7.5 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS9 for Exs. 5, 7, and 31
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 16, 17, 35, and 37

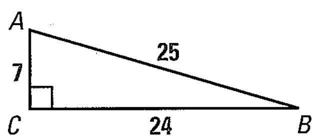
SKILL PRACTICE

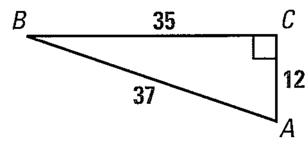
1. **VOCABULARY** Copy and complete: The tangent ratio compares the length of _?_ to the length of _?_.

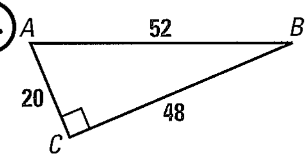
2. ★ **WRITING** *Explain* how you know that all right triangles with an acute angle measuring $n°$ are similar to each other.

EXAMPLE 1 on p. 467 for Exs. 3–5

FINDING TANGENT RATIOS Find tan A and tan B. Write each answer as a fraction and as a decimal rounded to four places.

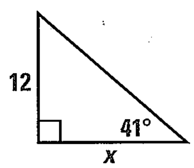
3. 

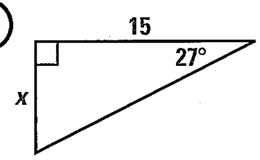
4. 

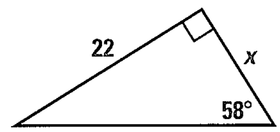
5. 

EXAMPLE 2 on p. 467 for Exs. 6–8

FINDING LEG LENGTHS Find the value of x to the nearest tenth.

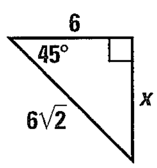
6. 

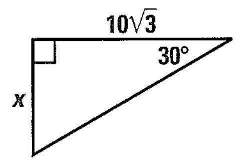
7. 

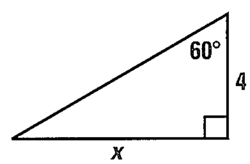
8. 

EXAMPLE 4 on p. 468 for Exs. 9–12

FINDING LEG LENGTHS Find the value of x using the definition of tangent. Then find the value of x using the 45°-45°-90° Theorem or the 30°-60°-90° Theorem. *Compare* the results.

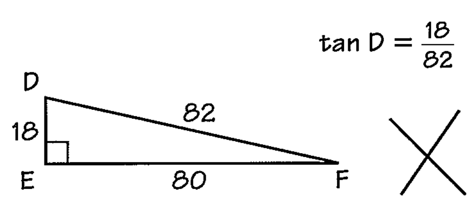
9. 

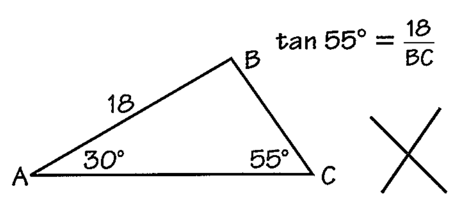
10. 

11. 

12. **SPECIAL RIGHT TRIANGLES** Find tan 30° and tan 45° using the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem.

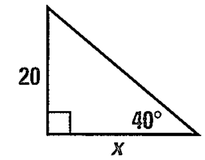
ERROR ANALYSIS *Describe* the error in the statement of the tangent ratio. Correct the statement, if possible. Otherwise, write *not possible*.

13. 

14. 

15. ★ **WRITING** *Describe* what you must know about a triangle in order to use the tangent ratio.

16. ★ **MULTIPLE CHOICE** Which expression can be used to find the value of x in the triangle shown?



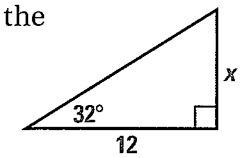
Ⓐ $x = 20 \cdot \tan 40°$

Ⓑ $x = \dfrac{\tan 40°}{20}$

Ⓒ $x = \dfrac{20}{\tan 40°}$

Ⓓ $x = \dfrac{20}{\tan 50°}$

17. ★ **MULTIPLE CHOICE** What is the approximate value of x in the triangle shown?



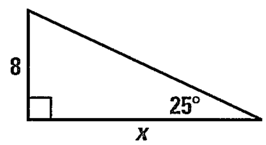
Ⓐ 0.4

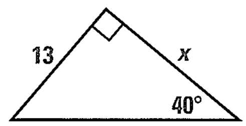
Ⓑ 2.7

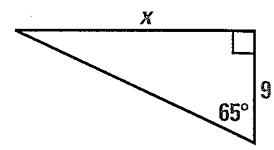
Ⓒ 7.5

Ⓓ 19.2

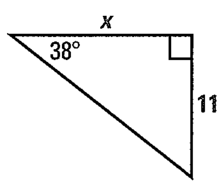
FINDING LEG LENGTHS **Use a tangent ratio to find the value of x. Round to the nearest tenth. Check your solution using the tangent of the other acute angle.**

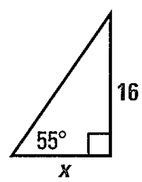
18.


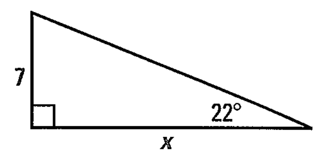
19.


20.


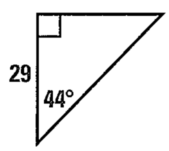
FINDING AREA **Find the area of the triangle. Round to the nearest tenth.**

21.


22.


23.


FINDING PERIMETER **Find the perimeter of the triangle. Round to the nearest tenth.**

24.


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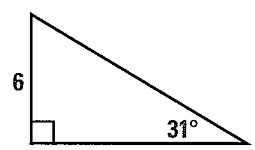

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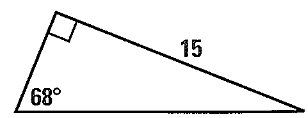
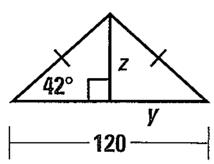
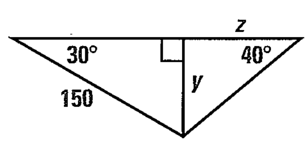


FINDING LENGTHS **Find y. Then find z. Round to the nearest tenth.**

27.


28.


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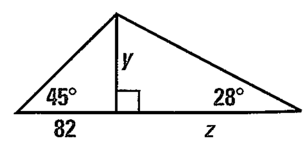
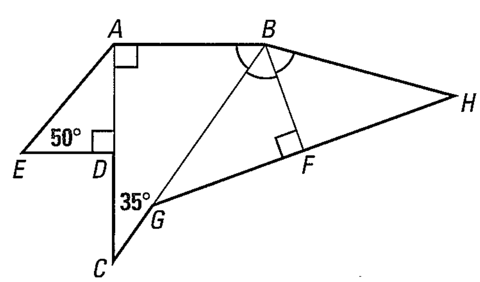


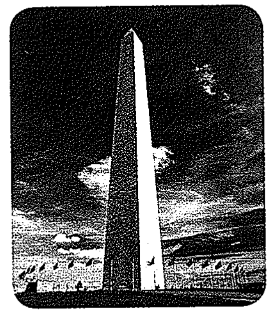
30. **CHALLENGE** Find the perimeter of the figure at the right, where $AC = 26$, $AD = BF$, and D is the midpoint of $\overline{AC}$.



PROBLEM SOLVING

EXAMPLE 3
on p. 468
for Exs. 31–32

31. **WASHINGTON MONUMENT** A surveyor is standing 118 feet from the base of the Washington Monument. The surveyor measures the angle between the ground and the top of the monument to be 78°. Find the height h of the Washington Monument to the nearest foot.



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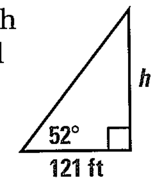
32. **ROLLER COASTERS** A roller coaster makes an angle of 52° with the ground. The horizontal distance from the crest of the hill to the bottom of the hill is about 121 feet, as shown. Find the height h of the roller coaster to the nearest foot.



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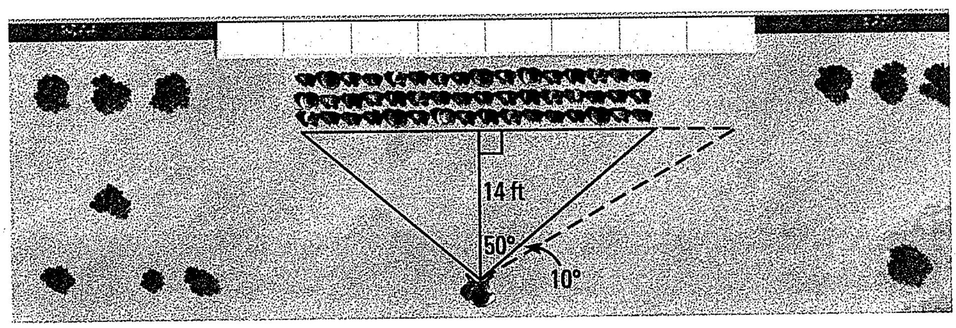
CLASS PICTURE **Use this information and diagram for Exercises 33 and 34.**

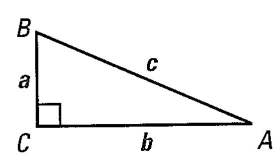
Your class is having a class picture taken on the lawn. The photographer is positioned 14 feet away from the center of the class. If she looks toward either end of the class, she turns 50°.



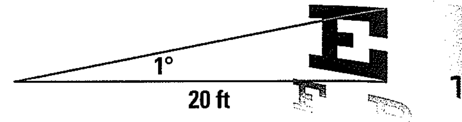
33. **ISOSCELES TRIANGLE** What is the distance between the ends of the class?

34. **MULTI-STEP PROBLEM** The photographer wants to estimate how many more students can fit at the end of the first row. The photographer turns 50° to see the last student and another 10° to see the end of the camera range.
    a. Find the distance from the center to the last student in the row.
    b. Find the distance from the center to the end of the camera range.
    c. Use the results of parts (a) and (b) to estimate the length of the empty space.
    d. If each student needs 2 feet of space, about how many more students can fit at the end of the first row? *Explain* your reasoning.

35. ★ **SHORT RESPONSE** Write expressions for the tangent of each acute angle in the triangle. *Explain* how the tangent of one acute angle is related to the tangent of the other acute angle. What kind of angle pair are $\angle A$ and $\angle B$?

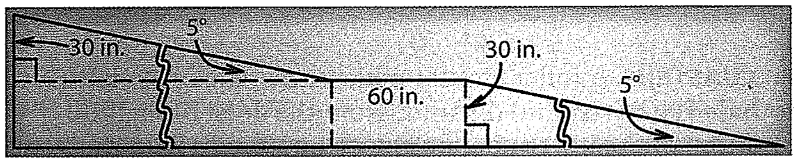


36. EYE CHART You are looking at an eye chart that is 20 feet away. Your eyes are level with the bottom of the "E" on the chart. To see the top of the "E," you look up 1°. How tall is the "E"?



Not drawn to scale

37. ★ EXTENDED RESPONSE According to the Americans with Disabilities Act, a ramp cannot have an incline that is greater than 5°. The regulations also state that the maximum rise of a ramp is 30 inches. When a ramp needs to reach a height greater than 30 inches, a series of ramps connected by 60 inch landings can be used, as shown below.

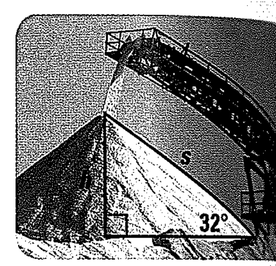


a. What is the maximum horizontal length of the base of one ramp, in feet? Round to the nearest foot.

b. If a doorway is 7.5 feet above the ground, what is the least number of ramps and landings you will need to lead to the doorway? Draw and label a diagram to *justify* your answer.

c. To the nearest foot, what is the total length of the base of the system of ramps and landings in part (b)?

38. CHALLENGE The road salt shown is stored in a cone-shaped pile. The base of the cone has a circumference of 80 feet. The cone rises at an angle of 32°. Find the height h of the cone. Then find the length s of the cone-shaped pile.



MIXED REVIEW

The expressions given represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles. *(p. 217)*

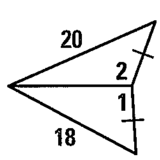
39. $m\angle A = x°$
$m\angle B = 4x°$
$m\angle C = 4x°$

40. $m\angle A = x°$
$m\angle B = x°$
$m\angle C = (5x - 60)°$

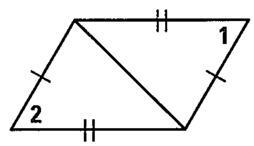
41. $m\angle A = (x + 20)°$
$m\angle B = (3x + 15)°$
$m\angle C = (x - 30)°$

Copy and complete the statement with <, >, or =. *Explain.* *(p. 335)*

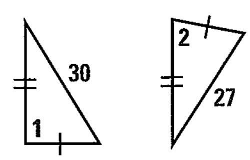
42. $m\angle 1$ __?__ $m\angle 2$



43. $m\angle 1$ __?__ $m\angle 2$



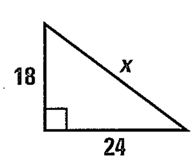
44. $m\angle 1$ __?__ $m\angle 2$



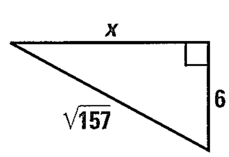
PREVIEW
Prepare for Lesson 7.6 in Exs. 45–47.

Find the unknown side length of the right triangle. *(p. 433)*

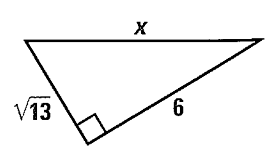
45.



46.



47.



7.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS9 for Exs. 5, 9, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 18, 29, 35, and 37

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

SKILL PRACTICE

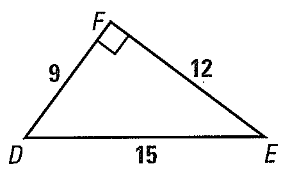
1. **VOCABULARY** Copy and complete: The sine ratio compares the length of __?__ to the length of __?__.

2. ★ **WRITING** *Explain* how to tell which side of a right triangle is adjacent to an angle and which side is the hypotenuse.

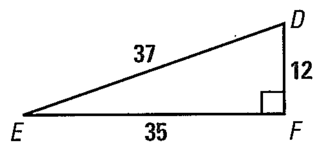
EXAMPLE 1 on p. 473 for Exs. 3–6

FINDING SINE RATIOS **Find sin D and sin E. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.**

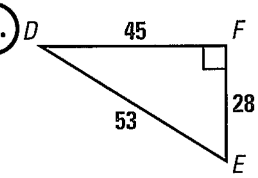
3.



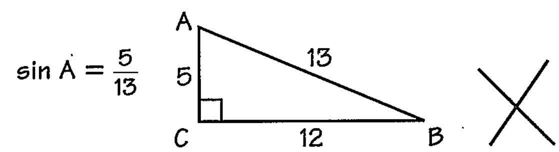
4.



(5.)



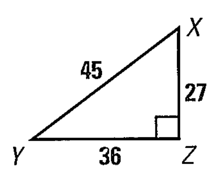
6. **ERROR ANALYSIS** *Explain* why the student's statement is incorrect. Write a correct statement for the sine of the angle.



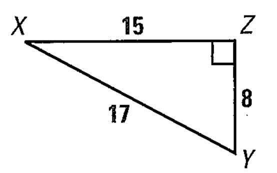
EXAMPLE 2 on p. 474 for Exs. 7–9

FINDING COSINE RATIOS **Find cos X and cos Y. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.**

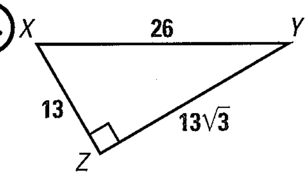
7.



8.



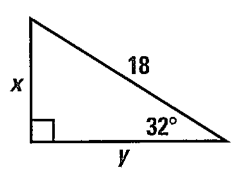
(9.)



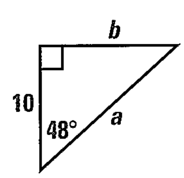
EXAMPLE 3 on p. 474 for Exs. 10–15

USING SINE AND COSINE RATIOS **Use a sine or cosine ratio to find the value of each variable. Round decimals to the nearest tenth.**

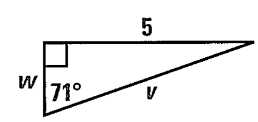
10.



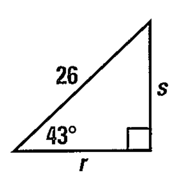
11.



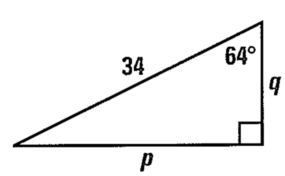
12.



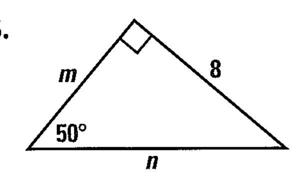
13.



14.



15.

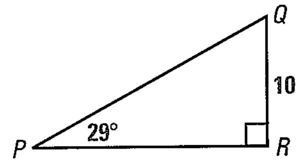


EXAMPLE 6 on p. 476 for Ex. 16

16. **SPECIAL RIGHT TRIANGLES** Use the 45°-45°-90° Triangle Theorem to find the sine and cosine of a 45° angle.

17. ★ **WRITING** *Describe* what you must know about a triangle in order to use the sine ratio and the cosine ratio.

18. ★ **MULTIPLE CHOICE** In $\triangle PQR$, which expression can be used to find PQ?

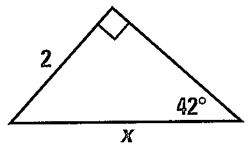


Ⓐ $10 \cdot \cos 29°$ Ⓑ $10 \cdot \sin 29°$

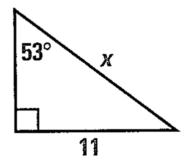
Ⓒ $\dfrac{10}{\sin 29°}$ Ⓓ $\dfrac{10}{\cos 29°}$

ALGEBRA **Find the value of x. Round decimals to the nearest tenth.**

19.



20.



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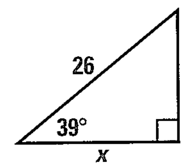

FINDING SINE AND COSINE RATIOS **Find the unknown side length. Then find $\sin X$ and $\cos X$. Write each answer as a fraction in simplest form and as a decimal. Round to four decimal places, if necessary.**

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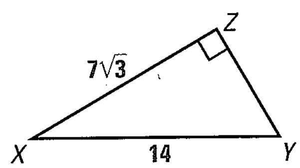

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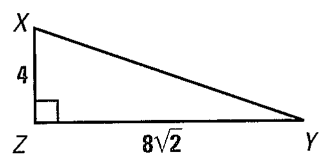

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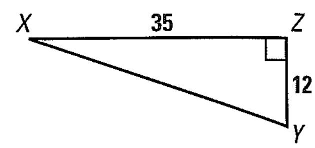

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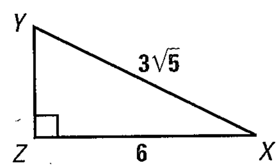

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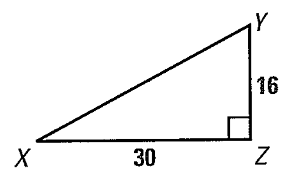

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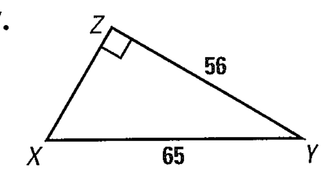


28. **ANGLE MEASURE** Make a prediction about how you could use trigonometric ratios to find angle measures in a triangle.

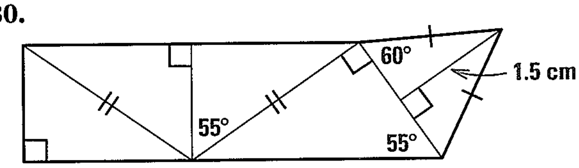
29. ★ **MULTIPLE CHOICE** In $\triangle JKL$, $m\angle L = 90°$. Which statement about $\triangle JKL$ *cannot* be true?

Ⓐ $\sin J = 0.5$ Ⓑ $\sin J = 0.1071$

Ⓒ $\sin J = 0.8660$ Ⓓ $\sin J = 1.1$

PERIMETER **Find the approximate perimeter of the figure.**

30.



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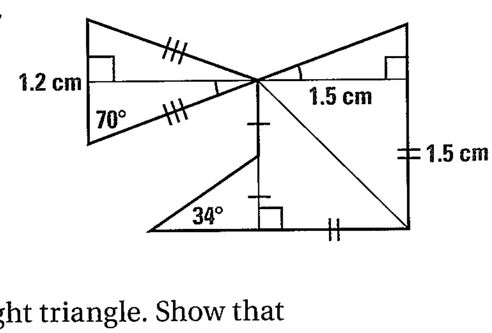

32. **CHALLENGE** Let A be any acute angle of a right triangle. Show that (a) $\tan A = \dfrac{\sin A}{\cos A}$ and (b) $(\sin A)^2 + (\cos A)^2 = 1$.

PROBLEM SOLVING

EXAMPLES 4 and 5 on pp. 475–476 for Exs. 33–36

33. **AIRPLANE RAMP** The airplane door is 19 feet off the ground and the ramp has a 31° angle of elevation. What is the length y of the ramp?

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34. **BLEACHERS** Find the horizontal distance h the bleachers cover. Round to the nearest foot.

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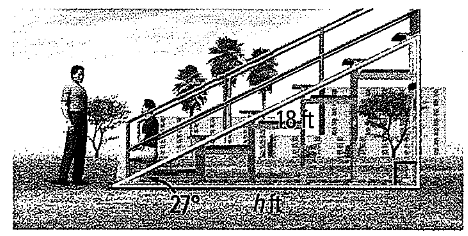
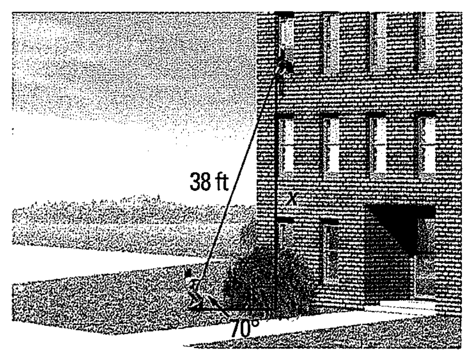
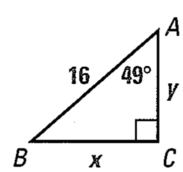


35. **SHORT RESPONSE** You are flying a kite with 20 feet of string extended. The angle of elevation from the spool of string to the kite is 41°.
   a. Draw and label a diagram to represent the situation.
   b. How far off the ground is the kite if you hold the spool 5 feet off the ground? *Describe* how the height where you hold the spool affects the height of the kite.

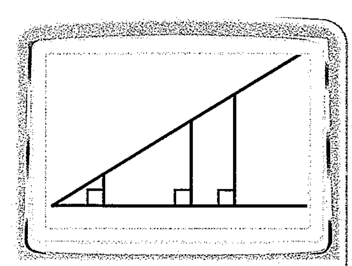
36. **MULTI-STEP PROBLEM** You want to hang a banner that is 29 feet tall from the third floor of your school. You need to know how tall the wall is, but there is a large bush in your way.
   a. You throw a 38 foot rope out of the window to your friend. She extends it to the end and measures the angle of elevation to be 70°. How high is the window?
   b. The bush is 6 feet tall. Will your banner fit above the bush?
   c. **What If?** Suppose you need to find how far from the school your friend needs to stand. Which trigonometric ratio should you use?



37. ★ **SHORT RESPONSE** Nick uses the equation $\sin 49° = \frac{x}{16}$ to find BC in $\triangle ABC$. Tim uses the equation $\cos 41° = \frac{x}{16}$. Which equation produces the correct answer? *Explain*.



38. **TECHNOLOGY** Use geometry drawing software to construct an angle. Mark three points on one side of the angle and construct segments perpendicular to that side at the points. Measure the legs of each triangle and calculate the sine of the angle. Is the sine the same for each triangle?



39. **MULTIPLE REPRESENTATIONS** You are standing on a cliff 30 feet above an ocean. You see a sailboat on the ocean.

a. Drawing a Diagram Draw and label a diagram of the situation.

b. Making a Table Make a table showing the angle of depression and the length of your line of sight. Use the angles 40°, 50°, 60°, 70°, and 80°.

c. Drawing a Graph Graph the values you found in part (b), with the angle measures on the x-axis.

d. Making a Prediction Predict the length of the line of sight when the angle of depression is 30°.

40. **ALGEBRA** If $\triangle EQU$ is equilateral and $\triangle RGT$ is a right triangle with $RG = 2$, $RT = 1$, and $m\angle T = 90°$, show that $\sin E = \cos G$.

41. **CHALLENGE** Make a conjecture about the relationship between sine and cosine values.

a. Make a table that gives the sine and cosine values for the acute angles of a 45°-45°-90° triangle, a 30°-60°-90° triangle, a 34°-56°-90° triangle, and a 17°-73°-90° triangle.

b. Compare the sine and cosine values. What pattern(s) do you notice?

c. Make a conjecture about the sine and cosine values in part (b).

d. Is the conjecture in part (c) true for right triangles that are not special right triangles? *Explain.*

MIXED REVIEW

Rewrite the equation so that x is a function of y. *(p. 877)*

42. $y = \sqrt{x}$

43. $y = 3x - 10$

44. $y = \frac{x}{9}$

PREVIEW
Prepare for Lesson 7.7 in Exs. 45–47.

Copy and complete the table. *(p. 884)*

45.

x	$\sqrt{x}$
?	0
?	1
?	$\sqrt{2}$
?	2
?	4

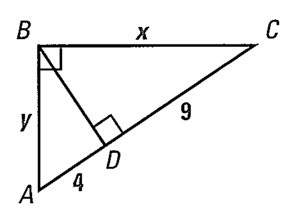
46.

x	$\frac{1}{x}$
?	1
?	$\frac{1}{2}$
?	3
?	$\frac{2}{7}$
?	7

47.

x	$\frac{2}{7}x + 4$
?	0
?	2
?	6
?	8
?	10

48. Find the values of x and y in the triangle at the right. *(p. 449)*



PROBLEM SOLVING WORKSHOP
LESSON 7.6

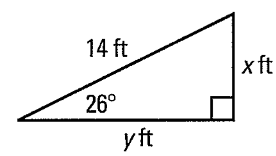
Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 476

MULTIPLE REPRESENTATIONS You can use the Pythagorean Theorem, tangent ratio, sine ratio, or cosine ratio to find the length of an unknown side of a right triangle. The decision of which method to use depends upon what information you have. In some cases, you can use more than one method to find the unknown length.

PROBLEM

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and base of the ramp.



METHOD 1

Using a Cosine Ratio and the Pythagorean Theorem

STEP 1 **Find** the measure of the third angle.

$26° + 90° + m\angle 3 = 180°$	**Triangle Sum Theorem**
$116° + m\angle 3 = 180°$	**Combine like terms.**
$m\angle 3 = 64°$	**Subtact 116° from each side.**

STEP 2 **Use** the cosine ratio to find the height of the ramp.

$\cos 64° = \frac{\text{adj.}}{\text{hyp.}}$	**Write ratio for cosine of 64°.**
$\cos 64° = \frac{x}{14}$	**Substitute.**
$14 \cdot \cos 64° = x$	**Multiply each side by 14.**
$6.1 \approx x$	**Use a calculator to simplify.**

▶ The height is about 6.1 feet.

STEP 3 **Use** the Pythagorean Theorem to find the length of the base of the ramp.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	**Pythagorean Theorem**
$14^2 = 6.1^2 + y^2$	**Substitute.**
$196 = 37.21 + y^2$	**Multiply.**
$158.79 = y^2$	**Subtract 37.21 from each side.**
$12.6 \approx y$	**Find the positive square root.**

▶ The length of the base is about 12.6 feet.

METHOD 2 **Using a Tangent Ratio**

Use the tangent ratio and $h = 6.1$ feet to find the length of the base of the ramp.

$\tan 26° = \frac{\text{opp.}}{\text{adj.}}$ **Write ratio for tangent of 26°.**

$\tan 26° = \frac{6.1}{y}$ **Substitute.**

$y \cdot \tan 26° = 6.1$ **Multiply each side by *y*.**

$y = \frac{6.1}{\tan 26°}$ **Divide each side by tan 26°.**

$y \approx 12.5$ **Use a calculator to simplify.**

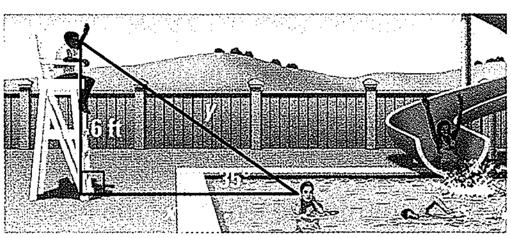
▶ The length of the base is about 12.5 feet.

Notice that when using the Pythagorean Theorem, the length of the base is 12.6 feet, but when using the tangent ratio, the length of the base is 12.5 feet. The tenth of a foot difference is due to the rounding error introduced when finding the height of the ramp and using that rounded value to calculate the length of the base.

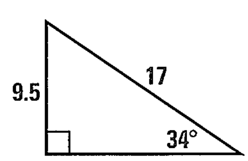
PRACTICE

1. **WHAT IF?** Suppose the length of the skateboard ramp is 20 feet. Find the height and base of the ramp.

2. **SWIMMER** The angle of elevation from the swimmer to the lifeguard is 35°. Find the distance x from the swimmer to the base of the lifeguard chair. Find the distance y from the swimmer to the lifeguard.

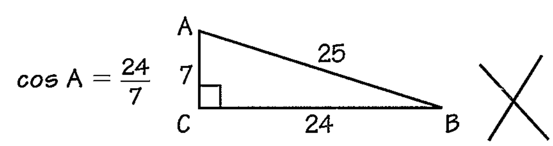


3. **ALGEBRA** Use the triangle below to write three different equations you can use to find the unknown leg length.



4. **SHORT RESPONSE** *Describe* how you would decide whether to use the Pythagorean Theorem or trigonometric ratios to find the lengths of unknown sides of a right triangle.

5. **ERROR ANALYSIS** *Explain* why the student's statement is incorrect. Write a correct statement for the cosine of the angle.

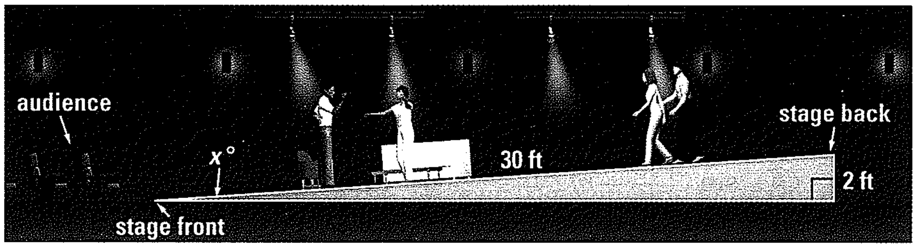


6. **EXTENDED RESPONSE** You want to find the height of a tree in your yard. The tree's shadow is 15 feet long and you measure the angle of elevation from the end of the shadow to the top of tree to be 75°.
   a. Find the height of the tree. *Explain* the method you chose to solve the problem.
   b. What else would you need to know to solve this problem using similar triangles.
   c. *Explain* why you cannot use the sine ratio to find the height of the tree.

EXAMPLE 4 Solve a real-world problem

READ VOCABULARY
A *raked stage* slants upward from front to back to give the audience a better view.

THEATER DESIGN Suppose your school is building a *raked stage*. The stage will be 30 feet long from front to back, with a total rise of 2 feet. A rake (angle of elevation) of 5° or less is generally preferred for the safety and comfort of the actors. Is the raked stage you are building within the range suggested?



Solution

Use the sine and inverse sine ratios to find the degree measure x of the rake.

$$\sin x° = \frac{\text{opp.}}{\text{hyp.}} = \frac{2}{30} \approx 0.0667$$

$$x \approx \sin^{-1} 0.0667 \approx 3.824$$

▶ The rake is about 3.8°, so it is within the suggested range of 5° or less.

✓ **GUIDED PRACTICE** for Examples 3 and 4

3. Solve a right triangle that has a 40° angle and a 20 inch hypotenuse.

4. **WHAT IF?** In Example 4, suppose another raked stage is 20 feet long from front to back with a total rise of 2 feet. Is this raked stage safe? *Explain.*

7.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS9 for Exs. 5, 13, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 29, 30, 35, 40, and 41

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

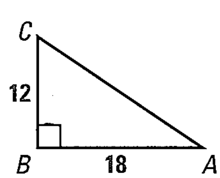
SKILL PRACTICE

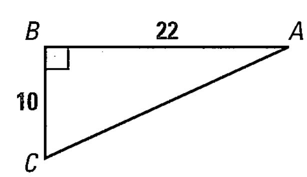
1. **VOCABULARY** Copy and complete: To solve a right triangle means to find the measures of all of its _?_ and _?_.

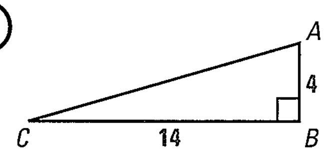
2. ★ **WRITING** *Explain* when to use a trigonometric ratio to find a side length of a right triangle and when to use the Pythagorean Theorem.

EXAMPLE 1 on p. 483 for Exs. 3–5

USING INVERSE TANGENTS **Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**

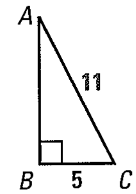
3. 

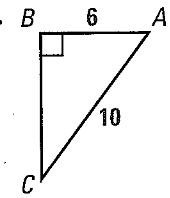
4. 

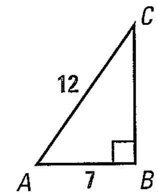
(5.) 

EXAMPLE 2
on p. 484
for Exs. 6–9

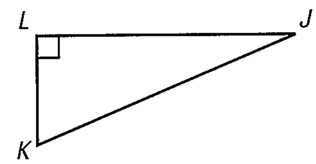
USING INVERSE SINES AND COSINES **Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**

6. 

7. 

8. 

9. ★ **MULTIPLE CHOICE** Which expression is correct?

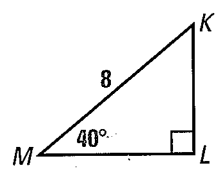


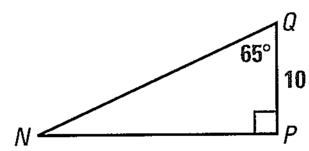
Ⓐ $\sin^{-1}\frac{JL}{JK} = m\angle J$ Ⓑ $\tan^{-1}\frac{KL}{JL} = m\angle J$

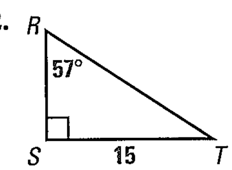
Ⓒ $\cos^{-1}\frac{JL}{JK} = m\angle K$ Ⓓ $\sin^{-1}\frac{JL}{KL} = m\angle K$

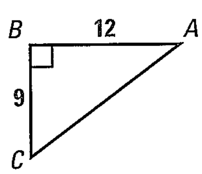
EXAMPLE 3
on p. 484
for Exs. 10–18

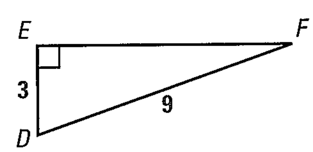
SOLVING RIGHT TRIANGLES **Solve the right triangle. Round decimal answers to the nearest tenth.**

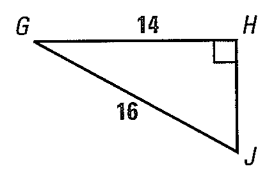
10. 

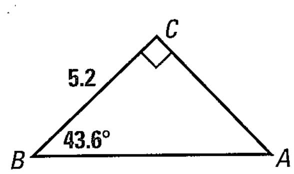
11. 

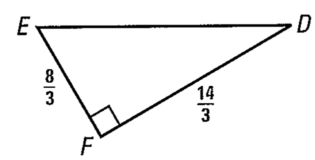
12. 

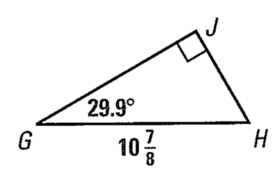
13. 

14. 

15. 

16. 

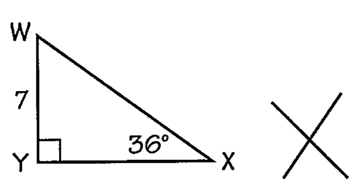
17. 

18. 

ERROR ANALYSIS ***Describe*** **and correct the student's error in using an inverse trigonometric ratio.**

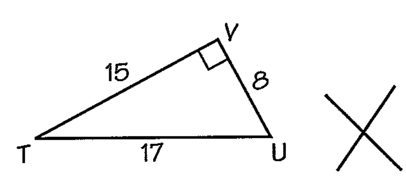
19.

$\sin^{-1}\frac{7}{WY} = 36°$



20.

$\cos^{-1}\frac{8}{15} = m\angle T$



CALCULATOR **Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree.**

21. $\sin A = 0.5$ **22.** $\sin A = 0.75$ **23.** $\cos A = 0.33$ **24.** $\cos A = 0.64$

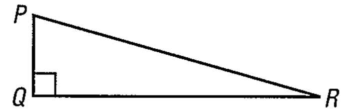
25. $\tan A = 1.0$ **26.** $\tan A = 0.28$ **27.** $\sin A = 0.19$ **28.** $\cos A = 0.81$

○ = **WORKED-OUT SOLUTIONS on p. WS1** ★ = **STANDARDIZED TEST PRACTICE**

29. ★ **MULTIPLE CHOICE** Which additional information would *not* be enough to solve $\triangle PRQ$?

Ⓐ $m\angle P$ and PR Ⓑ $m\angle P$ and $m\angle R$

Ⓒ PQ and PR Ⓓ $m\angle P$ and PQ



30. ★ **WRITING** *Explain* why it is incorrect to say that $\tan^{-1} x = \frac{1}{\tan x}$.

31. **SPECIAL RIGHT TRIANGLES** If $\sin A = \frac{1}{2}\sqrt{2}$, what is $m\angle A$? If $\sin B = \frac{1}{2}\sqrt{3}$, what is $m\angle B$?

32. **TRIGONOMETRIC VALUES** Use the *Table of Trigonometric Ratios* on page 925 to answer the questions.

   a. What angles have nearly the same sine and tangent values?

   b. What angle has the greatest difference in its sine and tangent value?

   c. What angle has a tangent value that is double its sine value?

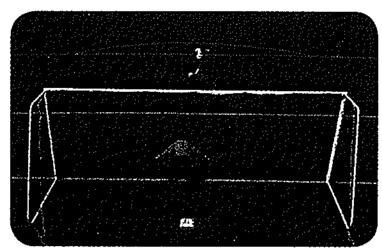
   d. Is $\sin 2x$ equal to $2 \cdot \sin x$?

33. **CHALLENGE** The perimeter of rectangle $ABCD$ is 16 centimeters, and the ratio of its width to its length is 1 : 3. Segment BD divides the rectangle into two congruent triangles. Find the side lengths and angle measures of one of these triangles.

PROBLEM SOLVING

EXAMPLE 4
on p. 485
for Exs. 34–36

34. **SOCCER** A soccer ball is placed 10 feet away from the goal, which is 8 feet high. You kick the ball and it hits the crossbar along the top of the goal. What is the angle of elevation of your kick?



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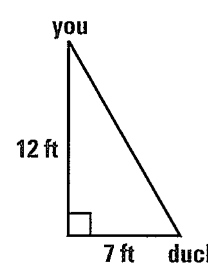
35. ★ **SHORT RESPONSE** You are standing on a footbridge in a city park that is 12 feet high above a pond. You look down and see a duck in the water 7 feet away from the footbridge. What is the angle of depression? *Explain* your reasoning.



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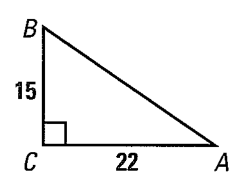
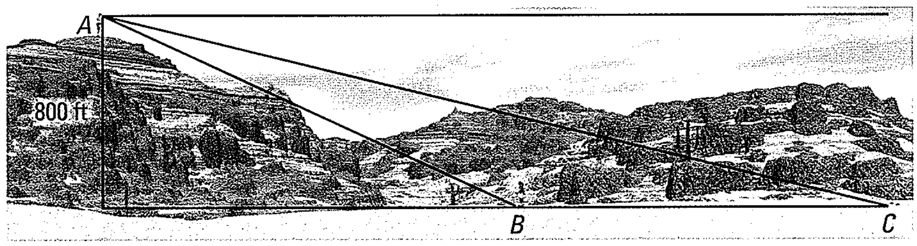
36. **CLAY** In order to unload clay easily, the body of a dump truck must be elevated to at least 55°. If the body of the dump truck is 14 feet long and has been raised 10 feet, will the clay pour out easily?

37. **REASONING** For $\triangle ABC$ shown, each of the expressions $\sin^{-1} \frac{BC}{AB}$, $\cos^{-1} \frac{AC}{AB}$, and $\tan^{-1} \frac{BC}{AC}$ can be used to approximate the measure of $\angle A$. Which expression would you choose? *Explain* your choice.



38. **MULTI-STEP PROBLEM** You are standing on a plateau that is 800 feet above a basin where you can see two hikers.

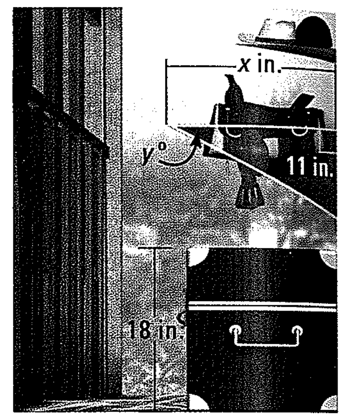


a. If the angle of depression from your line of sight to the hiker at B is 25°, how far is the hiker from the base of the plateau?

b. If the angle of depression from your line of sight to the hiker at C is 15°, how far is the hiker from the base of the plateau?

c. How far apart are the two hikers? *Explain.*

39. ◈ **MULTIPLE REPRESENTATIONS** A local ranch offers trail rides to the public. It has a variety of different sized saddles to meet the needs of horse and rider. You are going to build saddle racks that are 11 inches high. To save wood, you decide to make each rack fit each saddle.



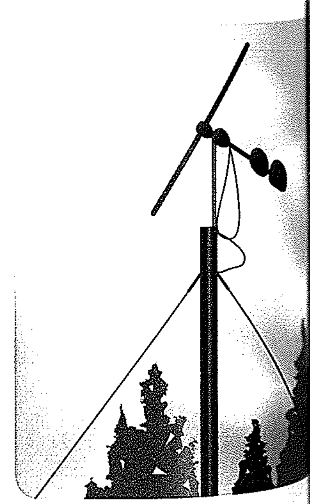
a. **Making a Table** The lengths of the saddles range from 20 inches to 27 inches. Make a table showing the saddle rack length x and the measure of the adjacent angle $y°$.

b. **Drawing a Graph** Use your table to draw a scatterplot.

c. **Making a Conjecture** Make a conjecture about the relationship between the length of the rack and the angle needed.

40. ★ **OPEN-ENDED MATH** *Describe* a real-world problem you could solve using a trigonometric ratio.

41. ★ **EXTENDED RESPONSE** Your town is building a wind generator to create electricity for your school. The builder wants your geometry class to make sure that the guy wires are placed so that the tower is secure. By safety guidelines, the distance along the ground from the tower to the guy wire's connection with the ground should be between 50% to 75% of the height of the guy wire's connection with the tower.



a. The tower is 64 feet tall. The builders plan to have the distance along the ground from the tower to the guy wire's connection with the ground be 60% of the height of the tower. How far apart are the tower and the ground connection of the wire?

b. How long will a guy wire need to be that is attached 60 feet above the ground?

c. How long will a guy wire need to be that is attached 30 feet above the ground?

d. Find the angle of elevation of each wire. Are the right triangles formed by the ground, tower, and wires *congruent, similar,* or *neither*? *Explain.*

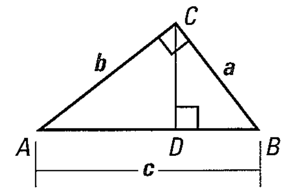
e. *Explain* which trigonometric ratios you used to solve the problem.

○ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE** ◈ = **MULTIPLE REPRESENTATIONS**

42. CHALLENGE Use the diagram of $\triangle ABC$.

GIVEN ▶ $\triangle ABC$ with altitude $\overline{CD}$.

PROVE ▶ $\frac{\sin A}{a} = \frac{\sin B}{b}$



MIXED REVIEW

PREVIEW
Prepare for Lesson 8.1 in Ex. 43.

43. Copy and complete the table. *(p. 42)*

Number of sides	Type of polygon
5	?
12	?
?	Octagon
?	Triangle
7	?

Number of sides	Type of polygon
?	*n*-gon
?	Quadrilateral
10	?
9	?
?	Hexagon

A point on an image and the transformation are given. Find the corresponding point on the original figure. *(p. 272)*

44. Point on image: (5, 1); translation: $(x, y) \to (x + 3, y - 2)$

45. Point on image: (4, −6); reflection: $(x, y) \to (x, -y)$

46. Point on image: (−2, 3); translation: $(x, y) \to (x - 5, y + 7)$

Draw a dilation of the polygon with the given vertices using the given scale factor *k*. *(p. 409)*

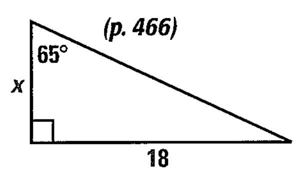
47. $A(2, 2), B(-1, -3), C(5, -3); k = 2$

48. $A(-4, -2), B(-2, 4), C(3, 6), D(6, 3); k = \frac{1}{2}$

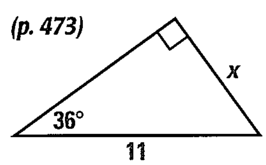
QUIZ for Lessons 7.5–7.7

Find the value of *x* to the nearest tenth.

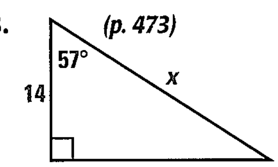
1. *(p. 466)*



2. *(p. 473)*

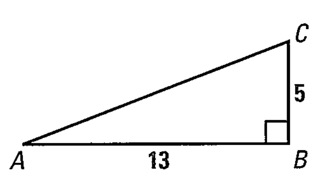


3. *(p. 473)*

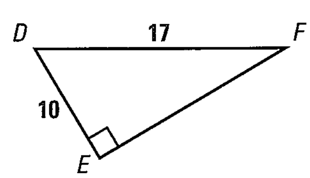


Solve the right triangle. Round decimal answers to the nearest tenth. *(p. 483)*

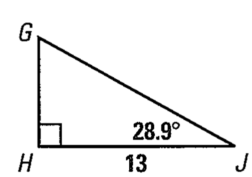
4.

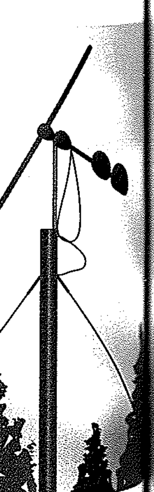


5.



6.





Extension
Use after Lesson 7.7

Law of Sines and Law of Cosines

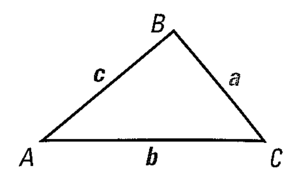
GOAL Use trigonometry with acute and obtuse triangles.

The trigonometric ratios you have seen so far in this chapter can be used to find angle and side measures in right triangles. You can use the Law of Sines to find angle and side measures in *any* triangle.

KEY CONCEPT *For Your Notebook*

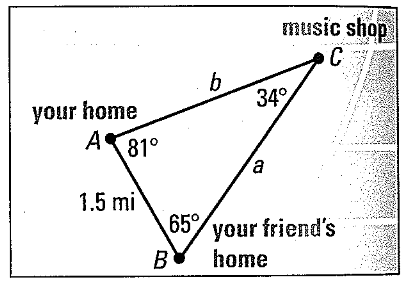
Law of Sines

If $\triangle ABC$ has sides of length a, b, and c as shown, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



EXAMPLE 1 Find a distance using Law of Sines

DISTANCE Use the information in the diagram to determine how much closer you live to the music store than your friend does.



Solution

STEP 1 **Use** the Law of Sines to find the distance a from your friend's home to the music store.

$\frac{\sin A}{a} = \frac{\sin C}{c}$ **Write Law of Sines.**

$\frac{\sin 81°}{a} = \frac{\sin 34°}{1.5}$ **Substitute.**

$a \approx 2.6$ **Solve for *a*.**

STEP 2 **Use** the Law of Sines to find the distance b from your home to the music store.

$\frac{\sin B}{b} = \frac{\sin C}{c}$ **Write Law of Sines.**

$\frac{\sin 65°}{b} = \frac{\sin 34°}{1.5}$ **Substitute.**

$b \approx 2.4$ **Solve for *b*.**

STEP 3 **Subtract** the distances.

$a - b \approx 2.6 - 2.4 = 0.2$

▶ You live about 0.2 miles closer to the music store.

LAW OF COSINES You can also use the Law of Cosines to solve any triangle.

KEY CONCEPT — *For Your Notebook*

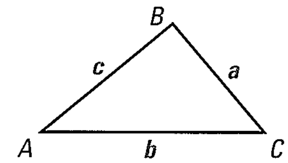
Law of Cosines

If $\triangle ABC$ has sides of length a, b, and c, then:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

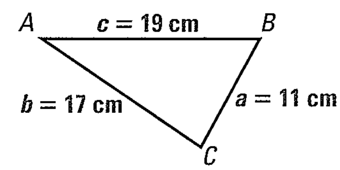
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



EXAMPLE 2 Find an angle measure using Law of Cosines

In $\triangle ABC$ at the right, $a = 11$ cm, $b = 17$ cm, and $c = 19$ cm. Find $m\angle C$.



Solution

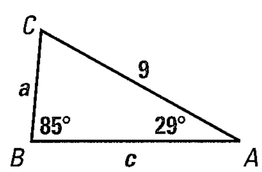
$c^2 = a^2 + b^2 - 2ab \cos C$	**Write Law of Cosines.**
$19^2 = 11^2 + 17^2 - 2(11)(17) \cos C$	**Substitute.**
$0.1310 = \cos C$	**Solve for cos *C*.**
$m\angle C \approx 82°$	**Find $\cos^{-1}$ (0.1310).**

PRACTICE

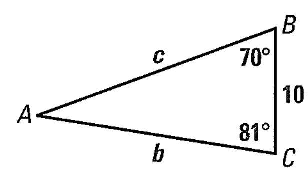
EXAMPLE 1 for Exs. 1–3

LAW OF SINES Use the Law of Sines to solve the triangle. Round decimal answers to the nearest tenth.

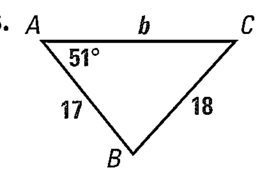
1.



2.



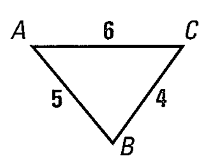
3.



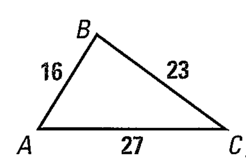
EXAMPLE 2 for Exs. 4–7

LAW OF COSINES Use the Law of Cosines to solve the triangle. Round decimal answers to the nearest tenth.

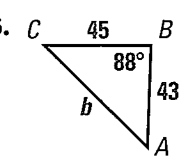
4.



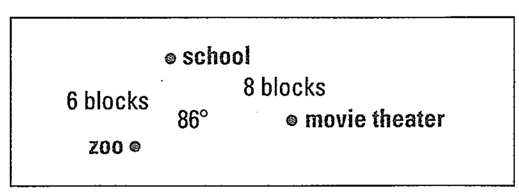
5.



6.



7. DISTANCE Use the diagram at the right. Find the straight distance between the zoo and movie theater.



MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
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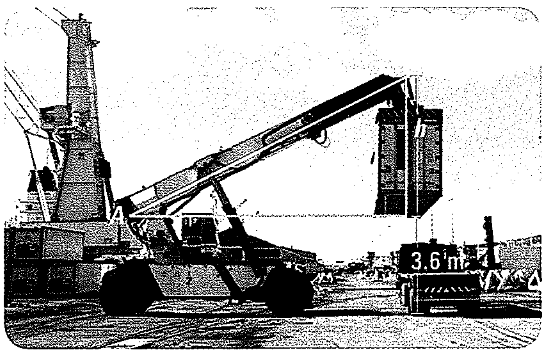
Lessons 7.5–7.7

1. **MULTI-STEP PROBLEM** A *reach stacker* is a vehicle used to lift objects and move them between ships and land.



   a. The vehicle's arm is 10.9 meters long. The maximum measure of $\angle A$ is 60°. What is the greatest height h the arm can reach if the vehicle is 3.6 meters tall?

   b. The vehicle's arm can extend to be 16.4 meters long. What is the greatest height its extended arm can reach?

   c. What is the difference between the two heights the arm can reach above the ground?

2. **EXTENDED RESPONSE** You and a friend are standing the same distance from the edge of a canyon. Your friend looks directly across the canyon at a rock. You stand 10 meters from your friend and estimate the angle between your friend and the rock to be 85°.

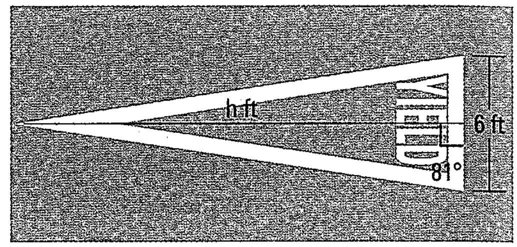
   a. Sketch the situation.

   b. *Explain* how to find the distance across the canyon.

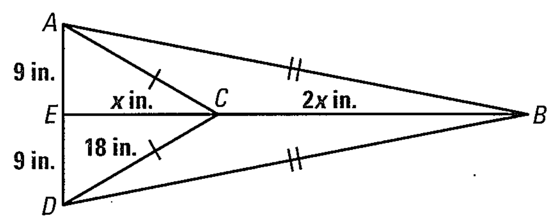
   c. Suppose the actual angle measure is 87°. How far off is your estimate of the distance?

3. **SHORT RESPONSE** The international rules of basketball state the rim of the net should be 3.05 meters above the ground. If your line of sight to the rim is 34° and you are 1.7 meters tall, what is the distance from you to the rim? *Explain* your reasoning.

4. **GRIDDED ANSWER** The specifications for a *yield ahead* pavement marking are shown. Find the height h in feet of this isosceles triangle to the nearest tenth.



5. **EXTENDED RESPONSE** Use the diagram to answer the questions.

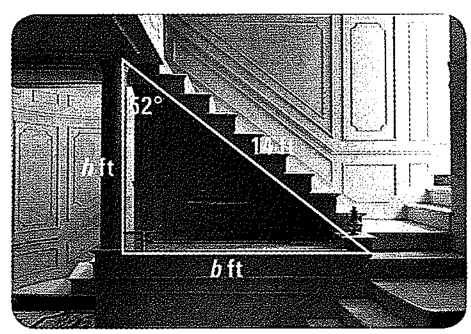


   a. Solve for x. *Explain* the method you chose.

   b. Find $m\angle ABC$. *Explain* the method you chose.

   c. *Explain* a different method for finding each of your answers in parts (a) and (b).

6. **SHORT RESPONSE** The triangle on the staircase below has a 52° angle and the distance along the stairs is 14 feet. What is the height h of the staircase? What is the length b of the base of the staircase?



7. **GRIDDED ANSWER** The base of an isosceles triangle is 70 centimeters long. The altitude to the base is 75 centimeters long. Find the measure of a base angle to the nearest degree.

7 CHAPTER SUMMARY

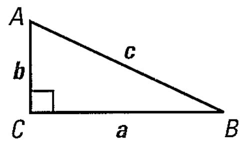
BIG IDEAS — For Your Notebook

Big Idea 1

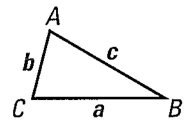
Using the Pythagorean Theorem and Its Converse

The Pythagorean Theorem states that in a right triangle the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b, so that $c^2 = a^2 + b^2$.

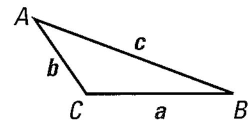
The Converse of the Pythagorean Theorem can be used to determine if a triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.



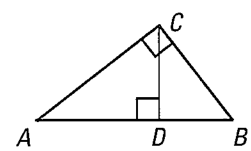
If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

Big Idea 2

Using Special Relationships in Right Triangles

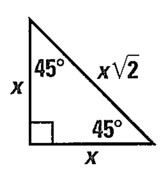
GEOMETRIC MEAN In right $\triangle ABC$, altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.

Also, $\frac{BD}{CD} = \frac{CD}{AD}$, $\frac{AB}{CB} = \frac{CB}{DB}$, and $\frac{AB}{AC} = \frac{AC}{AD}$.



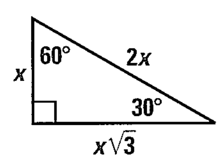
SPECIAL RIGHT TRIANGLES

45°-45°-90° Triangle



hypotenuse = leg • $\sqrt{2}$

30°-60°-90° Triangle



hypotenuse = 2 • shorter leg
longer leg = shorter leg • $\sqrt{3}$

Big Idea 3

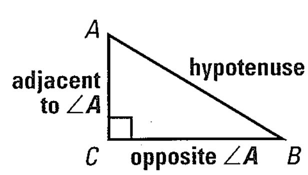
Using Trigonometric Ratios to Solve Right Triangles

The tangent, sine, and cosine ratios can be used to find unknown side lengths and angle measures of right triangles. The values of tan $x°$, sin $x°$, and cos $x°$ depend only on the angle measure and not on the side length.

$\tan A = \frac{\text{opp.}}{\text{adj.}} = \frac{BC}{AC}$ $\quad \tan^{-1}\frac{BC}{AC} = m\angle A$

$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{AB}$ $\quad \sin^{-1}\frac{BC}{AB} = m\angle A$

$\cos A = \frac{\text{adj.}}{\text{hyp.}} = \frac{AC}{AB}$ $\quad \cos^{-1}\frac{AC}{AB} = m\angle A$



7 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- Pythagorean triple, *p. 435*
- trigonometric ratio, *p. 466*
- tangent, *p. 466*
- sine, *p. 473*
- cosine, *p. 473*
- angle of elevation, *p. 475*
- angle of depression, *p. 475*
- solve a right triangle, *p. 483*
- inverse tangent, *p. 483*
- inverse sine, *p. 483*
- inverse cosine, *p. 483*

VOCABULARY EXERCISES

1. Copy and complete: A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation __?__.
2. **WRITING** What does it mean to solve a right triangle? What do you need to know to solve a right triangle?
3. **WRITING** *Describe* the difference between an angle of depression and an angle of elevation.

REVIEW EXAMPLES AND EXERCISES

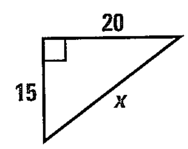
Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 7.

7.1 Apply the Pythagorean Theorem

pp. 433–439

EXAMPLE

Find the value of x.



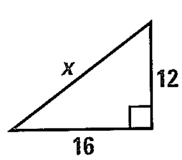
Because x is the length of the hypotenuse of a right triangle, you can use the Pythagorean Theorem to find its value.

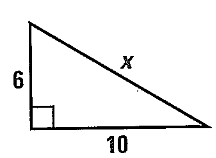
$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	**Pythagorean Theorem**
$x^2 = 15^2 + 20^2$	**Substitute.**
$x^2 = 625$	**Simplify.**
$x = 25$	**Find the positive square root.**

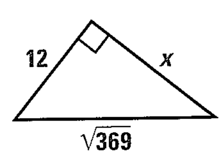
EXERCISES

EXAMPLES 1 and 2 on pp. 433–434 for Exs. 4–6

Find the unknown side length x.

4. 

5. 

6. 

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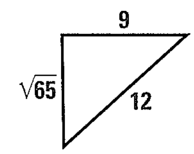
Chapter Review Practice

7.2 Use the Converse of the Pythagorean Theorem

pp. 441–447

EXAMPLE

Tell whether the given triangle is a right triangle.



Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

$12^2 \stackrel{?}{=} (\sqrt{65})^2 + 9^2$

$144 \stackrel{?}{=} 65 + 81$

$144 < 146$

The triangle is not a right triangle. It is an acute triangle.

EXERCISES

EXAMPLE 2 on p. 442 for Exs. 7–12

Classify the triangle formed by the side lengths as *acute*, *right*, or *obtuse*.

7. 6, 8, 9

8. 4, 2, 5

9. 10, $2\sqrt{2}$, $6\sqrt{3}$

10. 15, 20, 15

11. 3, 3, $3\sqrt{2}$

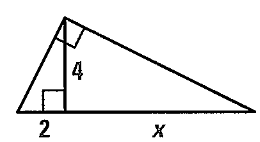
12. 13, 18, $3\sqrt{55}$

7.3 Use Similar Right Triangles

pp. 449–456

EXAMPLE

Find the value of x.



By Theorem 7.6, you know that 4 is the geometric mean of x and 2.

$\frac{x}{4} = \frac{4}{2}$ **Write a proportion.**

$2x = 16$ **Cross Products Property**

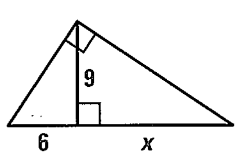
$x = 8$ **Divide.**

EXERCISES

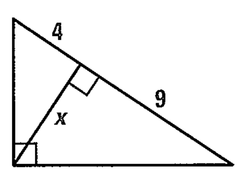
EXAMPLES 2 and 3 on pp. 450–451 for Exs. 13–18

Find the value of x.

13.



14.



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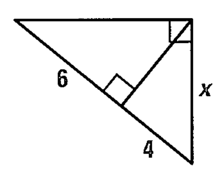

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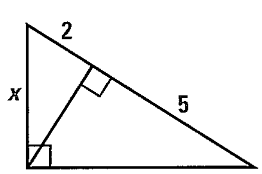
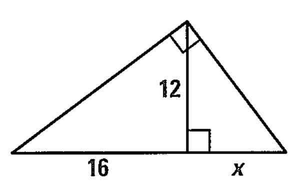


17.



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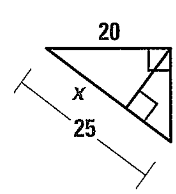


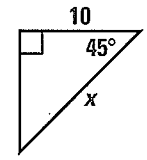
7 CHAPTER REVIEW

7.4 Special Right Triangles

pp. 457–464

EXAMPLE

Find the length of the hypotenuse.



By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle.

$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$ **45°-45°-90° Triangle Theorem**

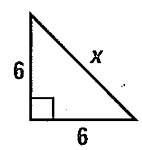
$x = 10\sqrt{2}$ **Substitute.**

EXERCISES

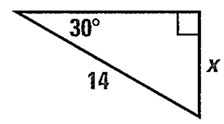
EXAMPLES 1, 2, and 5 on pp. 457–459 for Exs. 19–21

Find the value of x. Write your answer in simplest radical form.

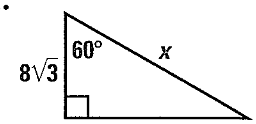
19.



20.



21.

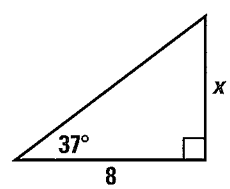


7.5 Apply the Tangent Ratio

pp. 466–472

EXAMPLE

Find the value of x.



$\tan 37° = \frac{\text{opp.}}{\text{adj.}}$ **Write ratio for tangent of 37°.**

$\tan 37° = \frac{x}{8}$ **Substitute.**

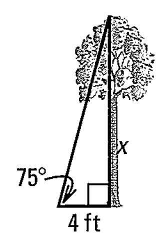
$8 \cdot \tan 37° = x$ **Multiply each side by 8.**

$6 \approx x$ **Use a calculator to simplify.**

EXERCISES

EXAMPLE 2 on p. 467 for Exs. 22–26

In Exercises 22 and 23, use the diagram.

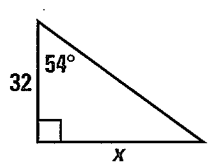


22. The angle between the bottom of a fence and the top of a tree is 75°. The tree is 4 feet from the fence. How tall is the tree? Round your answer to the nearest foot.

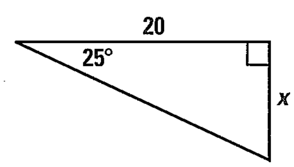
23. In Exercise 22, how tall is the tree if the angle is 55°?

Find the value of x to the nearest tenth.

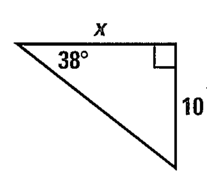
24.



25.



26.



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Chapter Review Practice

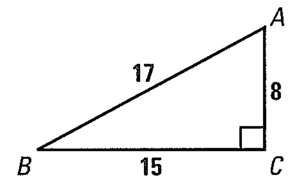
7.6 Apply the Sine and Cosine Ratios *pp. 473–480*

EXAMPLE

Find sin *A* and sin *B*.

$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{BA} = \frac{15}{17} \approx 0.8824$

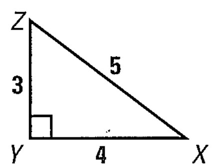
$\sin B = \frac{\text{opp.}}{\text{hyp.}} = \frac{AC}{AB} = \frac{8}{17} \approx 0.4706$

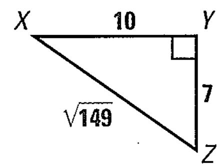


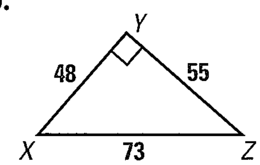
EXERCISES

EXAMPLES 1 and 2 on pp. 473–474 for Exs. 27–29

Find sin *X* and cos *X*. Write each answer as a fraction, and as a decimal. Round to four decimals places, if necessary.

27.


28.


29.


7.7 Solve Right Triangles *pp. 483–489*

EXAMPLE

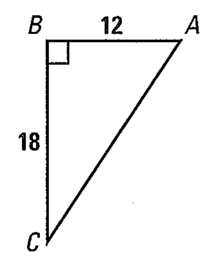
Use a calculator to approximate the measure of ∠*A* to the nearest tenth of a degree.

Because $\tan A = \frac{18}{12} = \frac{3}{2} = 1.5$, $\tan^{-1} 1.5 = m\angle A$.

Use a calculator to evaluate this expression.

$\tan^{-1} 1.5 \approx 56.3099324\ldots$

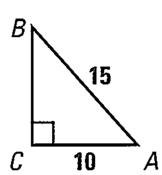
So, the measure of ∠*A* is approximately 56.3°.

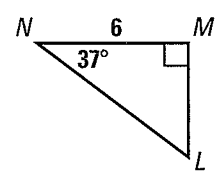


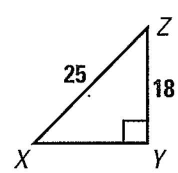
EXERCISES

EXAMPLE 3 on p. 484 for Exs. 30–33

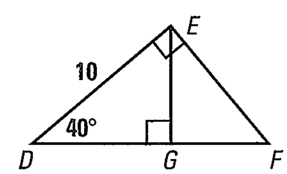
Solve the right triangle. Round decimal answers to the nearest tenth.

30.


31.


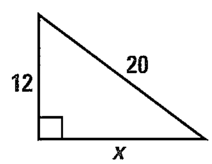
32.


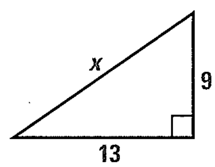
33. Find the measures of ∠*GED*, ∠*GEF*, and ∠*EFG*. Find the lengths of $\overline{EG}$, $\overline{DF}$, $\overline{EF}$.

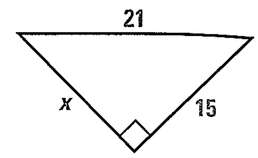


7 CHAPTER TEST

Find the value of x. Write your answer in simplest radical form.

1.


2.


3.


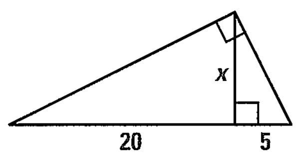
Classify the triangle as *acute, right,* or *obtuse.*

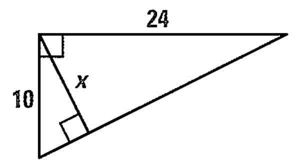
4. 5, 15, $5\sqrt{10}$

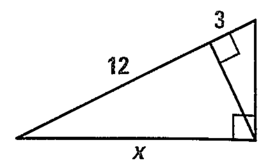
5. 4.3, 6.7, 8.2

6. 5, 7, 8

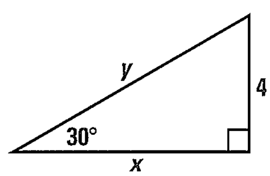
Find the value of x. Round decimal answers to the nearest tenth.

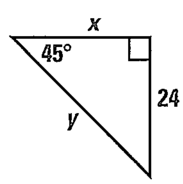
7.


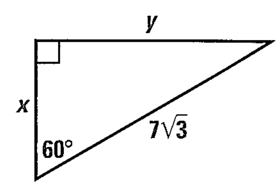
8.


9.


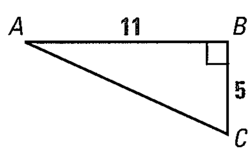
Find the value of each variable. Write your answer in simplest radical form.

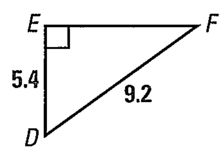
10.


11.


12.


Solve the right triangle. Round decimal answers to the nearest tenth.

13.


14.


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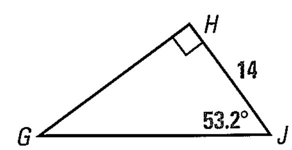
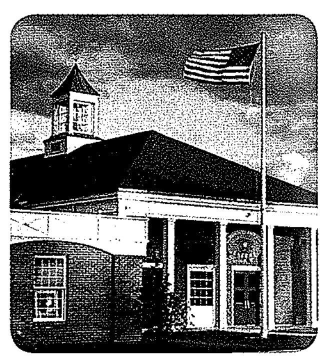
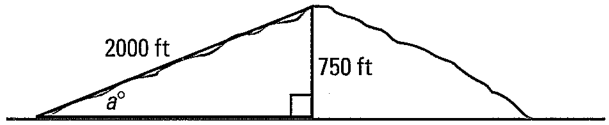


16. FLAGPOLE Julie is 6 feet tall. If she stands 15 feet from the flagpole and holds a cardboard square, the edges of the square line up with the top and bottom of the flagpole. Approximate the height of the flagpole.



17. HILLS The length of a hill in your neighborhood is 2000 feet. The height of the hill is 750 feet. What is the angle of elevation of the hill?



GRAPH AND SOLVE QUADRATIC EQUATIONS

The graph of $y = ax^2 + bx + c$ is a parabola that opens upward if $a > 0$ and opens downward if $a < 0$. The x-coordinate of the vertex is $-\frac{b}{2a}$. The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.

EXAMPLE 1 *Graph a quadratic function*

Graph the equation $y = -x^2 + 4x - 3$.

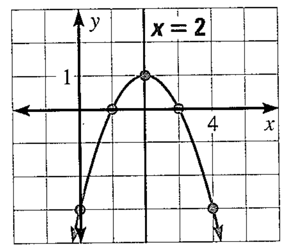
Because $a = -1$ and $-1 < 0$, the graph opens downward.

The vertex has x-coordinate $-\frac{b}{2a} = -\frac{4}{2(-1)} = 2$.

The y-coordinate of the vertex is $-(2)^2 + 4(2) - 3 = 1$.

So, the vertex is (2, 1) and the axis of symmetry is $x = 2$.

Use a table of values to draw a parabola through the plotted points.



EXAMPLE 2 *Solve a quadratic equation by graphing*

Solve the equation $x^2 - 2x = 3$.

Write the equation in the standard form $ax^2 + bx + c = 0$:

$x^2 - 2x - 3 = 0.$

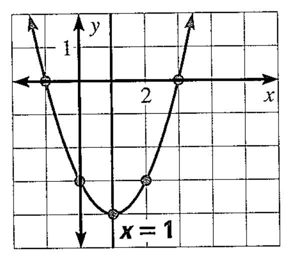
Graph the related quadratic function $y = x^2 - 2x - 3$, as shown.

The x-intercepts of the graph are -1 and 3.

So, the solutions of $x^2 - 2x = 3$ are -1 and 3.

Check the solution algebraically.

$(-1)^2 - 2(-1) \stackrel{?}{=} 3 \rightarrow 1 + 2 = 3$ $\qquad$ $(3)^2 - 2(3) \stackrel{?}{=} 3 \rightarrow 9 - 6 = 3$ ✓



EXERCISES

EXAMPLE 1 for Exs. 1–6

Graph the quadratic function. Label the vertex and axis of symmetry.

1. $y = x^2 - 6x + 8$
2. $y = -x^2 - 4x + 2$
3. $y = 2x^2 - x - 1$
4. $y = 3x^2 - 9x + 2$
5. $y = \frac{1}{2}x^2 - x + 3$
6. $y = -4x^2 + 6x - 5$

EXAMPLE 2 for Exs. 7–18

Solve the quadratic equation by graphing. Check solutions algebraically.

7. $x^2 = x + 6$
8. $4x + 4 = -x^2$
9. $2x^2 = -8$
10. $3x^2 + 2 = 14$
11. $-x^2 + 4x - 5 = 0$
12. $2x - x^2 = -15$
13. $\frac{1}{4}x^2 = 2x$
14. $x^2 + 3x = 4$
15. $x^2 + 8 = 6x$
16. $x^2 = 9x - 1$
17. $-25 = x^2 + 10x$
18. $x^2 + 6x = 0$