

Int  
Alg

# 7

# Systems of Equations and Inequalities

- 7.1 Solve Linear Systems by Graphing
- 7.2 Solve Linear Systems by Substitution
- 7.3 Solve Linear Systems by Adding or Subtracting
- 7.4 Solve Linear Systems by Multiplying First
- 7.5 Solve Special Types of Linear Systems
- 7.6 Solve Systems of Linear Inequalities

## Before

In previous chapters, you learned the following skills, which you'll use in Chapter 7: graphing linear equations, solving equations, determining whether lines are parallel, and graphing linear inequalities in two variables.

## Prerequisite Skills

### VOCABULARY CHECK

Copy and complete the statement.

1. The least common multiple of 10 and 15 is ?.
2. Two lines in the same plane are ? if they do not intersect.

### SKILLS CHECK

Graph the equation. (Review p. 225 for 7.1.)

3.  $x - y = 4$
4.  $6x - y = -1$
5.  $4x + 5y = 20$
6.  $3x - 2y = -12$

Solve the equation. (Review p. 148 for 7.2–7.4.)

7.  $5m + 4 - m = 20$
8.  $10(z + 5) + z = 6$

Tell whether the graphs of the two equations are parallel lines. Explain your reasoning. (Review p. 244 for 7.5.)

9.  $y = 2x - 3, y + 2x = -3$
10.  $y - 5x = -1, y - 5x = 1$
11.  $y = x + 10, x - y = -9$
12.  $6x - y = 4, 4x - y = 6$

Graph the inequality. (Review p. 405 for 7.6.)

13.  $y \leq -2x + 1$
14.  $x - y < 5$
15.  $x \geq -4$
16.  $y > 3$

@HomeTutor Prerequisite skills practice at [classzone.com](http://classzone.com)

## Now

In Chapter 7, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 474. You will also use the key vocabulary listed below.

### Big Ideas

- 1 Solving linear systems by graphing
- 2 Solving linear systems using algebra
- 3 Solving systems of linear inequalities

#### KEY VOCABULARY

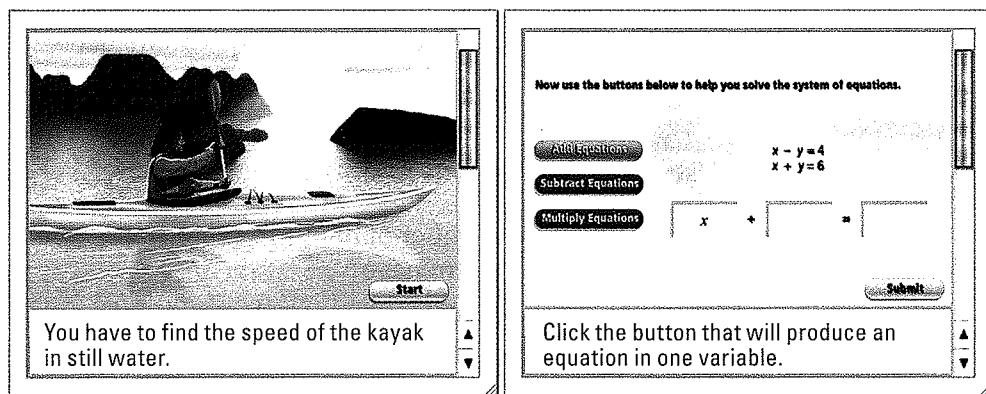
- system of linear equations, p. 427
- solution of a system of linear equations, p. 427
- consistent independent system, p. 427
- inconsistent system, p. 459
- consistent dependent system, p. 459
- system of linear inequalities, p. 466
- solution of a system of linear inequalities, p. 466
- graph of a system of linear inequalities, p. 466

## Why?

You can use a system of linear equations to solve problems about traveling with and against a current. For example, you can write and solve a system of linear equations to find the average speed of a kayak in still water.

### Animated Algebra

The animation illustrated below for Example 4 on page 446 helps you answer this question: What is the average speed of the kayak in still water?



You have to find the speed of the kayak in still water.

Now use the buttons below to help you solve the system of equations.

Add Equations Subtract Equations Multiply Equations

$$\begin{aligned}x - y &= 4 \\x + y &= 6\end{aligned}$$

x +  =

Submit

Click the button that will produce an equation in one variable.

**Animated Algebra** at [classzone.com](http://classzone.com)

**Other animations for Chapter 7:** pages 428, 435, 441, 446, 452, 459, and 466

# 7.1 Solving Linear Systems Using Tables

**MATERIALS** • pencil and paper

**QUESTION** How can you use a table to solve a linear system?

A *system of linear equations*, or *linear system*, consists of two or more linear equations in the same variables. A *solution of a linear system* is an ordered pair that satisfies each equation in the system. You can use a table to find a solution to a linear system.

**EXPLORE** Solve a linear system

Bill and his brother collect comic books. Bill currently has 15 books and adds 2 books to his collection every month. His brother currently has 7 books and adds 4 books to his collection every month. Use the equations below to find the number  $x$  of months after which Bill and his brother will have the same number  $y$  of comic books in their collections.

$y = 2x + 15$       Number of comic books in Bill's collection

$y = 4x + 7$       Number of comic books in his brother's collection

**STEP 1** *Make a table*

Copy and complete the table of values shown.

**STEP 2** *Find a solution*

Find an  $x$ -value that gives the same  $y$ -value for both equations.

**STEP 3** *Interpret the solution*

Use your answer to Step 2 to find the number of months after which Bill and his brother have the same number of comic books.

$x$	$y = 2x + 15$	$y = 4x + 7$
0	15	7
1	?	?
2	?	?
3	?	?
4	?	?
5	?	?

**DRAW CONCLUSIONS** Use your observations to complete these exercises

- When Bill and his brother have the same number of books in their collections, how many books will each of them have?
- Graph the equations above on the same coordinate plane. What do you notice about the graphs and the solution you found above?

Use a table to solve the linear system.

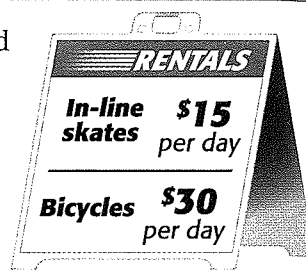
3.  $y = 2x + 3$   
 $y = -3x + 18$

4.  $y = -x + 1$   
 $y = 2x - 5$

5.  $y = -3x + 1$   
 $y = 5x - 31$

### EXAMPLE 4 Solve a multi-step problem

**RENTAL BUSINESS** A business rents in-line skates and bicycles. During one day, the business has a total of 25 rentals and collects \$450 for the rentals. Find the number of pairs of skates rented and the number of bicycles rented.



#### Solution

**STEP 1** Write a linear system. Let  $x$  be the number of pairs of skates rented, and let  $y$  be the number of bicycles rented.

$$x + y = 25 \quad \text{Equation for number of rentals}$$

$$15x + 30y = 450 \quad \text{Equation for money collected from rentals}$$

**STEP 2** Graph both equations.

**STEP 3** Estimate the point of intersection. The two lines appear to intersect at (20, 5).

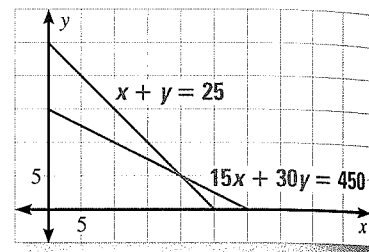
**STEP 4** Check whether (20, 5) is a solution.

$$20 + 5 \stackrel{?}{=} 25$$

$$25 = 25 \checkmark$$

$$15(20) + 30(5) \stackrel{?}{=} 450$$

$$450 = 450 \checkmark$$



► The business rented 20 pairs of skates and 5 bicycles.



#### GUIDED PRACTICE for Example 4

6. **WHAT IF?** In Example 4, suppose the business has a total of 20 rentals and collects \$420. Find the number of bicycles rented.

## 7.1 EXERCISES

#### HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS on p. WS16 for Exs. 15 and 31

★ = STANDARDIZED TEST PRACTICE Exs. 2, 6, 7, 27, 28, 29, and 32

◆ = MULTIPLE REPRESENTATIONS Ex. 35

### SKILL PRACTICE

- VOCABULARY** Copy and complete: A(n) ? of a system of linear equations in two variables is an ordered pair that satisfies each equation in the system.
- ★ WRITING** Explain how to use the graph-and-check method to solve a linear system of two equations in two variables.

**CHECKING SOLUTIONS** Tell whether the ordered pair is a solution of the linear system.

3.  $(-3, 1);$   
 $x + y = -2$   
 $x + 5y = 2$

4.  $(5, 2);$   
 $2x - 3y = 4$   
 $2x + 8y = 11$

5.  $(-2, 1);$   
 $6x + 5y = -7$   
 $x - 2y = 0$

**EXAMPLE 1**

on p. 427  
for Exs. 6–11

6. **★ MULTIPLE CHOICE** Which ordered pair is a solution of the linear system  $x + y = -2$  and  $7x - 4y = 8$ ?

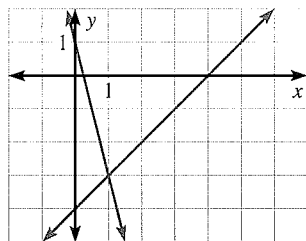
(A)  $(-2, 0)$       (B)  $(0, -2)$       (C)  $(2, 0)$       (D)  $(0, 2)$

7. **★ MULTIPLE CHOICE** Which ordered pair is a solution of the linear system  $2x + 3y = 12$  and  $10x + 3y = -12$ ?

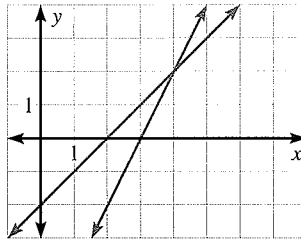
(A)  $(-3, 3)$       (B)  $(-3, 6)$       (C)  $(3, 3)$       (D)  $(3, 6)$

**SOLVING SYSTEMS GRAPHICALLY** Use the graph to solve the linear system. Check your solution.

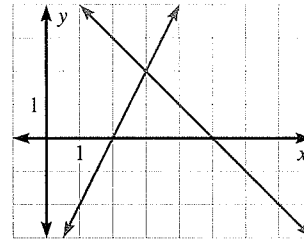
8.  $x - y = 4$   
 $4x + y = 1$



9.  $-x + y = -2$   
 $2x - y = 6$

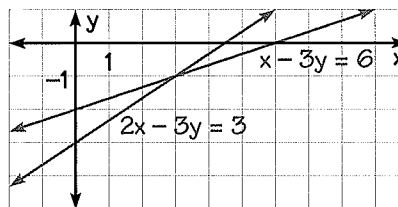


10.  $x + y = 5$   
 $-2x + y = -4$



11. **ERROR ANALYSIS** Describe and correct the error in solving the linear system below.

$x - 3y = 6$       Equation 1  
 $2x - 3y = 3$       Equation 2



The solution is  $(3, -1)$ .

**EXAMPLE 2**

on p. 428  
for Exs. 12–26

**GRAPH-AND-CHECK METHOD** Solve the linear system by graphing. Check your solution.

12.  $y = -x + 3$   
 $y = x + 1$

13.  $y = -x + 4$   
 $y = 2x - 8$

14.  $y = 2x + 2$   
 $y = 4x + 6$

15.  $x - y = 2$   
 $x + y = -8$

16.  $x + 2y = 1$   
 $-2x + y = -4$

17.  $3x + y = 15$   
 $y = -15$

18.  $2x - 3y = -1$   
 $5x + 2y = 26$

19.  $6x + y = 37$   
 $4x + 2y = 18$

20.  $7x + 5y = -3$   
 $-9x + y = -11$

21.  $6x + 12y = -6$   
 $2x + 5y = 0$

22.  $2x + y = 9$   
 $2x + 3y = 15$

23.  $-5x + 3y = 3$   
 $4x + 3y = 30$

24.  $\frac{3}{4}x + \frac{1}{4}y = \frac{13}{2}$   
 $x - \frac{3}{4}y = \frac{13}{2}$

25.  $\frac{1}{5}x - \frac{2}{5}y = -\frac{8}{5}$   
 $-\frac{3}{4}x + y = 3$

26.  $-1.6x - 3.2y = -24$   
 $2.6x + 2.6y = 26$

27. **★ OPEN-ENDED** Find values for  $m$  and  $b$  so that the system  $y = \frac{3}{5}x - 1$  and  $y = mx + b$  has  $(5, 2)$  as a solution.

28. **★ WRITING** Solve the linear system shown by graphing. Explain why it is important to check your solution.

$y = 4x - 1.5$       Equation 1  
 $y = -2x + 1.5$       Equation 2

29. ★ **EXTENDED RESPONSE** Consider the equation  $-\frac{1}{4}x + 6 = \frac{1}{2}x + 3$ .

- Solve the equation using algebra.
- Solve the linear system below using a graph.

$$y = -\frac{1}{4}x + 6 \quad \text{Equation 1}$$

$$y = \frac{1}{2}x + 3 \quad \text{Equation 2}$$

- How is the linear system in part (b) related to the original equation?
- Explain* how to use a graph to solve the equation  $-\frac{2}{5}x + 5 = \frac{1}{5}x + 2$ .

30. **CHALLENGE** The three lines given below form a triangle. Find the coordinates of the vertices of the triangle.

**Line 1:**  $-3x + 2y = 1$

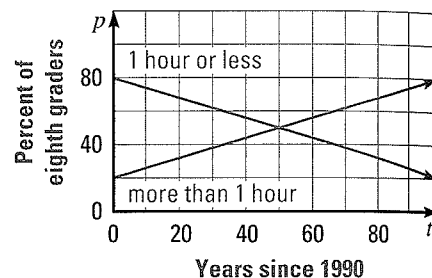
**Line 2:**  $2x + y = 11$

**Line 3:**  $x + 4y = 9$

## PROBLEM SOLVING

**EXAMPLES**  
3 and 4  
on pp. 429–430  
for Exs. 31–33

31. **TELEVISION** The graph shows a projection, from 1990 on, of the percent of eighth graders who watch 1 hour or less of television on a weekday and the percent of eighth graders who watch more than 1 hour of television on a weekday. Use the graph to predict the year when the percent of eighth graders who watch 1 hour or less will equal the percent who watch more than 1 hour.



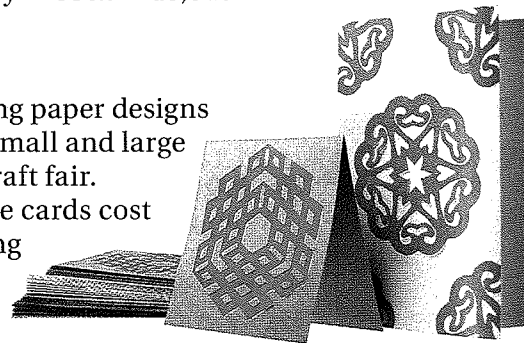
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32. ★ **MULTIPLE CHOICE** A car dealership is offering interest-free car loans for one day only. During this day, a salesperson at the dealership sells two cars. One of his clients decides to pay off his \$17,424 car in 36 monthly payments of \$484. His other client decides to pay off his \$15,840 car in 48 monthly payments of \$330. Which system of equations can be used to determine the number  $x$  of months after which both clients will have the same loan balance  $y$ ?

- |  |  |
|--|--|
| (A) $y = -484x$<br>$y = -330x$                   | (B) $y = -484x + 17,424$<br>$y = -330x + 15,840$ |
| (C) $y = -484x + 15,840$<br>$y = -330x + 17,424$ | (D) $y = 484x + 17,424$<br>$y = 330x + 15,840$   |

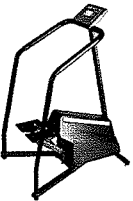
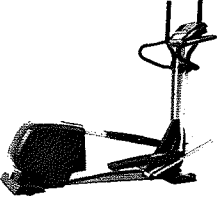
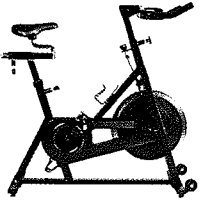
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33. **CRAFTS** Kirigami is the Japanese art of making paper designs by folding and cutting paper. A student sells small and large greeting cards decorated with kirigami at a craft fair. The small cards cost \$3 per card, and the large cards cost \$5 per card. The student collects \$95 for selling a total of 25 cards. How many of each type of card did the student sell?





34. **FITNESS** You want to burn 225 calories while exercising at a gym. The number of calories that you burn per minute on different machines at the gym is shown below.

<i>Stair machine</i>	<i>Elliptical trainer</i>	<i>Stationary bike</i>
You burn 5 Cal/min.	You burn 8 Cal/min.	You burn 6 Cal/min.
		

- a. Suppose you have 40 minutes to exercise at the gym and you want to use the stair machine and stationary bike. How many minutes should you spend on each machine so that you burn 225 calories?
- b. Suppose you have 30 minutes to exercise at the gym and you want to use the stair machine and the elliptical trainer. How many minutes should you spend on each machine so that you burn 225 calories?
35. **MULTIPLE REPRESENTATIONS** It costs \$15 for a yearly membership to a movie club at a movie theater. A movie ticket costs \$5 for club members and \$8 for nonmembers.
- a. **Writing a System of Equations** Write a system of equations that you can use to find the number  $x$  of movies viewed after which the total cost  $y$  for a club member, including the membership fee, is the same as the cost for a nonmember.
- b. **Making a Table** Make a table of values that shows the total cost for a club member and a nonmember after paying to see 1, 2, 3, 4, 5, and 6 movies.
- c. **Drawing a Graph** Use the table to graph the system of equations. Under what circumstances does it make sense to become a movie club member? *Explain* your answer by using the graph.
36. **CHALLENGE** With a minimum purchase of \$25, you can open a credit account with a clothing store. The store is offering either \$25 or 20% off of your purchase if you open a credit account. You decide to open a credit account. Should you choose \$25 or 20% off of your purchase? *Explain*.

## MIXED REVIEW

Solve the equation.

37.  $x + 5 = -14$  (p. 134)

38.  $-5x + 6 = 21$  (p. 141)

39.  $3(x + 2) = -6$  (p. 148)

40.  $11x + 9 = 13x - 3$  (p. 154)

41.  $3x - 8 = 11x + 12$  (p. 154)

42.  $4(x + 1) = -2x - 18$  (p. 154)

Write the equation so that  $y$  is a function of  $x$ . (p. 184)

43.  $5y + 25 = 3x$

44.  $7x = y + 9$

45.  $4y + 11 = 3y + 4x$

46.  $6x + 3y = 2x + 3$

47.  $y + 2x + 6 = -1$

48.  $4x - 12 = 5x + 2y$

### PREVIEW

Prepare for  
Lesson 7.2  
in Exs. 43–48.



# 7.1 Solving Linear Systems by Graphing

**QUESTION** How can you use a graphing calculator to solve a linear system?

**EXAMPLE** Solve a linear system

Solve the linear system using a graphing calculator.

$$5x + 2y = 6 \quad \text{Equation 1}$$

$$x - 3y = -5 \quad \text{Equation 2}$$

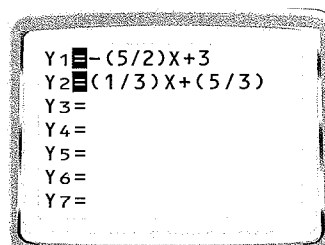
**STEP 1** Rewrite equations

Solve each equation for  $y$ .

Equation 1	Equation 2
$5x + 2y = 6$	$x - 3y = -5$
$2y = -5x + 6$	$-3y = -x - 5$
$y = -\frac{5}{2}x + 3$	$y = \frac{1}{3}x + \frac{5}{3}$

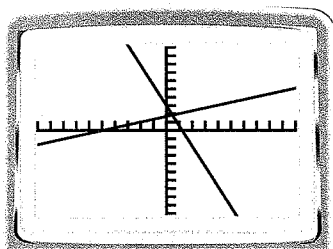
**STEP 2** Enter equations

Press  $\text{Y=}$  and enter the equations.



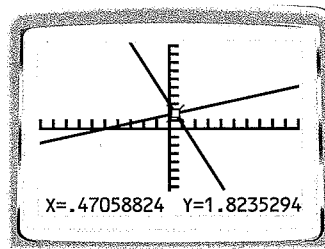
**STEP 3** Display graph

Graph the equations using a standard viewing window.



**STEP 4** Find point of intersection

Use the *intersect* feature to find the point where the graphs intersect.



The solution is about (0.47, 1.8).

**PRACTICE**

Solve the linear system using a graphing calculator.

1.  $y = x + 4$   
 $y = -3x - 2$

2.  $5x + y = -4$   
 $x - y = -2$

3.  $-0.45x - y = 1.35$   
 $-1.8x + y = -1.8$

4.  $-0.4x + 0.8y = -16$   
 $1.2x + 0.4y = 1$



# 7.2 EXERCISES

## HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS  
on p. WS16 for Exs. 13 and 33

★ = STANDARDIZED TEST PRACTICE  
Exs. 2, 18, 29, 33, and 37

### SKILL PRACTICE

1. **VOCABULARY** Give an example of a system of linear equations.

2. ★ **WRITING** If you are solving the linear system shown using the substitution method, which equation would you solve for which variable? Explain.

$$2x - 3y = 24 \quad \text{Equation 1}$$

$$2x + y = 8 \quad \text{Equation 2}$$

#### EXAMPLE 1

on p. 435  
for Exs. 3–8

**SOLVING LINEAR SYSTEMS** Solve the linear system using substitution.

3.  $x = 17 - 4y$   
 $y = x - 2$

4.  $y = 2x - 1$   
 $2x + y = 3$

5.  $x = y + 3$   
 $2x - y = 5$

6.  $4x - 7y = 10$   
 $y = x - 7$

7.  $x = 16 - 4y$   
 $3x + 4y = 8$

8.  $-5x + 3y = 51$   
 $y = 10x - 8$

9.  $2x = 12$   
 $x - 5y = -29$

10.  $2x - y = 23$   
 $x - 9 = -1$

11.  $x + y = 0$   
 $x - 2y = 6$

12.  $2x + y = 9$   
 $4x - y = -15$

13.  $5x + 2y = 9$   
 $x + y = -3$

14.  $5x + 4y = 32$   
 $9x - y = 33$

15.  $11x - 7y = -14$   
 $x - 2y = -4$

16.  $20x - 30y = -50$   
 $x + 2y = 1$

17.  $6x + y = 4$   
 $x - 4y = 19$

18. ★ **MULTIPLE CHOICE** Which ordered pair is a solution of the linear system  $4x - y = 17$  and  $-9x + 8y = 2$ ?

(A) (6, 7)

(B) (7, 6)

(C) (7, 11)

(D) (11, 7)

19. **ERROR ANALYSIS** Describe and correct the error in solving the linear system  $4x + 2y = 6$  and  $3x + y = 9$ .

Step 1

$$\begin{aligned} 3x + y &= 9 \\ y &= 9 - 3x \end{aligned}$$

Step 2

$$\begin{aligned} 4x + 2(9 - 3x) &= 6 \\ 4x + 18 - 6x &= 6 \\ -2x &= -12 \\ x &= 6 \end{aligned}$$

Step 3

$$\begin{aligned} y &= 9 - 3x \\ 6 &= 9 - 3x \\ -3 &= -3x \\ 1 &= x \end{aligned}$$

The solution is (6, 1).



**SOLVING LINEAR SYSTEMS** Solve the linear system using substitution.

20.  $4.5x + 1.5y = 24$   
 $x - y = 4$

21.  $35x + y = 20$   
 $1.5x - 0.1y = 18$

22.  $3x - 2y = 8$   
 $0.5x + y = 17$

23.  $0.5x + 0.6y = 5.7$   
 $2x - y = -1$

24.  $x - 9 = 0.5y$   
 $2.2x - 3.1y = -0.2$

25.  $0.2x + y = -1.8$   
 $1.8y + 5.5x = 27.6$

26.  $\frac{1}{2}x + \frac{1}{4}y = 5$   
 $x - \frac{1}{2}y = 1$

27.  $x + \frac{1}{3}y = -2$   
 $-8x - \frac{2}{3}y = 4$

28.  $\frac{3}{8}x + \frac{3}{4}y = 12$   
 $\frac{2}{3}x + \frac{1}{2}y = 13$

29. ★ **WRITING** Suppose you solve a linear system using substitution. Explain how you can use a graph to check your solution.
30. **CHALLENGE** Find values of  $a$  and  $b$  so that the linear system shown has a solution of  $(-9, 4)$ .
- $$ax + by = -16 \quad \text{Equation 1}$$
- $$ax - by = -56 \quad \text{Equation 2}$$

## PROBLEM SOLVING

**EXAMPLE 3**  
on p. 437  
for Exs. 31–33

31. **FUNDRAISING** During a football game, the parents of the football players sell pretzels and popcorn to raise money for new uniforms. They charge \$2.50 for a bag of popcorn and \$2 for a pretzel. The parents collect \$336 in sales during the game. They sell twice as many bags of popcorn as pretzels. How many bags of popcorn do they sell? How many pretzels do they sell?

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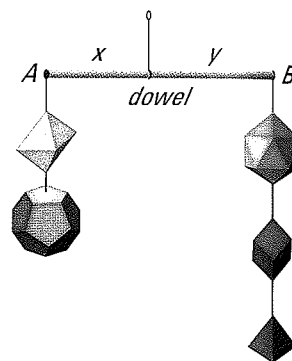
32. **TUBING COSTS** A group of friends takes a day-long tubing trip down a river. The company that offers the tubing trip charges \$15 to rent a tube for a person to use and \$7.50 to rent a “cooler” tube, which is used to carry food and water in a cooler. The friends spend \$360 to rent a total of 26 tubes. How many of each type of tube do they rent?

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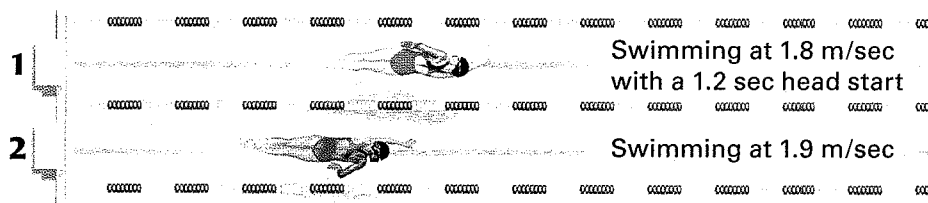
33. ★ **SHORT RESPONSE** In the mobile shown, objects are attached to each end of a dowel. For the dowel to balance, the following must be true:

$$x \cdot \text{Weight hanging from point A} = y \cdot \text{Weight hanging from point B}$$

The weight of the objects hanging from point A is 1.5 pounds, and the weight of the objects hanging from point B is 1.2 pounds. The length of the dowel is 9 inches. How far from point A should the string be placed? *Explain.*



34. **MULTI-STEP PROBLEM** Two swimming teams are competing in a 400 meter medley relay. During the last leg of the race, the swimmer in lane 1 has a 1.2 second head start on the swimmer in lane 2, as shown.




- a. Let  $t$  be the time since the swimmer in lane 2 started the last leg. After how many seconds into the leg will the swimmer in lane 2 catch up to the swimmer in lane 1?
- b. Does the swimmer in lane 2 catch up to the swimmer in lane 1 before the race ends? *Explain.*

**EXAMPLE 4**

on p. 438  
for Ex. 35

35. **CHEMISTRY** In your chemistry lab, you have a bottle of 1% hydrochloric acid solution and a bottle of 5% hydrochloric acid solution. You need 100 milliliters of a 3% hydrochloric acid solution for an experiment. How many milliliters of each solution do you need to mix together?
36. **MONEY** Laura has \$4.50 in dimes and quarters. She has 3 more dimes than quarters. How many quarters does she have?
37. **★ SHORT RESPONSE** A gazelle can run 73 feet per second for several minutes. A cheetah can run 88 feet per second, but it can sustain this speed for only 20 seconds. A gazelle is 350 feet from a cheetah when both animals start running. Can the gazelle stay ahead of the cheetah? *Explain.*

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38. **CHALLENGE** A gardener needs 6 bushels of a potting medium of 40% peat moss and 60% vermiculite. He decides to add 100% vermiculite to his current potting medium that is 50% peat moss and 50% vermiculite. The gardener has 5 bushels of the 50% peat moss and 50% vermiculite mix. Does he have enough of the 50% peat moss and 50% vermiculite mix to make 6 bushels of the 40% peat moss and 60% vermiculite mix? *Explain.*

**MIXED REVIEW**

Solve the proportion. Check your solution.

39.  $\frac{3}{7} = \frac{x}{21}$  (p. 162)

40.  $\frac{y}{30} = \frac{9}{10}$  (p. 162)

41.  $\frac{12}{16} = \frac{3z}{4}$  (p. 162)

42.  $\frac{35}{q} = \frac{5}{3}$  (p. 168)

43.  $\frac{3}{2r} = \frac{-6}{16}$  (p. 168)

44.  $\frac{4}{3} = \frac{s}{s-2}$  (p. 168)

Write two equations in standard form that are equivalent to the given equation. (p. 311)

45.  $x - 4y = 0$

46.  $-3x + 9y = 6$

47.  $-7x - y = 1$

48.  $5x - 10y = 5$

49.  $-2x - 12y = 8$

50.  $6x + 15y = -3$

**PREVIEW**

Prepare for  
Lesson 7.3  
in Exs. 45–50.

**QUIZ for Lessons 7.1–7.2**

Solve the linear system by graphing. Check your solution. (p. 427)

1.  $x + y = -2$   
 $-x + y = 6$

2.  $x - y = 0$   
 $5x + 2y = -7$

3.  $x - 2y = 12$   
 $-3x + y = -1$

Solve the linear system using substitution. (p. 435)

4.  $y = x - 4$   
 $-2x + y = 18$

5.  $y = 4 - 3x$   
 $5x - y = 22$

6.  $x = y + 9$   
 $5x - 3y = 7$

7.  $2y + x = -4$   
 $y - x = -5$

8.  $5x - 4y = 27$   
 $-2x + y = 3$

9.  $3x - 5y = 13$   
 $x + 4y = 10$

**Another Way to Solve Example 3, page 437**



**MULTIPLE REPRESENTATIONS** In Example 3 on page 437, you saw how to solve the problem about website hosting by solving a linear system algebraically. You can also solve the problem using a table.

**PROBLEM**

**WEBSITES** Many businesses pay website hosting companies to store and maintain the computer files that make up their websites. Internet service providers also offer website hosting. The costs for website hosting offered by a website hosting company and an Internet service provider are shown in the table. Find the number of months after which the total cost for website hosting will be the same for both companies.

Company	Set-up fee	Cost per month
Internet service provider	\$10	\$21.95
Website hosting company	None	\$22.45

**METHOD**

**Making a Table** An alternative approach is to make a table.

**STEP 1** Make a table for the total cost of website hosting for both companies.

Include the set-up fee in the cost for the first month.

**STEP 2** Look for the month in which the total cost of the service from the Internet service provider and the website hosting company is the same. This happens after 20 months.

Months	Internet service provider	Website hosting company
1	\$31.95	\$22.45
2	\$53.90	\$44.90
3	\$75.85	\$67.35
⋮	⋮	⋮
19	\$427.05	\$426.55
20	\$449.00	\$449.00
21	\$470.95	\$471.45

**PRACTICE**

- TAXI** A taxi company charges \$2.80 for the first mile and \$1.60 for each additional mile. Another taxi company charges \$3.20 for the first mile and \$1.50 for each additional mile. After how many miles will each taxi cost the same? Use a table to solve the problem.
- SCHOOL PLAY** An adult ticket to a school play costs \$5 and a student ticket costs \$3. A total of \$460 was collected from the sale of 120 tickets. How many student tickets were purchased? Solve the problem using algebra. Then use a table to check your answer.

# 7.3 Linear Systems and Elimination

**MATERIALS** • algebra tiles

**QUESTION** How can you solve a linear system using algebra tiles?

You can use the following algebra tiles to model equations.

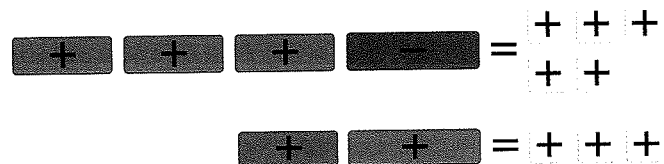


**EXPLORE** Solve a linear system using algebra tiles.

Solve the linear system:  $3x - y = 5$  Equation 1  
 $x + y = 3$  Equation 2

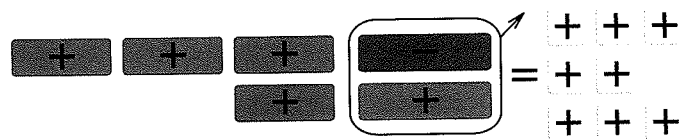
**STEP 1 Model equations**

Model each equation using algebra tiles. Arrange the algebra tiles so that one equation is directly below the other equation.



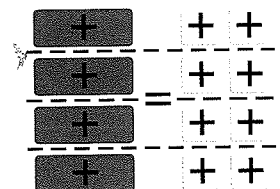
**STEP 2 Add equations**

Combine the two equations to form one equation. Notice that the new equation has one positive y-tile and one negative y-tile. The y-tiles can be removed because the pair of y-tiles has a value of 0.



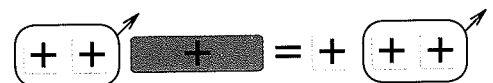
**STEP 3 Solve for x**

Divide the remaining tiles into four equal groups. Each x-tile is equal to two 1-tiles. So,  $x = 2$ .



**STEP 4 Solve for y**

To find the value of y, use the model for Equation 2. Because  $x = 2$ , you can replace the x-tile with two 1-tiles. Solve the new equation for y. So  $y = 1$ , and the solution of the system is (2, 1).



**DRAW CONCLUSIONS** Use your observations to complete these exercises

Use algebra tiles to model and solve the linear system.

1.  $x + 3y = 8$   
 $4x - 3y = 2$
2.  $2x + y = 5$   
 $-2x + 3y = 7$
3.  $5x - 2y = -2$   
 $x + 2y = 14$
4.  $x + 2y = 3$   
 $-x + 3y = 2$

5. **REASONING** Is it possible to solve the linear system  $3x - 2y = 6$  and  $2x + y = 11$  using the steps shown above? Explain your reasoning.

**GUIDED PRACTICE** for Example 4

7. **WHAT IF?** In Example 4, suppose it takes the kayaker 5 hours to travel 10 miles upstream and 2 hours to travel 10 miles downstream. The speed of the current remains constant during the trip. Find the average speed of the kayak in still water and the speed of the current.

**7.3 EXERCISES****HOMEWORK KEY**○ = **WORKED-OUT SOLUTIONS**  
on p. WS16 for Exs. 17 and 41★ = **STANDARDIZED TEST PRACTICE**  
Exs. 2, 15, 22, 36, and 44◆ = **MULTIPLE REPRESENTATIONS**  
Ex. 42**SKILL PRACTICE**

1. **VOCABULARY** Give an example of a linear system in two variables that can be solved by first adding the equations to eliminate one variable.

2. ★ **WRITING** Explain how to solve the linear system shown using the elimination method.
- $$2x - y = 2 \quad \text{Equation 1}$$
- $$2x + 3y = 22 \quad \text{Equation 2}$$

**EXAMPLE 1**on p. 444  
for Exs. 3–8**USING ADDITION** Solve the linear system using elimination.

- |                                    |                                      |                                       |
|------------------------------------|--------------------------------------|---------------------------------------|
| 3. $x + 2y = 13$<br>$-x + y = 5$   | 4. $9x + y = 2$<br>$-4x - y = -17$   | 5. $-3x - y = 8$<br>$7x + y = -12$    |
| 6. $3x - y = 30$<br>$-3x + 7y = 6$ | 7. $-9x + 4y = -17$<br>$9x - 6y = 3$ | 8. $-3x - 5y = -7$<br>$-4x + 5y = 14$ |

**EXAMPLE 2**on p. 445  
for Exs. 9–15**USING SUBTRACTION** Solve the linear system using elimination.

- |                                      |                                      |                                       |
|--------------------------------------|--------------------------------------|---------------------------------------|
| 9. $x + y = 1$<br>$-2x + y = 4$      | 10. $x - y = -4$<br>$x + 3y = 4$     | 11. $2x - y = 7$<br>$2x + 7y = 31$    |
| 12. $6x + y = -10$<br>$5x + y = -10$ | 13. $5x + 6y = 50$<br>$-x + 6y = 26$ | 14. $4x - 9y = -21$<br>$4x + 3y = -9$ |

15. ★ **MULTIPLE CHOICE** Which ordered pair is a solution of the linear system  $4x + 9y = -2$  and  $11x + 9y = 26$ ?

(A)  $(-2, 4)$       (B)  $(2, -4)$       (C)  $(4, -2)$       (D)  $(4, 2)$

**EXAMPLE 3**on p. 445  
for Exs. 16–22**ARRANGING LIKE TERMS** Solve the linear system using elimination.

- |                                    |  |                                      |
|------------------------------------|--|--------------------------------------|
| 16. $2x - y = 32$<br>$y - 5x = 13$ | 17. $-8y + 6x = 36$<br>$6x - y = 15$     | 18. $2x - y = -11$<br>$y = -2x - 13$ |
| 19. $-x - y = 14$<br>$x = 5y - 38$ | 20. $11y - 3x = 18$<br>$-3x = -16y + 33$ | 21. $-5x + y = -23$<br>$-y = 3x - 9$ |

22. ★ **MULTIPLE CHOICE** Which ordered pair is a solution of the linear system  $2x + y = 10$  and  $3y = 2x + 6$ ?

(A)  $(-3, -4)$       (B)  $(3, 4)$       (C)  $(-4, 3)$       (D)  $(4, 3)$



**ERROR ANALYSIS** Describe and correct the error in finding the value of one of the variables in the given linear system.

$$\begin{aligned} 23. \quad & 5x - 7y = 16 \\ & -x - 7y = 8 \end{aligned}$$

$$\begin{array}{r} 5x - 7y = 16 \\ -x - 7y = 8 \\ \hline 4x \quad = 24 \\ x = 6 \end{array}$$



$$\begin{aligned} 24. \quad & 3x - 2y = -3 \\ & 5y = 60 - 3x \end{aligned}$$

$$\begin{array}{r} 3x - 2y = -3 \\ -3x + 5y = 60 \\ \hline 3y = 57 \\ y = 19 \end{array}$$



**SOLVING LINEAR SYSTEMS** Solve the linear system using elimination.

$$\begin{aligned} 25. \quad & -x + \frac{1}{2}y = -19 \\ & x - y = 12 \end{aligned}$$

$$\begin{aligned} 26. \quad & \frac{1}{4}x - \frac{2}{3}y = 7 \\ & \frac{1}{2}x + \frac{2}{3}y = -4 \end{aligned}$$

$$\begin{aligned} 27. \quad & 8x - \frac{1}{2}y = -38 \\ & \frac{1}{4}x - \frac{1}{2}y = -7 \end{aligned}$$

$$\begin{aligned} 28. \quad & 5.2x + 3.5y = 54 \\ & -3.6x + 3.5y = 10 \end{aligned}$$

$$\begin{aligned} 29. \quad & 1.3x - 3y = -17.6 \\ & -1.3x + 4.5y = 25.1 \end{aligned}$$

$$\begin{aligned} 30. \quad & -2.6x - 3.2y = 4.8 \\ & 1.9x - 3.2y = -4.2 \end{aligned}$$


$$\begin{aligned} 31. \quad & \frac{4}{5}x + \frac{2}{5}y = 14 \\ & \frac{2}{5}y + \frac{1}{5}x = 11 \end{aligned}$$

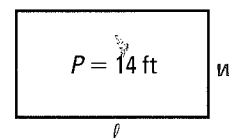
$$\begin{aligned} 32. \quad & 2.7x + 1.5y = 36 \\ & 3.5y = 2.7x - 6 \end{aligned}$$

$$\begin{aligned} 33. \quad & 4 - 4.8x = 1.7y \\ & 12.8 + 1.7y = -13.2x \end{aligned}$$

34. **WRITING AN EQUATION OF A LINE** Use the following steps to write an equation of the line that passes through the points (1, 2) and (-4, 12).

- Write a system of linear equations by substituting 1 for  $x$  and 2 for  $y$  in  $y = mx + b$  and  $-4$  for  $x$  and 12 for  $y$  in  $y = mx + b$ .
- Solve the system of linear equations from part (a). What is the slope of the line? What is the  $y$ -intercept?
- Write an equation of the line that passes through (1, 2) and (-4, 12).

35.  **GEOMETRY** The rectangle has a perimeter  $P$  of 14 feet, and twice its length  $l$  is equal to 1 less than 4 times its width  $w$ . Write and solve a system of linear equations to find the length and the width of the rectangle.



36. **★ SHORT RESPONSE** Find the solution of the system of linear equations below. Explain your steps.

$$\begin{array}{ll} x + 3y = 8 & \text{Equation 1} \\ x - 6y = -19 & \text{Equation 2} \\ 5x - 3y = -14 & \text{Equation 3} \end{array}$$

37. **CHALLENGE** For  $a \neq 0$ , what is the solution of the system  $ax + 2y = 4$  and  $ax - 3y = -6$ ?

38. **CHALLENGE** Solve for  $x$ ,  $y$ , and  $z$  in the system of equations below. Explain your steps.

$$\begin{array}{ll} x + 7y + 3z = 29 & \text{Equation 1} \\ 3z + x - 2y = -7 & \text{Equation 2} \\ 5y = 10 - 2x & \text{Equation 3} \end{array}$$

## PROBLEM SOLVING

**EXAMPLE 4**  
 on p. 446  
 for Exs. 39–41

- 39. ROWING** During a practice, a 4 person crew team rows a rowing shell upstream (against the current) and then rows the same distance downstream (with the current). The shell moves upstream at a speed of 4.3 meters per second and downstream at a speed of 4.9 meters per second. The speed of the current remains constant. Use the models below to write and solve a system of equations to find the average speed of the shell in still water and the speed of the current.

**Upstream**

$$\begin{array}{r} \text{Speed of shell} \\ \text{in still water} \end{array} - \begin{array}{r} \text{Speed of} \\ \text{current} \end{array} = \begin{array}{r} \text{Speed} \\ \text{of shell} \end{array}$$

**Downstream**

$$\begin{array}{r} \text{Speed of shell} \\ \text{in still water} \end{array} + \begin{array}{r} \text{Speed of} \\ \text{current} \end{array} = \begin{array}{r} \text{Speed} \\ \text{of shell} \end{array}$$

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- 40. OIL CHANGE** Two cars get an oil change at the same service center. Each customer is charged a fee  $x$  (in dollars) for the oil change plus  $y$  dollars per quart of oil used. The oil change for the car that requires 5 quarts of oil costs \$22.45. The oil change for the car that requires 7 quarts of oil costs \$25.45. Find the fee and the cost per quart of oil.

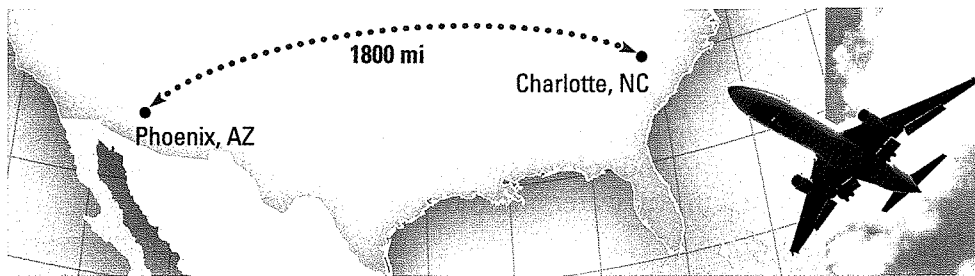
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- 41. PHONES** Cellular phone ring tones can be monophonic or polyphonic. Monophonic ring tones play one tone at a time, and polyphonic ring tones play multiple tones at a time. The table shows the ring tones downloaded from a website by two customers. Use the information to find the cost of a monophonic ring tone and a polyphonic ring tone, assuming that all monophonic ring tones cost the same and all polyphonic ring tones cost the same.

Customer	Monophonic ring tones	Polyphonic ring tones	Total cost (dollars)
Julie	3	2	12.85
Tate	1	2	8.95

- 42. ♦ MULTIPLE REPRESENTATIONS** For a floral arrangement class, Alicia has to create an arrangement of twigs and flowers that has a total of 9 objects. She has to pay for the twigs and flowers that she uses in her arrangement. Each twig costs \$1, and each flower costs \$3.
- Writing a System** Alicia spends \$15 on the twigs and flowers. Write and solve a linear system to find the number of twigs and the number of flowers she used.
  - Making a Table** Make a table showing the number of twigs in the arrangement and the total cost of the arrangement when the number of flowers purchased is 0, 1, 2, 3, 4, or 5. Use the table to check your answer to part (a).

43. **MULTI-STEP PROBLEM** On a typical day with light winds, the 1800 mile flight from Charlotte, North Carolina, to Phoenix, Arizona, takes longer than the return trip because the plane has to fly into the wind.



- a. The flight from Charlotte to Phoenix is 4 hours 30 minutes long, and the flight from Phoenix to Charlotte is 4 hours long. Find the average speed (in miles per hour) of the airplane on the way to Phoenix and on the return trip to Charlotte.
- b. Let  $s$  be the speed (in miles per hour) of the plane with no wind, and let  $w$  be the speed (in miles per hour) of the wind. Use your answer to part (a) to write and solve a system of equations to find the speed of the plane with no wind and the speed of the wind.
44. **★ SHORT RESPONSE** The students in the graduating classes at the three high schools in a school district have to pay for their caps and gowns. A cap-and-gown set costs  $x$  dollars, and an extra tassel costs  $y$  dollars. At one high school, students pay \$3262 for 215 cap-and-gown sets and 72 extra tassels. At another high school, students pay \$3346 for 221 cap-and-gown sets and 72 extra tassels. How much will students at the third high school pay for 218 cap-and-gown sets and 56 extra tassels? *Explain.*
45. **CHALLENGE** A clothing manufacturer makes men's dress shirts. For the production process, an ideal sleeve length  $x$  (in centimeters) for each shirt size and an allowable deviation  $y$  (in centimeters) from the ideal length are established. The deviation is expressed as  $\pm y$ . For a specific shirt size, the minimum allowable sleeve length is 62.2 centimeters and the maximum allowable sleeve length is 64.8 centimeters. Find the ideal sleeve length and the allowable deviation.

## MIXED REVIEW

Graph the equation.

46.  $x - 5y = -12$  (p. 225)

47.  $-2x + 3y = -15$  (p. 225)

48.  $y + 9 = -(x + 2)$  (p. 302)

49.  $y - 4 = \frac{2}{3}(x + 1)$  (p. 302)

Solve the linear system by graphing. Check your solution. (p. 427)

50.  $y = x - 3$   
 $y = -x + 1$

51.  $y = 2$   
 $2x - 3y = 6$

52.  $2x + y = 6$   
 $6x - 2y = -12$

Find the least common multiple of the pair of numbers. (p. 910)

53. 9, 12

54. 18, 24

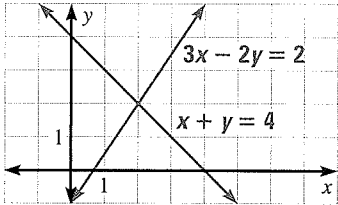
55. 15, 20

### PREVIEW

Prepare for  
Lesson 7.4 in  
Exs. 53–55.



Methods for Solving Linear Systems

Method	Example	When to Use												
Table (p. 426)	<table border="1"> <thead> <tr> <th>x</th> <th>y = 2x</th> <th>y = 3x - 1</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>-1</td> </tr> <tr> <td>1</td> <td>2</td> <td>2</td> </tr> <tr> <td>2</td> <td>4</td> <td>5</td> </tr> </tbody> </table>	x	y = 2x	y = 3x - 1	0	0	-1	1	2	2	2	4	5	When x-values are integers, so that equal values can be seen in the table
x	y = 2x	y = 3x - 1												
0	0	-1												
1	2	2												
2	4	5												
Graphing (p. 427)		When you want to see the lines that the equations represent												
Substitution (p. 435)	$y = 4 - 2x$ $4x + 2y = 8$	When one equation is already solved for x or y												
Addition (p. 444)	$4x + 7y = 15$ $6x - 7y = 5$	When the coefficients of one variable are opposites												
Subtraction (p. 445)	$3x + 5y = -13$ $3x + y = -5$	When the coefficients of one variable are the same												
Multiplication (p. 451)	$9x + 2y = 38$ $3x - 5y = 7$	When no corresponding coefficients are the same or opposites												

7.4 EXERCISES

HOMEWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS17 for Exs. 15 and 39
- ★ = STANDARDIZED TEST PRACTICE Exs. 2, 18, 34, 41, and 42
- ◆ = MULTIPLE REPRESENTATIONS Ex. 40

SKILL PRACTICE

- VOCABULARY** What is the least common multiple of 12 and 18?
- ★ WRITING** Explain how to solve the linear system using the elimination method.
 

$2x - 3y = -4$	<b>Equation 1</b>
$7x + 9y = -5$	<b>Equation 2</b>

**EXAMPLE 1**  
 on p. 451  
 for Exs. 3–8

**SOLVING LINEAR SYSTEMS** Solve the linear system using elimination.

- |                                       |                                    |                                       |
|---------------------------------------|------------------------------------|---------------------------------------|
| 3. $x + y = 2$<br>$2x + 7y = 9$       | 4. $3x - 2y = 3$<br>$-x + y = 1$   | 5. $4x + 3y = 8$<br>$x - 2y = 13$     |
| 6. $10x - 9y = 46$<br>$-2x + 3y = 10$ | 7. $8x - 5y = 11$<br>$4x - 3y = 5$ | 8. $11x - 20y = 28$<br>$3x + 4y = 36$ |

**EXAMPLE 2**on p. 452 for  
Exs. 9–20**SOLVING LINEAR SYSTEMS** Solve the linear system using elimination.

9.  $4x - 3y = 8$   
 $5x - 2y = -11$

10.  $-2x - 5y = 9$   
 $3x + 11y = 4$

11.  $7x - 6y = -1$   
 $5x - 4y = 1$

12.  $7x + 3y = -12$   
 $2x + 5y = 38$

13.  $9x - 8y = 4$   
 $2x - 3y = -4$

14.  $12x - 7y = -2$   
 $-8x + 11y = 14$

15.  $9x + 2y = 39$   
 $6x + 13y = -9$

16.  $-7x + 10y = 11$   
 $-8x + 15y = 34$

17.  $-14x + 15y = 15$   
 $21x - 20y = -10$

18. **★ MULTIPLE CHOICE** Which ordered pair is a solution of the linear system  
 $15x + 8y = 6$  and  $25x + 12y = 14$ ?

(A)  $(-3, -2)$

(B)  $(-3, 2)$

(C)  $(-2, -3)$

(D)  $(2, -3)$

**ERROR ANALYSIS** Describe and correct the error when solving the linear system.

19.

$$\begin{array}{r} 2x - 3y = -9 \quad \times 2 \rightarrow 4x - 6y = -18 \\ 5x - 6y = -9 \quad \times 1 \rightarrow 5x - 6y = -9 \\ \hline 9x \quad \quad = -27 \end{array}$$

$$x = -3$$

20.

$$\begin{array}{r} 9x + 8y = 11 \quad \times 3 \rightarrow 27x + 24y = 33 \\ 7x + 6y = 9 \quad \times 4 \rightarrow 28x + 24y = 36 \\ \hline -x \quad \quad = 2 \end{array}$$

$$x = -2$$

**SOLVING LINEAR SYSTEMS** Solve the linear system using any algebraic method.

21.  $3x + 2y = 4$   
 $2y = 8 - 5x$

22.  $4x - 5y = 18$   
 $3x = y + 11$

23.  $8x - 9y = -15$   
 $-4x = 19 + y$

24.  $0.3x + 0.1y = -0.1$   
 $-x + y = 3$

25.  $4.4x - 3.6y = 7.6$   
 $x - y = 1$

26.  $3x - 2y = -20$   
 $x + 1.2y = 6.4$

27.  $0.2x - 1.5y = -1$   
 $x - 4.5y = 1$

28.  $1.5x - 3.5y = -5$   
 $-1.2x + 2.5y = 1$

29.  $4.9x + 2.4y = 7.4$   
 $0.7x + 3.6y = -2.2$

30.  $x + y = 0$

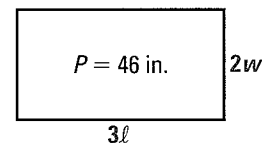
31.  $3x + y = \frac{1}{3}$

32.  $\frac{3}{5}x - \frac{3}{4}y = -3$

$\frac{1}{2}x - \frac{1}{2}y = 2$

$2x - 3y = \frac{8}{3}$

$\frac{2}{5}x + \frac{1}{3}y = 8$

33. **GEOMETRY** A rectangle has a perimeter of 18 inches. A new rectangle is formed by doubling the width  $w$  and tripling the length  $l$ , as shown. The new rectangle has a perimeter  $P$  of 46 inches.

- a. Write and solve a system of linear equations to find the length and width of the original rectangle.
- b. Find the length and width of the new rectangle.

34. **★ WRITING** For which values of  $a$  can you solve the linear system  
 $ax + 3y = 2$  and  $4x + 5y = 6$  without multiplying first? Explain.**CHALLENGE** Find the values of  $a$  and  $b$  so that the linear system has the given solution.

$ax - by = 4$  Equation 1

$bx - ay = 10$  Equation 2

35.  $(4, 2)$

36.  $(2, 1)$

## PROBLEM SOLVING

**EXAMPLE 3**

on p. 453  
for Exs. 37–39

- 37. BOOK SALE** A library is having a book sale to raise money. Hardcover books cost \$4 each and paperback books cost \$2 each. A person spends \$26 for 8 books. How many hardcover books did she purchase?

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- 38. MUSIC** A website allows users to download individual songs or an entire album. All individual songs cost the same to download, and all albums cost the same to download. Ryan pays \$14.94 to download 5 individual songs and 1 album. Seth pays \$22.95 to download 3 individual songs and 2 albums. How much does the website charge to download a song? an entire album?

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- 39. FARM PRODUCTS** The table shows the number of apples needed to make the apple pies and applesauce sold at a farm store. During a recent apple picking at the farm, 169 Granny Smith apples and 95 Golden Delicious apples were picked. How many apple pies and batches of applesauce can be made if every apple is used?

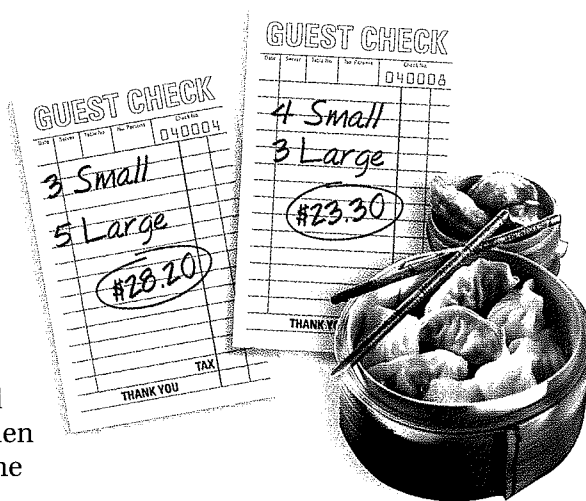
Type of apple	Granny Smith	Golden Delicious
<b>Needed for a pie</b>	5	3
<b>Needed for a batch of applesauce</b>	4	2

- 40. MULTIPLE REPRESENTATIONS** Tickets for admission to a high school football game cost \$3 for students and \$5 for adults. During one game, \$2995 was collected from the sale of 729 tickets.

- a. Writing a System** Write and solve a system of linear equations to find the number of tickets sold to students and the number of tickets sold to adults.
- b. Drawing a Graph** Graph the system of linear equations. Use the graph to determine whether your answer to part (a) is reasonable.

- 41. ★ SHORT RESPONSE** A dim sum restaurant offers two sizes of dishes: small and large. All small dishes cost the same and all large dishes cost the same. The bills show the cost of the food before the tip is included. What will 3 small and 2 large dishes cost before the tip is included? *Explain.*

- 42. ★ OPEN-ENDED** Describe a real-world problem that can be solved using a system of linear equations. Then solve the problem and explain what the solution means in this situation.





43. **INVESTMENTS** Matt invested \$2000 in stocks and bonds. This year the bonds paid 8% interest, and the stocks paid 6% in dividends. Matt received a total of \$144 in interest and dividends. How much money did he invest in stocks? in bonds?
44. **CHALLENGE** You drive a car 45 miles at an average speed  $r$  (in miles per hour) to reach your destination. Due to traffic, your average speed on the return trip is  $\frac{3}{4}r$ . The round trip took a total of 1 hour 45 minutes. Find the average speed for each leg of your trip.

## MIXED REVIEW

Graph the equation. (pp. 215, 225, 244)

45.  $x - 8y = -10$

46.  $-2x + 5y = -15$

47.  $\frac{1}{4}x + \frac{1}{2}y = 8$

48.  $y = \frac{2}{3}x - 1$

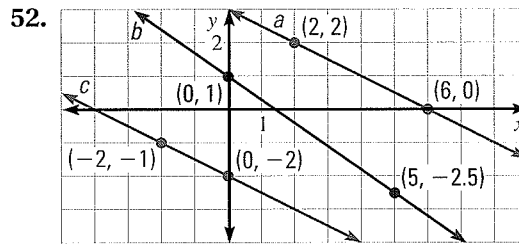
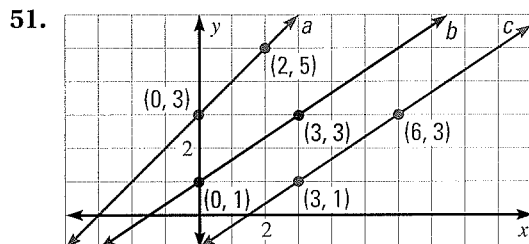
49.  $y = -9x + 2$

50.  $y - 1 = 2x - 7$

### PREVIEW

Prepare for  
Lesson 7.5 in  
Exs. 51–52.

Determine which lines are parallel. (p. 244)



Solve the linear system using any method. (pp. 427, 435, 444, 451)

53.  $y = 4x - 9$   
 $y = -8x + 15$

54.  $y = -2x + 20$   
 $y = -6x + 40$

55.  $x + 2y = 0$   
 $-x = y + 3$

56.  $x + 2y = 2$   
 $-x + y = -11$

57.  $7x - 8y = -15$   
 $5x + 8y = 3$

58.  $8x + y = 5$   
 $-2x + y = 0$

## QUIZ for Lessons 7.3–7.4

Solve the linear system using elimination. (pp. 444, 451)

1.  $x + y = 4$   
 $-3x + y = -8$

2.  $2x - y = 2$   
 $6x - y = -2$

3.  $x + y = 5$   
 $-x + y = -3$

4.  $x + 3y = -10$   
 $-x + 5y = -30$

5.  $x + 3y = 10$   
 $3x - y = 13$

6.  $x + 7y = 10$   
 $x + 2y = -8$

7.  $4x - y = -2$   
 $3x + 2y = 7$

8.  $x + 3y = 1$   
 $5x + 6y = 14$

9.  $3x + y = 21$   
 $x + y = 1$

10.  $2x - 3y = -5$   
 $5x + 2y = 16$

11.  $7x + 2y = 13$   
 $4x + 3y = 13$

12.  $\frac{1}{3}x + 5y = -3$   
 $-\frac{2}{3}x + 6y = -10$

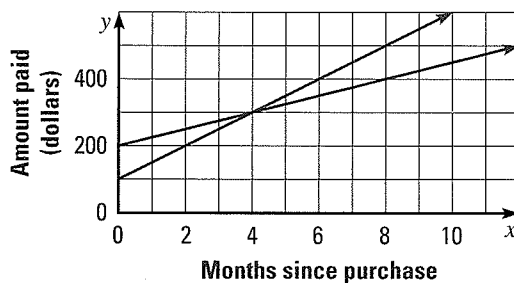


## Lessons 7.1–7.4

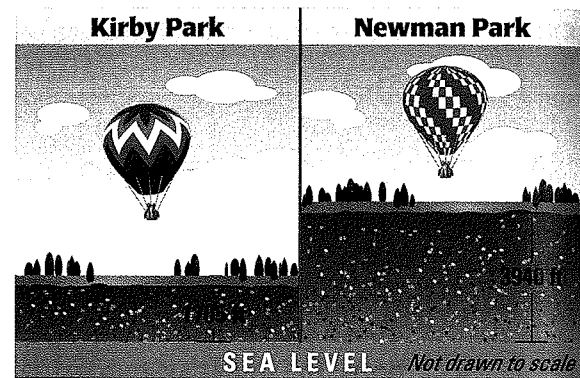
- MULTI-STEP PROBLEM** Flying into the wind, a helicopter takes 15 minutes to travel 15 kilometers. The return flight takes 12 minutes. The wind speed remains constant during the trip.
  - Find the helicopter's average speed (in kilometers per hour) for each leg of the trip.
  - Write a system of linear equations that represents the situation.
  - What is the helicopter's average speed in still air? What is the speed of the wind?



- SHORT RESPONSE** At a grocery store, a customer pays a total of \$9.70 for 1.8 pounds of potato salad and 1.4 pounds of coleslaw. Another customer pays a total of \$6.55 for 1 pound of potato salad and 1.2 pounds of coleslaw. How much do 2 pounds of potato salad and 2 pounds of coleslaw cost? *Explain.*
- GRIDDED ANSWER** During one day, two computers are sold at a computer store. The two customers each arrange payment plans with the salesperson. The graph shows the amount  $y$  of money (in dollars) paid for the computers after  $x$  months. After how many months will each customer have paid the same amount?



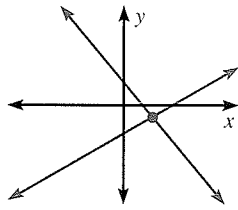
- OPEN-ENDED** Describe a real-world problem that can be modeled by a linear system. Then solve the system and interpret the solution in the context of the problem.
- SHORT RESPONSE** A hot air balloon is launched at Kirby Park, and it ascends at a rate of 7200 feet per hour. At the same time, a second hot air balloon is launched at Newman Park, and it ascends at a rate of 4000 feet per hour. Both of the balloons stop ascending after 30 minutes. The diagram shows the altitude of each park. Are the hot air balloons ever at the same height at the same time? *Explain.*



- EXTENDED RESPONSE** A chemist needs 500 milliliters of a 20% acid and 80% water mix for a chemistry experiment. The chemist combines  $x$  milliliters of a 10% acid and 90% water mix and  $y$  milliliters of a 30% acid and 70% water mix to make the 20% acid and 80% water mix.
  - Write a linear system that represents the situation.
  - How many milliliters of the 10% acid and 90% water mix and the 30% acid and 70% water mix are combined to make the 20% acid and 80% water mix?
  - The chemist also needs 500 milliliters of a 15% acid and 85% water mix. Does the chemist need more of the 10% acid and 90% water mix than the 30% acid and 70% water mix to make this new mix? *Explain.*

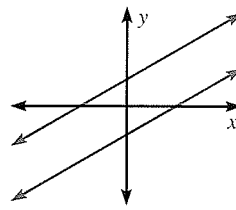
**Number of Solutions of a Linear System**

**One solution**



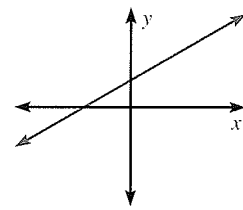
The lines intersect.  
The lines have different slopes.

**No solution**



The lines are parallel.  
The lines have the same slope and different y-intercepts.

**Infinitely many solutions**



The lines coincide.  
The lines have the same slope and the same y-intercept.

**7.5 EXERCISES**

**HOMEWORK KEY**

○ = WORKED-OUT SOLUTIONS on p. WS17 for Exs. 11 and 37

★ = STANDARDIZED TEST PRACTICE Exs. 3, 4, 24, 25, 32, 33, and 40

**SKILL PRACTICE**

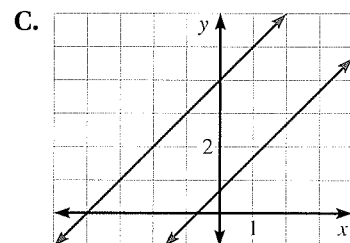
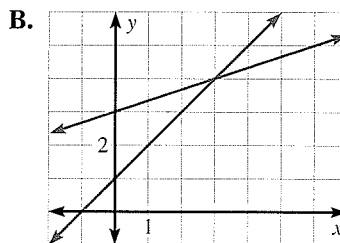
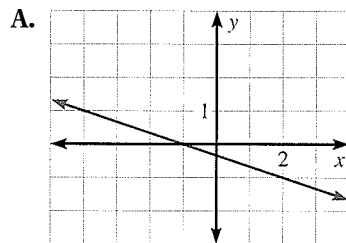
1. **VOCABULARY** Copy and complete: A linear system with no solution is called a(n)   ? system.
2. **VOCABULARY** Copy and complete: A linear system with infinitely many solutions is called a(n)   ? system.
3. ★ **WRITING** Describe the graph of a linear system that has no solution.
4. ★ **WRITING** Describe the graph of a linear system that has infinitely many solutions.

**INTERPRETING GRAPHS** Match the linear system with its graph. Then use the graph to tell whether the linear system has *one solution*, *no solution*, or *infinitely many solutions*.

5.  $x - 3y = -9$   
 $x - y = -1$

6.  $x - y = -4$   
 $-3x + 3y = 2$

7.  $x + 3y = -1$   
 $-2x - 6y = 2$



**EXAMPLES****1 and 2**on pp. 459–460  
for Exs. 8–25**INTERPRETING GRAPHS** Graph the linear system. Then use the graph to tell whether the linear system has *one solution*, *no solution*, or *infinitely many solutions*.

8.  $x + y = -2$   
 $y = -x + 5$

9.  $3x - 4y = 12$   
 $y = \frac{3}{4}x - 3$

10.  $3x - y = -9$   
 $3x + 5y = -15$

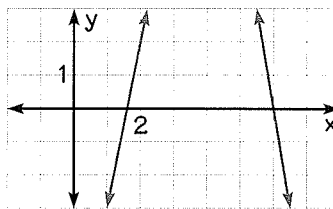
11.  $-2x + 2y = -16$   
 $3x - 6y = 30$

12.  $-9x + 6y = 18$   
 $6x - 4y = -12$

13.  $-3x + 4y = 12$   
 $-3x + 4y = 24$

14. **ERROR ANALYSIS** Describe and correct the error in solving the linear system below.

$$6x + y = 36$$
$$5x - y = 8$$



The lines do not intersect, so there is no solution.

**SOLVING LINEAR SYSTEMS** Solve the linear system using substitution or elimination.

15.  $2x + 5y = 14$   
 $6x + 7y = 10$

16.  $-16x + 2y = -2$   
 $y = 8x - 1$

17.  $3x - 2y = -5$   
 $4x + 5y = 47$

18.  $5x - 5y = -3$   
 $y = x + 0.6$

19.  $x - y = 0$   
 $5x - 2y = 6$

20.  $x - 2y = 7$   
 $-x + 2y = 7$

21.  $-18x + 6y = 24$   
 $3x - y = -2$

22.  $4y + 5x = 15$   
 $x = 8y + 3$

23.  $6x + 3y = 9$   
 $2x + 9y = 27$

24. **★ MULTIPLE CHOICE** Which of the linear systems has *exactly one* solution?

Ⓐ  $-x + y = 9$   
 $x - y = 9$

Ⓑ  $-x + y = 9$   
 $x - y = -9$

Ⓒ  $-x + y = 9$   
 $-x - y = 9$

Ⓓ  $x - y = -9$   
 $-x + y = -9$

25. **★ MULTIPLE CHOICE** Which of the linear systems has infinitely many solutions?

Ⓐ  $15x + 5y = 20$   
 $6x - 2y = 8$

Ⓑ  $15x - 5y = 20$   
 $6x - 2y = -8$

Ⓒ  $15x - 5y = -20$   
 $6x - 2y = 8$

Ⓓ  $15x - 5y = 20$   
 $6x - 2y = 8$

**EXAMPLE 3**on p. 461  
for Exs. 26–31**IDENTIFYING THE NUMBER OF SOLUTIONS** Without solving the linear system, tell whether the linear system has *one solution*, *no solution*, or *infinitely many solutions*.

26.  $y = -6x - 2$   
 $12x + 2y = -6$

27.  $y = 7x + 13$   
 $-21x + 3y = 39$

28.  $4x + 3y = 27$   
 $4x - 3y = -27$

29.  $9x - 15y = 24$   
 $6x - 10y = 16$

30.  $0.3x + 0.4y = 2.4$   
 $0.5x - 0.6y = 0.2$

31.  $0.9x - 2.1y = 12.3$   
 $1.5x - 3.5y = 20.5$

32. ★ **OPEN-ENDED** Write a linear system so that it has infinitely many solutions, and one of the equations is  $y = 3x + 2$ .
33. ★ **OPEN-ENDED** Write a linear system so that it has no solution and one of the equations is  $7x - 8y = -9$ .
34. **REASONING** Give a counterexample for the following statement:  
If the graphs of the equations of a linear system have the same slope, then the linear system has no solution.
35. **CHALLENGE** Find values of  $p$ ,  $q$ , and  $r$  that produce the solution(s).
- No solution
  - Infinitely many solutions
  - One solution of  $(4, 1)$

$$px + qy = r \quad \text{Equation 1}$$

$$2x - 3y = 5 \quad \text{Equation 2}$$

## PROBLEM SOLVING

**EXAMPLE 4**  
on p. 461  
for Exs. 36–38

36. **RECREATION** One admission to a roller skating rink costs  $x$  dollars and renting a pair of skates costs  $y$  dollars. A group pays \$243 for admission for 36 people and 21 skate rentals. Another group pays \$81 for admission for 12 people and 7 skate rentals. Is there enough information to determine the cost of one admission to the roller skating rink? *Explain.*

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37. **TRANSPORTATION** A passenger train travels from New York City to Washington, D.C., then back to New York City. The table shows the number of coach tickets and business class tickets purchased for each leg of the trip. Is there enough information to determine the cost of one coach ticket? *Explain.*

Destination	Coach tickets	Business class tickets	Money collected (dollars)
Washington, D.C.	150	80	22,860
New York City	170	100	27,280

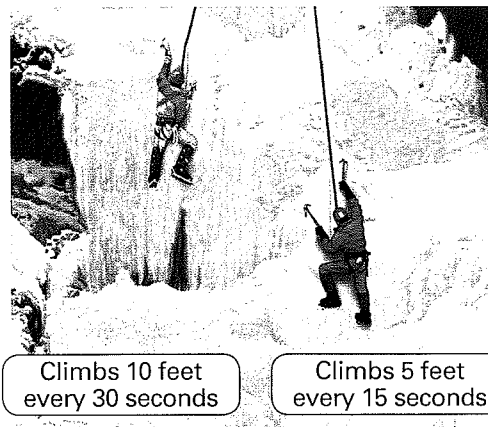
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38. **PHOTOGRAPHY** In addition to taking pictures on your digital camera, you can record 30 second movies. All pictures use the same amount of memory, and all 30 second movies use the same amount of memory. The number of pictures and 30 second movies on 2 memory cards is shown.

- Is there enough information given to determine the amount of memory used by a 30 second movie? *Explain.*
- Given that a 30 second movie uses 50 times the amount of memory that a digital picture uses, can you determine the amount of memory used by a 30 second movie? *Explain.*

Size of card (megabytes)	64	256
Pictures	450	1800
Movies	7	28

39. **MULTI-STEP PROBLEM** Two people are training for a speed ice-climbing event. During a practice climb, one climber starts 15 seconds after the first climber. The rates that the climbers ascend are shown.



- a. Let  $d$  be the distance (in feet) traveled by a climber  $t$  seconds after the first person starts climbing. Write a linear system that models the situation.
- b. Graph the linear system from part (a). Does the second climber catch up to the first climber? *Explain.*

40. **★ EXTENDED RESPONSE** Two employees at a banquet facility are given the task of folding napkins. One person starts folding napkins at a rate of 5 napkins per minute. The second person starts 10 minutes after the first person and folds napkins at a rate of 4 napkins per minute.

- a. **Model** Let  $y$  be the number of napkins folded  $x$  minutes after the first person starts folding. Write a linear system that models the situation.
- b. **Solve** Solve the linear system.
- c. **Interpret** Does the solution of the linear system make sense in the context of the problem? *Explain.*

41. **CHALLENGE** An airplane has an average air speed of 160 miles per hour. The airplane takes 3 hours to travel with the wind from Salem to Lancaster. The airplane has to travel against the wind on the return trip. After 3 hours into the return trip, the airplane is 120 miles from Salem. Find the distance from Salem to Lancaster. If the problem cannot be solved with the information given, *explain why.*

## MIXED REVIEW

Solve the equation, if possible. (p. 154)

42.  $61 + 5c = 7 - 4c$

43.  $3m - 2 = 7m - 50 + 8m$

44.  $11z + 3 = 10(2z + 3)$

45.  $-6(1 - w) = 14(w - 5)$

Solve the inequality. Then graph your solution.

46.  $x + 15 < 23$  (p. 356)

47.  $x - 1 \geq 10$  (p. 356)

48.  $\frac{x}{4} > -2.5$  (p. 363)

49.  $-7x \leq 84$  (p. 363)

50.  $2 - 5x \geq 27$  (p. 369)

51.  $3x - 9 > 3(x - 3)$  (p. 369)

52.  $-2 < x + 3 \leq 11$  (p. 380)

53.  $-7 \leq 5 - 2x \leq 7$  (p. 380)

54.  $|x - 3| \geq 5$  (p. 398)

55.  $2|2x - 1| - 9 \leq 1$  (p. 398)

Graph the inequality. (p. 405)

56.  $x + y < -3$

57.  $x - y \geq 1$

58.  $2x - y < 5$

59.  $-2x - 3y \leq 9$

60.  $y \leq -4$

61.  $x > 6.5$

### PREVIEW

Prepare for Lesson 7.6 in Exs. 46–61.



# 7.6 EXERCISES

**HOMWORK KEY**

○ = WORKED-OUT SOLUTIONS on p. WS18 for Exs. 13 and 39

★ = STANDARDIZED TEST PRACTICE Exs. 2, 21, 22, 33, and 40

## SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A(n)   ? of a system of linear inequalities is an ordered pair that is a solution of each inequality in the system.

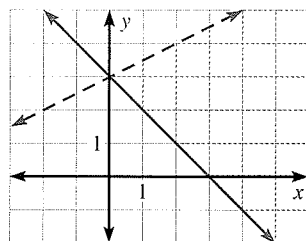
2. ★ **WRITING** Describe the steps you would take to graph the system of inequalities shown.

$$\begin{aligned} x - y &< 7 \\ y &\geq 3 \end{aligned}$$

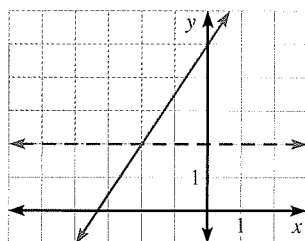
Inequality 1  
Inequality 2

**CHECKING A SOLUTION** Tell whether the ordered pair is a solution of the system of inequalities.

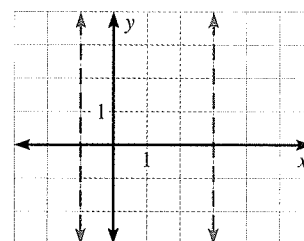
3. (1, 1)



4. (0, 6)



5. (3, -1)



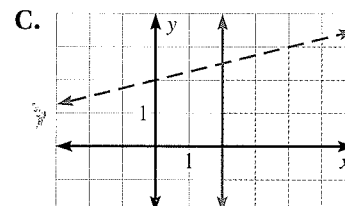
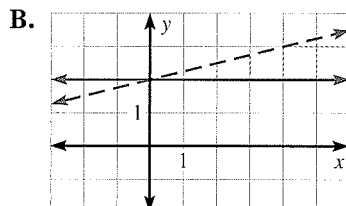
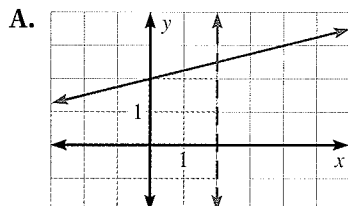
**EXAMPLE 1**  
on p. 466  
for Exs. 6–17

**MATCHING SYSTEMS AND GRAPHS** Match the system of inequalities with its graph.

6.  $x - 4y > -8$   
 $x \geq 2$

7.  $x - 4y \geq -8$   
 $x < 2$

8.  $x - 4y > -8$   
 $y \geq 2$



**GRAPHING A SYSTEM** Graph the system of inequalities.

9.  $x > -5$   
 $x < 2$

10.  $y \leq 10$   
 $y \geq 6$

11.  $x > 3$   
 $y > x$

12.  $y < -2x + 3$   
 $y \geq 4$

13.  $y \geq 0$   
 $y < 2.5x - 1$

14.  $y \geq 2x + 1$   
 $y < -x + 4$

15.  $x < 8$   
 $x - 4y \leq -8$

16.  $y \geq -2$   
 $2x + 3y > -6$

17.  $y - 2x < 7$   
 $y + 2x > -1$

**EXAMPLE 2**  
on p. 467  
for Exs. 18–21

18.  $x < 4$   
 $y > 1$   
 $y \geq -x + 1$

19.  $x \geq 0$   
 $y \geq 0$   
 $6x - y < 12$

20.  $x + y \leq 10$   
 $x - y \geq 2$   
 $y \geq 2$

21. ★ **MULTIPLE CHOICE** Which ordered pair is a solution of the system  $2x - y \leq 5$  and  $x + 2y > 2$ ?

(A) (1, -1)

(B) (4, 1)

(C) (2, 0)

(D) (3, 2)

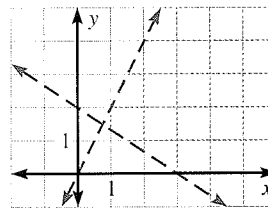
**EXAMPLE 2**

on p. 467

for Exs. 22–23

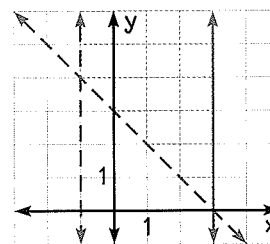
22. ★ **MULTIPLE CHOICE** The graph of which system of inequalities is shown?

- (A)  $y < 2x$   
 $2x + 3y < 6$
- (B)  $y < 2x$   
 $2x + 3y > 6$
- (C)  $y > 2x$   
 $2x + 3y < 6$
- (D)  $y > 2x$   
 $2x + 3y > 6$



23. **ERROR ANALYSIS** Describe and correct the error in graphing this system of inequalities:

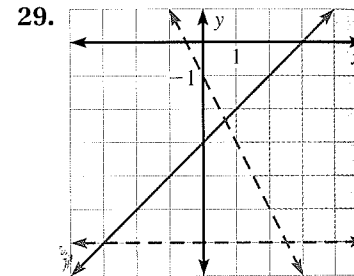
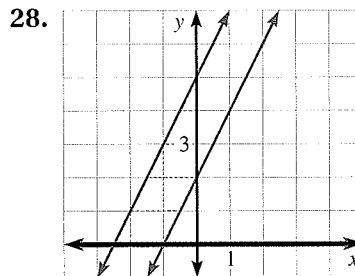
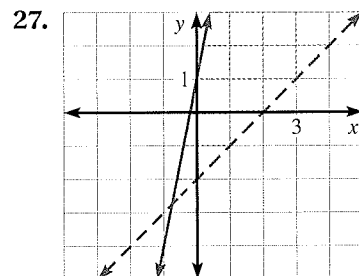
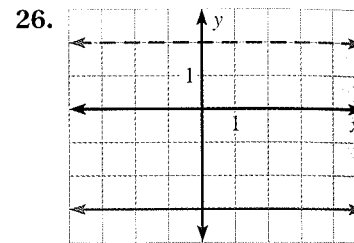
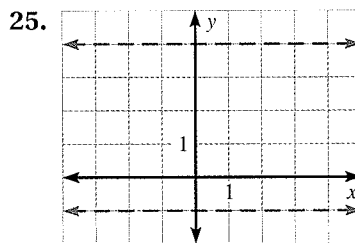
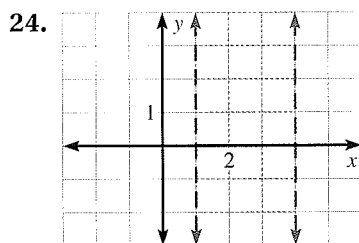
$x + y < 3$  Inequality 1  
 $x > -1$  Inequality 2  
 $x \leq 3$  Inequality 3

**EXAMPLE 3**

on p. 467

for Exs. 24–29

- WRITING A SYSTEM** Write a system of inequalities for the shaded region.



- GRAPHING A SYSTEM** Graph the system of inequalities.

30.  $x > 4$   
 $x < 9$   
 $y \leq 2$   
 $y > -2$
31.  $x + y < 4$   
 $x + y > -2$   
 $x - y \leq 3$   
 $x - y \geq -4$
32.  $x \leq 10$   
 $3x + 2y \geq 9$   
 $x - 2y \leq 6$   
 $x + y \leq 5$

33. ★ **SHORT RESPONSE** Does the system of inequalities have any solutions? Explain.

$x - y > 5$  Inequality 1  
 $x - y < 1$  Inequality 2

- CHALLENGE** Write a system of inequalities for the shaded region described.

34. The shaded region is a rectangle with vertices at (2, 1), (2, 4), (6, 4), and (6, 1).
35. The shaded region is a triangle with vertices at (-3, 0), (3, 2), and (0, -2).

## PROBLEM SOLVING

**EXAMPLE 4**  
on p. 468  
for Exs. 36–38

- 36. COMPETITION SCORES** In a marching band competition, scoring is based on a musical evaluation and a visual evaluation. The musical evaluation score cannot exceed 60 points, the visual evaluation score cannot exceed 40 points. Write and graph a system of inequalities for the scores that a marching band can receive.

**@HomeTutor** for problem solving help at classzone.com

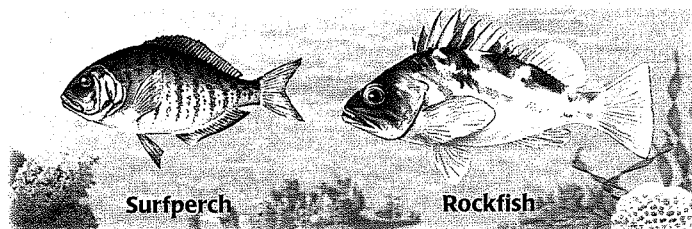
- 37. NUTRITION** For a hiking trip, you are making a mix of  $x$  ounces of peanuts and  $y$  ounces of chocolate pieces. You want the mix to have less than 70 grams of fat and weigh less than 8 ounces. An ounce of peanuts has 14 grams of fat, and an ounce of chocolate pieces has 7 grams of fat. Write and graph a system of inequalities that models the situation.

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- 38. FISHING LIMITS** You are fishing in a marina for surfperch and rockfish, which are two species of bottomfish. Gaming laws in the marina allow you to catch no more than 15 surfperch per day, no more than 10 rockfish per day, and no more than 15 total bottomfish per day.

a. Write and graph a system of inequalities that models the situation.

b. Use the graph to determine whether you can catch 11 surfperch and 9 rockfish in one day.



- 39. HEALTH** A person's maximum heart rate (in beats per minute) is given by  $220 - x$  where  $x$  is the person's age in years ( $20 \leq x \leq 65$ ). When exercising, a person should aim for a heart rate that is at least 70% of the maximum heart rate and at most 85% of the maximum heart rate.

a. Write and graph a system of inequalities that models the situation.

b. A 40-year-old person's heart rate varies from 104 to 120 beats per minute while exercising. Does his heart rate stay in the suggested target range for his age? *Explain.*

- 40. ★ SHORT RESPONSE** A photography shop has a self-service photo center that allows you to make prints of pictures. Each sheet of printed pictures costs \$8. The number of pictures that fit on each sheet is shown.

a. You want at least 16 pictures of any size, and you are willing to spend up to \$48. Write and graph a system of inequalities that models the situation.

b. Will you be able to purchase 12 pictures that are 3 inches by 5 inches and 6 pictures that are 4 inches by 6 inches? *Explain.*

Four 3 inch by 5 inch pictures fit on one sheet.



Two 4 inch by 6 inch pictures fit on one sheet.

41. **CHALLENGE** You make necklaces and keychains to sell at a craft fair. The table shows the time that it takes to make each necklace and keychain, the cost of materials for each necklace and keychain, and the time and money that you can devote to making necklaces and keychains.

	Necklace	Keychain	Available
Time to make (hours)	0.5	0.25	20
Cost to make (dollars)	2	3	120

- Write and graph a system of inequalities for the number  $x$  of necklaces and the number  $y$  of keychains that you can make under the given constraints.
- Find the vertices (corner points) of the graph.
- You sell each necklace for \$10 and each keychain for \$8. The revenue  $R$  is given by the equation  $R = 10x + 8y$ . Find the revenue for each ordered pair in part (b). Which vertex results in the maximum revenue?

## MIXED REVIEW

### PREVIEW

Prepare for Lesson 8.1 in Exs. 42–47.

Evaluate the expression.

- $13x^2$  when  $x = 2$  (p. 8)
- $64 \div z^3$  when  $z = 2$  (p. 8)
- $-|-c| - 3^2$  when  $c = 8$  (p. 64)
- $-8 + 3y - 16$  when  $y = -6$  (p. 88)
- $\frac{7 - 8w}{11w}$  when  $w = 5$  (p. 103)
- $21 + \sqrt{x}$  when  $x = 144$  (p. 110)

Use any method to solve the linear system. (pp. 427, 435, 444, 451)

- $y = 4x - 1$   
 $y = -8x + 23$
- $y = -2x - 6$   
 $y = -5x - 12$
- $x + 2y = -1$   
 $-x = y - 2$
- $4x + y = 0$   
 $-x + y = 5$
- $2x - y = -5$   
 $y = -x - 5$
- $3x + 2y = 2$   
 $-3x + y = -11$

## QUIZ for Lessons 7.5–7.6

Graph the linear system. Then use the graph to tell whether the linear system has *one solution*, *no solution*, or *infinitely many solutions*. (p. 459)

- $x - y = 1$   
 $x - y = 6$
- $6x + 2y = 16$   
 $2x - y = 2$
- $3x - 3y = -2$   
 $-6x + 6y = 4$

Graph the system of linear inequalities. (p. 466)

- $x > -3$   
 $x < 7$
- $y \leq 2$   
 $y < 6x + 2$
- $4x \geq y$   
 $-x + 4y < 4$
- $x + y < 2$   
 $2x + y > -3$   
 $y \geq 0$
- $y \geq 3x - 4$   
 $y \leq x$   
 $y \geq -5x - 15$
- $x > -5$   
 $x < 0$   
 $y \leq 2x + 7$



## Lessons 7.5–7.6

**1. MULTI-STEP PROBLEM** A minimum of 600 bricks and 12 bags of sand are needed for a construction job. Each brick weighs 2 pounds, and each bag of sand weighs 50 pounds. The maximum weight that a delivery truck can carry is 3000 pounds.

- Let  $x$  be the number of bricks, and let  $y$  be the number of bags of sand. Write a system of linear inequalities that models the situation.
- Graph the system of inequalities.
- Use the graph to determine whether 700 bricks and 20 bags of sand can be delivered in one trip.

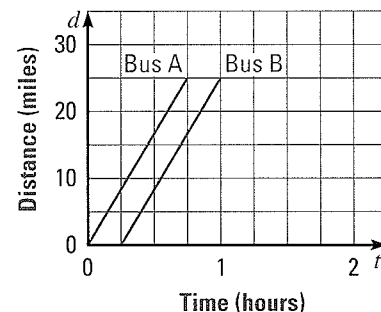
**2. MULTI-STEP PROBLEM** Dana decides to paint the ceiling and the walls of a room. She spends \$120 on 2 gallons of paint for the ceiling and 4 gallons of paint for the walls. Then she decides to paint the ceiling and the walls of another room using the same kinds of paint. She spends \$60 for 1 gallon of paint for the ceiling and 2 gallons of paint for the walls.

- Write a system of linear equations that models the situation.
- Is there enough information given to determine the cost of one gallon of each type of paint? *Explain.*

**c.** A gallon of ceiling paint costs \$3 more than a gallon of wall paint. What is the cost of one gallon of each type of paint?

**3. SHORT RESPONSE** During a sale at a music and video store, all CDs are priced the same and all DVDs are priced the same. Karen buys 4 CDs and 2 DVDs for \$78. The next day, while the sale is still in progress, Karen goes back and buys 2 CDs and 1 DVD for \$39. Is there enough information to determine the cost of 1 CD? *Explain.*

**4. SHORT RESPONSE** Two airport shuttles, bus A and bus B, take passengers to the airport from the same bus stop. The graph shows the distance  $d$  (in miles) traveled by each bus  $t$  hours after bus A leaves the station. The distance from the bus stop to the airport is 25 miles. If bus A and bus B continue at the same rates, will bus B ever catch up to bus A? *Explain.*



**5. EXTENDED RESPONSE** During the summer, you want to earn at least \$200 per week. You earn \$10 per hour working as a lifeguard, and you earn \$8 per hour working at a retail store. You can work at most 30 hours per week.

- Write and graph a system of linear inequalities that models the situation.
- If you work 5 hours per week as a lifeguard and 15 hours per week at the retail store, will you earn at least \$200 per week? *Explain.*
- You are scheduled to work 20 hours per week at the retail store. What is the range of hours you can work as a lifeguard to earn at least \$200 per week?

**6. OPEN-ENDED** Describe a real-world situation that can be modeled by a system of linear inequalities. Then write and graph the system of inequalities.

**7. GRIDDED ANSWER** What is the area (in square feet) of the triangular garden defined by the system of inequalities below?

$$\begin{aligned} y &\geq 0 \\ x &\geq 0 \\ 4x + 5y &\leq 60 \end{aligned}$$



## BIG IDEAS

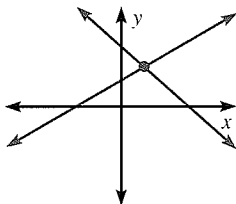
For Your Notebook

## Big Idea 1

## Solving Linear Systems by Graphing

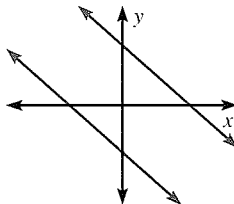
The graph of a system of two linear equations tells you how many solutions the system has.

## One solution



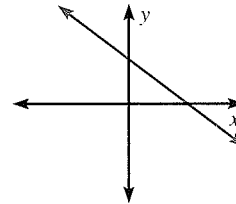
The lines intersect.

## No solution



The lines are parallel.

## Infinitely many solutions



The lines coincide.

## Big Idea 2

## Solving Linear Systems Using Algebra

You can use any of the following algebraic methods to solve a system of linear equations. Sometimes it is easier to use one method instead of another.

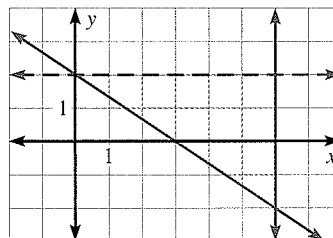
Method	Procedure	When to use
Substitution	Solve one equation for $x$ or $y$ . Substitute the expression for $x$ or $y$ into the other equation.	When one equation is already solved for $x$ or $y$
Addition	Add the equations to eliminate $x$ or $y$ .	When the coefficients of one variable are opposites
Subtraction	Subtract the equations to eliminate $x$ or $y$ .	When the coefficients of one variable are the same
Multiplication	Multiply one or both equations by a constant so that adding or subtracting the equations will eliminate $x$ or $y$ .	When no corresponding coefficients are the same or opposites

## Big Idea 3

## Solving Systems of Linear Inequalities

The graph of a system of linear inequalities is the intersection of the half-planes of each inequality in the system. For example, the graph of the system of inequalities below is the shaded region.

$$\begin{array}{ll} x \leq 6 & \text{Inequality 1} \\ y < 2 & \text{Inequality 2} \\ 2x + 3y \geq 6 & \text{Inequality 3} \end{array}$$





## REVIEW KEY VOCABULARY

- system of linear equations, p. 427
- solution of a system of linear equations, p. 427
- consistent independent system, p. 427
- inconsistent system, p. 459
- consistent dependent system, p. 459
- system of linear inequalities, p. 466
- solution of a system of linear inequalities, p. 466
- graph of a system of linear inequalities, p. 466

## VOCABULARY EXERCISES

1. Copy and complete: A(n)   ?   consists of two or more linear inequalities in the same variables.
2. Copy and complete: A(n)   ?   consists of two or more linear equations in the same variables.
3. Describe how you would graph a system of two linear inequalities.
4. Give an example of a consistent dependent system. Explain why the system is a consistent dependent system.

## REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 7.

### 7.1

## Solve Linear Systems by Graphing

pp. 427–433

### EXAMPLE

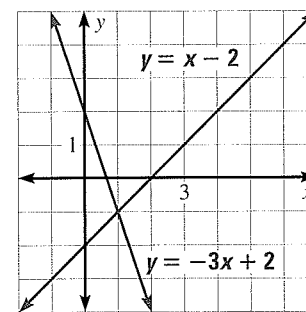
Solve the linear system by graphing. Check your solution.

$$y = x - 2 \quad \text{Equation 1}$$

$$y = -3x + 2 \quad \text{Equation 2}$$

Graph both equations. The lines appear to intersect at  $(1, -1)$ . Check the solution by substituting 1 for  $x$  and  $-1$  for  $y$  in each equation.

$y = x - 2$	$y = -3x + 2$
$-1 \stackrel{?}{=} 1 - 2$	$-1 \stackrel{?}{=} -3(1) + 2$
$-1 = -1 \checkmark$	$-1 = -1 \checkmark$



### EXERCISES

Solve the linear system by graphing. Check your solution.

5.  $y = -3x + 1$   
 $y = x - 7$

6.  $y = 3x + 4$   
 $y = -2x - 1$

7.  $x + y = 3$   
 $x - y = 5$

EXAMPLES  
1 and 2  
on pp. 427–428  
for Exs. 5–7

## 7

## CHAPTER REVIEW

## 7.2 Solve Linear Systems by Substitution

pp. 435–441

## EXAMPLE

Solve the linear system:  $3x + y = -9$  Equation 1  
 $y = 5x + 7$  Equation 2

**STEP 1** Substitute  $5x + 7$  for  $y$  in Equation 1 and solve for  $x$ .

$$3x + y = -9 \quad \text{Write Equation 1.}$$

$$3x + 5x + 7 = -9 \quad \text{Substitute } 5x + 7 \text{ for } y.$$

$$x = -2 \quad \text{Solve for } x.$$

**STEP 2** Substitute  $-2$  for  $x$  in Equation 2 to find the value of  $y$ .

$$y = 5x + 7 = 5(-2) + 7 = -10 + 7 = -3$$

► The solution is  $(-2, -3)$ . Check the solution by substituting  $-2$  for  $x$  and  $-3$  for  $y$  in each of the original equations.

## EXERCISES

Solve the linear system using substitution.

8.  $y = 2x - 7$   
 $x + 2y = 1$

9.  $x + 4y = 9$   
 $x - y = 4$

10.  $2x + y = -15$   
 $y - 5x = 6$

11. **ART** Kara spends \$16 on tubes of paint and disposable brushes for an art project. Each tube of paint costs \$3, and each disposable brush costs \$.50. Kara purchases twice as many brushes as tubes of paint. Find the number of brushes and the number of tubes of paint that she purchases.

EXAMPLES  
 1, 2, and 3  
 on pp. 435–437  
 for Exs. 8–11

## 7.3 Solve Linear Systems by Adding or Subtracting

pp. 444–450

## EXAMPLE

Solve the linear system:  $5x - y = 8$  Equation 1  
 $-5x + 4y = -17$  Equation 2

**STEP 1** Add the equations to eliminate one variable.

$$\begin{array}{r} 5x - y = 8 \\ -5x + 4y = -17 \\ \hline 3y = -9 \end{array}$$

**STEP 2** Solve for  $y$ .

$$y = -3$$

**STEP 3** Substitute  $-3$  for  $y$  in either equation and solve for  $x$ .

$$5x - y = 8 \quad \text{Write Equation 1.}$$

$$5x - (-3) = 8 \quad \text{Substitute } -3 \text{ for } y.$$

$$x = 1 \quad \text{Solve for } x.$$

► The solution is  $(1, -3)$ . Check the solution by substituting 1 for  $x$  and  $-3$  for  $y$  in each of the original equations.

**EXAMPLES**  
1, 2, and 3  
on pp. 444–445  
for Exs. 12–17

**EXERCISES**

Solve the linear system using elimination.

12.  $x + 2y = 13$   
 $x - 2y = -7$

13.  $4x - 5y = 14$   
 $-4x + y = -6$

14.  $x + 7y = 12$   
 $-2x + 7y = 18$

15.  $9x - 2y = 34$   
 $5x - 2y = 10$

16.  $3x = y + 1$   
 $2x - y = 9$

17.  $4y = 11 - 3x$   
 $3x + 2y = -5$

**7.4 Solve Linear Systems by Multiplying First**

pp. 451–457

**EXAMPLE**

Solve the linear system:  $x - 2y = -7$  Equation 1  
 $3x - y = 4$  Equation 2

**STEP 1** Multiply the first equation by  $-3$ .

$$\begin{array}{r} x - 2y = -7 \quad \times (-3) \quad -3x + 6y = 21 \\ 3x - y = 4 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 3x - y = 4 \\ \hline \end{array}$$

**STEP 2** Add the equations.  $5y = 25$

**STEP 3** Solve for  $y$ .  $y = 5$

**STEP 4** Substitute 5 for  $y$  in either of the original equations and solve for  $x$ .

$x - 2y = -7$  Write Equation 1.

$x - 2(5) = -7$  Substitute 5 for  $y$ .

$x = 3$  Solve for  $x$ .

► The solution is (3, 5).

**CHECK** Substitute 3 for  $x$  and 5 for  $y$  in each of the original equations.

Equation 1	Equation 2
$x - 2y = -7$	$3x - y = 4$
$3 - 2(5) \stackrel{?}{=} -7$	$3(3) - 5 \stackrel{?}{=} 4$
$-7 = -7 \checkmark$	$4 = 4 \checkmark$

**EXERCISES**

Solve the linear system using elimination.

18.  $-x + y = -4$   
 $2x - 3y = 5$

19.  $x + 6y = 28$   
 $2x - 3y = -19$

20.  $3x - 5y = -7$   
 $-4x + 7y = 8$

21.  $8x - 7y = -3$   
 $6x - 5y = -1$

22.  $5x = 3y - 2$   
 $3x + 2y = 14$

23.  $11x = 2y - 1$   
 $3y = 10 + 8x$

24. **CAR MAINTENANCE** You pay \$24.50 for 10 gallons of gasoline and 1 quart of oil at a gas station. Your friend pays \$22 for 8 gallons of the same gasoline and 2 quarts of the same oil. Find the cost of 1 quart of oil.

**EXAMPLES**  
1 and 2  
on pp. 451–452  
for Exs. 18–24

# 7 CHAPTER REVIEW

## 7.5 Solve Special Types of Linear Systems

pp. 459–465

### EXAMPLE

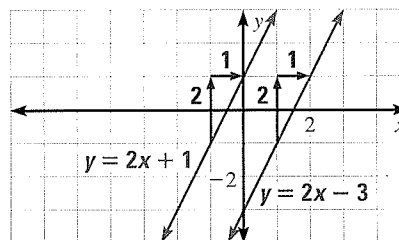
Show that the linear system has no solution.

$$-2x + y = -3 \quad \text{Equation 1}$$

$$y = 2x + 1 \quad \text{Equation 2}$$

Graph the linear system.

The lines are parallel because they have the same slope but different  $y$ -intercepts. Parallel lines do not intersect, so the system has no solution.



### EXERCISES

Tell whether the linear system has *one solution*, *no solution*, or *infinitely many solutions*. Explain.

25.  $x = 2y - 3$   
 $1.5x - 3y = 0$

26.  $-x + y = 8$   
 $x + 8 = y$

27.  $4x = 2y + 6$   
 $4x + 2y = 10$

EXAMPLES  
 1, 2, and 3  
 on pp. 459–461  
 for Exs. 25–27

## 7.6 Solve Systems of Linear Inequalities

pp. 466–472

### EXAMPLE

Graph the system of linear inequalities.

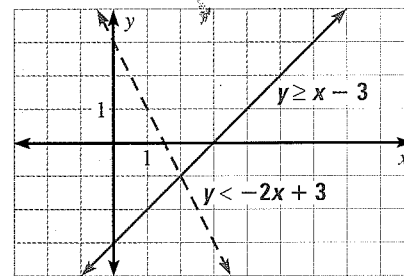
$$y < -2x + 3 \quad \text{Inequality 1}$$

$$y \geq x - 3 \quad \text{Inequality 2}$$

The graph of  $y < -2x + 3$  is the half-plane *below* the *dashed* line  $y = -2x + 3$ .

The graph of  $y \geq x - 3$  is the half-plane *on and above* the *solid* line  $y = x - 3$ .

The graph of the system is the intersection of the two half-planes shown as the darker shade of blue.



### EXERCISES

Graph the system of linear inequalities.

28.  $y < x + 3$   
 $y > -3x - 2$

29.  $y \leq -x - 2$   
 $y > 4x + 1$

30.  $y \geq 0$   
 $x \leq 2$   
 $y < x + 4$

EXAMPLES  
 1, 2, 3, and 4  
 on pp. 466–468  
 for Exs. 28–31

31. **MOVIE COSTS** You receive a \$40 gift card to a movie theater. A ticket to a matinee movie costs \$5, and a ticket to an evening movie costs \$8. Write and graph a system of inequalities for the number of tickets you can purchase using the gift card.

Solve the linear system by graphing. Check your solution.

1.  $3x - y = -6$   
 $x + y = 2$

2.  $-2x + y = 5$   
 $x + y = -1$

3.  $y = 4x + 4$   
 $3x + 2y = 12$

4.  $5x - 4y = 20$   
 $x + 2y = 4$

5.  $x + 3y = 9$   
 $2x - y = 4$

6.  $2x + 7y = 14$   
 $5x + 7y = -7$

Solve the linear system using substitution.

7.  $y = 5x - 7$   
 $-4x + y = -1$

8.  $x = y - 11$   
 $x - 3y = 1$

9.  $3x + y = -19$   
 $x - y = 7$

10.  $15x + y = 70$   
 $3x - 2y = -8$

11.  $3y + x = 17$   
 $x + y = 8$

12.  $0.5x + y = 9$   
 $1.6x + 0.2y = 13$

Solve the linear system using elimination.

13.  $8x + 3y = -9$   
 $-8x + y = 29$

14.  $x - 5y = -3$   
 $3x - 5y = 11$

15.  $4x + y = 17$   
 $7y = 4x - 9$

16.  $3x + 2y = -5$   
 $x - y = 10$

17.  $3y = x + 5$   
 $-3x + 8y = 8$

18.  $6x - 5y = 9$   
 $9x - 7y = 15$

Tell whether the linear system has *one solution*, *no solution*, or *infinitely many solutions*.

19.  $15x - 3y = 12$   
 $y = 5x - 4$

20.  $4x - y = -4$   
 $-8x + 2y = 2$

21.  $-12x + 3y = 18$   
 $4x + y = -6$

22.  $6x - 7y = 5$   
 $-12x + 14y = 10$

23.  $3x - 4y = 24$   
 $3x + 4y = 24$

24.  $10x - 2y = 14$   
 $15x - 3y = 21$


Graph the system of linear inequalities.

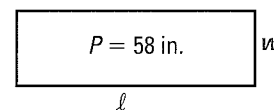
25.  $y < 2x + 2$   
 $y \geq -x - 1$

26.  $y \leq 3x - 2$   
 $y > x + 4$

27.  $y \leq 3$   
 $x > -1$   
 $y > 3x - 3$

28. **TRUCK RENTALS** Carrie and Dave each rent the same size moving truck for one day. They pay a fee of  $x$  dollars for the truck and  $y$  dollars per mile they drive. Carrie drives 150 miles and pays \$215. Dave drives 120 miles and pays \$176. Find the amount of the fee and the cost per mile.

29.  **GEOMETRY** The rectangle has a perimeter  $P$  of 58 inches. The length  $l$  is one more than 3 times the width  $w$ . Write and solve a system of linear equations to find the length and width of the rectangle.



30. **COMMUNITY SERVICE** A town committee has a budget of \$75 to spend on snacks for the volunteers participating in a clean-up day. The committee chairperson decides to purchase granola bars and at least 50 bottles of water. Granola bars cost \$.50 each, and bottles of water cost \$.75 each. Write and graph a system of linear inequalities for the number of bottles of water and the number of granola bars that can be purchased.