing point on the horizon. To make the train cars in the drawing appear equal in length, they are drawn so that the lines connecting the opposite corners of each car are parallel.



- Use the dimensions given and the red parallel lines to find the length of the bottom edge of the drawing of Car 2.
  - What other set of parallel lines exist in the figure? *Explain* how these can be used to form a set of similar triangles.
  - Find the length of the top edge of the drawing of Car 2.
29. **CHALLENGE** Prove *Ceva's Theorem*: If  $P$  is any point inside  $\triangle ABC$ , then  $\frac{AY}{YC} \cdot \frac{CZ}{XB} \cdot \frac{BZ}{ZA} = 1$ . (*Hint*: Draw lines parallel to  $\overline{BY}$  through  $A$  and  $C$ . Apply Theorem 6.4 to  $\triangle ACM$ . Show that  $\triangle APN \sim \triangle MPC$ ,  $\triangle CXM \sim \triangle BXP$ , and  $\triangle BZP \sim \triangle AZN$ .)



## MIXED REVIEW

### PREVIEW

Prepare for  
Lesson 6.7 in  
Exs. 30–36.

Perform the following operations. Then simplify.

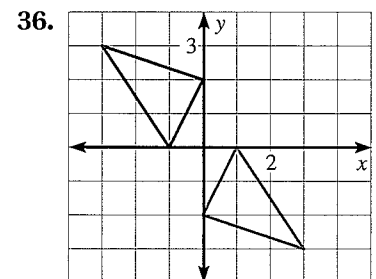
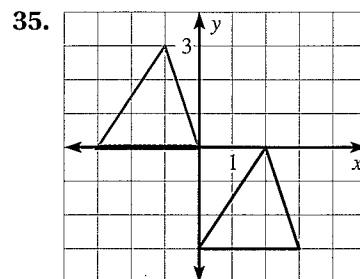
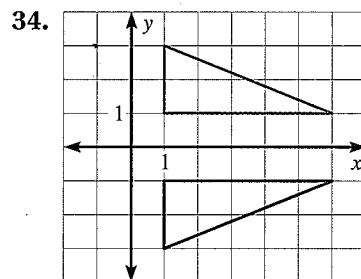
30.  $(-3) \cdot \frac{7}{2}$  (p. 869)

31.  $\frac{4}{3} \cdot \frac{1}{2}$  (p. 869)

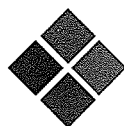
32.  $5\left(\frac{1}{2}\right)^2$  (p. 871)

33.  $\left(\frac{5}{4}\right)^3$  (p. 871)

*Describe* the translation in words and write the coordinate rule for the translation. (p. 272)



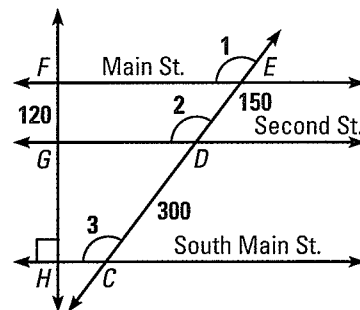
**Another Way to Solve Example 3, page 399**



**MULTIPLE REPRESENTATIONS** In Lesson 6.6, you used proportionality theorems to find lengths of segments formed when transversals intersect two or more parallel lines. Now, you will learn two different ways to solve Example 3 on page 399.

**PROBLEM**

**CITY TRAVEL** In the diagram,  $\angle 1$ ,  $\angle 2$ , and  $\angle 3$  are all congruent and  $GF = 120$  yards,  $DE = 150$  yards, and  $CD = 300$  yards. Find the distance  $HF$  between Main Street and South Main Street.



**METHOD 1**

**Applying a Ratio** One alternative approach is to look for ratios in the diagram.

**STEP 1** Read the problem. Because Main Street, Second Street, and South Main Street are all parallel, the lengths of the segments of the cross streets will be in proportion, so they have the same ratio.

**STEP 2** Apply a ratio. Notice that on  $\overleftrightarrow{CE}$ , the distance  $CD$  between South Main Street and Second Street is twice the distance  $DE$  between Second Street and Main Street. So the same will be true for the distances  $HG$  and  $GF$ .

$HG = 2 \cdot GF$	<b>Write equation.</b>
$= 2 \cdot 120$	<b>Substitute.</b>
$= 240$	<b>Simplify.</b>

**STEP 3** Calculate the distance. Line  $HF$  is perpendicular to both Main Street and South Main Street, so the distance between Main Street and South Main Street is this perpendicular distance,  $HF$ .

$HF = HG + GF$	<b>Segment Addition Postulate</b>
$= 120 + 240$	<b>Substitute.</b>
$= 360$	<b>Simplify.</b>

**STEP 4** Check page 399 to verify your answer, and confirm that it is the same.

## METHOD 2

**Writing a Proportion** Another alternative approach is to use a graphic organizer to set up a proportion.

**STEP 1** Make a table to compare the distances.

	$\overline{CE}$	$\overline{HF}$
<b>Total distance</b>	300 + 150, or 450	$x$
<b>Partial distance</b>	150	120

**STEP 2** Write and solve a proportion.

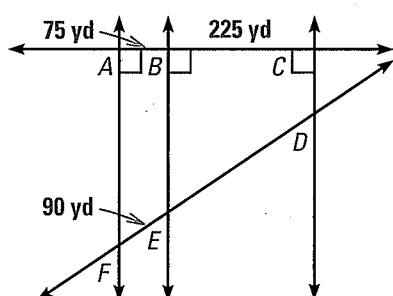
$$\frac{450}{150} = \frac{x}{120} \quad \text{Write proportion.}$$

$$360 = x \quad \text{Multiply each side by 120 and simplify.}$$

► The distance is 360 yards.

## PRACTICE

1. **MAPS** Use the information on the map.



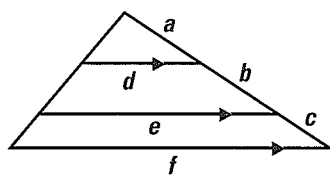
- Find  $DE$ .
- What if?** Suppose there is an alley one fourth of the way from  $\overline{BE}$  to  $\overline{CD}$  and parallel to  $\overline{BE}$ . What is the distance from  $E$  to the alley along  $\overline{FD}$ ?

2. **REASONING** Given the diagram below, explain why the three given proportions are true.

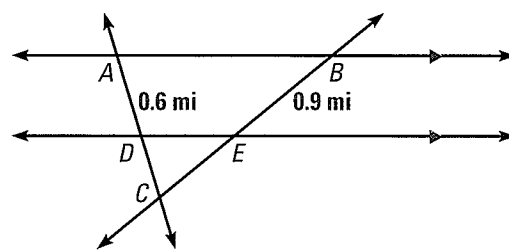
$$\frac{a}{a+b} = \frac{d}{e}$$

$$\frac{a}{a+b+c} = \frac{d}{f}$$

$$\frac{a+b}{a+b+c} = \frac{e}{f}$$

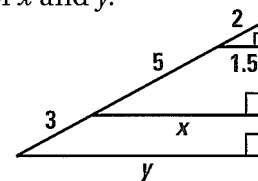


3. **WALKING** Two people leave points  $A$  and  $B$  at the same time. They intend to meet at point  $C$  at the same time. The person who leaves point  $A$  walks at a speed of 3 miles per hour. How fast must the person who leaves point  $B$  walk?



4. **ERROR ANALYSIS** A student who attempted to solve the problem in Exercise 3 claims that you need to know the length of  $AC$  to solve the problem. Describe and correct the error that the student made.

5. **ALGEBRA** Use the diagram to find the values of  $x$  and  $y$ .





## Extension

Use after Lesson 6.6

# Fractals

**GOAL** Explore the properties of fractals.

### Key Vocabulary

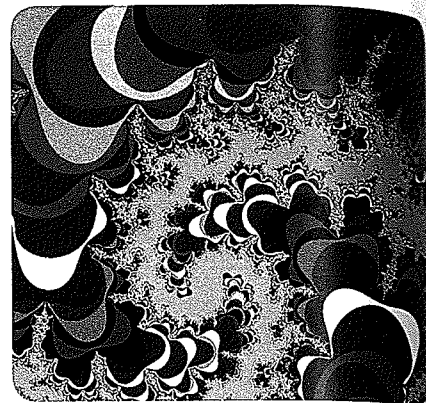
- fractal
- self-similarity
- iteration

### HISTORY NOTE

Computers made it easier to study mathematical iteration by reducing the time needed to perform calculations. Using fractals, mathematicians have been able to create better models of coastlines, clouds, and other natural objects.

A **fractal** is an object that is *self-similar*. An object is **self-similar** if one part of the object can be enlarged to look like the whole object. In nature, fractals can be found in ferns and branches of a river. Scientists use fractals to map out clouds in order to predict rain.

Many fractals are formed by a repetition of a sequence of the steps called **iteration**. The first stage of drawing a fractal is considered Stage 0. Helge van Koch (1870–1924) described a fractal known as the *Koch snowflake*, shown in Example 1.



A Mandelbrot fractal

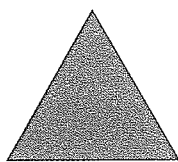
### EXAMPLE 1 Draw a fractal

Use the directions below to draw a Koch snowflake.

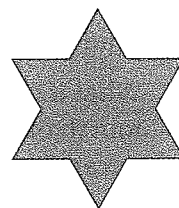
Starting with an equilateral triangle, at each stage each side is divided into thirds and a new equilateral triangle is formed using the middle third as the triangle side length.

#### Solution

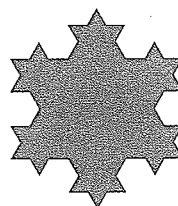
**STAGE 0** Draw an equilateral triangle with a side length of one unit.



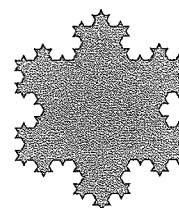
**STAGE 1** Replace the middle third of each side with an equilateral triangle.



**STAGE 2** Repeat Stage 1 with the six smaller equilateral triangles.



**STAGE 3** Repeat Stage 1 with the eighteen smaller equilateral triangles.



**MEASUREMENT** Benoit Mandelbrot (b. 1924) was the first mathematician to formalize the idea of fractals when he observed methods used to measure the lengths of coastlines. Coastlines cannot be measured as straight lines because of the inlets and rocks. Mandelbrot used fractals to model coastlines.

### EXAMPLE 2 Find lengths in a fractal

Make a table to study the lengths of the sides of a Koch snowflake at different stages.

Stage number	Edge length	Number of edges	Perimeter
0	1	3	3
1	$\frac{1}{3}$	$3 \cdot 4 = 12$	4
2	$\frac{1}{9}$	$12 \cdot 4 = 48$	$\frac{48}{9} = 5\frac{1}{3}$
3	$\frac{1}{27}$	$48 \cdot 4 = 192$	$\frac{192}{27} = 7\frac{1}{9}$
$n$	$\frac{1}{3^n}$	$3 \cdot 4^n$	$\frac{4^n}{3^{n-1}}$

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### PRACTICE

**EXAMPLES**  
1 and 2  
for Exs. 1–3

- PERIMETER** Find the ratio of the edge length of the triangle in Stage 0 of a Koch snowflake to the edge length of the triangle in Stage 1. How is the perimeter of the triangle in Stage 0 related to the perimeter of the triangle in Stage 1? *Explain.*
- MULTI-STEP PROBLEM** Use the *Cantor set*, which is a fractal whose iteration consists of dividing a segment into thirds and erasing the middle third.
  - Draw Stage 0 through Stage 5 of the Cantor set. Stage 0 has a length of one unit.
  - Make a table showing the stage number, number of segments, segment length, and total length of the Cantor set.
  - What is the total length of the Cantor set at Stage 10? Stage 20? Stage  $n$ ?
- EXTENDED RESPONSE** A *Sierpinski carpet* starts with a square with side length one unit. At each stage, divide the square into nine equal squares with the middle square shaded a different color.
  - Draw Stage 0 through Stage 3 of a Sierpinski carpet.
  - Explain* why the carpet is said to be *self-similar* by comparing the upper left hand square to the whole square.
  - Make a table to find the total area of the colored squares at Stage 3.

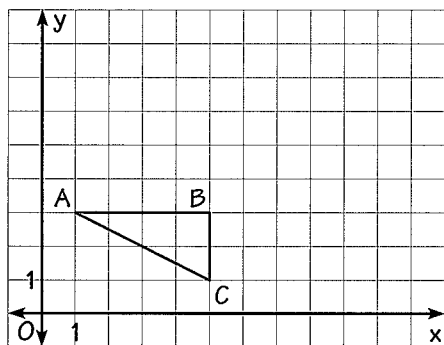
## 6.7 Dilations

**MATERIALS** • graph paper • straightedge • compass • ruler

**QUESTION** How can you construct a similar figure?

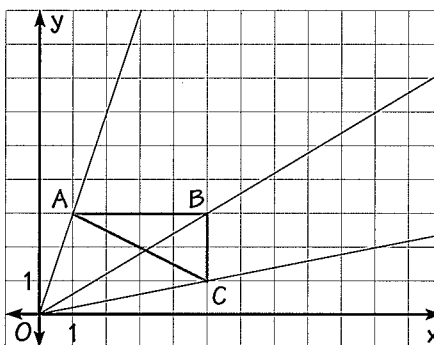
**EXPLORE** Construct a similar triangle

**STEP 1**



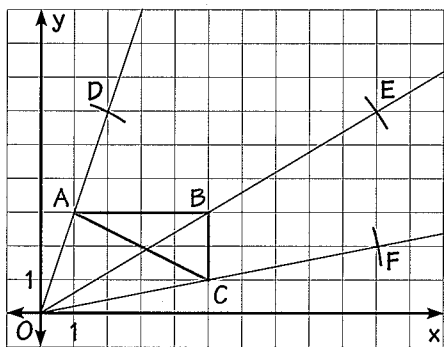
**Draw a triangle** Plot the points  $A(1, 3)$ ,  $B(5, 3)$ , and  $C(5, 1)$  in a coordinate plane. Draw  $\triangle ABC$ .

**STEP 2**



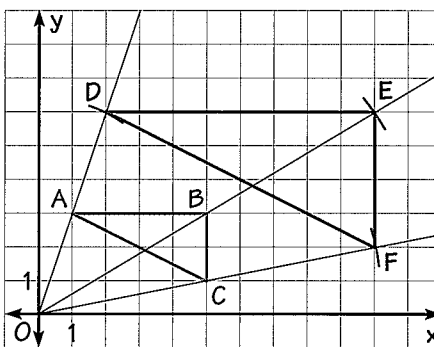
**Draw rays** Using the origin as an endpoint  $O$ , draw  $\overrightarrow{OA}$ ,  $\overrightarrow{OB}$ , and  $\overrightarrow{OC}$ .

**STEP 3**



**Draw equal segments** Use a compass to mark a point  $D$  on  $\overrightarrow{OA}$  so  $OA = AD$ . Mark a point  $E$  on  $\overrightarrow{OB}$  so  $OB = BE$ . Mark a point  $F$  on  $\overrightarrow{OC}$  so  $OC = CF$ .

**STEP 4**



**Draw the image** Connect points  $D$ ,  $E$ , and  $F$  to form a right triangle.

**DRAW CONCLUSIONS** Use your observations to complete these exercises

1. Measure  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{DE}$ , and  $\overline{EF}$ . Calculate the ratios  $\frac{DE}{AB}$  and  $\frac{EF}{BC}$ . Using this information, show that the two triangles are similar.
2. Repeat the steps in the Explore to construct  $\triangle GHJ$  so that  $3 \cdot OA = AG$ ,  $3 \cdot OB = BH$ , and  $3 \cdot OC = CJ$ .

# 6.7 EXERCISES

**HOMEWORK KEY**

○ = WORKED-OUT SOLUTIONS  
on p. WS8 for Exs. 5, 11, and 27

★ = STANDARDIZED TEST PRACTICE  
Exs. 2, 13, 21, 22, 28, 30, and 31

## SKILL PRACTICE

- VOCABULARY** Copy and complete: In a dilation, the image is ? to the original figure.
- ★ **WRITING** Explain how to find the scale factor of a dilation. How do you know whether a dilation is an enlargement or a reduction?

### EXAMPLES 1 and 2

on pp. 409–410  
for Exs. 3–8

**DRAWING DILATIONS** Draw a dilation of the polygon with the given vertices using the given scale factor  $k$ .

3.  $A(-2, 1), B(-4, 1), C(-2, 4); k = 2$

4.  $A(-5, 5), B(-5, -10), C(10, 0); k = \frac{3}{5}$

5.  $A(1, 1), B(6, 1), C(6, 3); k = 1.5$

6.  $A(2, 8), B(8, 8), C(16, 4); k = 0.25$

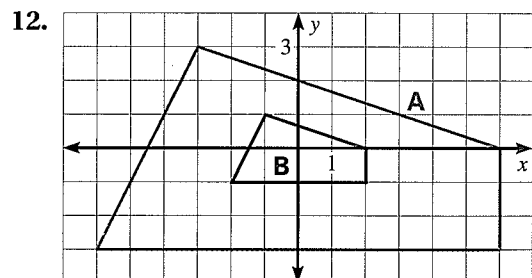
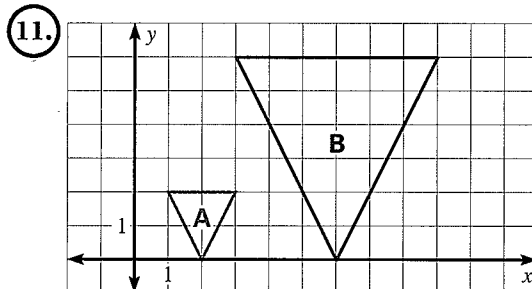
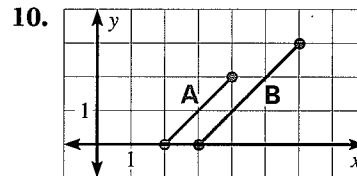
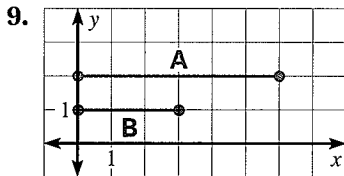
7.  $A(-8, 0), B(0, 8), C(4, 0), D(0, -4); k = \frac{3}{8}$

8.  $A(0, 0), B(0, 3), C(2, 4), D(2, -1); k = \frac{13}{2}$

### EXAMPLE 3

on p. 410  
for Exs. 9–12

**IDENTIFYING DILATIONS** Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.

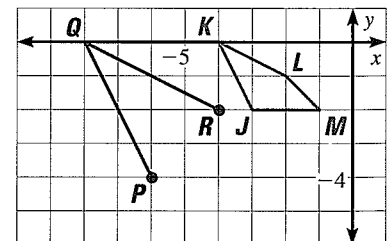


### EXAMPLE 4

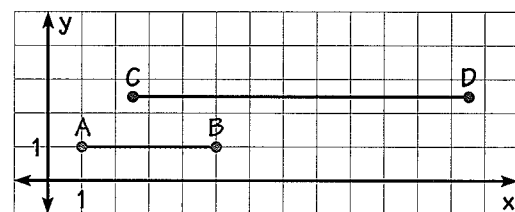
on p. 411  
for Ex. 13

13. ★ **MULTIPLE CHOICE** You want to create a quadrilateral  $PQRS$  that is similar to quadrilateral  $JKLM$ . What are the coordinates of  $S$ ?

- (A) (2, 4)                      (B) (4, -2)  
(C) (-2, -4)                  (D) (-4, -2)



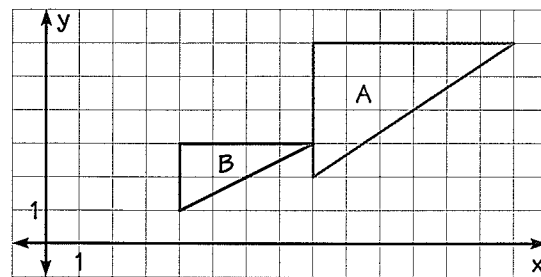
14. **ERROR ANALYSIS** A student found the scale factor of the dilation from  $\overline{AB}$  to  $\overline{CD}$  to be  $\frac{2}{5}$ . Describe and correct the student's error.



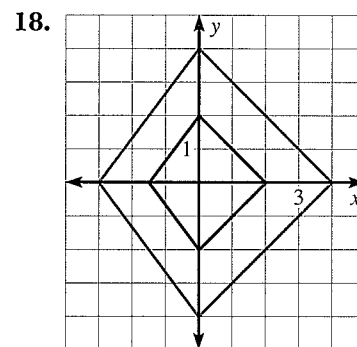
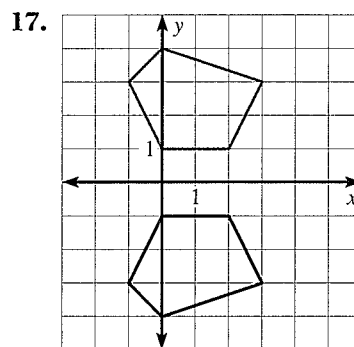
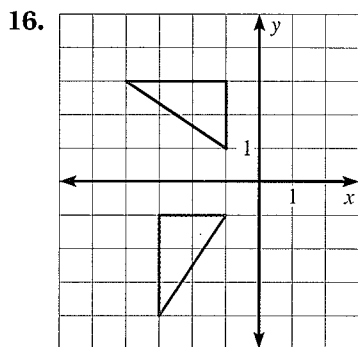
$\frac{AB}{CD} = \frac{2}{5}$



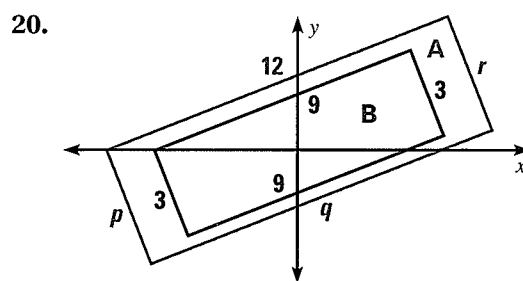
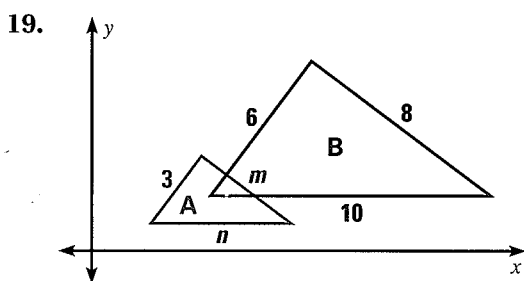
15. **ERROR ANALYSIS** A student says that the figure shown represents a dilation. What is wrong with this statement?



**IDENTIFYING TRANSFORMATIONS** Determine whether the transformation shown is a *translation*, *reflection*, *rotation*, or *dilation*.

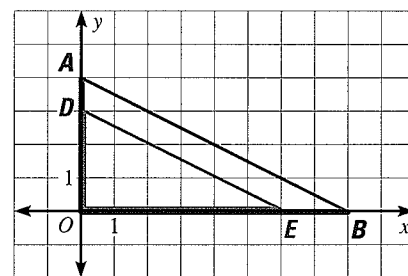


**FINDING SCALE FACTORS** Find the scale factor of the dilation of Figure A to Figure B. Then give the unknown lengths of Figure A.



21. **★ MULTIPLE CHOICE** In the diagram shown,  $\triangle ABO$  is a dilation of  $\triangle DEO$ . The length of a median of  $\triangle ABO$  is what percent of the length of the corresponding median of  $\triangle DEO$ ?

- (A) 50%                      (B) 75%  
 (C)  $133\frac{1}{3}\%$                 (D) 200%



22. **★ SHORT RESPONSE** Suppose you dilate a figure using a scale factor of 2. Then, you dilate the image using a scale factor of  $\frac{1}{2}$ . Describe the size and shape of this new image.

**CHALLENGE** Describe the two transformations, the first followed by the second, that combined will transform  $\triangle ABC$  into  $\triangle DEF$ .


23.  $A(-3, 3), B(-3, 1), C(0, 1)$   
 $D(6, 6), E(6, 2), F(0, 2)$

24.  $A(6, 0), B(9, 6), C(12, 6)$   
 $D(0, 3), E(1, 5), F(2, 5)$

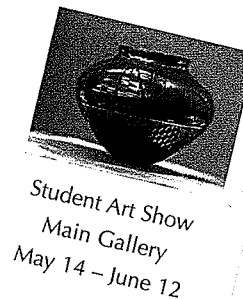
## PROBLEM SOLVING


**EXAMPLE 3**  
on p. 410 for  
Exs. 25–27

- 25. BILLBOARD ADVERTISEMENT** A billboard advertising agency requires each advertisement to be drawn so that it fits in a 12-inch by 6-inch rectangle. The agency uses a scale factor of 24 to enlarge the advertisement to create the billboard. What are the dimensions of a billboard, in feet?

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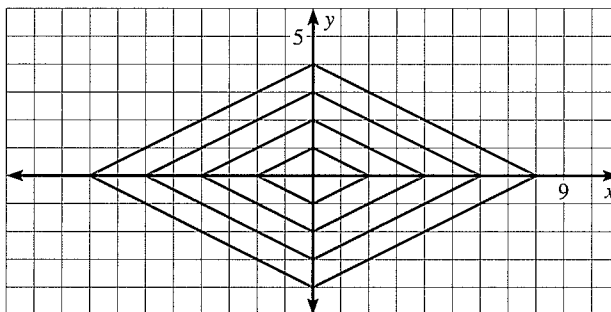
- 26. POTTERY** Your pottery is used on a poster for a student art show. You want to make postcards using the same image. On the poster, the image is 8 inches in width and 6 inches in height. If the image on the postcard can be 5 inches wide, what scale should you use for the image on the postcard?



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- 27. SHADOWS** You and your friend are walking at night. You point a flashlight at your friend, and your friend's shadow is cast on the building behind him. The shadow is an enlargement, and is 15 feet tall. Your friend is 6 feet tall. What is the scale factor of the enlargement?

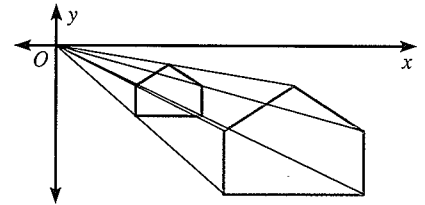
- 28. ★ OPEN-ENDED MATH** Describe how you can use dilations to create the figure shown below.



 at classzone.com

- 29. MULTI-STEP PROBLEM**  $\triangle ABC$  has vertices  $A(3, -3)$ ,  $B(3, 6)$ , and  $C(15, 6)$ .
- Draw a dilation of  $\triangle ABC$  using a scale factor of  $\frac{2}{3}$ .
  - Find the ratio of the perimeter of the image to the perimeter of the original figure. How does this ratio compare to the scale factor?
  - Find the ratio of the area of the image to the area of the original figure. How does this ratio compare to the scale factor?
- 30. ★ EXTENDED RESPONSE** Look at the coordinate notation for a dilation on page 409. Suppose the definition of dilation allowed  $k < 0$ .
- Describe the dilation if  $-1 < k < 0$ .
  - Describe the dilation if  $k < -1$ .
  - Use a rotation to describe a dilation with  $k = -1$ .

31. ★ **SHORT RESPONSE** Explain how you can use dilations to make a perspective drawing with the center of dilation as a vanishing point. Draw a diagram.



32. **MIDPOINTS** Let  $\overline{XY}$  be a dilation of  $\overline{PQ}$  with scale factor  $k$ . Show that the image of the midpoint of  $\overline{PQ}$  is the midpoint of  $\overline{XY}$ .

33. **REASONING** In Exercise 32, show that  $\overline{XY} \parallel \overline{PQ}$ .

34. **CHALLENGE** A rectangle has vertices  $A(0, 0)$ ,  $B(0, 6)$ ,  $C(9, 6)$ , and  $D(9, 0)$ . Explain how to dilate the rectangle to produce an image whose area is twice the area of the original rectangle. Make a conjecture about how to dilate any polygon to produce an image whose area is  $n$  times the area of the original polygon.

### MIXED REVIEW

Simplify the expression. (p. 873)

35.  $(3x + 2)^2 + (x - 5)^2$

36.  $4\left(\frac{1}{2}ab\right) + (b - a)^2$

37.  $(a + b)^2 - (a - b)^2$

Find the distance between each pair of points. (p. 15)

38.  $(0, 5)$  and  $(4, 3)$

39.  $(-3, 0)$  and  $(2, 4)$

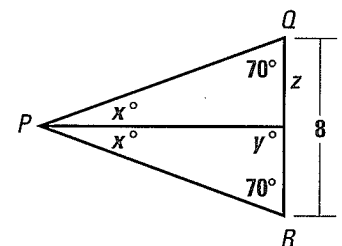
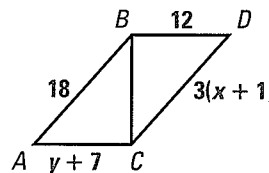
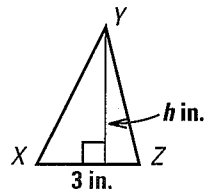
40.  $(-2, -4)$  and  $(3, -2)$

Find the value(s) of the variable(s).

41. Area = 6 in.<sup>2</sup> (p. 49)

42.  $\triangle ABC \cong \triangle DCB$  (p. 256)

43.  $\triangle PQR$  is isosceles. (p. 303)

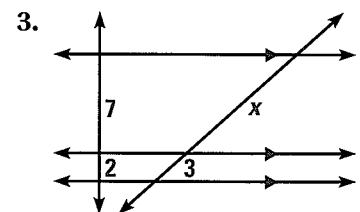
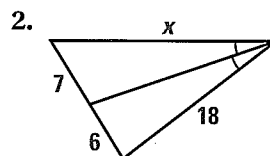
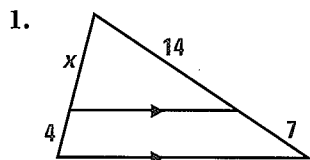


#### PREVIEW

Prepare for Lesson 7.1 in Exs. 41–43.

### QUIZ for Lessons 6.6–6.7

Find the value of  $x$ . (p. 397)



Draw a dilation of  $\triangle ABC$  with the given vertices and scale factor  $k$ . (p. 409)

4.  $A(-5, 5)$ ,  $B(-5, -10)$ ,  $C(10, 0)$ ;  $k = 0.4$

5.  $A(-2, 1)$ ,  $B(-4, 1)$ ,  $C(-2, 4)$ ;  $k = 2.5$

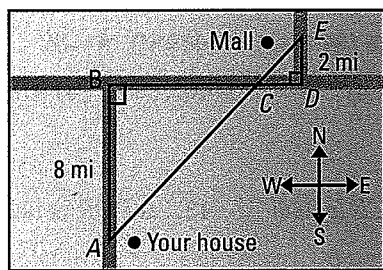


## Lessons 6.4–6.7

1. **OPEN-ENDED** The diagram shows the front of a house. What information would you need in order to show that  $\triangle WXY \sim \triangle VXZ$  using the SAS Similarity Theorem?

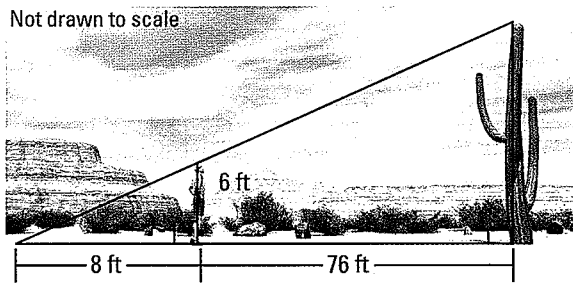


2. **EXTENDED RESPONSE** You leave your house to go to the mall. You drive due north 8 miles, due east 7.5 miles, and due north again 2 miles.

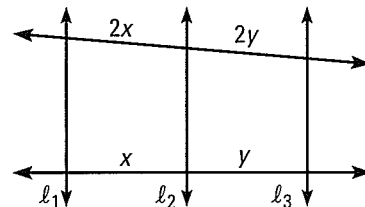


- Explain how to prove that  $\triangle ABC \sim \triangle EDC$ .
- Find  $CD$ .
- Find  $AE$ , the distance between your house and the mall.

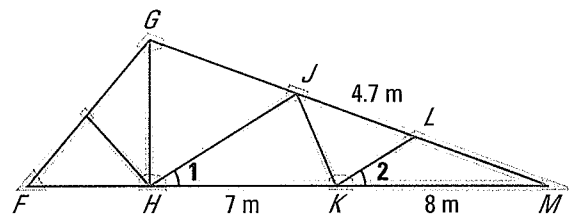
3. **SHORT RESPONSE** The Cardon cactus found in the Sonoran Desert in Mexico is the tallest type of cactus in the world. Marco stands 76 feet from the cactus so that his shadow coincides with the cactus' shadow. Marco is 6 feet tall and his shadow is 8 feet long. How tall is the Cardon cactus? *Explain.*



4. **SHORT RESPONSE** In the diagram, is it *always*, *sometimes*, or *never* true that  $l_1 \parallel l_2 \parallel l_3$ ? *Explain.*



5. **GRIDDED ANSWER** In the diagram of the roof truss,  $HK = 7$  meters,  $KM = 8$  meters,  $JL = 4.7$  meters, and  $\angle 1 \cong \angle 2$ . Find  $LM$  to the nearest tenth of a meter.



6. **GRIDDED ANSWER** You are designing a catalog for a greeting card company.

The catalog features a  $2\frac{4}{5}$  inch by 2 inch photograph of each card. The actual dimensions of a greeting card are 7 inches by 5 inches. What is the scale factor of the reduction?

7. **MULTI-STEP PROBLEM** Rectangle  $ABCD$  has vertices  $A(2, 2)$ ,  $B(4, 2)$ ,  $C(4, -4)$ , and  $D(2, -4)$ .

- Draw rectangle  $ABCD$ . Then draw a dilation of rectangle  $ABCD$  using a scale factor of  $\frac{5}{4}$ . Label the image  $PQRS$ .
- Find the ratio of the perimeter of the image to the perimeter of the original figure. How does this ratio compare to the scale factor?
- Find the ratio of the area of the image to the area of the original figure. How does this ratio compare to the scale factor?



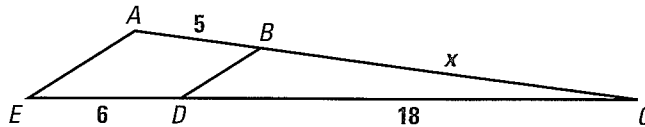
## BIG IDEAS

For Your Notebook

## Big Idea 1

## Using Ratios and Proportions to Solve Geometry Problems

You can use properties of proportions to solve a variety of algebraic and geometric problems.



For example, in the diagram above, suppose you know that  $\frac{AB}{BC} = \frac{ED}{DC}$ . Then you can write any of the following relationships.

$$\frac{5}{x} = \frac{6}{18}$$

$$5 \cdot 18 = 6x$$

$$\frac{x}{5} = \frac{18}{6}$$

$$\frac{5}{6} = \frac{x}{18}$$

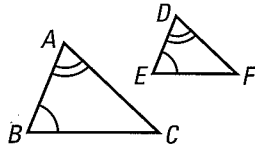
$$\frac{5+x}{x} = \frac{6+18}{18}$$

## Big Idea 2

## Showing that Triangles are Similar

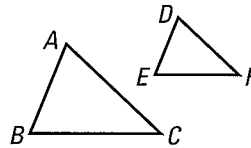
You learned three ways to prove two triangles are similar.

## AA Similarity Postulate



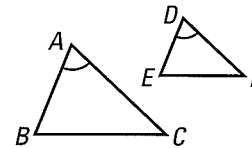
If  $\angle A \cong \angle D$  and  $\angle B \cong \angle E$ , then  $\triangle ABC \sim \triangle DEF$ .

## SSS Similarity Theorem



If  $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ , then  $\triangle ABC \sim \triangle DEF$ .

## SAS Similarity Theorem



If  $\angle A \cong \angle D$  and  $\frac{AB}{DE} = \frac{AC}{DF}$ , then  $\triangle ABC \sim \triangle DEF$ .

## Big Idea 3

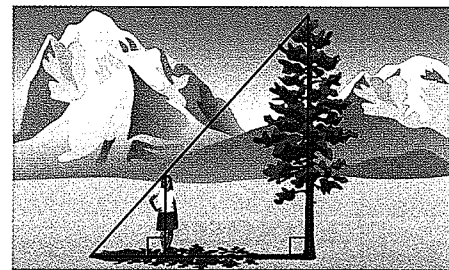
## Using Indirect Measurement and Similarity

You can use triangle similarity theorems to apply indirect measurement in order to find lengths that would be inconvenient or impossible to measure directly.

Consider the diagram shown. Because the two triangles formed by the person and the tree are similar by the AA Similarity Postulate, you can write the following proportion to find the height of the tree.

$$\frac{\text{height of person}}{\text{length of person's shadow}} = \frac{\text{height of tree}}{\text{length of tree's shadow}}$$

You also learned about dilations, a type of similarity transformation. In a dilation, a figure is either enlarged or reduced in size.



# 6

# CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

## REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- ratio, p. 356
- proportion, p. 358  
means, extremes
- geometric mean, p. 359
- scale drawing, p. 365
- scale, p. 365
- similar polygons, p. 372
- scale factor of two similar polygons, p. 373
- dilation, p. 409
- center of dilation, p. 409
- scale factor of a dilation, p. 409
- reduction, p. 409
- enlargement, p. 409

## VOCABULARY EXERCISES

Copy and complete the statement.

1. A ? is a transformation in which the original figure and its image are similar.
2. If  $\triangle PQR \sim \triangle XYZ$ , then  $\frac{PQ}{XY} = \frac{?}{YZ} = \frac{?}{?}$ .
3. **WRITING** Describe the relationship between a ratio and a proportion. Give an example of each.

## REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 6.

### 6.1 Ratios, Proportions, and the Geometric Mean

pp. 356–363

#### EXAMPLE

The measures of the angles in  $\triangle ABC$  are in the extended ratio of 3:4:5. Find the measures of the angles.

Use the extended ratio of 3:4:5 to label the angle measures as  $3x^\circ$ ,  $4x^\circ$ , and  $5x^\circ$ .

$$3x^\circ + 4x^\circ + 5x^\circ = 180^\circ \quad \text{Triangle Sum Theorem}$$

$$12x = 180 \quad \text{Combine like terms.}$$

$$x = 15 \quad \text{Divide each side by 12.}$$

So, the angle measures are  $3(15^\circ) = 45^\circ$ ,  $4(15^\circ) = 60^\circ$ , and  $5(15^\circ) = 75^\circ$ .

#### EXERCISES

4. The length of a rectangle is 20 meters and the width is 15 meters. Find the ratio of the width to the length of the rectangle. Then simplify the ratio.
5. The measures of the angles in  $\triangle UVW$  are in the extended ratio of 1:1:2. Find the measures of the angles.
6. Find the geometric mean of 8 and 12.

**EXAMPLES**  
**1, 3, and 6**  
on pp. 356–359  
for Exs. 4–6

## 6.2 Use Proportions to Solve Geometry Problems

pp. 364–370

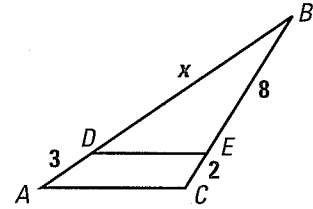
### EXAMPLE

In the diagram,  $\frac{BA}{DA} = \frac{BC}{EC}$ . Find  $BD$ .

$$\frac{x+3}{3} = \frac{8+2}{2} \quad \text{Substitution Property of Equality}$$

$$2x + 6 = 30 \quad \text{Cross Products Property}$$

$$x = 12 \quad \text{Solve for } x.$$

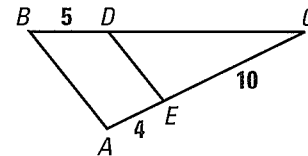
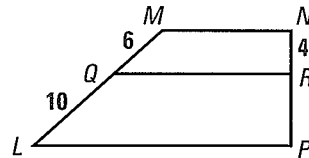


### EXERCISES

Use the diagram and the given information to find the unknown length.

7. Given  $\frac{RN}{RP} = \frac{QM}{QL}$ , find  $RP$ .

8. Given  $\frac{CD}{DB} = \frac{CE}{EA}$ , find  $CD$ .



### EXAMPLE 2

on p. 365  
for Exs. 7–8

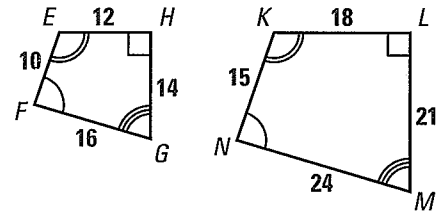
## 6.3 Use Similar Polygons

pp. 372–379

### EXAMPLE

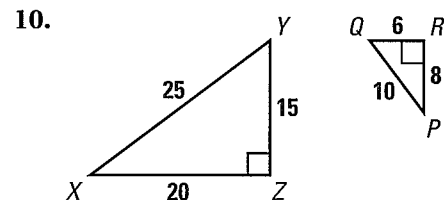
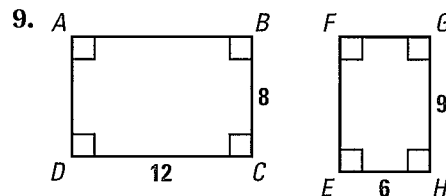
In the diagram,  $EHGF \sim KLMN$ . Find the scale factor.

From the diagram, you can see that  $\overline{EH}$  and  $\overline{KL}$  correspond. So, the scale factor of  $EHGF$  to  $KLMN$  is  $\frac{EH}{KL} = \frac{12}{18} = \frac{2}{3}$ .



### EXERCISES

In Exercises 9 and 10, determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.



### EXAMPLES

2 and 4

on pp. 373–374  
for Exs. 9–11

11. **POSTERS** Two similar posters have a scale factor of 4:5. The large poster's perimeter is 85 inches. Find the small poster's perimeter.

# 6

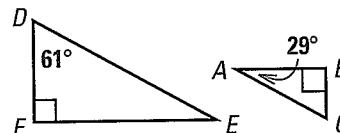
## CHAPTER REVIEW

### 6.4 Prove Triangles Similar by AA

pp. 381–387

#### EXAMPLE

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.

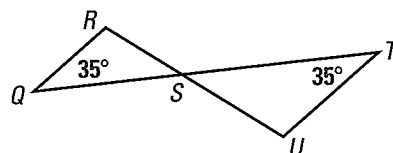


Because they are right angles,  $\angle F \cong \angle B$ . By the Triangle Sum Theorem,  $61^\circ + 90^\circ + m\angle E = 180^\circ$ , so  $m\angle E = 29^\circ$  and  $\angle E \cong \angle A$ . Then, two angles of  $\triangle DFE$  are congruent to two angles of  $\triangle CBA$ . So,  $\triangle DFE \sim \triangle CBA$ .

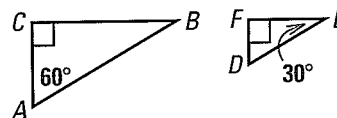
#### EXERCISES

Use the AA Similarity Postulate to show that the triangles are similar.

12.



13.



14. **CELL TOWER** A cellular telephone tower casts a shadow that is 72 feet long, while a tree nearby that is 27 feet tall casts a shadow that is 6 feet long. How tall is the tower?

#### EXAMPLES

2 and 3

on pp. 382–383  
for Exs. 12–14

### 6.5 Prove Triangles Similar by SSS and SAS

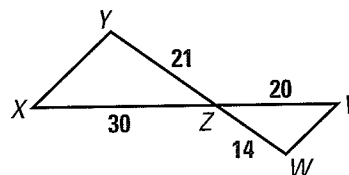
pp. 388–395

#### EXAMPLE

Show that the triangles are similar.

Notice that the lengths of two pairs of corresponding sides are proportional.

$$\frac{WZ}{YZ} = \frac{14}{21} = \frac{2}{3} \quad \frac{VZ}{XZ} = \frac{20}{30} = \frac{2}{3}$$

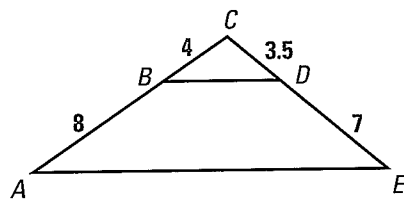


The included angles for these sides,  $\angle XZY$  and  $\angle VZW$ , are vertical angles, so  $\angle XZY \cong \angle VZW$ . Then  $\triangle XYZ \sim \triangle VWZ$  by the SAS Similarity Theorem.

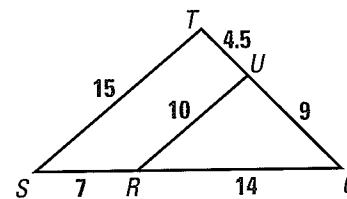
#### EXERCISES

Use the SSS Similarity Theorem or SAS Similarity Theorem to show that the triangles are similar.

15.



16.



#### EXAMPLE 4

on p. 391  
for Exs. 15–16

## 6.6 Use Proportionality Theorems

pp. 397–403

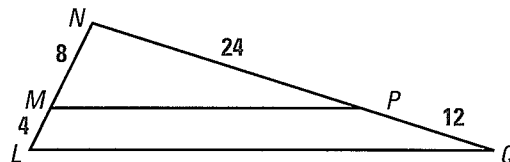
### EXAMPLE

Determine whether  $\overline{MP} \parallel \overline{LQ}$ .

Begin by finding and simplifying ratios of lengths determined by  $\overline{MP}$ .

$$\frac{NM}{ML} = \frac{8}{4} = \frac{2}{1} \qquad \frac{NP}{PQ} = \frac{24}{12} = \frac{2}{1}$$

Because  $\frac{NM}{ML} = \frac{NP}{PQ}$ ,  $\overline{MP}$  is parallel to  $\overline{LQ}$  by Theorem 6.5, the Triangle Proportionality Converse.

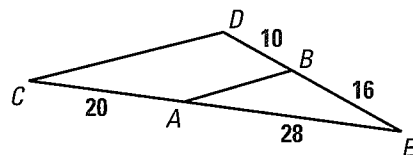


**EXAMPLE 2**  
on p. 398  
for Exs. 17–18

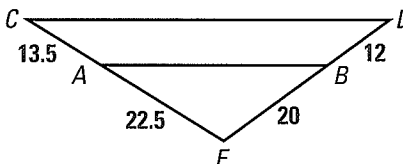
### EXERCISES

Use the given information to determine whether  $\overline{AB} \parallel \overline{CD}$ .

17.



18.



## 6.7 Perform Similarity Transformations

pp. 409–415

### EXAMPLE

Draw a dilation of quadrilateral  $FGHJ$  with vertices  $F(1, 1)$ ,  $G(2, 2)$ ,  $H(4, 1)$ , and  $J(2, -1)$ . Use a scale factor of 2.

First draw  $FGHJ$ . Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.

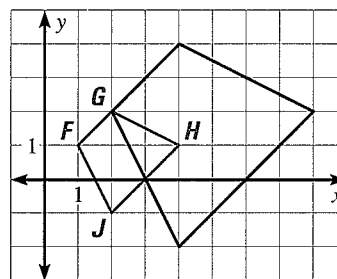
$$(x, y) \rightarrow (2x, 2y)$$

$$F(1, 1) \rightarrow (2, 2)$$

$$G(2, 2) \rightarrow (4, 4)$$

$$H(4, 1) \rightarrow (8, 2)$$

$$J(2, -1) \rightarrow (4, -2)$$



**EXAMPLE 1**  
on p. 409  
for Exs. 19–21

### EXERCISES

Draw a dilation of the polygon with the given vertices using the given scale factor  $k$ .

19.  $T(0, 8)$ ,  $U(6, 0)$ ,  $V(0, 0)$ ;  $k = \frac{3}{2}$

20.  $A(6, 0)$ ,  $B(3, 9)$ ,  $C(0, 0)$ ,  $D(3, 1)$ ;  $k = 4$

21.  $P(8, 2)$ ,  $Q(4, 0)$ ,  $R(3, 1)$ ,  $S(6, 4)$ ;  $k = 0.5$

# 6

# CHAPTER TEST

Solve the proportion.

1.  $\frac{6}{x} = \frac{9}{24}$

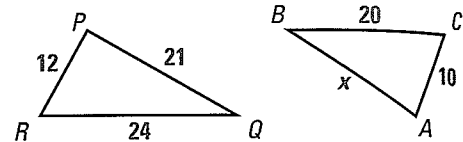
2.  $\frac{5}{4} = \frac{y-5}{12}$

3.  $\frac{3-2b}{4} = \frac{3}{2}$

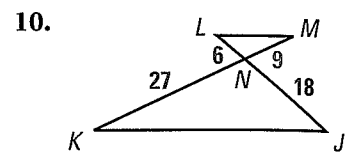
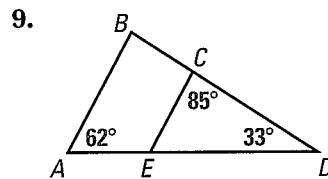
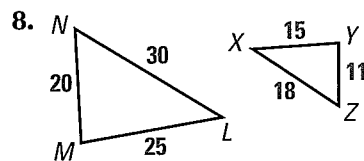
4.  $\frac{7}{2a+8} = \frac{1}{a-1}$

In Exercises 5–7, use the diagram where  $\triangle PQR \sim \triangle ABC$ .

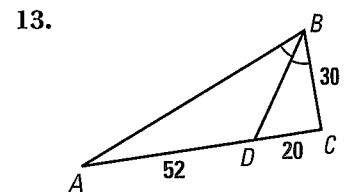
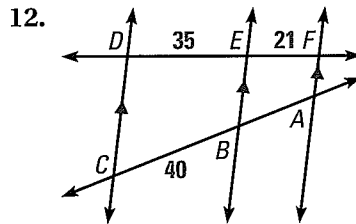
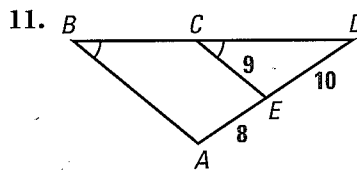
5. List all pairs of congruent angles.
6. Write the ratios of the corresponding sides in a statement of proportionality.
7. Find the value of  $x$ .



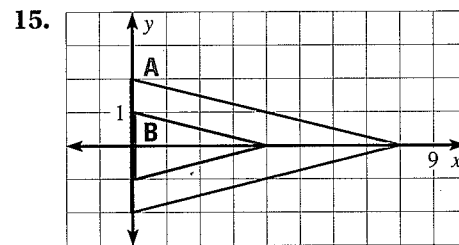
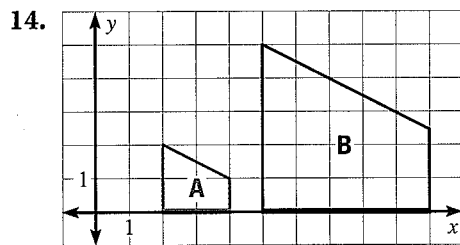
Determine whether the triangles are similar. If so, write a similarity statement and the postulate or theorem that justifies your answer.



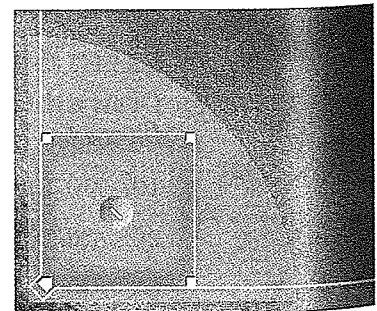
In Exercises 11–13, find the length of  $\overline{AB}$ .



Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.



16. **SCALE MODEL** You are making a scale model of your school's baseball diamond as part of an art project. The distance between two consecutive bases is 90 feet. If you use a scale factor of  $\frac{1}{180}$  to build your model, what will be the distance around the bases on your model?



## SOLVE QUADRATIC EQUATIONS AND SIMPLIFY RADICALS

A radical expression is *simplified* when the radicand has no perfect square factor except 1, there is no fraction in the radicand, and there is no radical in a denominator.

xy **EXAMPLE 1** Solve quadratic equations by finding square roots

Solve the equation  $4x^2 - 3 = 109$ .

$$4x^2 - 3 = 109 \quad \text{Write original equation.}$$

$$4x^2 = 112 \quad \text{Add 3 to each side.}$$

$$x^2 = 28 \quad \text{Divide each side by 4.}$$

$$x = \pm\sqrt{28} \quad \sqrt{ab} = \sqrt{a} \cdot \sqrt{b}, \text{ so } \sqrt{28} = \pm\sqrt{4} \cdot \sqrt{7}.$$

$$x = \pm 2\sqrt{7} \quad \text{Simplify.}$$

xy **EXAMPLE 2** Simplify quotients with radicals

Simplify the expression.

a.  $\sqrt{\frac{10}{8}}$

b.  $\sqrt{\frac{1}{5}}$

**Solution**

a.  $\sqrt{\frac{10}{8}} = \sqrt{\frac{5}{4}} \quad \text{Simplify fraction.}$

$$= \frac{\sqrt{5}}{\sqrt{4}} \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$= \frac{\sqrt{5}}{2} \quad \text{Simplify.}$$

b.  $\sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$

$$= \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{5}}{5}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ and } \sqrt{1} = 1.$$

Multiply numerator and denominator by  $\sqrt{5}$ .

Multiply fractions.  
 $\sqrt{a} \cdot \sqrt{a} = a$ .

## EXERCISES

**EXAMPLE 1**  
for Exs. 1–9

Solve the equation or write *no solution*.

1.  $x^2 + 8 = 108$

2.  $2x^2 - 1 = 49$

3.  $x^2 - 9 = 8$

4.  $5x^2 + 11 = 1$

5.  $2(x^2 - 7) = 6$

6.  $9 = 21 + 3x^2$

7.  $3x^2 - 17 = 43$

8.  $56 - x^2 = 20$

9.  $-3(-x^2 + 5) = 39$

**EXAMPLE 2**  
for Exs. 10–17

Simplify the expression.

10.  $\sqrt{\frac{7}{81}}$

11.  $\sqrt{\frac{3}{5}}$

12.  $\sqrt{\frac{24}{27}}$

13.  $\frac{3\sqrt{7}}{\sqrt{12}}$

14.  $\sqrt{\frac{75}{64}}$

15.  $\frac{\sqrt{2}}{\sqrt{200}}$

16.  $\frac{9}{\sqrt{27}}$

17.  $\sqrt{\frac{21}{42}}$