Geo

7 Right Triangles and Trigonometry

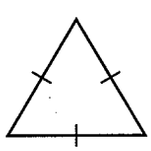
Before

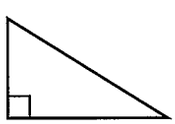
In previous courses and in Chapters 1–6, you learned the following skills, which you'll use in Chapter 7: classifying triangles, simplifying radicals, and solving proportions.

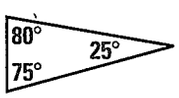
Prerequisite Skills

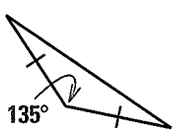
VOCABULARY CHECK

Classify the triangle shown.

1. 

2. 

3. 

4. 

SKILLS AND ALGEBRA CHECK

Simplify the radical. *(Review p. 874 for 7.1, 7.2, 7.4.)*

5. $\sqrt{45}$

6. $(3\sqrt{7})^2$

7. $\sqrt{3} \cdot \sqrt{5}$

8. $\frac{7}{\sqrt{2}}$

Solve the proportion. *(Review p. 356 for 7.3, 7.5–7.7.)*

9. $\frac{3}{x} = \frac{12}{16}$

10. $\frac{2}{3} = \frac{x}{18}$

11. $\frac{x+5}{4} = \frac{1}{2}$

12. $\frac{x+4}{x-4} = \frac{6}{5}$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 7, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 493. You will also use the key vocabulary listed below.

Big Ideas

1. **Using the Pythagorean Theorem and its converse**
2. **Using special relationships in right triangles**
3. **Using trigonometric ratios to solve right triangles**

KEY VOCABULARY

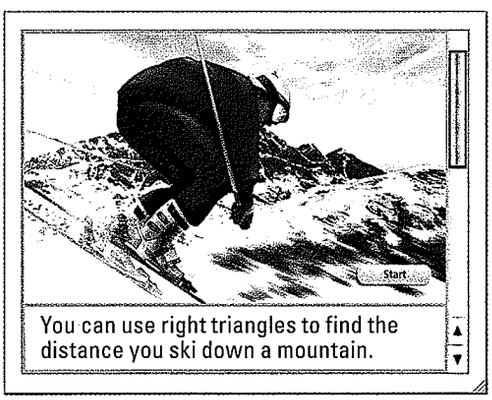
- Pythagorean triple, *p. 435*
- trigonometric ratio, *p. 466*
- tangent, *p. 466*
- sine, *p. 473*
- cosine, *p. 473*
- angle of elevation, *p. 475*
- angle of depression, *p. 475*
- solve a right triangle, *p. 483*
- inverse tangent, *p. 483*
- inverse sine, *p. 483*
- inverse cosine, *p. 483*

Why?

You can use trigonometric ratios to find unknown side lengths and angle measures in right triangles. For example, you can find the length of a ski slope.

Animated Geometry

The animation illustrated below for Example 4 on page 475 helps you answer this question: How far will you ski down the mountain?



Animated Geometry at classzone.com

Other animations for Chapter 7: pages 434, 442, 450, 460, and 462

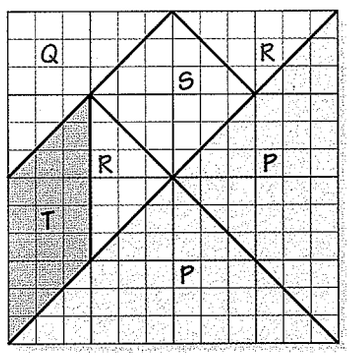
7.1 Pythagorean Theorem

MATERIALS • graph paper • ruler • pencil • scissors

QUESTION **What relationship exists among the sides of a right triangle?**

Recall that a square is a four sided figure with four right angles and four congruent sides.

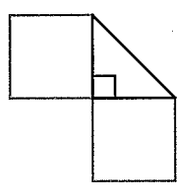
EXPLORE **Make and use a tangram set**



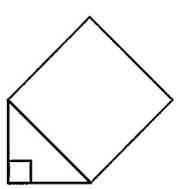
STEP 1 ***Make a tangram set*** On your graph paper, copy the tangram set as shown. Label each piece with the given letters. Cut along the solid black lines to make seven pieces.

STEP 2 ***Trace a triangle*** On another piece of paper, trace one of the large triangles P of the tangram set.

STEP 3 ***Assemble pieces along the legs*** Use all of the tangram pieces to form two squares along the legs of your triangle so that the length of each leg is equal to the side length of the square. Trace all of the pieces.



STEP 4 ***Assemble pieces along the hypotenuse*** Use all of the tangram pieces to form a square along the hypotenuse so that the side length of the square is equal to the length of the hypotenuse. Trace all of the pieces.

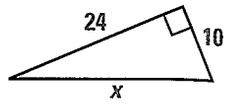


DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Find the sum of the areas of the two squares formed in Step 3. Let the letters labeling the figures represent the area of the figure. How are the side lengths of the squares related to Triangle P?
2. Find the area of the square formed in Step 4. How is the side length of the square related to Triangle P?
3. Compare your answers from Exercises 1 and 2. Make a conjecture about the relationship between the legs and hypotenuse of a right triangle.
4. The triangle you traced in Step 2 is an isosceles right triangle. Why? Do you think that your conjecture is true for all isosceles triangles? Do you think that your conjecture is true for all right triangles? *Justify* your answers.

EXAMPLE 4 Find the length of a hypotenuse using two methods

Find the length of the hypotenuse of the right triangle.



Solution

Method 1: Use a Pythagorean triple.

A common Pythagorean triple is **5**, **12**, **13**. Notice that if you multiply the lengths of the legs of the Pythagorean triple by 2, you get the lengths of the legs of this triangle: $\mathbf{5} \cdot 2 = 10$ and $\mathbf{12} \cdot 2 = 24$. So, the length of the hypotenuse is $\mathbf{13} \cdot 2 = 26$.

Method 2: Use the Pythagorean Theorem.

$x^2 = 10^2 + 24^2$ **Pythagorean Theorem**

$x^2 = 100 + 576$ **Multiply.**

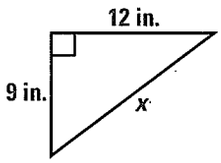
$x^2 = 676$ **Add.**

$x = 26$ **Find the positive square root.**

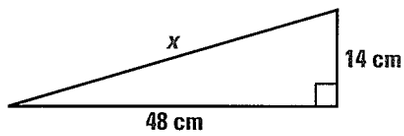
✓ GUIDED PRACTICE for Example 4

Find the unknown side length of the right triangle using the Pythagorean Theorem. Then use a Pythagorean triple.

7.



8.



7.1 EXERCISES

HOMEWORK KEY

- ○ = **WORKED-OUT SOLUTIONS** on p. WS8 for Exs. 9, 11, and 33
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 27, 33, and 36
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 35

SKILL PRACTICE

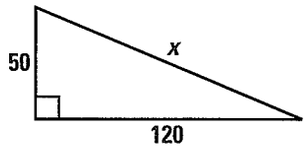
1. **VOCABULARY** Copy and complete: A set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$ is called a ___?___.

2. ★ **WRITING** *Describe* the information you need to have in order to use the Pythagorean Theorem to find the length of a side of a triangle.

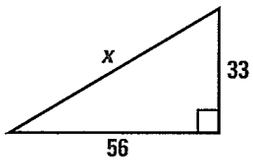
EXAMPLE 1 on p. 433 for Exs. 3–7

ALGEBRA Find the length of the hypotenuse of the right triangle.

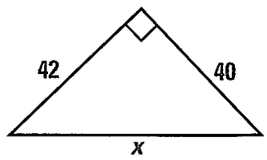
3.



4.

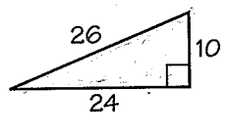


5.



ERROR ANALYSIS *Describe* **and correct the error in using the Pythagorean Theorem.**

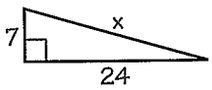
6.



$a^2 + b^2 = c^2$

$10^2 + 26^2 = 24^2$

7.



$x^2 = 7^2 + 24^2$

$x^2 = (7 + 24)^2$

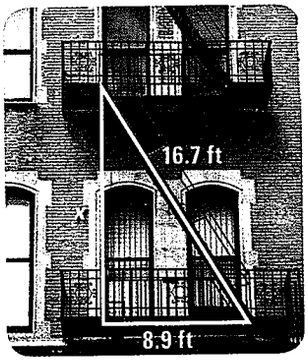
$x^2 = 31^2$

$x = 31$

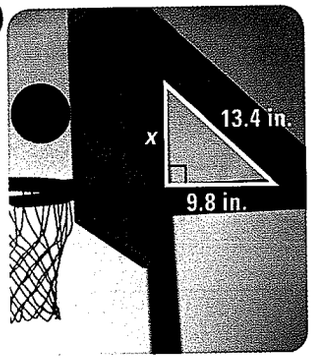
EXAMPLE 2 on p. 434 for Exs. 8–10

FINDING A LENGTH **Find the unknown leg length x.**

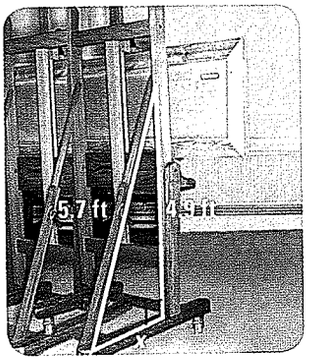
8.



9.



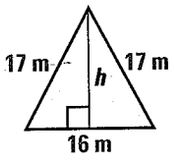
10.



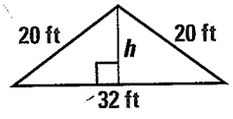
EXAMPLE 3 on p. 435 for Exs. 11–13

FINDING THE AREA **Find the area of the isosceles triangle.**

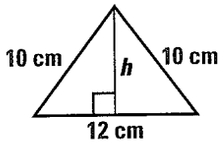
11.



12.



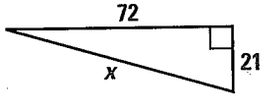
13.



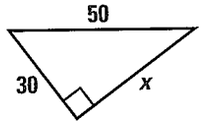
EXAMPLE 4 on p. 436 for Exs. 14–17

FINDING SIDE LENGTHS **Find the unknown side length of the right triangle using the Pythagorean Theorem or a Pythagorean triple.**

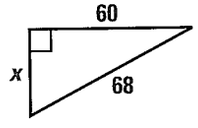
14.



15.



16.



17. ★ **MULTIPLE CHOICE** What is the length of the hypotenuse of a right triangle with leg lengths of 8 inches and 15 inches?

Ⓐ 13 inches Ⓑ 17 inches Ⓒ 21 inches Ⓓ 25 inches

PYTHAGOREAN TRIPLES **The given lengths are two sides of a right triangle. All three side lengths of the triangle are integers and together form a Pythagorean triple. Find the length of the third side and tell whether it is a leg or the hypotenuse.**

18. 24 and 51

19. 20 and 25

20. 28 and 96

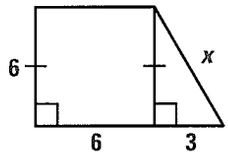
21. 20 and 48

22. 75 and 85

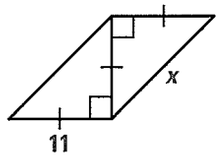
23. 72 and 75

FINDING SIDE LENGTHS **Find the unknown side length x. Write your answer in simplest radical form.**

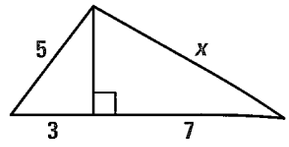
24.



25.



26.



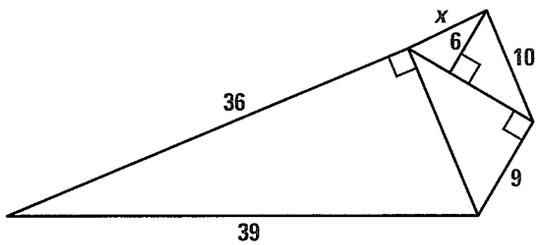
27. ★ **MULTIPLE CHOICE** What is the area of a right triangle with a leg length of 15 feet and a hypotenuse length of 39 feet?

Ⓐ 270 ft^2 Ⓑ 292.5 ft^2 Ⓒ 540 ft^2 Ⓓ 585 ft^2

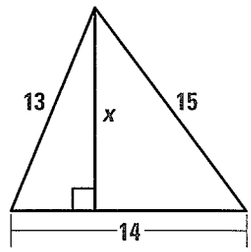
28. **ALGEBRA** Solve for x if the lengths of the two legs of a right triangle are $2x$ and $2x + 4$, and the length of the hypotenuse is $4x - 4$.

CHALLENGE **In Exercises 29 and 30, solve for x.**

29.



30.



PROBLEM SOLVING

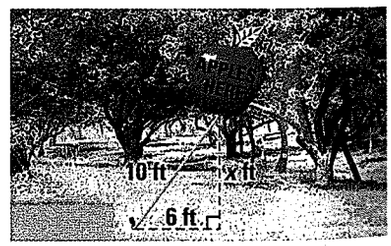
EXAMPLE 2 on p. 434 for Exs. 31–32

31. **BASEBALL DIAMOND** In baseball, the distance of the paths between each pair of consecutive bases is 90 feet and the paths form right angles. How far does the ball need to travel if it is thrown from home plate directly to second base?

@HomeTutor for problem solving help at classzone.com

32. **APPLE BALLOON** You tie an apple balloon to a stake in the ground. The rope is 10 feet long. As the wind picks up, you observe that the balloon is now 6 feet away from the stake. How far above the ground is the balloon now?

@HomeTutor for problem solving help at classzone.com



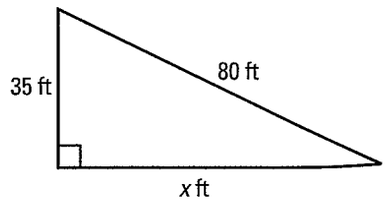
33. ★ **SHORT RESPONSE** Three side lengths of a right triangle are 25, 65, and 60. *Explain* how you know which side is the hypotenuse.

34. **MULTI-STEP PROBLEM** In your town, there is a field that is in the shape of a right triangle with the dimensions shown.

a. Find the perimeter of the field.

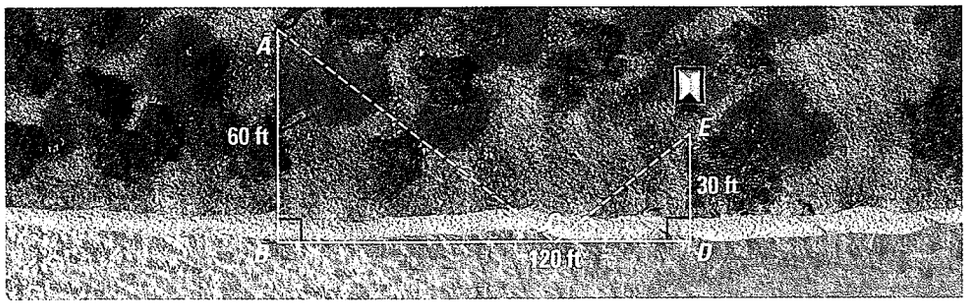
b. You are going to plant dogwood seedlings about every ten feet around the field's edge. How many trees do you need?

c. If each dogwood seedling sells for $12, how much will the trees cost?



○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

35. ◆ **MULTIPLE REPRESENTATIONS** As you are gathering leaves for a science project, you look back at your campsite and see that the campfire is not completely out. You want to get water from a nearby river to put out the flames with the bucket you are using to collect leaves. Use the diagram and the steps below to determine the shortest distance you must travel.



a. Making a Table Make a table with columns labeled *BC*, *AC*, *CE*, and *AC* + *CE*. Enter values of *BC* from 10 to 120 in increments of 10.

b. Calculating Values Calculate *AC*, *CE*, and *AC* + *CE* for each value of *BC*, and record the results in the table. Then, use your table of values to determine the shortest distance you must travel.

c. Drawing a Picture Draw an accurate picture to scale of the shortest distance.

36. ★ **SHORT RESPONSE** *Justify* the Distance Formula using the Pythagorean Theorem.

37. **PROVING THEOREM 4.5** Find the Hypotenuse-Leg (HL) Congruence Theorem on page 241. Assign variables for the side lengths in the diagram. Use your variables to write GIVEN and PROVE statements. Use the Pythagorean Theorem and congruent triangles to prove Theorem 4.5.

38. **CHALLENGE** Trees grown for sale at nurseries should stand at least five feet from one another while growing. If the trees are grown in parallel rows, what is the smallest allowable distance between rows?

MIXED REVIEW

PREVIEW
Prepare for Lesson 7.2 in Exs. 39–42.

Evaluate the expression. *(p. 874)*

39. $(\sqrt{7})^2$ **40.** $(4\sqrt{3})^2$ **41.** $(-6\sqrt{81})^2$ **42.** $(-8\sqrt{2})^2$

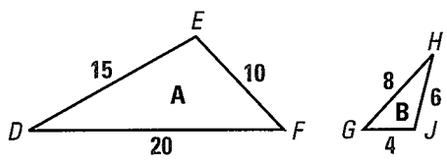
Describe **the possible lengths of the third side of the triangle given the lengths of the other two sides.** *(p. 328)*

43. 3 feet, 6 feet **44.** 5 inches, 11 inches **45.** 14 meters, 21 meters

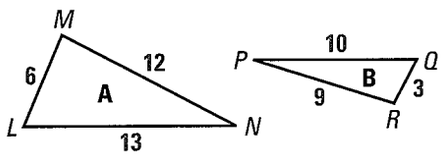
46. 12 inches, 27 inches **47.** 18 yards, 18 yards **48.** 27 meters, 39 meters

Determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A. *(p. 388)*

49.



50.



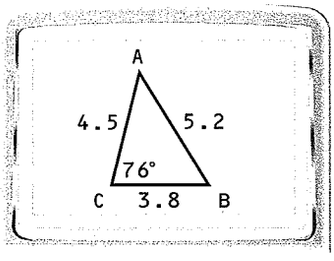
7.2 Converse of the Pythagorean Theorem

MATERIALS • graphing calculator or computer

QUESTION **How can you use the side lengths in a triangle to classify the triangle by its angle measures?**

You can use geometry drawing software to construct and measure triangles.

EXPLORE **Construct a triangle**



STEP 1 ***Draw a triangle*** Draw any $\triangle ABC$ with the largest angle at C. Measure $\angle C$, $\overline{AB}$, $\overline{AC}$, and $\overline{CB}$.

STEP 2 ***Calculate*** Use your measurements to calculate AB^2, AC^2, CB^2, and $(AC^2 + CB^2)$.

STEP 3 ***Complete a table*** Copy the table below and record your results in the first row. Then move point A to different locations and record the values for each triangle in your table. Make sure $\overline{AB}$ is always the longest side of the triangle. Include triangles that are acute, right, and obtuse.

$m\angle C$	AB	AB^2	AC	CB	$AC^2 + CB^2$
76°	5.2	27.04	4.5	3.8	34.69
?	?	?	?	?	?
?	?	?	?	?	?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. The Pythagorean Theorem states that "In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs." Write the Pythagorean Theorem in if-then form. Then write its converse.
2. Is the converse of the Pythagorean Theorem true? *Explain.*
3. Make a conjecture about the relationship between the measure of the largest angle in a triangle and the squares of the side lengths.

Copy and complete the statement.

4. If $AB^2 > AC^2 + CB^2$, then the triangle is a(n) __?__ triangle.
5. If $AB^2 < AC^2 + CB^2$, then the triangle is a(n) __?__ triangle.
6. If $AB^2 = AC^2 + CB^2$, then the triangle is a(n) __?__ triangle.

7.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS8 for Exs. 7, 17, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 24, 25, 32, 38, 39, and 43

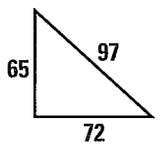
SKILL PRACTICE

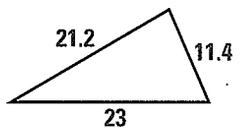
1. **VOCABULARY** What is the longest side of a right triangle called?

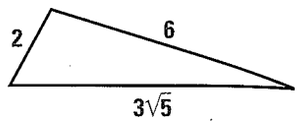
2. ★ **WRITING** *Explain* how the side lengths of a triangle can be used to classify it as acute, right, or obtuse.

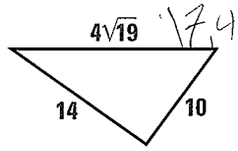
EXAMPLE 1 on p. 441 for Exs. 3–14

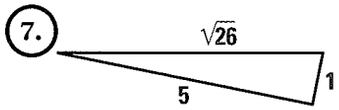
VERIFYING RIGHT TRIANGLES **Tell whether the triangle is a right triangle.**

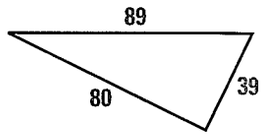
3. 

4. 

5. 

6. 

7. 

8. 

VERIFYING RIGHT TRIANGLES **Tell whether the given side lengths of a triangle can represent a right triangle.**

9. 9, 12, and 15
10. 9, 10, and 15
11. 36, 48, and 60
12. 6, 10, and $2\sqrt{34}$
13. 7, 14, and $7\sqrt{5}$
14. 10, 12, and 20

EXAMPLE 2 on p. 442 for Exs. 15–23

CLASSIFYING TRIANGLES **In Exercises 15–23, decide if the segment lengths form a triangle. If so, would the triangle be *acute, right,* or *obtuse?***

15. 10, 11, and 14
16. 10, 15, and $5\sqrt{13}$
17. 24, 30, and $6\sqrt{43}$
18. 5, 6, and 7
19. 12, 16, and 20
20. 8, 10, and 12
21. 15, 20, and 36
22. 6, 8, and 10
23. 8.2, 4.1, and 12.2

24. ★ **MULTIPLE CHOICE** Which side lengths do not form a right triangle?

Ⓐ 5, 12, 13 Ⓑ 10, 24, 28 Ⓒ 15, 36, 39 Ⓓ 50, 120, 130

25. ★ **MULTIPLE CHOICE** What type of triangle has side lengths of 4, 7, and 9?

Ⓐ Acute scalene Ⓑ Right scalene

Ⓒ Obtuse scalene Ⓓ None of the above

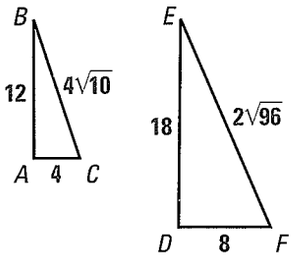
26. **ERROR ANALYSIS** A student tells you that if you double all the sides of a right triangle, the new triangle is obtuse. *Explain* why this statement is incorrect.

GRAPHING TRIANGLES **Graph points *A*, *B*, and *C*. Connect the points to form $\triangle ABC$. Decide whether $\triangle ABC$ is *acute, right,* or *obtuse.***

27. $A(-2, 4)$, $B(6, 0)$, $C(-5, -2)$
28. $A(0, 2)$, $B(5, 1)$, $C(1, -1)$

29. **ALGEBRA** Tell whether a triangle with side lengths $5x$, $12x$, and $13x$ (where $x > 0$) is *acute*, *right*, or *obtuse*.

USING DIAGRAMS In Exercises 30 and 31, copy and complete the statement with <, >, or =, if possible. If it is not possible, *explain* why.



30. $m\angle A$? $m\angle D$

31. $m\angle B + m\angle C$? $m\angle E + m\angle F$

32. ★ **OPEN-ENDED MATH** The side lengths of a triangle are 6, 8, and x (where $x > 0$). What are the values of x that make the triangle a right triangle? an acute triangle? an obtuse triangle?

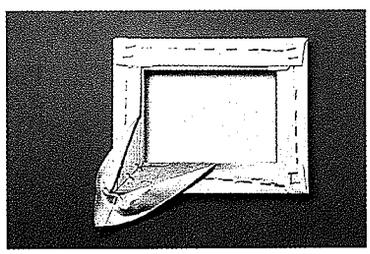
33. **ALGEBRA** The sides of a triangle have lengths x, $x + 4$, and 20. If the length of the longest side is 20, what values of x make the triangle acute?

34. **CHALLENGE** The sides of a triangle have lengths $4x + 6$, $2x + 1$, and $6x - 1$. If the length of the longest side is $6x - 1$, what values of x make the triangle obtuse?

PROBLEM SOLVING

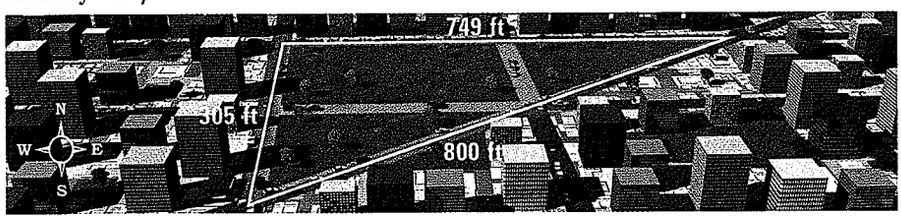
EXAMPLE 3 on p. 443 for Ex. 35

35. **PAINTING** You are making a canvas frame for a painting using stretcher bars. The rectangular painting will be 10 inches long and 8 inches wide. Using a ruler, how can you be certain that the corners of the frame are 90°?



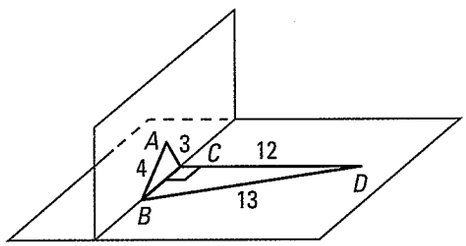
@HomeTutor for problem solving help at classzone.com

36. **WALKING** You walk 749 feet due east to the gym from your home. From the gym you walk 800 feet southwest to the library. Finally, you walk 305 feet from the library back home. Do you live directly north of the library? *Explain.*



@HomeTutor for problem solving help at classzone.com

37. **MULTI-STEP PROBLEM** Use the diagram shown.

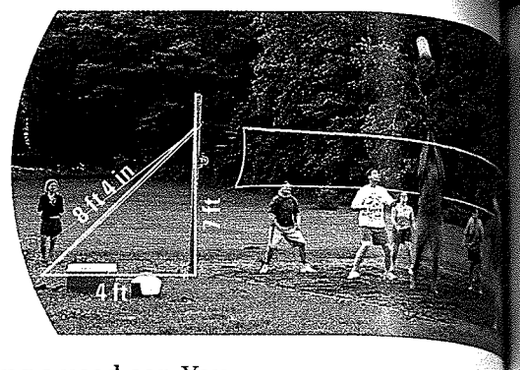


a. Find BC.

b. Use the Converse of the Pythagorean Theorem to show that $\triangle ABC$ is a right triangle.

c. Draw and label a similar diagram where $\triangle DBC$ remains a right triangle, but $\triangle ABC$ is not.

38. ★ **SHORT RESPONSE** You are setting up a volleyball net. To stabilize the pole, you tie one end of a rope to the pole 7 feet from the ground. You tie the other end of the rope to a stake that is 4 feet from the pole. The rope between the pole and stake is about 8 feet 4 inches long. Is the pole perpendicular to the ground? *Explain.* If it is not, how can you fix it?



39. ★ **EXTENDED RESPONSE** You are considering buying a used car. You would like to know whether the frame is sound. A sound frame of the car should be rectangular, so it has four right angles. You plan to measure the shadow of the car on the ground as the sun shines directly on the car.

a. You make a triangle with three tape measures on one corner. It has side lengths 12 inches, 16 inches, and 20 inches. Is this a right triangle? *Explain.*

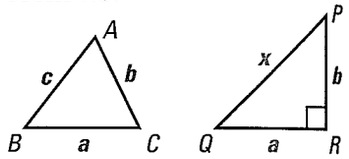
b. You make a triangle on a second corner with side lengths 9 inches, 12 inches, and 18 inches. Is this a right triangle? *Explain.*

c. The car owner says the car was never in an accident. Do you believe this claim? *Explain.*

40. **PROVING THEOREM 7.3** Copy and complete the proof of Theorem 7.3.

GIVEN ▶ In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side.

PROVE ▶ $\triangle ABC$ is an acute triangle.



Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x, where $\angle R$ is a right angle and x is the length of the longest side. Compare lengths c and x.

STATEMENTS	REASONS
1. In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side. In $\triangle PQR$, $\angle R$ is a right angle.	**1.** ?
2. $a^2 + b^2 = x^2$	**2.** ?
3. $c^2 < x^2$	**3.** ?
4. $c < x$	**4.** A property of square roots
5. $m\angle R = 90°$	**5.** ?
6. $m\angle C < m\angle$?	**6.** Converse of the Hinge Theorem
7. $m\angle C < 90°$	**7.** ?
8. $\angle C$ is an acute angle.	**8.** ?
9. $\triangle ABC$ is an acute triangle.	**9.** ?

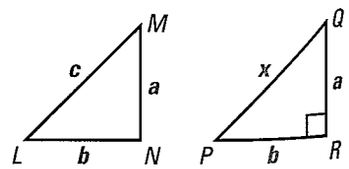
41. **PROVING THEOREM 7.4** Prove Theorem 7.4. Include a diagram and GIVEN and PROVE statements. (*Hint*: Look back at Exercise 40.)

42. **PROVING THEOREM 7.2** Prove the Converse of the Pythagorean Theorem.

GIVEN ▶ In $\triangle LMN$, $\overline{LM}$ is the longest side, and $c^2 = a^2 + b^2$.

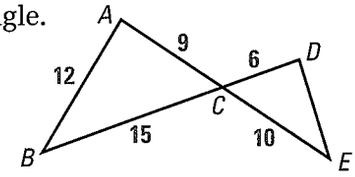
PROVE ▶ $\triangle LMN$ is a right triangle.

Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x. Compare lengths c and x.



★ = **STANDARDIZED TEST PRACTICE**

43. ★ **SHORT RESPONSE** *Explain* why $\angle D$ must be a right angle.



44. **COORDINATE PLANE** Use graph paper.

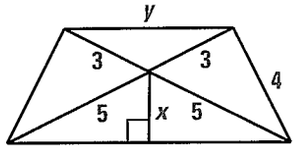
a. Graph $\triangle ABC$ with $A(-7, 2)$, $B(0, 1)$ and $C(-4, 4)$.

b. Use the slopes of the sides of $\triangle ABC$ to determine whether it is a right triangle. *Explain.*

c. Use the lengths of the sides of $\triangle ABC$ to determine whether it is a right triangle. *Explain.*

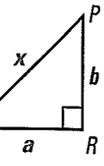
d. Did you get the same answer in parts (b) and (c)? If not, *explain* why.

45. **CHALLENGE** Find the values of x and y.



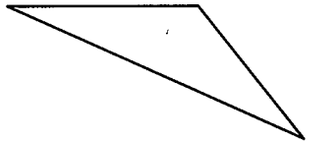
MIXED REVIEW

PREVIEW
Prepare for Lesson 7.3 in Exs. 46–48.

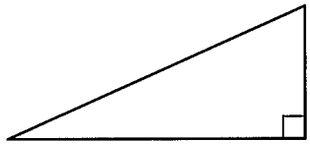


In Exercises 46–48, copy the triangle and draw one of its altitudes. *(p. 319)*

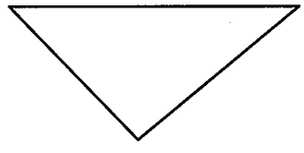
46.



47.



48.



Copy and complete the statement. *(p. 364)*

49. If $\frac{10}{x} = \frac{7}{y}$, then $\frac{10}{7} = \frac{?}{?}$.

50. If $\frac{x}{15} = \frac{y}{2}$, then $\frac{x}{y} = \frac{?}{?}$.

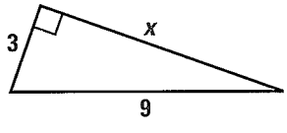
51. If $\frac{x}{8} = \frac{y}{9}$, then $\frac{x+8}{8} = \frac{?}{?}$.

52. The perimeter of a rectangle is 135 feet. The ratio of the length to the width is 8 : 1. Find the length and the width. *(p. 372)*

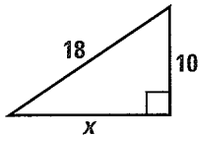
QUIZ for Lessons 7.1–7.2

Find the unknown side length. Write your answer in simplest radical form. *(p. 433)*

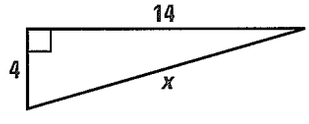
1.



2.



3.



Classify the triangle formed by the side lengths as *acute*, *right*, or *obtuse*. *(p. 441)*

4. 6, 7, and 9

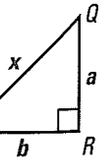
5. 10, 12, and 16

6. 8, 16, and $8\sqrt{6}$

7. 20, 21, and 29

8. 8, 3, $\sqrt{73}$

9. 8, 10, and 12



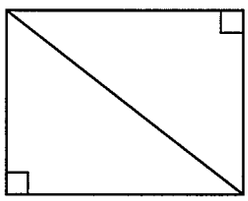
7.3 Similar Right Triangles

MATERIALS • rectangular piece of paper • ruler • scissors • colored pencils

QUESTION **How are geometric means related to the altitude of a right triangle?**

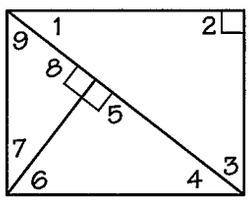
EXPLORE **Compare right triangles**

STEP 1



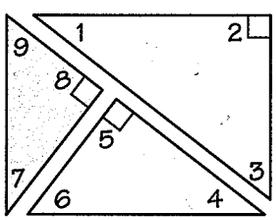
Draw a diagonal Draw a diagonal on your rectangular piece of paper to form two congruent right triangles.

STEP 2



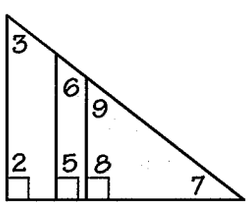
Draw an altitude Fold the paper to make an altitude to the hypotenuse of one of the triangles.

STEP 3



Cut and label triangles Cut the rectangle into the three right triangles that you drew. Label the angles and color the triangles as shown.

STEP 4



Arrange the triangles Arrange the triangles so $\angle 1$, $\angle 4$, and $\angle 7$ are on top of each other as shown.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. How are the two smaller right triangles related to the large triangle?
2. *Explain* how you would show that the green triangle is similar to the red triangle.
3. *Explain* how you would show that the red triangle is similar to the blue triangle.
4. The *geometric mean* of a and b is x if $\frac{a}{x} = \frac{x}{b}$. Write a proportion involving the side lengths of two of your triangles so that one side length is the geometric mean of the other two lengths in the proportion.

7.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS8 for Exs. 5, 15, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 20, 31, and 34

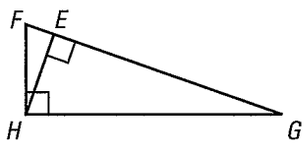
SKILL PRACTICE

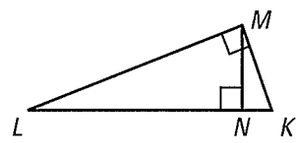
1. **VOCABULARY** Copy and complete: Two triangles are _?_ if their corresponding angles are congruent and their corresponding side lengths are proportional.

2. ★ **WRITING** In your own words, explain *geometric mean*.

EXAMPLE 1 on p. 449 for Exs. 3–4

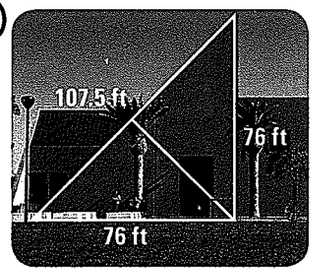
IDENTIFYING SIMILAR TRIANGLES **Identify the three similar right triangles in the given diagram.**

3. 

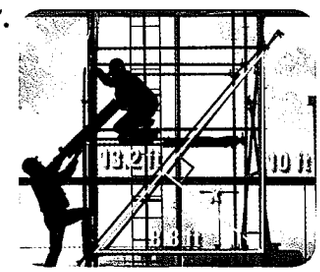
4. 

EXAMPLE 2 on p. 450 for Exs. 5–7

FINDING ALTITUDES **Find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.**

5. 

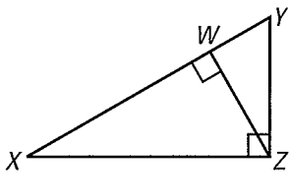
6. 

7. 

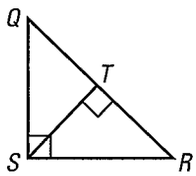
EXAMPLES 3 and 4 on pp. 451–452 for Exs. 8–18

COMPLETING PROPORTIONS **Write a similarity statement for the three similar triangles in the diagram. Then complete the proportion.**

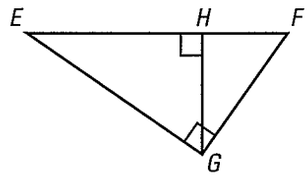
8. $\frac{XW}{?} = \frac{ZW}{YW}$



9. $\frac{?}{SQ} = \frac{SQ}{TQ}$

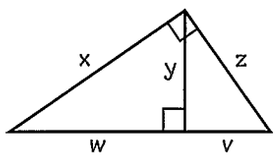


10. $\frac{EF}{EG} = \frac{EG}{?}$



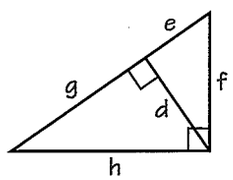
ERROR ANALYSIS ***Describe*** **and correct the error in writing a proportion for the given diagram.**

11.



$\frac{w}{z} = \frac{z}{w+v}$ ✗

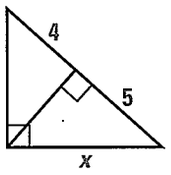
12.



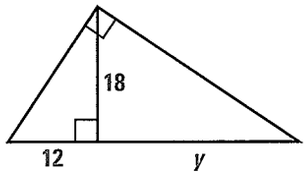
$\frac{e}{d} = \frac{d}{f}$ ✗

FINDING LENGTHS **Find the value of the variable. Round decimal answers to the nearest tenth.**

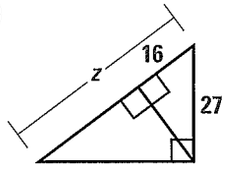
13.



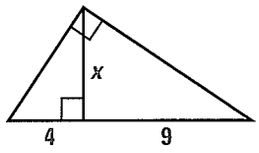
14.



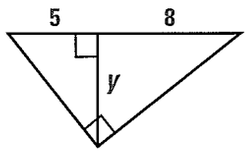
15.



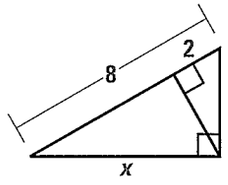
16.



17.



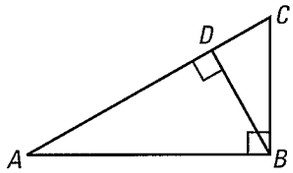
18.



19. ★ **MULTIPLE CHOICE** Use the diagram at the right. Decide which proportion is false.

Ⓐ $\frac{DB}{DC} = \frac{DA}{DB}$ Ⓑ $\frac{CA}{AB} = \frac{AB}{AD}$

Ⓒ $\frac{CA}{BA} = \frac{BA}{CA}$ Ⓓ $\frac{DC}{BC} = \frac{BC}{CA}$

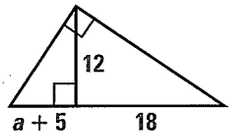


20. ★ **MULTIPLE CHOICE** In the diagram in Exercise 19 above, $AC = 36$ and $BC = 18$. Find AD. If necessary, round to the nearest tenth.

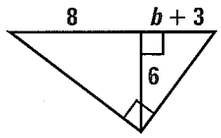
Ⓐ 9 Ⓑ 15.6 Ⓒ 27 Ⓓ 31.2

ALGEBRA **Find the value(s) of the variable(s).**

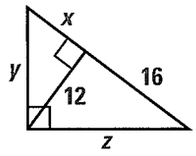
21.



22.

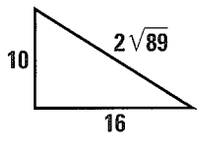


23.

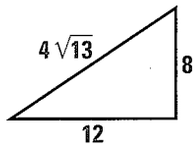


USING THEOREMS **Tell whether the triangle is a right triangle. If so, find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.**

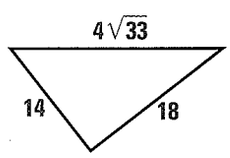
24.



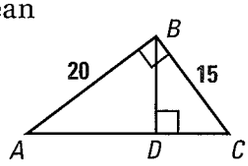
25.



26.



27. **FINDING LENGTHS** Use the Geometric Mean Theorems to find AC and BD.



28. **CHALLENGE** Draw a right isosceles triangle and label the two leg lengths x. Then draw the altitude to the hypotenuse and label its length y. Now draw the three similar triangles and label any side length that is equal to either x or y. What can you conclude about the relationship between the two smaller triangles? *Explain.*

○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

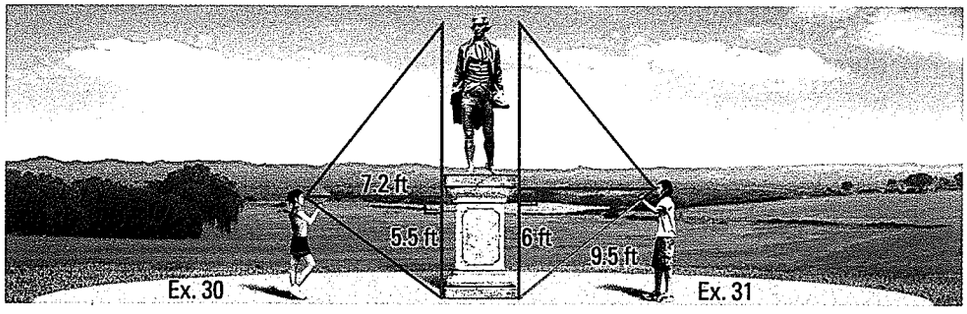
PROBLEM SOLVING

29. **DOGHOUSE** The peak of the doghouse shown forms a right angle. Use the given dimensions to find the height of the roof.

@HomeTutor for problem solving help at classzone.com

EXAMPLE 4 on p. 452 for Exs. 30–31

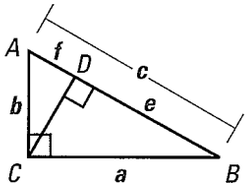
30. **MONUMENT** You want to determine the height of a monument at a local park. You use a cardboard square to line up the top and bottom of the monument. Mary measures the vertical distance from the ground to your eye and the distance from you to the monument. Approximate the height of the monument (as shown at the left below).



@HomeTutor for problem solving help at classzone.com

31. ★ **SHORT RESPONSE** Paul is standing on the other side of the monument in Exercise 30 (as shown at the right above). He has a piece of rope staked at the base of the monument. He extends the rope to the cardboard square he is holding lined up to the top and bottom of the monument. Use the information in the diagram above to approximate the height of the monument. Do you get the same answer as in Exercise 30? *Explain.*

32. **PROVING THEOREM 7.1** Use the diagram of $\triangle ABC$. Copy and complete the proof of the Pythagorean Theorem.

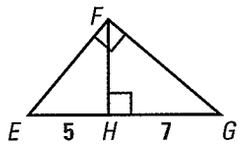


GIVEN ▶ In $\triangle ABC$, $\angle BCA$ is a right angle.

PROVE ▶ $c^2 = a^2 + b^2$

STATEMENTS	REASONS
1. Draw $\triangle ABC$. $\angle BCA$ is a right angle.	1. ?
2. Draw a perpendicular from C to $\overline{AB}$.	2. Perpendicular Postulate
3. $\frac{c}{a} = \frac{a}{e}$ and $\frac{c}{b} = \frac{b}{f}$	3. ?
4. $ce = a^2$ and $cf = b^2$	4. ?
5. $ce + b^2 =$? $+ b^2$	5. Addition Property of Equality
6. $ce + cf = a^2 + b^2$	6. ?
7. $c(e + f) = a^2 + b^2$	7. ?
8. $e + f =$?	8. Segment Addition Postulate
9. $c \cdot c = a^2 + b^2$	9. ?
10. $c^2 = a^2 + b^2$	10. Simplify.

33. MULTI-STEP PROBLEM Use the diagram.

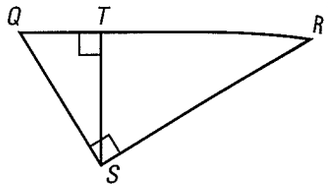


a. Name all the altitudes in $\triangle EGF$. *Explain.*

b. Find FH.

c. Find the area of the triangle.

34. ★ EXTENDED RESPONSE Use the diagram.

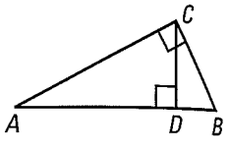


a. Sketch the three similar triangles in the diagram. Label the vertices. *Explain* how you know which vertices correspond.

b. Write similarity statements for the three triangles.

c. Which segment's length is the geometric mean of RT and RQ? *Explain* your reasoning.

PROVING THEOREMS In Exercises 35–37, use the diagram and GIVEN statements below.



GIVEN ▶ $\triangle ABC$ is a right triangle.
Altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

35. Prove Theorem 7.5 by using the Plan for Proof on page 449.

36. Prove Theorem 7.6 by showing $\frac{BD}{CD} = \frac{CD}{AD}$.

37. Prove Theorem 7.7 by showing $\frac{AB}{CB} = \frac{CB}{DB}$ and $\frac{AB}{AC} = \frac{AC}{AD}$.

38. CHALLENGE The *harmonic mean* of a and b is $\frac{2ab}{a+b}$. The Greek mathematician Pythagoras found that three equally taut strings on stringed instruments will sound harmonious if the length of the middle string is equal to the harmonic mean of the lengths of the shortest and longest string.

a. Find the harmonic mean of 10 and 15.

b. Find the harmonic mean of 6 and 14.

c. Will equally taut strings whose lengths have the ratio 4 : 6 : 12 sound harmonious? *Explain* your reasoning.

MIXED REVIEW

PREVIEW
Prepare for Lesson 7.4 in Exs. 39–46.

Simplify the expression. *(p. 874)*

39. $\sqrt{27} \cdot \sqrt{2}$ **40.** $\sqrt{8} \cdot \sqrt{10}$ **41.** $\sqrt{12} \cdot \sqrt{7}$ **42.** $\sqrt{18} \cdot \sqrt{12}$

43. $\frac{5}{\sqrt{7}}$ **44.** $\frac{8}{\sqrt{11}}$ **45.** $\frac{15}{\sqrt{27}}$ **46.** $\frac{12}{\sqrt{24}}$

Tell whether the lines through the given points are *parallel, perpendicular,* or *neither. Justify* your answer. *(p. 171)*

47. **Line 1:** (2, 4), (4, 2)
Line 2: (3, 5), (−1, 1)

48. **Line 1:** (0, 2), (−1, −1)
Line 2: (3, 1), (1, −5)

49: **Line 1:** (1, 7), (4, 7)
Line 2: (5, 2), (7, 4)

7.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS8 for Exs. 5, 9, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 19, 22, 29, and 34

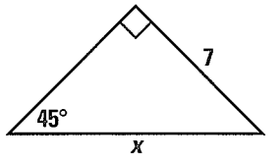
SKILL PRACTICE

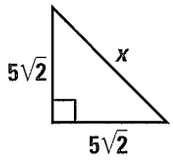
1. **VOCABULARY** Copy and complete: A triangle with two congruent sides and a right angle is called _?_.

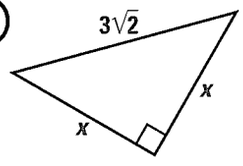
2. ★ **WRITING** *Explain* why the acute angles in an isosceles right triangle always measure 45°.

EXAMPLES 1 and 2 on pp. 457–458 for Exs. 3–5

45°-45°-90° TRIANGLES **Find the value of x. Write your answer in simplest radical form.**

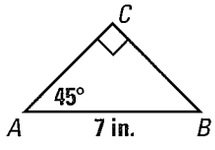
3. 

4. 

5. 

EXAMPLE 3 on p. 458 for Exs. 6–7

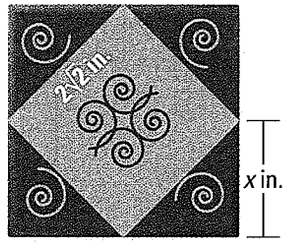
6. ★ **MULTIPLE CHOICE** Find the length of $\overline{AC}$.



Ⓐ $7\sqrt{2}$ in. Ⓑ $2\sqrt{7}$ in.

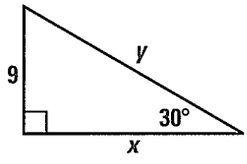
Ⓒ $\frac{7\sqrt{2}}{2}$ in. Ⓓ $\sqrt{14}$ in.

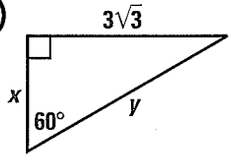
7. **ISOSCELES RIGHT TRIANGLE** The square tile shown has painted corners in the shape of congruent 45°-45°-90° triangles. What is the value of x? What is the side length of the tile?

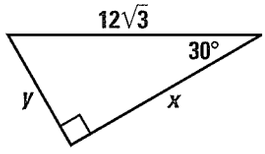


EXAMPLES 4 and 5 on p. 459 for Exs. 8–10

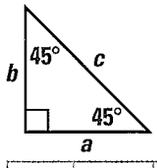
30°-60°-90° TRIANGLES **Find the value of each variable. Write your answers in simplest radical form.**

8. 

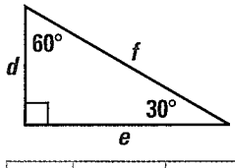
9. 

10. 

SPECIAL RIGHT TRIANGLES **Copy and complete the table.**

11. 

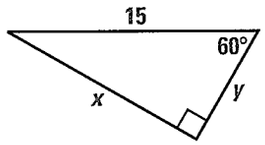
a	7	?	?	?	$\sqrt{5}$
b	?	11	?	?	?
c	?	?	10	$6\sqrt{2}$	?

12. 

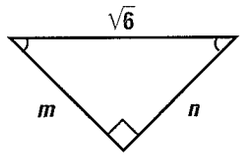
d	5	?	?	?	?
e	?	?	$8\sqrt{3}$	?	12
f	?	14	?	$18\sqrt{3}$	?

ALGEBRA **Find the value of each variable. Write your answers in simplest radical form.**

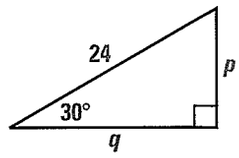
13.



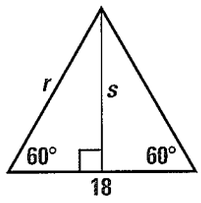
14.



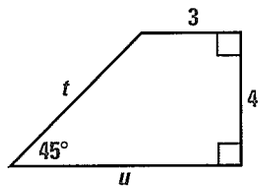
15.



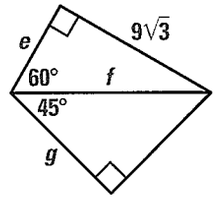
16.



17.



18.



Animated Geometry at classzone.com

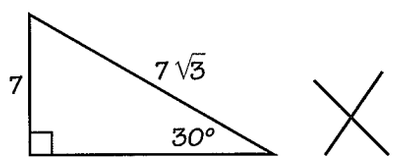
19. ★ **MULTIPLE CHOICE** Which side lengths do *not* represent a 30°-60°-90° triangle?

(A) $\frac{1}{2}, \frac{\sqrt{3}}{2}, 1$ (B) $\sqrt{2}, \sqrt{6}, 2\sqrt{2}$

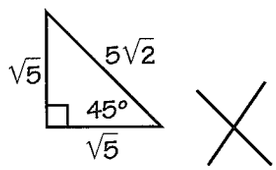
(C) $\frac{5}{2}, \frac{5\sqrt{3}}{2}, 10$ (D) $3, 3\sqrt{3}, 6$

ERROR ANALYSIS ***Describe*** **and correct the error in finding the length of the hypotenuse.**

20.



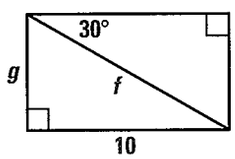
21.



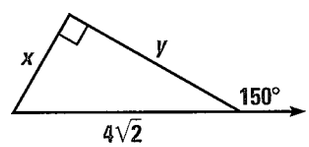
22. ★ **WRITING** Abigail solved Example 5 on page 459 in a different way. Instead of dividing each side by $\sqrt{3}$, she multiplied each side by $\sqrt{3}$. Does her method work? *Explain* why or why not.

ALGEBRA **Find the value of each variable. Write your answers in simplest radical form.**

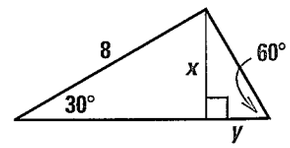
23.



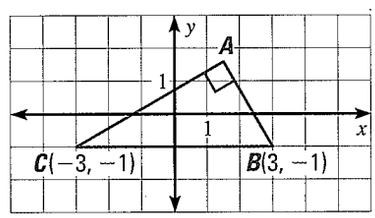
24.



25.



26. **CHALLENGE** $\triangle ABC$ is a 30°-60°-90° triangle. Find the coordinates of A.



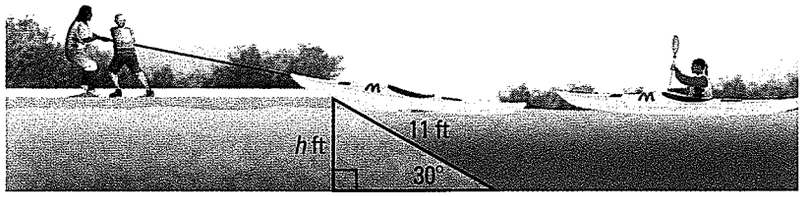
○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

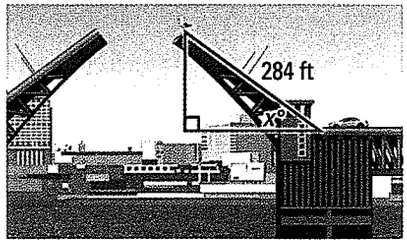
EXAMPLE 6 on p. 460 for Ex. 27

27. KAYAK RAMP A ramp is used to launch a kayak. What is the height of an 11 foot ramp when its angle is 30° as shown?



@HomeTutor for problem solving help at classzone.com

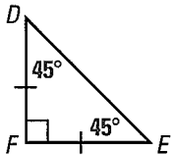
28. DRAWBRIDGE Each half of the drawbridge is about 284 feet long, as shown. How high does a seagull who is on the end of the drawbridge rise when the angle with measure $x°$ is 30°? 45°? 60°?



@HomeTutor for problem solving help at classzone.com

29. ★ SHORT RESPONSE *Describe* two ways to show that all isosceles right triangles are similar to each other.

30. PROVING THEOREM 7.8 Write a paragraph proof of the 45°-45°-90° Triangle Theorem.

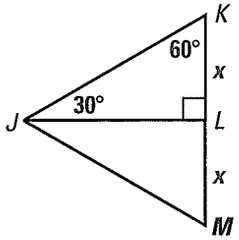


GIVEN ▶ $\triangle DEF$ is a 45°-45°-90° triangle.

PROVE ▶ The hypotenuse is $\sqrt{2}$ times as long as each leg.

31. EQUILATERAL TRIANGLE If an equilateral triangle has a side length of 20 inches, find the height of the triangle.

32. PROVING THEOREM 7.9 Write a paragraph proof of the 30°-60°-90° Triangle Theorem.

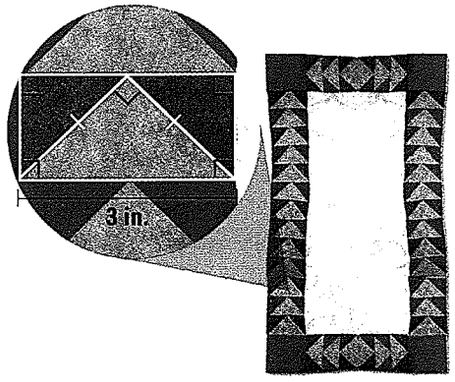


GIVEN ▶ $\triangle JKL$ is a 30°-60°-90° triangle.

PROVE ▶ The hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

Plan for Proof Construct $\triangle JML$ congruent to $\triangle JKL$. Then prove that $\triangle JKM$ is equilateral. Express the lengths of $\overline{JK}$ and $\overline{JL}$ in terms of x.

33. MULTI-STEP PROBLEM You are creating a quilt that will have a traditional "flying geese" border, as shown below.

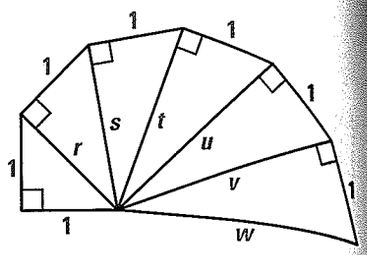


a. Find all the angle measures of the small blue triangles and the large orange triangles.

b. The width of the border is to be 3 inches. To create the large triangle, you cut a square of fabric in half. Not counting any extra fabric needed for seams, what size square do you need?

c. What size square do you need to create each small triangle?

34. ★ EXTENDED RESPONSE Use the figure at the right. You can use the fact that the converses of the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem are true.



a. Find the values of r, s, t, u, v, and w. *Explain* the procedure you used to find the values.

b. Which of the triangles, if any, is a 45°-45°-90° triangle? *Explain.*

c. Which of the triangles, if any, is a 30°-60°-90° triangle? *Explain.*

35. CHALLENGE In quadrilateral $QRST$, $m\angle R = 60°$, $m\angle T = 90°$, $QR = RS$, $ST = 8$, $TQ = 8$, and $\overline{RT}$ and $\overline{QS}$ intersect at point Z.

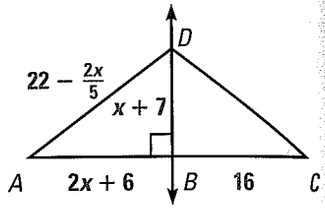
a. Draw a diagram.

b. *Explain* why $\triangle RQT \cong \triangle RST$.

c. Which is longer, QS or RT? *Explain.*

MIXED REVIEW

In the diagram, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$. *(p. 303)*



36. Which pairs of segment lengths are equal?

37. What is the value of x?

38. Find CD.

Is it possible to build a triangle using the given side lengths? *(p. 328)*

39. 4, 4, and 7 **40.** 3, 3, and $9\sqrt{2}$ **41.** 7, 15, and 21

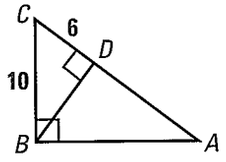
Tell whether the given side lengths form a right triangle. *(p. 441)*

42. 21, 22, and $5\sqrt{37}$ **43.** $\frac{3}{2}$, 2, and $\frac{5}{2}$ **44.** 8, 10, and 14

PREVIEW
Prepare for Lesson 7.5 in Exs. 42–44.

QUIZ for Lessons 7.3–7.4

In Exercises 1 and 2, use the diagram. *(p. 449)*

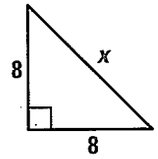


1. Which segment's length is the geometric mean of AC and CD?

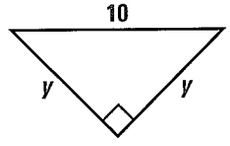
2. Find BD, AD, and AB.

Find the values of the variable(s). Write your answer(s) in simplest radical form. *(p. 457)*

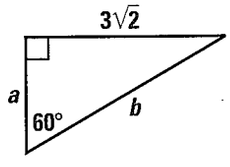
3.



4.



5.



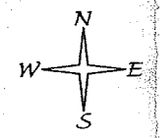
MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
classzone.com

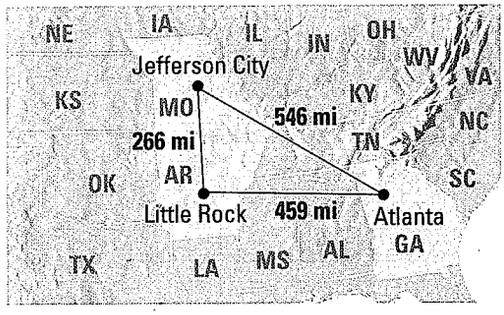
Lessons 7.1–7.4

1. **GRIDDED ANSWER** Find the direct distance, in paces, from the treasure to the stump.

From the old stump, take 30 paces east, then 20 paces north, 6 paces west, and then another 25 paces north to find the hidden treasure.



2. **MULTI-STEP PROBLEM** On a map of the United States, you put a pushpin on three state capitols you want to visit: Jefferson City, Missouri; Little Rock, Arkansas; and Atlanta, Georgia.

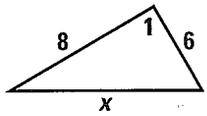


   a. Draw a diagram to model the triangle.

   b. Do the pushpins form a right triangle? If not, what type of triangle do they form?

3. **SHORT RESPONSE** Bob and John started running at 10 A.M. Bob ran east at 4 miles per hour while John ran south at 5 miles per hour. How far apart were they at 11:30 A.M.? *Describe* how you calculated the answer.

4. **EXTENDED RESPONSE** Give all values of x that make the statement true for the given diagram.



   a. $\angle 1$ is a right angle. *Explain.*

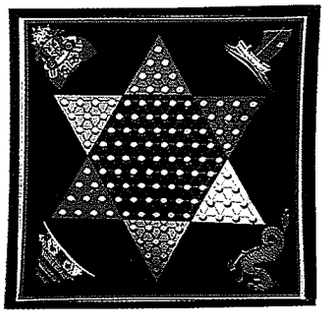
   b. $\angle 1$ is an obtuse angle. *Explain.*

   c. $\angle 1$ is an acute angle. *Explain.*

   d. The triangle is isosceles. *Explain.*

   e. No triangle is possible. *Explain.*

5. **EXTENDED RESPONSE** A Chinese checker board is made of triangles. Use the picture below to answer the questions.



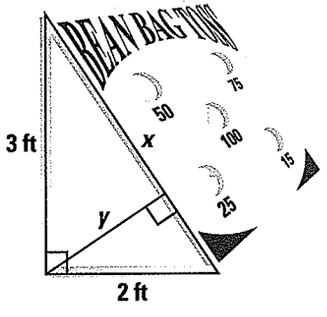
   a. Count the marble holes in the purple triangle. What kind of triangle is it?

   b. If a side of the purple triangle measures 8 centimeters, find the area of the purple triangle.

   c. How many marble holes are in the center hexagon? Assuming each marble hole takes up the same amount of space, what is the relationship between the purple triangle and center hexagon?

   d. Find the area of the center hexagon. *Explain* your reasoning.

6. **MULTI-STEP PROBLEM** You build a beanbag toss game. The game is constructed from a sheet of plywood supported by two boards. The two boards form a right angle and their lengths are 3 feet and 2 feet.



   a. Find the length x of the plywood.

   b. You put in a support that is the altitude y to the hypotenuse of the right triangle. What is the length of the support?

   c. Where does the support attach to the plywood? *Explain.*