8 Exponents and Exponential Functions

Before

In previous chapters and courses, you learned the following skills, which you'll use in Chapter 8: evaluating expressions involving exponents, ordering numbers, writing percents as decimals, and writing function rules.

Prerequisite Skills

VOCABULARY CHECK

1. Identify the exponent and the base in the expression 13^8.
2. Copy and complete: An expression that represents repeated multiplication of the same factor is called a(n) _?_.

SKILLS CHECK

Evaluate the expression. ***(Review p. 2 for 8.1–8.3.)***

3. x^2 when $x = 10$ **4.** a^3 when $a = 3$ **5.** r^2 when $r = \frac{5}{6}$ **6.** z^3 when $z = \frac{1}{2}$

Order the numbers from least to greatest. ***(Review p. 909 for 8.4.)***

7. 6.12, 6.2, 6.01 **8.** 0.073, 0.101, 0.0098

Write the percent as a decimal. ***(Review p. 916 for 8.5 and 8.6.)***

9. 4% **10.** 0.5% **11.** 13.8% **12.** 145%

13. Write a rule for the function. ***(Review p. 35 for 8.5 and 8.6.)***

Input	0	1	4	6	10
Output	2	3	6	8	12

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 8, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 542. You will also use the key vocabulary listed below.

Big Ideas

1. **Applying properties of exponents to simplify expressions**
2. **Working with numbers in scientific notation**
3. **Writing and graphing exponential functions**

KEY VOCABULARY

- order of magnitude, *p. 491*
- scientific notation, *p. 512*
- exponential function, *p. 520*
- exponential growth, *p. 522*
- compound interest, *p. 523*
- exponential decay, *p. 533*

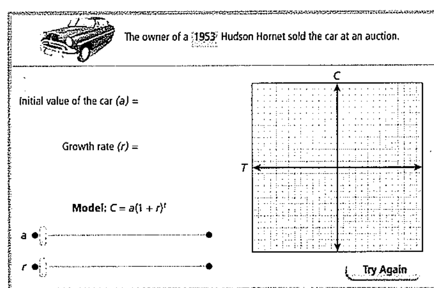
Why?

You can use exponents to explore exponential growth and decay. For example, you can write an exponential function to find the value of a collector car over time.

Animated Algebra

The animation illustrated below for Example 4 on page 522 helps you answer this question: If you know the growth rate of the value of a collector car over time, can you predict what the car will sell for at an auction?

Find the value of the collector car over time.



Click on the boxes to enter the initial value and growth rate.

Animated Algebra at classzone.com

Other animations for Chapter 8: pages 491, 505, 512, 534, and 536

8.1 Products and Powers

MATERIALS • paper and pencil

QUESTION **How can you find a product of powers and a power of a power?**

EXPLORE 1 **Find products of powers**

STEP 1 ***Copy and complete*** Copy and complete the table.

Expression	Expression as repeated multiplication	Number of factors	Simplified expression
$7^4 \cdot 7^5$	$(7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7 \cdot 7 \cdot 7 \cdot 7)$	9	7^9
$(-4)^2 \cdot (-4)^3$	$[(-4) \cdot (-4)] \cdot [(-4) \cdot (-4) \cdot (-4)]$	?	?
$x^1 \cdot x^5$	?	?	?

STEP 2 ***Analyze results*** Find a pattern that relates the exponents of the factors in the first column and the exponent of the expression in the last column.

EXPLORE 2 **Find powers of powers**

STEP 1 ***Copy and complete*** Copy and complete the table.

Expression	Expanded expression	Expression as repeated multiplication	Number of factors	Simplified expression
$(5^3)^2$	$(5^3) \cdot (5^3)$	$(5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$	6	5^6
$[(-6)^2]^4$	$[(-6)^2] \cdot [(-6)^2] \cdot [(-6)^2] \cdot [(-6)^2]$	?	?	?
$(a^3)^3$	?	?	?	?

STEP 2 ***Analyze results*** Find a pattern that relates the exponents of the expression in the first column and the exponent of the expression in the last column.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

Simplify the expression. Write your answer using exponents.

1. $5^2 \cdot 5^3$
2. $(-6)^1 \cdot (-6)^4$
3. $m^6 \cdot m^4$
4. $(10^3)^3$
5. $[(-2)^3]^4$
6. $(c^2)^6$

In Exercises 7 and 8, copy and complete the statement.

7. If a is a real number and m and n are positive integers, then $a^m \cdot a^n =$ __?__.
8. If a is a real number and m and n are positive integers, then $(a^m)^n =$ __?__.

8.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS18 for Exs. 31 and 55

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 40, 41, 50, and 58

◆ = **MULTIPLE REPRESENTATIONS** Ex. 55

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The _?_ of the quantity 93,534,004 people is the power of 10 nearest the quantity, or 10^8 people.

2. ★ **WRITING** *Explain* when and how to use the product of powers property.

EXAMPLES 1,2,3, and 4 on pp. 489–491 for Exs. 3–41

SIMPLIFYING EXPRESSIONS **Simplify the expression. Write your answer using exponents.**

3. $4^2 \cdot 4^6$
4. $8^5 \cdot 8^2$
5. $3^3 \cdot 3$
6. $9 \cdot 9^5$
7. $(-7)^4(-7)^5$
8. $(-6)^6(-6)$
9. $2^4 \cdot 2^9 \cdot 2$
10. $(-3)^2(-3)^{11}(-3)$
11. $(3^5)^2$
12. $(7^4)^3$
13. $[(-5)^3]^4$
14. $[(-8)^9]^2$
15. $(15 \cdot 29)^3$
16. $(17 \cdot 16)^4$
17. $(132 \cdot 9)^6$
18. $((-14) \cdot 22)^5$

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

19. $x^4 \cdot x^2$
20. $y^9 \cdot y$
21. $z^2 \cdot z \cdot z^3$
22. $a^4 \cdot a^3 \cdot a^{10}$
23. $(x^5)^2$
24. $(y^4)^6$
25. $[(b-2)^2]^6$
26. $[(d+9)^7]^3$
27. $(-5x)^2$
28. $-(5x)^2$
29. $(7xy)^2$
30. $(5pq)^3$
31. $(-10x^6)^2 \cdot x^2$
32. $(-8m^4)^2 \cdot m^3$
33. $6d^2 \cdot (2d^5)^4$
34. $(-20x^3)^2(-x^7)$
35. $-(2p^4)^3(-1.5p^7)$
36. $\left(\frac{1}{2}y^5\right)^3(2y^2)^4$
37. $(3x^5)^3(2x^7)^2$
38. $(-10n)^2(-4n^3)^3$

39. **ERROR ANALYSIS** *Describe* and correct the error in simplifying $c \cdot c^4 \cdot c^5$.

$$c \cdot c^4 \cdot c^5 = c^1 \cdot c^4 \cdot c^5$$
$$= c^{1 \cdot 4 \cdot 5}$$
$$= c^{20}$$
✗

40. ★ **MULTIPLE CHOICE** Which expression is equivalent to $(-9)^6$?

Ⓐ $(-9)^2(-9)^3$ Ⓑ $(-9)(-9)^5$ Ⓒ $[(-9)^4]^2$ Ⓓ $[(-9)^3]^3$

41. ★ **MULTIPLE CHOICE** Which expression is equivalent to $36x^{12}$?

Ⓐ $(6x^3)^4$ Ⓑ $12x^4 \cdot 3x^3$ Ⓒ $3x^3 \cdot (4x^3)^3$ Ⓓ $(6x^5)^2 \cdot x^2$

SIMPLIFYING EXPRESSIONS **Find the missing exponent.**

42. $x^4 \cdot x^? = x^5$
43. $(y^8)^? = y^{16}$
44. $(2z^?)^3 = 8z^{15}$
45. $(3a^3)^? \cdot 2a^3 = 18a^9$

46. **POPULATION** The population of New York City in 2000 was 8,008,278. What was the order of magnitude of the population of New York City?

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

47. $(-3x^2y)^3(11x^3y^5)^2$
48. $-(-xy^2z^3)^5(x^4yz)^2$
49. $(-2s)(-5r^3st)^3(-2r^4st^7)^2$

50. ★ **OPEN-ENDED** Write three expressions involving products of powers, powers of powers, or powers of products that are equivalent to $12x^8$.

51. **CHALLENGE** Show that when a and b are real numbers and n is a positive integer, $(ab)^n = a^n b^n$.

PROBLEM SOLVING

EXAMPLE 5
on p. 491
for Exs. 52–56

52. **ICE CREAM COMPOSITION** There are about 954,930 air bubbles in 1 cubic centimeter of ice cream. There are about 946 cubic centimeters in 1 quart. Use order of magnitude to find the approximate number of air bubbles in 1 quart of ice cream.

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53. **ASTRONOMY** The order of magnitude of the radius of our solar system is 10^{13} meters. The order of magnitude of the radius of the visible universe is 10^{13} times as great. Find the approximate radius of the visible universe.

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54. **COASTAL LANDSLIDE** There are about 1 billion grains of sand in 1 cubic foot of sand. In 1995 a stretch of beach at Sleeping Bear Dunes National Lakeshore in Michigan slid into Lake Michigan. Scientists believe that around 35 million cubic feet of sand fell into the lake. Use order of magnitude to find about how many grains of sand slid into the lake.

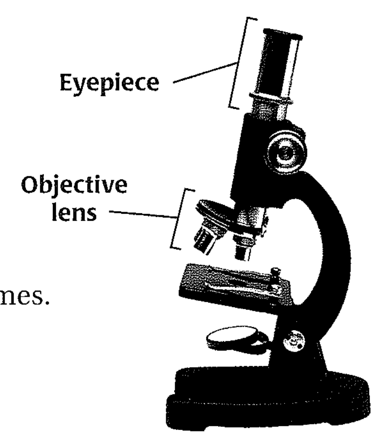
55. ◆ **MULTIPLE REPRESENTATIONS** There are about 10^{23} atoms of gold in 1 ounce of gold.

a. **Making a Table** Copy and complete the table by finding the number of atoms of gold for the given amounts of gold (in ounces).

Gold (ounces)	10	100	1000	10,000	100,000
Number of atoms	?	?	?	?	?

b. **Writing an Expression** A particular mine in California extracted about 96,000 ounces of gold in 1 year. Use order of magnitude to write an expression you can use to find the approximate number of atoms of gold extracted in the mine that year. Simplify the expression. Verify your answer using the table.

56. **MULTI-STEP PROBLEM** A microscope has two lenses, the objective lens and the eyepiece, that work together to magnify an object. The total magnification of the microscope is the product of the magnification of the objective lens and the magnification of the eyepiece.



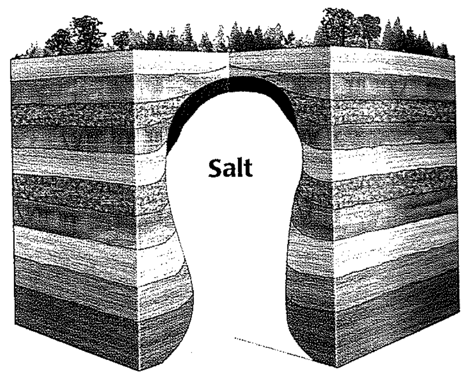
a. Your microscope's objective lens magnifies an object 10^2 times, and the eyepiece magnifies an object 10 times. What is the total magnification of your microscope?

b. You magnify an object that is 10^2 nanometers long. How long is the magnified image?

57. **VOLUME OF THE SUN** The radius of the sun is about 695,000,000 meters. The formula for the volume of a sphere, such as the sun, is $V = \frac{4}{3}\pi r^3$.

Because the order of magnitude of $\frac{4}{3}\pi$ is 1, it does not contribute to the formula in a significant way. So, you can find the order of magnitude of the volume of the sun by cubing its radius. Find the order of magnitude of the volume of the sun.

58. ★ **EXTENDED RESPONSE** Rock salt can be mined from large deposits of salt called salt domes. A particular salt dome is roughly cylindrical in shape. The order of magnitude of the radius of the salt dome is 10^3 feet. The order of magnitude of the height of the salt dome is about 10 times that of its radius. The formula for the volume of a cylinder is $V = \pi r^2 h$.



a. **Calculate** What is the order of magnitude of the height of the salt dome?

b. **Calculate** What is the order of magnitude of the volume of the salt dome?

c. **Explain** The order of magnitude of the radius of a salt dome can be 10 times the radius of the salt dome described in this exercise. What effect does multiplying the order of magnitude of the radius of the salt dome by 10 have on the volume of the salt dome? *Explain.*

59. **CHALLENGE** Your school is conducting a poll that has two parts, one part that has 13 questions and a second part that has 10 questions. Students can answer the questions in either part with "agree" or "disagree." What power of 2 represents the number of ways there are to answer the questions in the first part of the poll? What power of 2 represents the number of ways there are to answer the questions in the second part of the poll? What power of 2 represents the number of ways there are to answer all of the questions on the poll?

MIXED REVIEW

PREVIEW
Prepare for Lesson 8.2 in Exs. 60–65.

Find the product. *(p. 88)*

60. $\left(\frac{1}{2}\right)\left(-\frac{4}{5}\right)$
61. $\left(-\frac{2}{3}\right)\left(\frac{7}{4}\right)$
62. $\left(-\frac{6}{5}\right)\left(-\frac{3}{8}\right)$

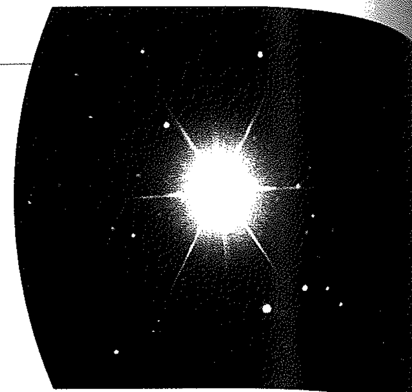
Evaluate the expression for the given value of the variable. *(p. 2)*

63. x^4 when $x = 3$
64. x^2 when $x = -2.2$
65. x^3 when $x = \frac{3}{4}$

Graph the equation or inequality.

66. $y = -4$ *(p. 215)*
67. $3x - y = 15$ *(p. 225)*
68. $7x - 6y = 84$ *(p. 225)*
69. $y = -5x + 3$ *(p. 244)*
70. $y = \frac{1}{2}x - 5$ *(p. 244)*
71. $x \geq -3$ *(p. 405)*
72. $y < 1.5$ *(p. 405)*
73. $x + y \leq 7$ *(p. 405)*
74. $2x - y < 3$ *(p. 405)*

EXAMPLE 5 Solve a real-world problem



Canopus

ASTRONOMY The luminosity (in watts) of a star is the total amount of energy emitted from the star per unit of time. The order of magnitude of the luminosity of the sun is 10^{26} watts. The star Canopus is one of the brightest stars in the sky. The order of magnitude of the luminosity of Canopus is 10^{30} watts. How many times more luminous is Canopus than the sun?

Solution

$$\frac{\text{Luminosity of Canopus (watts)}}{\text{Luminosity of the sun (watts)}} = \frac{10^{30}}{10^{26}} = 10^{30-26} = 10^4$$

▶ Canopus is about 10^4 times as luminous as the sun.

✓ **GUIDED PRACTICE** for Example 5

10. **WHAT IF?** Sirius is considered the brightest star in the sky. Sirius is less luminous than Canopus, but Sirius appears to be brighter because it is much closer to Earth. The order of magnitude of the luminosity of Sirius is 10^{28} watts. How many times more luminous is Canopus than Sirius?

8.2 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS18 for Exs. 33 and 51

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 37, 46, and 54

◆ = **MULTIPLE REPRESENTATIONS** Ex. 49

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In the power 4^3, 4 is the __?__ and 3 is the __?__.

2. ★ **WRITING** *Explain* when and how to use the quotient of powers property.

EXAMPLES 1 and 2 on pp. 495–496 for Exs. 3–20

SIMPLIFYING EXPRESSIONS Simplify the expression. Write your answer using exponents.

3. $\frac{5^6}{5^2}$
4. $\frac{2^{11}}{2^6}$
5. $\frac{3^9}{3^5}$
6. $\frac{(-6)^8}{(-6)^5}$
7. $\frac{(-4)^7}{(-4)^4}$
8. $\frac{(-12)^9}{(-12)^3}$
9. $\frac{10^5 \cdot 10^5}{10^4}$
10. $\frac{6^7 \cdot 6^4}{6^6}$
11. $\left(\frac{1}{3}\right)^5$
12. $\left(\frac{3}{2}\right)^4$
13. $\left(-\frac{5}{4}\right)^4$
14. $\left(-\frac{2}{5}\right)^5$
15. $7^9 \cdot \frac{1}{7^2}$
16. $\frac{1}{9^5} \cdot 9^{11}$
17. $\left(\frac{1}{3}\right)^4 \cdot 3^{12}$
18. $4^9 \cdot \left(-\frac{1}{4}\right)^5$

19. ★ **MULTIPLE CHOICE** Which expression is equivalent to 16^6?

Ⓐ $\dfrac{16^4}{16^2}$ Ⓑ $\dfrac{16^{12}}{16^2}$ Ⓒ $\left(\dfrac{16^6}{16^3}\right)^2$ Ⓓ $\left(\dfrac{16^9}{16^6}\right)^3$

20. **ERROR ANALYSIS** *Describe* and correct the error in simplifying $\dfrac{9^5 \cdot 9^3}{9^4}$.

$$\frac{9^5 \cdot 9^3}{9^4} = \frac{9^8}{9^4} = 9^{12} \quad \times$$

EXAMPLES 1, 2, and 3 on pp. 495–496 for Exs. 21–37

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

21. $\dfrac{1}{y^8} \cdot y^{15}$
22. $z^8 \cdot \dfrac{1}{z^7}$
23. $\left(\dfrac{a}{y}\right)^9$
24. $\left(\dfrac{j}{k}\right)^{11}$
25. $\left(\dfrac{p}{q}\right)^4$
26. $\left(-\dfrac{1}{x}\right)^5$
27. $\left(-\dfrac{4}{x}\right)^3$
28. $\left(-\dfrac{a}{b}\right)^4$
29. $\left(\dfrac{4c}{d^2}\right)^3$
30. $\left(\dfrac{a^7}{2b}\right)^5$
31. $\left(\dfrac{x^2}{3y^3}\right)^2$
32. $\left(\dfrac{3x^5}{7y^2}\right)^3$
33. $\left(\dfrac{3x^3}{2y}\right)^2 \cdot \dfrac{1}{x^2}$
34. $\left(\dfrac{2x^3}{y}\right)^3 \cdot \dfrac{1}{6x^3}$
35. $\dfrac{3}{8m^5} \cdot \left(\dfrac{m^4}{n^2}\right)^3$
36. $\left(-\dfrac{5}{x}\right)^2 \cdot \left(\dfrac{2x^4}{y^3}\right)^2$

37. ★ **MULTIPLE CHOICE** Which expression is equivalent to $\left(\dfrac{7x^3}{2y^4}\right)^2$?

Ⓐ $\dfrac{7x^5}{2y^6}$ Ⓑ $\dfrac{7x^6}{2y^8}$ Ⓒ $\dfrac{49x^5}{4y^6}$ Ⓓ $\dfrac{49x^6}{4y^8}$

SIMPLIFYING EXPRESSIONS **Find the missing exponent.**

38. $\dfrac{(-8)^7}{(-8)^?} = (-8)^3$
39. $\dfrac{7^? \cdot 7^2}{7^4} = 7^6$
40. $\dfrac{1}{p^5} \cdot p^? = p^9$
41. $\left(\dfrac{2c^3}{d^2}\right)^? = \dfrac{16c^{12}}{d^8}$

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

42. $\left(\dfrac{2f^2g^3}{3fg}\right)^4$
43. $\dfrac{2s^3t^3}{st^2} \cdot \dfrac{(3st)^3}{s^2t}$
44. $\left(\dfrac{2m^5n}{4m^2}\right)^2 \cdot \left(\dfrac{mn^4}{5n}\right)^2$
45. $\left(\dfrac{3x^3y}{x^2}\right)^3 \cdot \left(\dfrac{y^2x^4}{5y}\right)^2$

46. ★ **OPEN-ENDED** Write three expressions involving quotients that are equivalent to 14^7.

47. **REASONING** Name the definition or property that justifies each step to show that $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}$ for $m < n$.

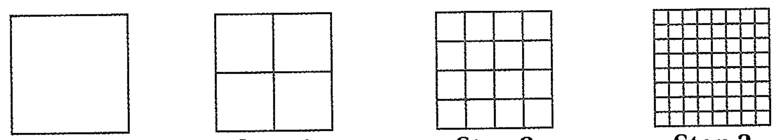
Let $m < n$.	Given
$\dfrac{a^m}{a^n} = \dfrac{a^m}{a^n}\left(\dfrac{\frac{1}{a^m}}{\frac{1}{a^m}}\right)$	?
$= \dfrac{1}{\frac{a^n}{a^m}}$	?
$= \dfrac{1}{a^{n-m}}$	?

48. **CHALLENGE** Find the values of x and y if you know that $\dfrac{b^x}{b^y} = b^9$ and $\dfrac{b^x \cdot b^2}{b^{3y}} = b^{13}$. *Explain* how you found your answer.

PROBLEM SOLVING

EXAMPLES 4 and 5 on pp. 497–498 for Exs. 49–51

49. ◈ **MULTIPLE REPRESENTATIONS** Draw a square with side lengths that are 1 unit long. Divide it into four new squares with side lengths that are one half the side length of the original square, as shown in Step 1. Keep dividing the squares into new squares, as shown in Steps 2 and 3.



Step 0 **Step 1** **Step 2** **Step 3**

a. **Making a Table** Make a table showing the number of new squares and the side length of a new square at each step for Steps 1–4. Write the number of new squares as a power of 4. Write the side length of a new square as a power of $\frac{1}{2}$.

b. **Writing an Expression** Write and simplify an expression to find by how many times the number of new squares increased from Step 2 to Step 4.

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50. **GROSS DOMESTIC PRODUCT** In 2003 the gross domestic product (GDP) for the United States was about 11 trillion dollars, and the order of magnitude of the population of the U.S. was 10^8. Use order of magnitude to find the approximate per capita (per person) GDP.

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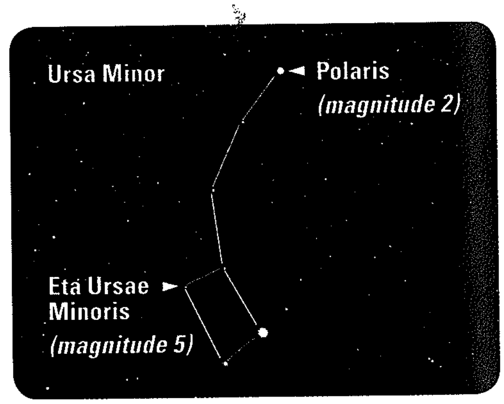
(51.) **SPACE TRAVEL** Alpha Centauri is the closest star system to Earth. Alpha Centauri is about 10^{13} kilometers away from Earth. A spacecraft leaves Earth and travels at an average speed of 10^4 meters per second. About how many years would it take the spacecraft to reach Alpha Centauri?

52. **ASTRONOMY** The brightness of one star relative to another star can be measured by comparing the magnitudes of the stars. For every increase in magnitude of 1, the relative brightness is diminished by a factor of 2.512. For instance, a star of magnitude 8 is 2.512 times less bright than a star of magnitude 7.

The constellation Ursa Minor (the Little Dipper) is shown. How many times less bright is Eta Ursae Minoris than Polaris?



53. **EARTHQUAKES** The energy released by one earthquake relative to another earthquake can be measured by comparing the magnitudes (as determined by the Richter scale) of the earthquakes. For every increase of 1 in magnitude, the energy released is multiplied by a factor of about 31. How many times greater is the energy released by an earthquake of magnitude 7 than the energy released by an earthquake of magnitude 4?

54. ★ **EXTENDED RESPONSE** A byte is a unit used to measure computer memory. Other units are based on the number of bytes they represent. The table shows the number of bytes in certain units. For example, from the table you can calculate that 1 terabyte is equivalent to 2^{10} gigabytes.

a. **Calculate** How many kilobytes are there in 1 terabyte?

b. **Calculate** How many megabytes are there in 1 petabyte?

c. **CHALLENGE** Another unit used to measure computer memory is a bit. There are 8 bits in a byte. *Explain* how you can convert the number of bytes per unit given in the table to the number of bits per unit.

Unit	Number of bytes
Kilobyte	2^{10}
Megabyte	2^{20}
Gigabyte	2^{30}
Terabyte	2^{40}
Petabyte	2^{50}

MIXED REVIEW

PREVIEW Prepare for Lesson 8.3 in Exs. 55–60.

Solve the equation. Check your solution. *(p. 134)*

55. $\frac{3}{4}k = 9$
56. $\frac{2}{5}t = -4$
57. $-\frac{2}{3}v = 14$
58. $-\frac{5}{2}y = -35$
59. $-\frac{7}{5}z = \frac{14}{3}$
60. $-\frac{3}{2}z = -\frac{3}{4}$

Write an equation of the line that passes through the given points. *(p. 292)*

61. $(-2, 1), (0, -5)$
62. $(0, 3), (-4, 1)$
63. $(0, -3), (7, -3)$
64. $(4, 3), (5, 6)$
65. $(4, 1), (-2, 4)$
66. $(-1, -3), (-3, 1)$

QUIZ *for Lessons 8.1–8.2*

Simplify the expression. Write your answer using exponents.

1. $3^2 \cdot 3^6$ *(p. 489)*
2. $(5^4)^3$ *(p. 489)*
3. $(32 \cdot 14)^7$ *(p. 489)*
4. $7^2 \cdot 7^6 \cdot 7$ *(p. 489)*
5. $(-4)(-4)^9$ *(p. 489)*
6. $\frac{7^{12}}{7^4}$ *(p. 495)*
7. $\frac{(-9)^9}{(-9)^7}$ *(p. 495)*
8. $\frac{3^7 \cdot 3^4}{3^6}$ *(p. 495)*
9. $\left(\frac{5}{4}\right)^4$ *(p. 495)*

Simplify the expression.

10. $x^2 \cdot x^5$ *(p. 489)*
11. $(3x^3)^2$ *(p. 489)*
12. $-(7x)^2$ *(p. 489)*
13. $(6x^5)^3 \cdot x$ *(p. 489)*
14. $(2x^5)^3(7x^7)^2$ *(p. 489)*
15. $\frac{1}{x^9} \cdot x^{21}$ *(p. 495)*
16. $\left(-\frac{4}{x}\right)^3$ *(p. 495)*
17. $\left(\frac{w}{v}\right)^6$ *(p. 495)*
18. $\left(\frac{x^3}{4}\right)^2$ *(p. 495)*

19. **AGRICULTURE** In 2004 the order of magnitude of the number of pounds of oranges produced in the United States was 10^{10}. The order of magnitude of the number of acres used for growing oranges was 10^6. About how many pounds of oranges per acre were produced in the United States in 2004? *(p. 495)*

8.3 Zero and Negative Exponents

MATERIALS • paper and pencil

QUESTION **How can you simplify expressions with zero or negative exponents?**

EXPLORE **Evaluate powers with zero and negative exponents**

STEP 1 Find a pattern

Copy and complete the tables for the powers of 2 and 3.

Exponent, n	Value of 2^n
4	16
3	?
2	?
1	?

Exponent, n	Value of 3^n
4	81
3	?
2	?
1	?

As you read the tables from the *bottom up*, you see that each time the exponent is increased by 1, the value of the power is multiplied by the base. What can you say about the exponents and the values of the powers as you read the table from the *top down*?

STEP 2 Extend the pattern

Copy and complete the tables using the pattern you observed in Step 1.

Exponent, n	Power, 2^n
3	8
2	?
1	?
0	?
−1	?
−2	?

Exponent, n	Power, 3^n
3	27
2	?
1	?
0	?
−1	?
−2	?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Find 2^n and 3^n for $n = -3, -4,$ and -5.
2. What appears to be the value of a^0 for any nonzero number a?
3. Write each power in the tables above as a power with a positive exponent. For example, you can write 3^{-1} as $\frac{1}{3^1}$.

8.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS19 for Exs. 11 and 53

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 44, 45, 54, and 57

◆ = **MULTIPLE REPRESENTATIONS** Ex. 55

SKILL PRACTICE

1. **VOCABULARY** Which definitions or properties would you use to simplify the expression $3^5 \cdot 3^{-5}$? *Explain.*

2. ★ **WRITING** *Explain* why the expression 0^{-4} is undefined.

EXAMPLE 1 on p. 503 for Exs. 3–14

EVALUATING EXPRESSIONS **Evaluate the expression.**

3. 4^{-3}
4. 7^{-3}
5. $(-3)^{-1}$
6. $(-2)^{-6}$
7. 2^0
8. $(-4)^0$
9. $\left(\frac{3}{4}\right)^0$
10. $\left(\frac{-9}{16}\right)^0$
11. $\left(\frac{2}{7}\right)^{-2}$
12. $\left(\frac{4}{3}\right)^{-3}$
13. 0^{-3}
14. 0^{-2}

EXAMPLE 2 on p. 504 for Exs. 15–27

15. $2^{-2} \cdot 2^{-3}$
16. $7^{-6} \cdot 7^4$
17. $(2^{-1})^5$
18. $(3^{-2})^2$
19. $\frac{1}{3^{-3}}$
20. $\frac{1}{6^{-2}}$
21. $\frac{3^{-3}}{3^2}$
22. $\frac{6^{-3}}{6^{-5}}$
23. $4\left(\frac{3}{2}\right)^{-1}$
24. $16\left(\frac{2^{-3}}{2^2}\right)$
25. $6^0 \cdot \left(\frac{1}{4^{-2}}\right)$
26. $3^{-2} \cdot \left(\frac{5}{7^0}\right)$

27. **ERROR ANALYSIS** *Describe* and correct the error in evaluating the expression $-6 \cdot 3^0$.

$$-6 \cdot 3^0 = -6 \cdot 0$$
$$= 0$$

EXAMPLE 3 on p. 505 for Exs. 28–43

SIMPLIFYING EXPRESSIONS **Simplify the expression. Write your answer using only positive exponents.**

28. x^{-4}
29. $2y^{-3}$
30. $(4g)^{-3}$
31. $(-11h)^{-2}$
32. x^2y^{-3}
33. $5m^{-3}n^{-4}$
34. $(6x^{-2}y^3)^{-3}$
35. $(-15fg^2)^0$
36. $\frac{r^{-2}}{s^{-4}}$
37. $\frac{x^{-5}}{y^2}$
38. $\frac{1}{8x^{-2}y^{-6}}$
39. $\frac{1}{15x^{10}y^{-8}}$
40. $\frac{1}{(-2z)^{-2}}$
41. $\frac{9}{(3d)^{-3}}$
42. $\frac{(3x)^{-3}y^4}{-x^2y^{-6}}$
43. $\frac{12x^8y^{-7}}{(4x^{-2}y^{-6})^2}$

44. ★ **MULTIPLE CHOICE** Which expression simplifies to $2x^4$?

(A) $2x^{-4}$ (B) $\frac{32}{(2x)^{-4}}$ (C) $\frac{1}{2x^{-4}}$ (D) $\frac{8}{4x^{-4}}$

45. ★ **MULTIPLE CHOICE** Which expression is equivalent to $(-4 \cdot 2^0 \cdot 3)^{-2}$?

(A) -12 (B) $-\frac{1}{144}$ (C) 0 (D) $\frac{1}{144}$

CHALLENGE In Exercises 46–48, tell whether the statement is true for all nonzero values of *a* and *b*. If it is not true, give a counterexample.

46. $\frac{a^{-3}}{a^{-4}} = \frac{1}{a}$ **47.** $\frac{a^{-1}}{b^{-1}} = \frac{b}{a}$ **48.** $a^{-1} + b^{-1} = \frac{1}{a+b}$

49. REASONING For $n > 0$, what happens to the value of a^{-n} as n increases?

PROBLEM SOLVING

EXAMPLE 4 on p. 505 for Exs. 50–54

50. MASS The mass of a grain of salt is about 10^{-4} gram. About how many grains of salt are in a box containing 100 grams of salt?

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51. MASS The mass of a grain of a certain type of rice is about 10^{-2} gram. About how many grains of rice are in a box containing 10^3 grams of rice?

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52. BOTANY The average mass of the fruit of the wolffia angusta plant is about 10^{-4} gram. The largest pumpkin ever recorded had a mass of about 10^4 kilograms. About how many times greater is the mass of the largest pumpkin than the mass of the fruit of the wolffia angusta plant?

53. MEDICINE A doctor collected about 10^{-2} liter of blood from a patient to run some tests. The doctor determined that a drop of the patient's blood, or about 10^{-6} liter, contained about 10^7 red blood cells. How many red blood cells did the entire sample contain?

54. ★ SHORT RESPONSE One of the smallest plant seeds comes from an orchid, and one of the largest plant seeds comes from a giant fan palm. A seed from an orchid has a mass of 10^{-9} gram and is 10^{13} times less massive than a seed from a giant fan palm. A student says that the seed from the giant fan palm has a mass of about 1 kilogram. Is the student correct? *Explain.*

Orchid

Giant fan palm

55. MULTIPLE REPRESENTATIONS Consider folding a piece of paper in half a number of times.

a. Making a Table Each time the paper is folded, record the number of folds and the fraction of the original area in a table like the one shown.

Number of folds	0	1	2	3
Fraction of original area	?	?	?	?

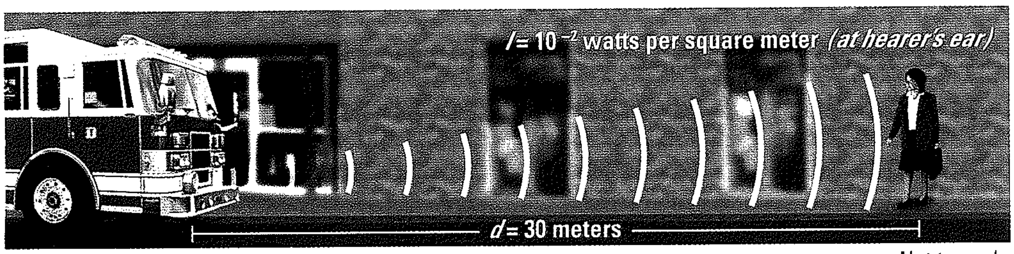
b. Writing an Expression Write an exponential expression for the fraction of the original area of the paper using a base of $\frac{1}{2}$.

56. **SCIENCE** Diffusion is the movement of molecules from one location to another. The time t (in seconds) it takes molecules to diffuse a distance of x centimeters is given by $t = \frac{x^2}{2D}$ where D is the diffusion coefficient.

a. You can examine a cross section of a drop of ink in water to see how the ink diffuses. The diffusion coefficient for the molecules in the drop of ink is about 10^{-5} square centimeter per second. How long will it take the ink to diffuse 1 micrometer (10^{-4} centimeter)?

b. Check your answer to part (a) using unit analysis.

57. ★ **EXTENDED RESPONSE** The intensity of sound I (in watts per square meter) can be modeled by $I = 0.08Pd^{-2}$ where P is the power (in watts) of the sound's source and d is the distance (in meters) that you are from the source of the sound.



Not to scale

a. What is the power (in watts) of the siren of the firetruck shown in the diagram?

b. Using the power of the siren you found in part (a), simplify the formula for the intensity of sound from the siren.

c. *Explain* what happens to the intensity of the siren when you double your distance from it.

58. **CHALLENGE** Coal can be burned to generate energy. The heat energy in 1 pound of coal is about 10^4 BTU (British Thermal Units). Suppose you have a stereo. It takes about 10 pounds of coal to create the energy needed to power the stereo for 1 year.

a. About how many BTUs does your stereo use in 1 year?

b. Suppose the power plant that delivers energy to your home produces 10^{-1} pound of sulfur dioxide for each 10^6 BTU of energy that it creates. How much sulfur dioxide is added to the air by generating the energy needed to power your stereo for 1 year?

MIXED REVIEW

PREVIEW
Prepare for Lesson 8.4 in Exs. 59–62.

Evaluate the expression.

59. $10^3 \cdot 10^3$ *(p. 489)*
60. $10^2 \cdot 10^5$ *(p. 489)*
61. $\frac{10^9}{10^7}$ *(p. 495)*
62. $\frac{10^6}{10^3}$ *(p. 495)*

Solve the linear system. Then check your answer. *(pp. 427, 435, 444, 451)*

63. $y = 3x - 6$, $y = -7x - 1$
64. $y = -2x + 12$, $y = -5x + 24$
65. $5x + y = 40$, $-x + y = -8$
66. $-x - 2y = -6.5$, $3x - 6y = 16.5$
67. $3x + 4y = -5$, $x - 2y = 5$
68. $2x + 6y = 5$, $-2x - 3y = 2$

EXTRA PRACTICE for Lesson 8.3, p. 945 **ONLINE QUIZ** at classzone.com

Extension
Use after Lesson 8.3

Define and Use Fractional Exponents

GOAL Use fractional exponents.

Key Vocabulary
- cube root

In Lesson 2.7, you learned to write the square root of a number using a radical sign. You can also write a square root of a number using exponents.

For any $a \geq 0$, suppose you want to write $\sqrt{a}$ as a^k. Recall that a number b (in this case, a^k) is a square root of a number a provided $b^2 = a$. Use this definition to find a value for k as follows.

$b^2 = a$ **Definition of square root**

$(a^k)^2 = a$ **Substitute a^k for b.**

$a^{2k} = a^1$ **Product of powers property**

Because the bases are the same in the equation $a^{2k} = a^1$, the exponents must be equal:

$2k = 1$ **Set exponents equal.**

$k = \frac{1}{2}$ **Solve for k.**

So, for a nonnegative number a, $\sqrt{a} = a^{1/2}$.

You can work with exponents of $\frac{1}{2}$ and multiples of $\frac{1}{2}$ just as you work with integer exponents.

EXAMPLE 1 Evaluate expressions involving square roots

a. $16^{1/2} = \sqrt{16}$
$= 4$

b. $25^{-1/2} = \frac{1}{25^{1/2}}$
$= \frac{1}{\sqrt{25}}$
$= \frac{1}{5}$

c. $9^{5/2} = 9^{(1/2) \cdot 5}$
$= (9^{1/2})^5$
$= (\sqrt{9})^5$
$= 3^5$
$= 243$

d. $4^{-3/2} = 4^{(1/2) \cdot (-3)}$
$= (4^{1/2})^{-3}$
$= (\sqrt{4})^{-3}$
$= 2^{-3}$
$= \frac{1}{2^3}$
$= \frac{1}{8}$

FRACTIONAL EXPONENTS You can work with other fractional exponents just as you did with $\frac{1}{2}$.

CUBE ROOTS If $b^3 = a$, then b is the **cube root** of a. For example, $2^3 = 8$, so 2 is the cube root of 8. The cube root of a can be written as $\sqrt[3]{a}$ or $a^{1/3}$.

EXAMPLE 2 Evaluate expressions involving cube roots

a. $27^{1/3} = \sqrt[3]{27} = \sqrt[3]{3^3} = 3$

b. $8^{-1/3} = \frac{1}{8^{1/3}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

c. $64^{4/3} = 64^{(1/3) \cdot 4} = (64^{1/3})^4 = (\sqrt[3]{64})^4 = 4^4 = 256$

d. $125^{-2/3} = 125^{(1/3) \cdot (-2)} = (125^{1/3})^{-2} = (\sqrt[3]{125})^{-2} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

PROPERTIES OF EXPONENTS The properties of exponents for integer exponents also apply to fractional exponents.

EXAMPLE 3 Use properties of exponents

a. $12^{-1/2} \cdot 12^{5/2} = 12^{(-1/2) + (5/2)} = 12^{4/2} = 12^2 = 144$

b. $\frac{6^{4/3} \cdot 6}{6^{1/3}} = \frac{6^{(4/3) + 1}}{6^{1/3}} = \frac{6^{7/3}}{6^{1/3}} = 6^{(7/3) - (1/3)} = 6^2 = 36$

PRACTICE

EXAMPLES 1, 2, and 3 on pp. 509–510 for Exs. 1–12

Evaluate the expression.

1. $100^{3/2}$
2. $121^{-1/2}$
3. $81^{-3/2}$
4. $216^{2/3}$
5. $27^{-1/3}$
6. $343^{-2/3}$
7. $9^{7/2} \cdot 9^{-3/2}$
8. $\left(\frac{1}{16}\right)^{1/2}\left(\frac{1}{16}\right)^{-1/2}$
9. $36^{5/2} \cdot \frac{36^{-1/2}}{(36^{-1})^{-7/2}}$
10. $(27^{-1/3})^3$
11. $(-64)^{-5/3}(-64)^{4/3}$
12. $(-8)^{1/3}(-8)^{-2/3}(-8)^{1/3}$
13. **REASONING** Show that the cube root of a can be written as $a^{1/3}$ using an argument similar to the one given for square roots on the previous page.

MIXED REVIEW of Problem Solving

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Lessons 8.1–8.3

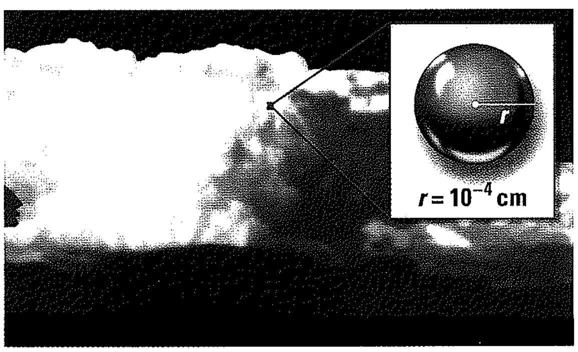
1. **GRIDDED ANSWER** In 2004 the fastest computers could record about 10^9 bits per second. (A bit is the smallest unit of memory storage for computers.) Scientists believed that the speed limit at the time was about 10^{12} bits per second. About how many times more bits per second was the speed limit than the fastest computers?

2. **MULTI-STEP PROBLEM** An office supply store sells cubical containers that can be used to store paper clips, rubber bands, or other supplies.

   a. One of the containers has a side length of $4\frac{1}{2}$ inches. Find the container's volume by writing the side length as an improper fraction and substituting the length into the formula for the volume of a cube.

   b. Identify the property of exponents you used to find the volume in part (a).

3. **SHORT RESPONSE** Clouds contain millions of tiny spherical water droplets. The radius of one droplet is shown.



   a. Find the order of magnitude of the volume of the droplet.

   b. Droplets combine to form raindrops. The radius of a raindrop is about 10^2 times greater than the droplet's radius. Find the order of magnitude of the volume of the raindrop.

   c. *Explain* how you can find the number of droplets that combine to form the raindrop. Then find the number of droplets and identify any properties of exponents you used.

4. **GRIDDED ANSWER** The least intense sound that is audible to the human ear has an intensity of about 10^{-12} watt per square meter. The intensity of sound from a jet engine at a distance of 30 meters is about 10^{15} times greater than the least intense sound. Find the intensity of sound from the jet engine.

5. **EXTENDED RESPONSE** For an experiment, a scientist dropped a spoonful, or about 10^{-1} cubic inch, of biodegradable olive oil into a pond to see how the oil would spread out over the surface of the pond. The scientist found that the oil spread until it covered an area of about 10^5 square inches.

   a. About how thick was the layer of oil that spread out across the pond? Check your answer using unit analysis.

   b. The pond has a surface area of 10^7 square inches. If the oil spreads to the same thickness as in part (a), how many cubic inches of olive oil would be needed to cover the entire surface of the pond?

   c. *Explain* how you could find the amount of oil needed to cover a pond with a surface area of 10^x square inches.

6. **OPEN-ENDED** The table shows units of measurement of time and the durations of the units in seconds.

Name of unit	Duration (seconds)
Gigasecond	10^9
Megasecond	10^6
Millisecond	10^{-3}
Nanosecond	10^{-9}

   a. Use the table to write a conversion problem that can be solved by applying a property of exponents involving products.

   b. Use the table to write a conversion problem that can be solved by applying a property of exponents involving quotients.

8.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS19 for Exs. 3, 17, and 53

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 48, 49, 54, and 59

◆ = **MULTIPLE REPRESENTATIONS** Ex. 58

SKILL PRACTICE

1. **VOCABULARY** Is 0.5×10^6 written in scientific notation? *Explain* why or why not.

2. ★ **WRITING** Is 7.89×10^6 between 0 and 1 or greater than 1? *Explain* how you know.

EXAMPLE 1 on p. 512 for Exs. 3–15

WRITING IN SCIENTIFIC NOTATION **Write the number in scientific notation.**

3. 8.5	4. 0.72	5. 82.4
6. 0.005	7. 72,000,000	8. 0.00406
9. 1,065,250	10. 0.000045	11. 1,060,000,000
12. 0.00000526	13. 900,000,000,000,000	14. 0.00000007008

15. ★ **MULTIPLE CHOICE** Which number represents 54,004,000,000 written in scientific notation?

(A) 54004×10^6 (B) 54.004×10^9

(C) 5.4004×10^{10} (D) 0.54004×10^{11}

EXAMPLE 2 on p. 512 for Exs. 16–28

WRITING IN STANDARD FORM **Write the number in standard form.**

16. 2.6×10^3	17. 7.5×10^7	18. 1.11×10^2
19. 3.03×10^4	20. 4.709×10^6	21. 1.544×10^{10}
22. 6.1×10^{-3}	23. 4.4×10^{-10}	24. 2.23×10^{-6}
25. 8.52×10^{-8}	26. 6.4111×10^{-10}	27. 1.2034×10^{-6}

28. **ERROR ANALYSIS** *Describe* and correct the error in writing 1.24×10^{-3} in standard form.

$1.24 \times 10^{-3} = 1240$ ✗

EXAMPLE 3 on p. 513 for Exs. 29–32

ORDERING NUMBERS **Order the numbers from least to greatest.**

29. 45,000; 6.7×10^3; 12,439; 2×10^4

30. 65,000,000; 6.2×10^6; 3.557×10^7; 55,004,000; 6.07×10^6

31. 0.0005; 9.8×10^{-6}; 5×10^{-3}; 0.00008; 0.04065; 8.2×10^{-3}

32. 0.0000395; 0.00010068; 2.4×10^{-5}; 5.08×10^{-6}; 0.000005

COMPARING NUMBERS **Copy and complete the statement using <, >, or =.**

33. 5.6×10^3 _?_ 56,000	34. 404,000.1 _?_ 4.04001×10^5
35. 9.86×10^{-3} _?_ 0.00986	36. 0.003309 _?_ 3.309×10^{-3}
37. 2.203×10^{-4} _?_ 0.0000203	38. 604,589,000 _?_ 6.04589×10^7

EXAMPLE 4
on p. 513
for Exs. 39–48

EVALUATING EXPRESSIONS Evaluate the expression. Write your answer in scientific notation.

39. $(4.4 \times 10^3)(1.5 \times 10^{-7})$

40. $(7.3 \times 10^{-5})(5.8 \times 10^2)$

41. $(8.1 \times 10^{-4})(9 \times 10^{-6})$

42. $\frac{6 \times 10^{-3}}{8 \times 10^{-6}}$

43. $\frac{5.4 \times 10^{-5}}{1.8 \times 10^{-2}}$

44. $\frac{4.1 \times 10^4}{8.2 \times 10^8}$

45. $(5 \times 10^{-8})^3$

46. $(7 \times 10^{-5})^4$

47. $(1.4 \times 10^3)^2$

48. ★ **MULTIPLE CHOICE** Which number is the value of $\frac{1.235 \times 10^4}{9.5 \times 10^7}$?

(A) 0.13×10^{-4} (B) 1.3×10^{-4} (C) 1.3×10^{-3} (D) 0.13×10^3

49. ★ **OPEN-ENDED** Write two numbers in scientific notation whose product is 2.8×10^4. Write two numbers in scientific notation whose quotient is 2.8×10^4.

50. **CHALLENGE** Add the numbers 3.6×10^5 and 6.7×10^4 *without* writing the numbers in standard form. Write your answer in scientific notation. *Describe* the steps you take.

PROBLEM SOLVING

EXAMPLE 3
on p. 513
for Exs. 51–52

51. **INSECT LENGTHS** The lengths of several insects are shown in the table.

a. List the lengths of the insects in order from least to greatest.

b. Which insects are longer than the fringed ant beetle?

Insect	Length (millimeters)
Fringed ant beetle	2.5×10^{-1}
Walking stick	555
Parasitic wasp	1.4×10^{-4}
Elephant beetle	1.67×10^2

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52. **ASTRONOMY** The spacecrafts *Voyager 1* and *Voyager 2* were launched in 1977 to gather data about our solar system. As of March 12, 2004, *Voyager 1* had traveled a total distance of about 9,643,000,000 miles, and *Voyager 2* had traveled a total distance of about 9.065×10^9 miles. Which spacecraft had traveled the greater distance at that time?

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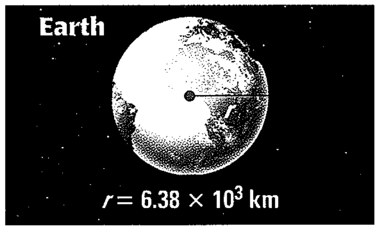
EXAMPLE 4
on p. 513
for Ex. 53

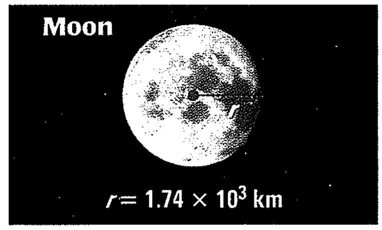
(53.) **AGRICULTURE** In 2002, about 9.7×10^8 pounds of cotton were produced in California. The cotton was planted on 6.9×10^5 acres of land. What was the average number of pounds of cotton produced per acre? Round your answer to the nearest whole number.

EXAMPLE 5
on p. 514
for Exs. 54–55

54. ★ **SHORT RESPONSE** The average flow rate of the Amazon River is about 7.6×10^6 cubic feet per second. The average flow rate of the Mississippi River is about 5.53×10^5 cubic feet per second. Find the ratio of the flow rate of the Amazon to the flow rate of the Mississippi. Round to the nearest whole number. What does the ratio tell you?

55. **ASTRONOMY** The radius of Earth and the radius of the moon are shown.





a. Find the ratio of the radius of Earth to the radius of the moon. Round to the nearest hundredth. What does the ratio tell you?

b. Assume Earth and the moon are spheres. Find the ratio of the volume of Earth to the volume of the moon. Round to the nearest hundredth. What does the ratio tell you?

c. What is the relationship between the ratios of the radii and the ratios of the volumes?

56. **MULTI-STEP PROBLEM** In 1954, 50 swarms of locusts were observed in Kenya. The largest swarm covered an area of 200 square kilometers. The average number of locusts in a swarm is about 5×10^7 locusts per square kilometer.

a. About how many locusts were in Kenya's largest swarm? Write your answer in scientific notation.

b. The average mass of a desert locust is 2 grams. What was the total mass (in kilograms) of Kenya's largest swarm? Write your answer in scientific notation.

57. **DIGITAL PHOTOGRAPHY** When a picture is taken with a digital camera, the resulting image is made up of square pixels (the smallest unit that can be displayed on a monitor). For one image, the side length of a pixel is 4×10^{-3} inch. A print of the image measures 1×10^3 pixels by 1.5×10^3 pixels. What are the dimensions of the print in inches?

58. **MULTIPLE REPRESENTATIONS** The speed of light is 1.863×10^5 miles per second.

a. Writing an Expression Assume 1 year is 365 days. Write an expression to convert the speed of light from miles per second to miles per year.

b. Making a Table Make a table that shows the distance light travels in 1, 10, 100, 1000, 10,000, and 100,000 years. Our galaxy has a diameter of about 5.875×10^{17} miles. Based on the table, about how long would it take for light to travel across our galaxy?

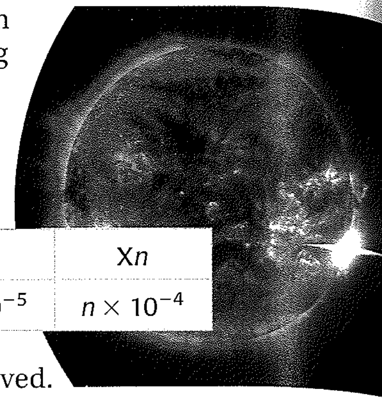
59. ★ **EXTENDED RESPONSE** When a person is at rest, approximately 7×10^{-2} liter of blood flows through the heart with each heartbeat. The human heart beats about 70 times per minute.

a. Calculate About how many liters of blood flow through the heart each minute when a person is at rest?

b. Estimate There are approximately 5.265×10^5 minutes in a year. Use your answer from part (a) to estimate the number of liters of blood that flow through the human heart in 1 year, in 10 years, and in 80 years. Write your answers in scientific notation.

c. Explain Are your answers to part (b) underestimates or overestimates? *Explain.*

60. CHALLENGE A solar flare is a sudden eruption of energy in the sun's atmosphere. Solar flares are classified according to their peak X-ray intensity (in watts per meter squared) and are denoted with a capital letter and a number, as shown in the table. For example, a C4 flare has a peak intensity of 4×10^{-6} watt per square meter.



Class	B*n*	C*n*	M*n*	X*n*
Peak intensity (w/m^2)	$n \times 10^{-7}$	$n \times 10^{-6}$	$n \times 10^{-5}$	$n \times 10^{-4}$

a. In November 2003, a massive X45 solar flare was observed. In April 2004, a C9 flare was observed. How many times greater was the intensity of the X45 flare than that of the C9 flare?

b. A solar flare may be accompanied by a coronal mass ejection (CME), a bubble of mass ejected from the sun. A CME related to the X45 flare was estimated to be traveling at 8.2 million kilometers per hour. At that rate, how long would it take the CME to travel from the sun to Earth, a distance of about 1.5×10^{11} meters?

MIXED REVIEW

PREVIEW
Prepare for Lesson 8.5 in Exs. 61–68.

Write the percent as a decimal. *(p. 916)*

61. 33% **62.** 62.7% **63.** 0.9% **64.** 0.04%

65. 3.95% **66.** $\frac{1}{4}\%$ **67.** $\frac{5}{2}\%$ **68.** 133%

Graph the equation.

69. $x = -5$ *(p. 215)* **70.** $y = 4$ *(p. 215)* **71.** $3x - 7y = 42$ *(p. 225)*

72. $y - 2x = 12$ *(p. 225)* **73.** $y = -2x + 6$ *(p. 244)* **74.** $y = 1.5x - 9$ *(p. 244)*

QUIZ *for Lessons 8.3–8.4*

Simplify the expression. Write your answer using only positive exponents. *(p. 503)*

1. $(-4x)^4 \cdot (-4)^{-6}$ **2.** $(-3x^7y^{-2})^{-3}$ **3.** $\frac{1}{(5z)^{-3}}$ **4.** $\frac{(6x)^{-2}y^5}{-x^3y^{-7}}$

Write the number in standard form. *(p. 512)*

5. 6.02×10^6 **6.** 5.41×10^{11} **7.** 8.007×10^{-5} **8.** 9.253×10^{-7}

9. DINOSAURS The estimated masses of several dinosaurs are shown in the table. *(p. 512)*

a. List the masses of the dinosaurs in order from least to greatest.

b. Which dinosaurs are more massive than Brachiosaurus?

Dinosaur	Mass (kilograms)
Brachiosaurus	77,100
Diplodocus	1.06×10^4
Apatosaurus	29,900
Ultrasaurus	1.36×10^5

Graphing Calculator ACTIVITY *Use after Lesson 8.4*

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Keystrokes

8.4 Use Scientific Notation

QUESTION **How can you use a graphing calculator to solve problems that involve numbers in scientific notation?**

EXAMPLE Use numbers in scientific notation

Gold is one of many trace elements dissolved in seawater. There is about 1.1×10^{-8} gram of gold per kilogram of seawater. The mass of the oceans is about 1.4×10^{21} kilograms. About how much gold is present in the oceans?

STEP 1 ***Write a verbal model***

Amount of gold present in oceans (grams)	=	Amount of gold in 1 kilogram of seawater (gram/kilogram)	•	Amount of seawater in oceans (kilograms)

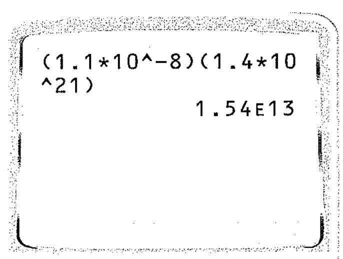
STEP 2 ***Find product*** The product is $(1.1 \times 10^{-8}) \cdot (1.4 \times 10^{21})$.

[( ] 1.1 [×] 10 [^] [(–)] 8 [ )] [( ] 1.4 [×] 10 [^] 21 [ )] [ENTER]

STEP 3 ***Read result***

The calculator indicates that a number is in scientific notation by using "E." You can read the calculator's result 1.54E13 as 1.54×10^{13}.

There are about 1.54×10^{13} grams of gold present in the oceans.



PRACTICE

Evaluate the expression. Write the result in scientific notation.

1. $(1.5 \times 10^{4})(1.8 \times 10^{9})$
2. $(2.6 \times 10^{-14})(1.4 \times 10^{20})$
3. $(7.0 \times 10^{25}) \div (2.8 \times 10^{6})$
4. $(4.5 \times 10^{15}) \div (9.0 \times 10^{-2})$
5. **GASOLINE** A scientist estimates that it takes about 4.45×10^{7} grams of carbon from ancient plant matter to produce 1 gallon of gasoline. In 2002 motor vehicles in the U.S. used about 1.37×10^{11} gallons of gasoline.
   a. If all of the gasoline used in 2002 by motor vehicles in the U.S. came from carbon from ancient plant matter, how many grams of carbon were used to produce the gasoline?
   b. There are about 5.0×10^{22} atoms of carbon in 1 gram of carbon. How many atoms of carbon were used?

COMPOUND INTEREST **Compound interest** is interest earned on both an initial investment and on previously earned interest. Compounding of interest can be modeled by exponential growth where a is the initial investment, r is the annual interest rate, and t is the number of years the money is invested.

★ EXAMPLE 5 Standardized Test Practice

You put $250 in a savings account that earns 4% annual interest compounded yearly. You do not make any deposits or withdrawals. How much will your investment be worth in 5 years?

Ⓐ $300 Ⓑ $304.16 Ⓒ $1344.56 Ⓓ $781,250

ESTIMATE

You can use the simple interest formula, $I = prt$, to estimate the amount of interest earned: $(250)(0.04)(5) = 50$. Compounding interest will result in slightly more than $50.

Solution

$y = a(1 + r)^t$	**Write exponential growth model.**
$= 250(1 + 0.04)^5$	**Substitute 250 for *a*, 0.04 for *r*, and 5 for *t*.**
$= 250(1.04)^5$	**Simplify.**
≈ 304.16	**Use a calculator.**

You will have $304.16 in 5 years.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 4 and 5

5. **WHAT IF?** In Example 4, suppose the owner of the car sold it in 1994. Find the value of the car to the nearest dollar.

6. **WHAT IF?** In Example 5, suppose the annual interest rate is 3.5%. How much will your investment be worth in 5 years?

8.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS19 for Exs. 13 and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 3, 8, 34, 35, 42, 43, 46, and 50

◆ = **MULTIPLE REPRESENTATIONS** Ex. 41

SKILL PRACTICE

1. **VOCABULARY** In the exponential growth model $y = a(1 + r)^t$, the quantity $1 + r$ is called the _?_.

2. **VOCABULARY** For what values of b does the exponential function $y = ab^x$ (where $a > 0$) represent exponential growth?

3. ★ **WRITING** How does the graph of $y = 2 \cdot 5^x$ compare with the graph of $y = 5^x$? *Explain.*

EXAMPLE 1
on p. 520
for Exs. 4–8

WRITING FUNCTIONS Write a rule for the function.

4.

x	−2	−1	0	1	2
y	1	2	4	8	16

5.

x	−2	−1	0	1	2
y	5	25	125	625	3125

6.

x	−2	−1	0	1	2
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2

7.

x	−2	−1	0	1	2
y	$\frac{1}{81}$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1

8. ★ **WRITING** Given a table of values, describe how can you tell if the table represents a linear function or an exponential function.

EXAMPLE 2
on p. 521
for Exs. 9–21

GRAPHING FUNCTIONS Graph the function and identify its domain and range.

9. $y = 4^x$ **10.** $y = 7^x$ **11.** $y = 8^x$ **12.** $y = 9^x$

13. $y = (1.5)^x$ **14.** $y = (2.5)^x$ **15.** $y = (1.2)^x$ **16.** $y = (4.3)^x$

17. $y = \left(\frac{4}{3}\right)^x$ **18.** $y = \left(\frac{7}{2}\right)^x$ **19.** $y = \left(\frac{5}{3}\right)^x$ **20.** $y = \left(\frac{5}{4}\right)^x$

21. **ERROR ANALYSIS** The price P (in dollars) of a pound of flour was $.27 in 1999. The price has increased by about 2% each year. Let t be the number of years since 1999. *Describe* and correct the error in finding the price of a pound of flour in 2002.

$P = a(1 + r)^t$
$= 0.27(1 + 2)^3 = 0.27(3)^3 = 7.29$

In 2002 the price of a pound of flour was $7.29.

EXAMPLE 3
on p. 521
for Exs. 22–34

COMPARING GRAPHS OF FUNCTIONS Graph the function. Compare the graph with the graph of $y = 3^x$.

22. $y = 2 \cdot 3^x$ **23.** $y = 4 \cdot 3^x$ **24.** $y = \frac{1}{4} \cdot 3^x$ **25.** $y = \frac{2}{3} \cdot 3^x$

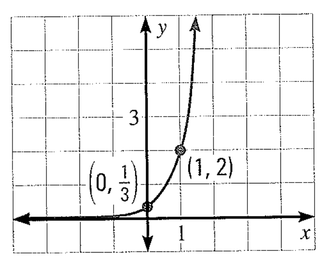
26. $y = 0.5 \cdot 3^x$ **27.** $y = 2.5 \cdot 3^x$ **28.** $y = -2 \cdot 3^x$ **29.** $y = -4 \cdot 3^x$

30. $y = -\frac{1}{4} \cdot 3^x$ **31.** $y = -\frac{2}{3} \cdot 3^x$ **32.** $y = -0.5 \cdot 3^x$ **33.** $y = -2.5 \cdot 3^x$

34. ★ **MULTIPLE CHOICE** The graph of which function is shown?

Ⓐ $f(x) = 6^x$ Ⓑ $f(x) = \left(\frac{1}{3}\right)^x$

Ⓒ $f(x) = \frac{1}{3} \cdot 6^x$ Ⓓ $f(x) = 6 \cdot \left(\frac{1}{3}\right)^x$



35. ★ **WRITING** If a population triples each year, what is the population's growth rate (as a percent)? *Explain.*

36. **CHALLENGE** Write a linear function and an exponential function whose graphs pass through the points (0, 2) and (1, 6).

37. **CHALLENGE** Compare the graph of the function $f(x) = 2^{x+2}$ with the graph of the function $g(x) = 4 \cdot 2^x$. Use properties of exponents to explain your observations.

○ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE** ◆ = **MULTIPLE REPRESENTATIONS**

PROBLEM SOLVING

GRAPHING CALCULATOR You may wish to use a graphing calculator to complete the following Problem Solving exercises.

EXAMPLES 4 and 5 on pp. 522–523 for Exs. 38–41

38. INVESTMENTS You deposit \$125 in a savings account that earns 5% annual interest compounded yearly. Find the balance in the account after the given amounts of time.

a. 1 year **b.** 2 years **c.** 5 years **d.** 20 years

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39. MULTI-STEP PROBLEM One computer industry expert reported that there were about 600 million computers in use worldwide in 2001 and that the number was increasing at an annual rate of about 10%.

a. Write a function that models the number of computers in use over time.

b. Use the function to predict the number of computers that will be in use worldwide in 2009.

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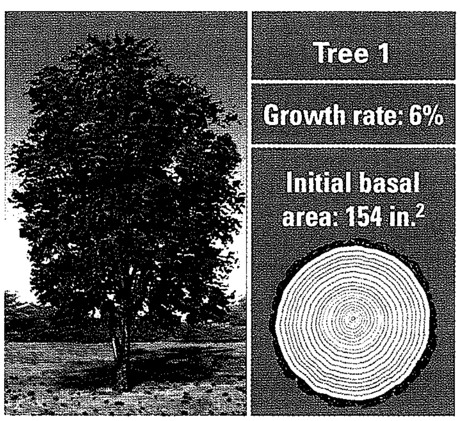
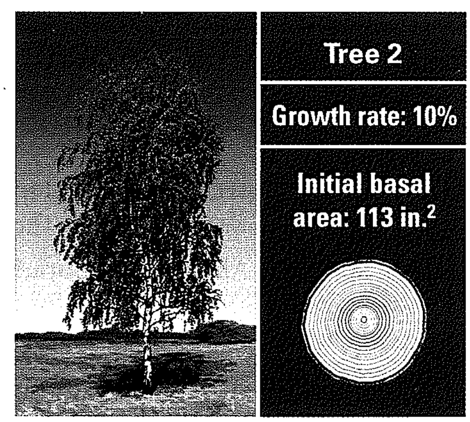
40. MULTI-STEP PROBLEM A research association reported that 3,173,000 gas grills were shipped by various manufacturers in the U.S. in 1985. Shipments increased by about 7% per year from 1985 to 2002.

a. Write a function that models the number of gas grills shipped over time.

b. About how many gas grills were shipped in 2002?

41. MULTIPLE REPRESENTATIONS A tree's cross-sectional area taken at a height of 4.5 feet from the ground is called its basal area and is measured in square inches. Tree growth can be measured by the growth of the tree's basal area. The initial basal area and annual growth rate for two particular trees are shown.





a. Writing a Model Write a function that models the basal area A of each tree over time.

b. Graphing a Function Use a graphing calculator to graph the functions from part (a) in the same coordinate plane. In about how many years will the trees have the same basal area?

42. ★ **SHORT RESPONSE** A company sells advertising blimps. The table shows the costs of advertising blimps of different lengths. Does the table represent an exponential function? *Explain.*

Length, ℓ (feet)	10	15	20	25
Cost, c (dollars)	400.00	700.00	1225.00	2143.75

43. ★ **MULTIPLE CHOICE** A weblog, or blog, refers to a website that contains a personal journal. According to one analyst, over one 18 month period, the number of blogs in existence doubled about every 6 months. The analyst estimated that there were about 600,000 blogs at the beginning of the period. How many blogs were there at the end of the period?

(A) 660,000 (B) 1,200,000 (C) 4,800,000 (D) 16,200,000

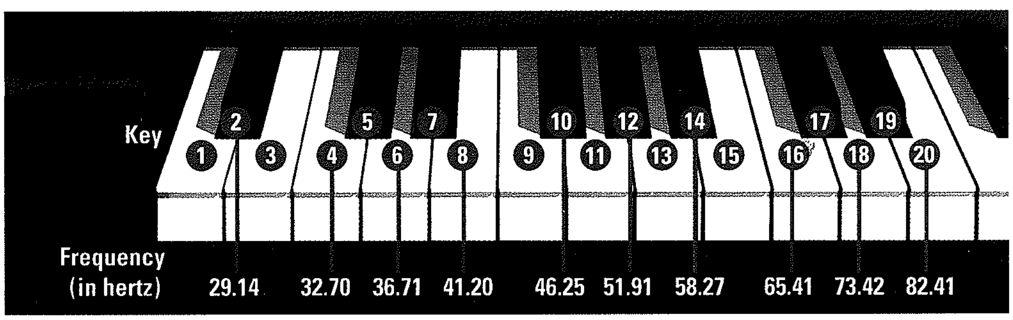
44. **TELECOMMUNICATIONS** For the period 1991–2001, the number y (in millions) of Internet users worldwide can be modeled by the function $y = 4.67(1.65)^x$ where x is the number of years since 1991.

a. Identify the initial amount, the growth factor, and the growth rate.

b. Graph the function. Identify its domain and range.

c. Use the graph to estimate the year in which the number of Internet users worldwide was about 21 million.

45. **GRAPHING CALCULATOR** The frequency (in hertz) of a note played on a piano is a function of the position of the key that creates the note. The position of some piano keys and the frequencies of the notes created by the keys are shown below. Use the exponential regression feature on a graphing calculator to find an exponential model for the frequency of piano notes. What is the frequency of the note created by the 30th key?



46. ★ **EXTENDED RESPONSE** In 1830, the population of the United States was 12,866,020. By 1890, the population was 62,947,714.

a. Model Assume the population growth from 1830 to 1890 was linear. Write a linear model for the U.S. population from 1830 to 1890. By about how much did the population grow per year from 1830 to 1890?

b. Model Assume the population growth from 1830 to 1890 was exponential. Write an exponential model for the U.S. population from 1830 to 1890. By approximately what percent did the population grow per year from 1830 to 1890?

c. Explain The U.S. population was 23,191,876 in 1850 and 38,558,371 in 1870. Which of the models in parts (a) and (b) is a better approximation of actual U.S. population for the time period 1850–1890? *Explain.*

★ = **STANDARDIZED TEST PRACTICE**

COMPOUND INTEREST In Exercises 47–49, use the example below to find the balance of the account compounded with the given frequency.

EXAMPLE **Use the general compound interest formula**

FINANCE You deposit $1000 in an account that pays 3% annual interest. Find the balance after 8 years if the interest is compounded monthly.

Solution

The general formula for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$. In this formula, P is the initial amount, called principal, in an account that pays interest at an annual rate r and that is compounded n times per year. The amount A (in dollars) is the amount in the account after t years.

Here, the interest is compounded monthly. So, $n = 12$.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ **Write compound interest formula.**

$= 1000\left(1 + \frac{0.03}{12}\right)^{12(8)}$ **Substitute 1000 for *P*, 0.03 for *r*, 12 for *n*, and 8 for *t*.**

$= 1000(1.0025)^{96}$ **Simplify.**

≈ 1270.868467 **Use a calculator.**

▶ The account balance after 8 years will be about $1270.87.

47. Yearly **48.** Quarterly **49.** Daily ($n = 365$)

50. ★ **WRITING** Which compounding frequency yields the highest balance in the account in the example above: monthly, yearly, quarterly, or daily? *Explain* why this is so.

51. **CHALLENGE** You invest $500 in an account that earns interest compounded monthly. Use a table or graph to find the least annual interest rate (to the nearest tenth of a percent) that the account would have to earn if you want to have a balance of $600 in 4 years.

MIXED REVIEW

PREVIEW Prepare for Lesson 8.6 in Exs. 52–59.

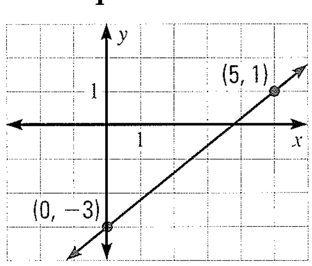
Evaluate the expression.

52. $\left(\frac{1}{3}\right)^2$ *(p. 495)* **53.** $\left(\frac{1}{8}\right)^2$ *(p. 495)* **54.** $\left(\frac{1}{4}\right)^3$ *(p. 495)* **55.** $\left(\frac{1}{2}\right)^6$ *(p. 495)*

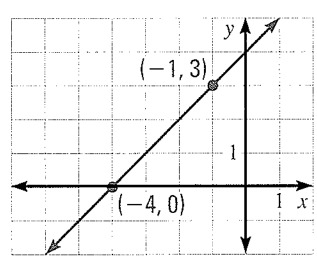
56. $\left(\frac{2}{3}\right)^{-2}$ *(p. 503)* **57.** $\left(\frac{7}{5}\right)^{-2}$ *(p. 503)* **58.** $\left(\frac{4}{3}\right)^{-3}$ *(p. 503)* **59.** $\left(\frac{3}{2}\right)^{-4}$ *(p. 503)*

Write an equation of the line shown. *(p. 283)*

60.



61.



PROBLEM SOLVING WORKSHOP
LESSON 8.5

Using ALTERNATIVE METHODS

Another Way to Solve Example 4, page 522

MULTIPLE REPRESENTATIONS In Example 4 on page 522, you saw how to solve a problem about the value of a collector car over time by using an exponential model. You can also solve the problem by using a spreadsheet.

PROBLEM

COLLECTOR CAR The owner of a 1953 Hudson Hornet convertible sold the car at an auction. The owner bought it in 1984 when its value was \$11,000. The value of the car increased at a rate of 6.9% per year.

a. Write a function that models the value of the car over time.

b. The auction took place in 2004. What was the approximate value of the car at the time of the auction? Round your answer to the nearest dollar.

METHOD

Using a Spreadsheet An alternative approach is to use a spreadsheet.

a. The model for the value of the car over time is $C = 11{,}000(1.069)^t$, as shown in Example 4 on page 522.

b. You can find the value of the car in 2004 by creating a spreadsheet.

STEP 1 **Create** a table showing the years since 1984 and the value of the car. Enter the car's value in 1984. To find the value in any year after 1984, multiply the car's value in the preceding year by the growth factor, as shown in cell B3 below.

FORMAT A SPREADSHEET
Format the spreadsheet so that calculations are rounded to 2 decimal places.

	A	B
1	Years since 1984, *t*	Value, *C* (dollars)
2	0	11000
3	1	=B2*1.069

STEP 2 **Find** the value of the car in 2004 by using the *fill down* feature until you get to the desired cell.

	A	B
1	Years since 1984, *t*	Value, *C* (dollars)
2	0	11000
3	1	11759
...	...	...
21	19	39081.31
22	20	41777.92

▶ From the spreadsheet, you can see the value of the car was about \$41,778 in 2004.

PROBLEM

WHAT IF? Suppose the owner decided to sell the car when it was worth about \$28,000. In what year did the owner sell the car?

METHOD

Using a Spreadsheet To solve the equation algebraically, you need to substitute 28,000 for C and solve for t, but you have not yet learned how to solve this type of equation. An alternative to the algebraic approach is using a spreadsheet.

STEP 1 **Use** the same spreadsheet as on the previous page.

STEP 2 **Find** when the value of the car is about \$28,000.

	A	B
1	Years since 1984, *t*	Value, *C* (dollars)
2	0	11000
...	...	...
15	13	26188.03
16	14	27995.01

The value of the car is about \$28,000 when $t = 14$.

▶ The owner sold the car in 1998.

PRACTICE

1. **TRANSPORTATION** In 1997 the average intercity bus fare for a particular state was \$20. For the period 1997–2000, the bus fare increased at a rate of about 12% each year.
   - **a.** Write a function that models the intercity bus fare for the period 1997–2000.
   - **b.** Find the intercity bus fare in 1998. Use two different methods to solve the problem.
   - **c.** In what year was the intercity bus fare \$28.10? *Explain* how you found your answer.

2. **ERROR ANALYSIS** *Describe* and correct the error in writing the function for part (a) of Exercise 1.

   Let b be the bus fare (in dollars) and t be the number of years since 1997.
   $b = 20(0.12)^t$

3. **TECHNOLOGY** A computer's Central Processing Unit (CPU) is made up of transistors. One manufacturer released a CPU in May 1997 that had 7.5 million transistors. The number of transistors in the CPUs sold by the company increased at a rate of 3.9% per month.
   - **a.** Write a function that models the number T (in millions) of transistors in the company's CPUs t months after May 1997.
   - **b.** Use a spreadsheet to find the number of transistors in a CPU released by the company in November 2000.

4. **HOUSING** The value of a home in 2002 was \$150,000. The value of the home increased at a rate of about 6.5% per year.
   - **a.** Write a function that models the value of the home over time.
   - **b.** Use a spreadsheet to find the year in which the value of the home was about \$200,000.

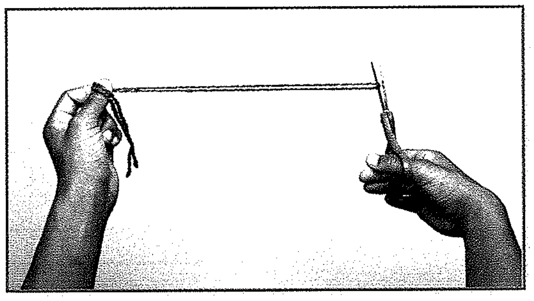
8.6 Exponential Models

MATERIALS • yarn • scissors

QUESTION **How can you model a situation using an exponential function?**

EXPLORE **Collect data so that you can write exponential models**

STEP 1 ***Fold and cut*** Take about 1 yard of yarn and consider it to be 1 unit long. Fold it in half and cut, as shown. You are left with two pieces of yarn, each half the length of the original piece of yarn.



STEP 2 ***Copy and complete*** Copy the table. Notice that the row for stage 1 has the data from Step 1. For each successive stage, fold *all* the pieces of yarn in half and cut. Then record the number of new pieces and the length of each new piece until the table is complete.

Stage	Number of pieces	Length of each new piece
1	2	$\frac{1}{2}$
2	?	?
3	?	?
4	?	?
5	?	?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Use the data in the first and second columns of the table.

a. Do the data represent an exponential function? *Explain* how you know.

b. Write a function that models the number of pieces of yarn at stage x.

c. Use the function to find the number of pieces of yarn at stage 10.

2. Use the data in the first and third columns of the table.

a. Do the data represent an exponential function? *Explain* how you know.

b. Write a function that models the length of each new piece of yarn at stage x.

c. Use the function to find the length of each new piece of yarn at stage 10.

8.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS19 for Exs. 7 and 49

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 36, 45, and 49

◆ = **MULTIPLE REPRESENTATIONS** Ex. 50

SKILL PRACTICE

1. **VOCABULARY** What is the decay factor in the exponential decay model $y = a(1 - r)^t$?

2. ★ **WRITING** *Explain* how you can tell if a graph represents *exponential growth* or *exponential decay.*

EXAMPLE 1 on p. 531 for Exs. 3–6

WRITING FUNCTIONS Tell whether the table represents an exponential function. If so, write a rule for the function.

3.

x	−1	0	1	2
y	2	8	32	128

4.

x	−1	0	1	2
y	50	10	2	0.4

5.

x	−1	0	1	2
y	6	2	$\frac{2}{3}$	$\frac{2}{9}$

6.

x	−1	0	1	2
y	−11	−7	−3	1

EXAMPLE 2 on p. 532 for Exs. 7–18

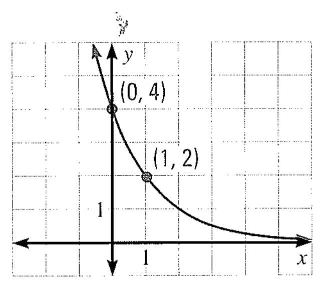
GRAPHING FUNCTIONS Graph the function and identify its domain and range.

7. $y = \left(\frac{1}{5}\right)^x$
8. $y = \left(\frac{1}{6}\right)^x$
9. $y = \left(\frac{2}{3}\right)^x$
10. $y = \left(\frac{3}{4}\right)^x$
11. $y = \left(\frac{4}{5}\right)^x$
12. $y = \left(\frac{3}{5}\right)^x$
13. $y = (0.3)^x$
14. $y = (0.5)^x$
15. $y = (0.1)^x$
16. $y = (0.9)^x$
17. $y = (0.7)^x$
18. $y = (0.25)^x$

EXAMPLE 3 on p. 532 for Exs. 19–31

19. ★ **MULTIPLE CHOICE** The graph of which function is shown?

(A) $y = (0.25)^x$
(B) $y = (0.5)^x$
(C) $y = 0.25 \cdot (0.5)^x$
(D) $y = 4 \cdot (0.5)^x$



COMPARING FUNCTIONS Graph the function. Compare the graph with the graph of $y = \left(\frac{1}{4}\right)^x$.

20. $y = 5 \cdot \left(\frac{1}{4}\right)^x$
21. $y = 3 \cdot \left(\frac{1}{4}\right)^x$
22. $y = \frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$
23. $y = \frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$
24. $y = 0.2 \cdot \left(\frac{1}{4}\right)^x$
25. $y = 1.5 \cdot \left(\frac{1}{4}\right)^x$
26. $y = -5 \cdot \left(\frac{1}{4}\right)^x$
27. $y = -3 \cdot \left(\frac{1}{4}\right)^x$
28. $y = -\frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$
29. $y = -\frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$
30. $y = -0.2 \cdot \left(\frac{1}{4}\right)^x$
31. $y = -1.5 \cdot \left(\frac{1}{4}\right)^x$

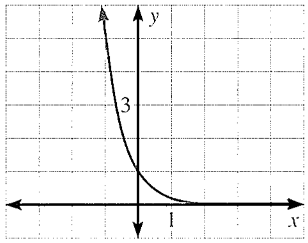
MATCHING Match the function with its graph.

32. $y = (0.2)^x$

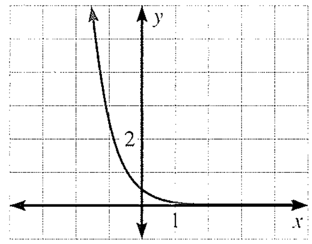
33. $y = 5(0.2)^x$

34. $y = \frac{1}{2}(0.2)^x$

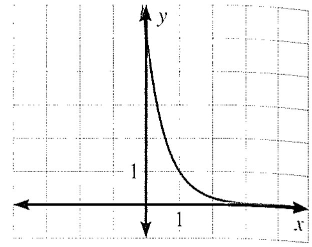
A.



B.



C.



35. **POPULATION** A population of 90,000 decreases by 2.5% per year. Identify the initial amount, the decay factor, and the decay rate. Then write a function that models the population over time.

36. ★ **MULTIPLE CHOICE** What is the decay rate of the function $y = 4(0.97)^t$?

Ⓐ 4 Ⓑ 0.97 Ⓒ 0.3 Ⓓ 0.03

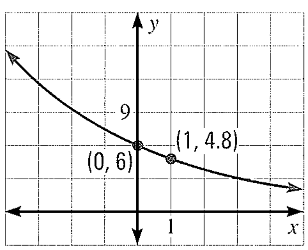
37. **ERROR ANALYSIS** In 2004 a person purchased a car for $25,000. The value of the car decreased by 14% annually. *Describe* and correct the error in writing a function that models the value of the car since 2004.

$y = a(1 - r)^t$
$= 25{,}000(0.14)^t$

EXAMPLE 4
on p. 533
for Exs. 38–40

RECOGNIZING EXPONENTIAL MODELS Tell whether the graph represents *exponential growth* or *exponential decay*. Then write a rule for the function.

38.



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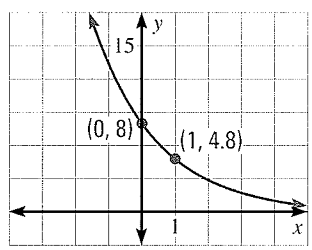

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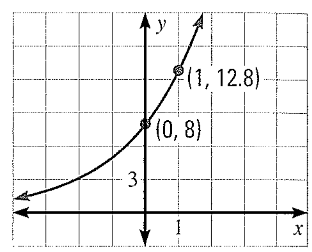

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41. **REASONING** Without graphing, explain how the graphs of the given functions are related to the graph of $f(x) = (0.5)^x$.

a. $m(x) = \frac{1}{3} \cdot (0.5)^x$ **b.** $n(x) = -4 \cdot (0.5)^x$ **c.** $p(x) = (0.5)^x + 1$

CHALLENGE Write an exponential function of the form $y = ab^x$ whose graph passes through the given points.

42. $(0, 1), \left(2, \frac{1}{4}\right)$

43. $(1, 20), (2, 4)$

44. $\left(1, \frac{3}{2}\right), \left(2, \frac{3}{4}\right)$

45. ★ **WRITING** The initial amount of a quantity is a units and the quantity is decaying at a rate of r (a percent per time period). Show that the amount of the quantity after one time period is $a(1 - r)$. *Explain* how you found your answer.

46. **CHALLENGE** Compare the graph of the function $f(x) = 4^{x-2}$ with the graph of the function $g(x) = \frac{1}{16} \cdot 4^x$. Use properties of exponents to explain your observation.

PROBLEM SOLVING

GRAPHING CALCULATOR You may wish to use a graphing calculator to complete the following Problem Solving exercises.

EXAMPLE 5
on p. 534
for Exs. 47–50

47. CELL PHONES You purchase a cell phone for \$125. The value of the cell phone decreases by about 20% annually. Write a function that models the value of the cell phone over time. Then find the value of the cell phone after 3 years.

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48. ANIMAL POPULATION Scientists studied the population of a species of bat in some caves in Missouri from 1983 to 2003. In 1983, there were 141,200 bats living in the caves. That number decreased by about 11% annually until 2003.

a. Identify the initial amount, the decay factor, and the decay rate.

b. Write a function that models the number of bats since 1983. Then find the number of bats in 2003.

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49. ★ **SHORT RESPONSE** In 2003 a family bought a boat for \$4000. The boat depreciates (loses value) at a rate of 7% annually. In 2006 a person offers to buy the boat for \$3000. Should the family sell the boat? *Explain.*

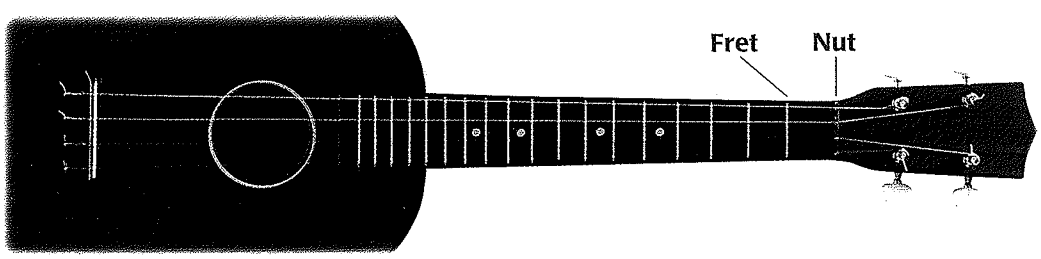
50. ◆ **MULTIPLE REPRESENTATIONS** There are a total of 128 teams at the start of a citywide 3-on-3 basketball tournament. Half of the teams are eliminated after each round.

a. Writing a Model Write a function for the number of teams left after x rounds.

b. Making a Table Make a table for the function using $x = 0, 1, 2, \dots, 7$.

c. Drawing a Graph Use the table in part (b) to graph the function. After which round are there 4 teams left in the tournament?

51. GUITARS The frets on a guitar are the small metal bars that divide the fingerboard. The distance d (in inches) between the nut and the first fret or any two consecutive frets can be modeled by the function $d = 1.516(0.9439)^f$ where f is the number of the fret farthest from the nut.



a. Identify the decay factor and the decay rate for the model.

b. What is the distance between the nut and the first fret?

c. The distance between the 12th and 13th frets is about half the distance between the nut and the first fret. Use this fact to find the distance between the 12th and 13th frets. Use the model to verify your answer.

52. **CHALLENGE** A college student finances a computer that costs \$1850. The financing plan states that as long as a minimum monthly payment of 2.25% of the remaining balance is made, the student does not have to pay interest for 24 months. The student makes only the minimum monthly payments until the last payment. What is the amount of the last payment if the student buys the computer without paying interest? Round your answer to the nearest cent.

53. **MULTI-STEP PROBLEM** Maximal oxygen consumption is the maximum volume of oxygen (in liters per minute) that the body uses during exercise. Maximal oxygen consumption varies from person to person and decreases with age by about 0.5% per year after age 25 for active adults.

a. **Model** A 25-year-old female athlete has a maximal oxygen consumption of 4 liters per minute. Another 25-year-old female athlete has a maximal oxygen consumption of 3.5 liters per minute. Write a function for each athlete that models the maximal consumption each year after age 25.

b. **Graph** Graph the models in the same coordinate plane.

c. **Estimate** About how old will the first athlete be when her maximal oxygen consumption is equal to what the second athlete's maximal oxygen consumption is at age 25?

MIXED REVIEW

PREVIEW
Prepare for Lesson 9.1 in Exs. 54–62.

Simplify the expression. *(p. 96)*

54. $-12x + (-3x)$

55. $8x - 3x$

56. $14 + x + 2x$

57. $7(2x + 1) - 5$

58. $13x + (x - 4)5$

59. $3x + 6(x + 9)$

60. $(5 - x) + x$

61. $(3x - 4)7 + 21$

62. [illegible]$(x - 1) - x^2$

Solve the equation.

63. $x + 14 = 8$ *(p. 134)*

64. $8x - 7 = 17$ *(p. 141)*

65. $4x + 2x - 6 = 18$ *(p. 148)*

66. $2x - 7(x + 5) = 20$ *(p. 148)*

QUIZ *for Lessons 8.5–8.6*

Graph the function.

1. $y = \left(\frac{5}{2}\right)^x$ *(p. 520)*

2. $y = 3 \cdot \left(\frac{1}{4}\right)^x$ *(p. 531)*

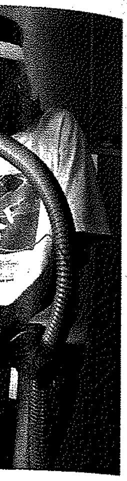
3. $y = \frac{1}{4} \cdot 3^x$ *(p. 520)*

4. $y = (0.1)^x$ *(p. 531)*

5. $y = 10 \cdot 5^x$ *(p. 520)*

6. $y = 7(0.4)^x$ *(p. 531)*

7. **COINS** You purchase a coin from a coin collector for \$25. Each year the value of the coin increases by 8%. Write a function that models the value of the coin over time. Then find the value of the coin after 10 years. Round to the nearest cent. *(p. 520)*



Extension

Use after Lesson 8.6

Relate Geometric Sequences to Exponential Functions

GOAL Identify, graph, and write geometric sequences.

Key Vocabulary
- **geometric sequence**
- **common ratio**

In a **geometric sequence**, the ratio of any term to the previous term is constant. This constant ratio is called the **common ratio** and is denoted by r.

A geometric sequence with first term a_1 and common ratio r has the form a_1, a_1r, a_1r^2, a_1r^3, ... For instance, if $a_1 = 5$ and $r = 2$, the sequence 5, $5 \cdot 2$, $5 \cdot 2^2$, $5 \cdot 2^3$, . . . , or 5, 10, 20, 40, . . . , is geometric.

EXAMPLE 1 Identify a geometric sequence

Tell whether the sequence is *arithmetic* or *geometric*. Then write the next term of the sequence.

a. 3, 6, 9, 12, 15, . . .

b. 128, 64, 32, 16, 8, . . .

Solution

a. The first term is $a_1 = 3$. Find the ratios of consecutive terms:

$$\frac{a_2}{a_1} = \frac{6}{3} = 2 \qquad \frac{a_3}{a_2} = \frac{9}{6} = 1\frac{1}{2} \qquad \frac{a_4}{a_3} = \frac{12}{9} = 1\frac{1}{3} \qquad \frac{a_5}{a_4} = \frac{15}{12} = 1\frac{1}{4}$$

Because the ratios are not constant, the sequence is not geometric. To see if the sequence is arithmetic, find the differences of consecutive terms.

$$a_2 - a_1 = 6 - 3 = 3 \qquad a_3 - a_2 = 9 - 6 = 3$$

$$a_4 - a_3 = 12 - 9 = 3 \qquad a_5 - a_4 = 15 - 12 = 3$$

The common difference is 3, so the sequence is arithmetic. The next term of the sequence is $a_6 = a_5 + 3 = 18$.

b. The first term is $a_1 = 128$. Find the ratios of consecutive terms:

$$\frac{a_2}{a_1} = \frac{64}{128} = \frac{1}{2} \qquad \frac{a_3}{a_2} = \frac{32}{64} = \frac{1}{2} \qquad \frac{a_4}{a_3} = \frac{16}{32} = \frac{1}{2} \qquad \frac{a_5}{a_4} = \frac{8}{16} = \frac{1}{2}$$

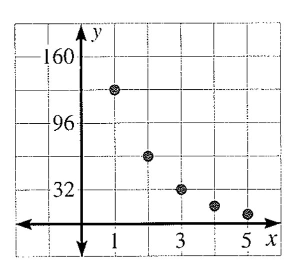
Because the ratios are constant, the sequence is geometric. The common ratio is $\frac{1}{2}$. The next term of the sequence is $a_6 = a_5 \cdot \frac{1}{2} = 4$.

REVIEW ARITHMETIC SEQUENCES
For help with identifying an arithmetic sequence and finding a common difference, see p. 309.

EXAMPLE 2 Graph a geometric sequence

To graph the sequence from part (b) of Example 1, let each term's position number in the sequence be the x-value. The term is the corresponding y-value. Then make and plot the points.

Position, x	1	2	3	4	5
Term, y	128	64	32	16	8



ANALYZE A GRAPH
Notice that the graph in Example 2 appears to be exponential.

FUNCTIONS The table shows that a rule for finding the nth term of a geometric sequence is $a_n = a_1r^{n-1}$. Notice that the rule is an exponential function.

	+1	+1	+1	+1		
Position, n	1	2	3	4	...	n
Term, a_n	a_1	a_1r	a_1r^2	a_1r^3	...	a_1r^{n-1}
	$\cdot r$	$\cdot r$	$\cdot r$	$\cdot r$		

For the nth term, you multiply a_1 by r (n − 1) times.

KEY CONCEPT — *For Your Notebook*

General Rule for a Geometric Sequence

The nth term of a geometric sequence with first term a_1 and common ratio r is given by: $a_n = a_1r^{n-1}$.

EXAMPLE 3 Write a rule for a geometric sequence

Write a rule for the nth term of the geometric sequence in Example 1. Then find a_{10}.

Solution

To write a rule for the nth term of the sequence, substitute the values for a_1 and r in the general rule $a_n = a_1r^{n-1}$. Because $a_1 = 128$ and $r = \frac{1}{2}$, $a_n = 128 \cdot \left(\frac{1}{2}\right)^{n-1}$. The 10th term of the sequence is $a_{10} = 128 \cdot \left(\frac{1}{2}\right)^{10-1} = \frac{1}{4}$.

PRACTICE

EXAMPLES 1, 2, and 3 on pp. 539–540 for Exs. 1–10

Tell whether the sequence is *arithmetic* or *geometric*. Then graph the sequence.

1. 3, 12, 48, 192, . . .
2. 7, 16, 25, 34, . . .
3. 34, 28, 22, 16, . . .
4. 1024, 128, 16, 2, . . .
5. 9, −18, 36, −72, . . .
6. 29, 43, 57, 71, . . .

Write a rule for the nth term of the geometric sequence. Then find a_7.

7. 1, −5, 25, −125, . . .
8. 13, 26, 52, 104, . . .
9. 432, 72, 12, 2, . . .

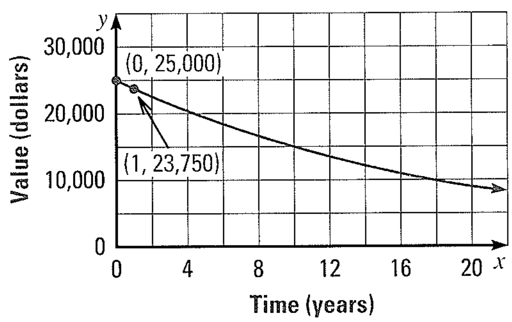
10. **E-MAIL** A chain e-mail instructs the recipient to forward the e-mail to four more people. The table shows the number of rounds of sending the e-mail and the number of new e-mails generated. Write a rule for the nth term of the sequence. Then graph the first six terms of the sequence.

Number of rounds sending e-mail, n	1	2	3	4
Number of new e-mails generated, a_n	1	4	16	64

Lessons 8.4–8.6

1. **MULTI-STEP PROBLEM** The radius of the sun is about 96,600,000 kilometers. The radius of Earth is about 6370 kilometers.
   - **a.** Write each radius in scientific notation.
   - **b.** The surface area S of a sphere with radius r is given by $S = 4\pi r^2$. Assume the sun and Earth are perfect spheres. Find their surface areas. Write your answers in scientific notation.
   - **c.** What is the ratio of the surface area of the sun to the surface area of Earth? What does the ratio tell you?

2. **SHORT RESPONSE** The graph shows the value of a truck over time.

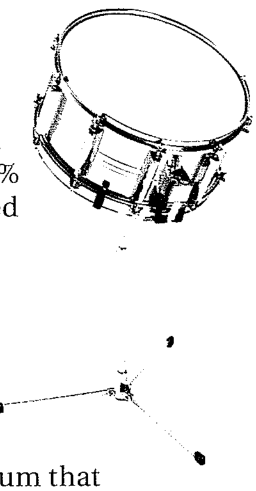


   - **a.** Write an equation for the function whose graph is shown.
   - **b.** At what rate is the truck losing value? *Explain.*

3. **GRIDDED ANSWER** A new laptop computer costs \$2000. The value of the computer decreases over time. The value V (in dollars) of the computer after t years is given by the function $V = 2000(0.82)^t$. What is the decay rate, written as a decimal, of the value of the computer?

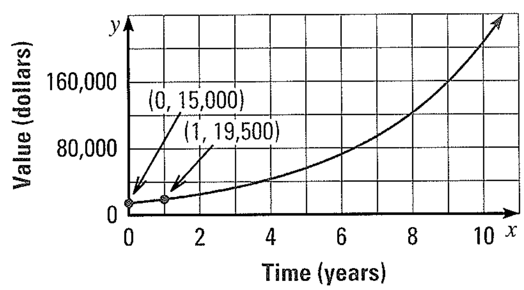
4. **OPEN-ENDED** The value of a house in Iowa increased, on average, at a rate of about 4% per quarter from the first quarter in 2001 to the last quarter in 2004. Write a function that models the value of the house over time. Choose an initial value of the house and a quarter such that the value of the house is about \$275,000.

5. **EXTENDED RESPONSE** A musician is saving money to buy a new snare drum. The musician puts \$100 in a savings account that pays 3% annual interest compounded yearly.



   - **a.** Write a function that models the amount of money in the account over time.
   - **b.** Graph the function.
   - **c.** The musician wants a drum that costs \$149.95. Will there be enough in the account after 3 years? *Explain.*

6. **MULTI-STEP PROBLEM** The graph shows the value of a business over time.



   - **a.** Does the graph represent *exponential growth* or *exponential decay*?
   - **b.** Write a function that models the value of the business over time.
   - **c.** How much is the business worth after 4 years?

7. **MULTI-STEP PROBLEM** The half-life of a medication is the time it takes for the medication to reduce to half of its original amount in a patient's bloodstream. A certain antibiotic has a half-life of about 8 hours.
   - **a.** A patient is administered 500 milligrams of the medication. Write a function that models the amount of the medication in the patient's bloodstream over time.
   - **b.** How much of the 500 milligram dose will be in the patient's bloodstream after 24 hours?

8 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

Applying Properties of Exponents to Simplify Expressions

You can use the properties of exponents to simplify expressions. For the properties listed below, a and b are real numbers, and m and n are integers.

Expression	Property
$a^m \cdot a^n = a^{m+n}$	**Product of powers property**
$(a^m)^n = a^{mn}$	**Power of a power property**
$(ab)^m = a^m b^m$	**Power of a product property**
$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	**Quotient of powers property**
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	**Power of a quotient property**

Big Idea 2

Working with Numbers in Scientific Notation

You can write numbers in scientific notation.

Number	Standard form	Scientific notation
Four billion	4,000,000,000	4×10^9
Thirty-two thousandths	0.032	3.2×10^{-2}

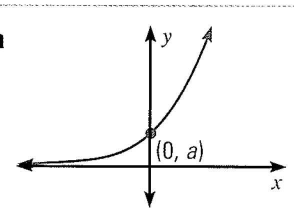
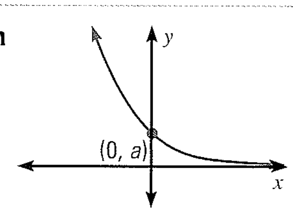
You can also compute with numbers in scientific notation. For example:

$$(4 \times 10^9) \times (3.2 \times 10^{-2}) = 12.8 \times 10^7 = 1.28 \times 10^8, \text{ or } 128{,}000{,}000$$

Big Idea 3

Writing and Graphing Exponential Functions

You can write and graph exponential growth and decay functions. You can also model real-world situations involving exponential growth and exponential decay.

Exponential growth	Exponential decay
Function $y = ab^x, a > 0$ and $b > 1$	**Function** $y = ab^x, a > 0$ and $0 < b < 1$
Graph 	**Graph** 
Model $y = a(1 + r)^t$	**Model** $y = a(1 - r)^t$

CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

ebook

REVIEW KEY VOCABULARY

- order of magnitude, *p. 491*
- zero exponent, *p. 503*
- negative exponent, *p. 503*
- scientific notation, *p. 512*
- exponential function, *p. 520*
- exponential growth, *p. 522*
- growth factor, growth rate, *p. 522*
- compound interest, *p. 523*
- exponential decay, *p. 533*
- decay factor, decay rate, *p. 534*

VOCABULARY EXERCISES

1. Copy and complete: The function $y = 1200(0.3)^t$ is an exponential _?_ function, and the base 0.3 is called the _?_.
2. **WRITING** *Explain* how you can tell whether a table represents a linear function or an exponential function.

Tell whether the function represents exponential growth or exponential decay. *Explain.*

3. $y = 3(0.85)^x$
4. $y = \frac{1}{2}(1.01)^x$
5. $y = 2(2.1)^x$

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 8.

8.1 Apply Exponent Properties Involving Products

pp. 489–494

EXAMPLE

Simplify $(3y^3)^4 \cdot y^5$.

$(3y^3)^4 \cdot y^5 = 3^4 \cdot (y^3)^4 \cdot y^5$ **Power of a product property**

$= 81 \cdot y^{12} \cdot y^5$ **Power of a power property**

$= 81y^{17}$ **Product of powers property**

EXERCISES

EXAMPLES 1, 2, 3, 4, and 5 on pp. 489–491 for Exs. 6–15

Simplify the expression.

6. $4^4 \cdot 4^3$
7. $(-3)^7(-3)$
8. $z^3 \cdot z^5 \cdot z^5$
9. $(y^4)^5$
10. $[(-7)^4]^4$
11. $[(b + 2)^8]^3$
12. $(6^4 \cdot 31)^5$
13. $-(8xy)^2$
14. $(2x^2)^4 \cdot x^5$
15. **EARTH SCIENCE** The order of magnitude of the mass of Earth's atmosphere is 10^{18} kilograms. The order of magnitude of the mass of Earth's oceans is 10^3 times greater. What is the order of magnitude of the mass of Earth's oceans?

8 CHAPTER REVIEW

8.2 Apply Exponent Properties Involving Quotients

pp. 495–501

EXAMPLE

Simplify $\left(\frac{x^3}{y}\right)^4 \cdot \frac{2}{x^5}$.

$$\left(\frac{x^3}{y}\right)^4 \cdot \frac{2}{x^5} = \frac{(x^3)^4}{y^4} \cdot \frac{2}{x^5}$$ **Power of a quotient property**

$$= \frac{x^{12}}{y^4} \cdot \frac{2}{x^5}$$ **Power of a power property**

$$= \frac{2x^{12}}{y^4x^5}$$ **Multiply fractions.**

$$= \frac{2x^7}{y^4}$$ **Quotient of powers property**

EXERCISES

EXAMPLES 1, 2, and 3 on pp. 495–496 for Exs. 16–24

Simplify the expression.

16. $\frac{(-3)^7}{(-3)^3}$ **17.** $\frac{5^2 \cdot 5^4}{5^3}$ **18.** $\left(\frac{m}{n}\right)^3$ **19.** $\frac{17^{12}}{17^8}$

20. $\left(-\frac{1}{x}\right)^4$ **21.** $\left(\frac{7x^5}{y^2}\right)^2$ **22.** $\frac{1}{p^2} \cdot p^6$ **23.** $\frac{6}{7r^{10}} \cdot \left(\frac{r^5}{s}\right)^5$

24. PER CAPITA INCOME The order of magnitude of the population of Montana in 2003 was 10^6 people. The order of magnitude of the total personal income (in dollars) for Montana in 2003 was 10^{10}. What was the order of magnitude of the mean personal income in Montana in 2003?

8.3 Define and Use Zero and Negative Exponents

pp. 503–508

EXAMPLE

Evaluate $(2x^0y^{-5})^3$.

$$(2x^0y^{-5})^3 = 2^3 \cdot x^0 \cdot y^{-15}$$ **Power of a power property**

$$= 8 \cdot 1 \cdot y^{-15}$$ **Definition of zero exponent**

$$= \frac{8}{y^{15}}$$ **Definition of negative exponents**

EXERCISES

EXAMPLES 1, 2, and 4 on pp. 503–505 for Exs. 25–29

Evaluate the expression.

25. 14^0 **26.** 3^{-4} **27.** $\left(\frac{2}{3}\right)^{-3}$ **28.** $7^{-5} \cdot 7^5$

29. UNITS OF MEASURE Use the fact that 1 femtogram = 10^{-18} kilogram and 1 nanogram = 10^{-12} kilogram to complete the following statement: 1 nanogram = ? femtogram(s).

8.4 Use Scientific Notation

pp. 512–518

EXAMPLE

Write the number in scientific notation.

a. $2097 = 2.097 \times 10^3$ **Move decimal point left 3 places. Exponent is 3.**

b. $0.00032 = 3.2 \times 10^{-4}$ **Move decimal point right 4 places. Exponent is −4.**

Write the number in standard form.

a. $4.3201 \times 10^2 = 432.01$ **Exponent is 2. Move decimal point right 2 places.**

b. $2.068 \times 10^{-3} = 0.002068$ **Exponent is −3. Move decimal point left 3 places.**

EXERCISES

EXAMPLES 1, 2, 4, and 5 on pp. 512–514 for Exs. 30–34

30. Write 78,120 in scientific notation.

31. Write 7.5×10^{-5} in standard form.

Evaluate the expression. Write your answer in scientific notation.

32. $(6.3 \times 10^3)(1.9 \times 10^{-5})$

33. $\dfrac{6.5 \times 10^9}{1.6 \times 10^{-4}}$

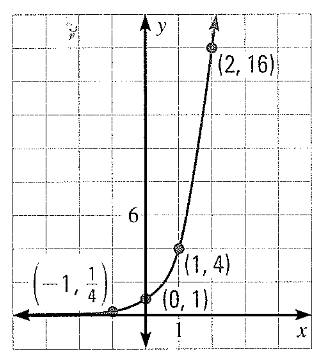
34. MASS The mass m_1 of a gate of the Thames Barrier in London is about 1.5×10^6 kilograms. The mass m_2 of the Great Pyramid of Giza is about 6×10^9 kilograms. Find the ratio of m_1 to m_2. What does the ratio tell you?

8.5 Write and Graph Exponential Growth Functions

pp. 520–527

EXAMPLE

Graph the function $y = 4^x$ and identify its domain and range.



STEP 1 **Make** a table. The domain is all real numbers.

x	−1	0	1	2
y	$\frac{1}{4}$	1	4	16

STEP 2 **Plot** the points.

STEP 3 **Draw** a smooth curve through the points.

STEP 4 **Identify** the range. As you can see from the graph, the range is all positive real numbers.

EXERCISES

EXAMPLES 2 and 3 on p. 521 for Exs. 35–39

Graph the function and identify its domain and range.

35. $y = 6^x$

36. $y = (1.1)^x$

37. $y = (3.5)^x$

38. $y = \left(\frac{5}{2}\right)^x$

39. Graph the function $y = -5 \cdot 2^x$. Compare the graph with the graph of $y = 2^x$.

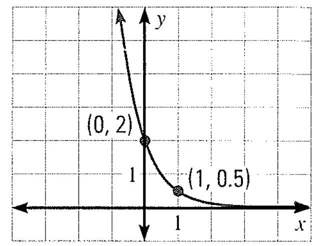
8 CHAPTER REVIEW

8.6 Write and Graph Exponential Decay Functions

pp. 531–538

EXAMPLE 1

Tell whether the graph represents *exponential growth* or *exponential decay*. Then write a rule for the function.



The graph represents exponential decay ($y = ab^x$ where $0 < b < 1$). The y-intercept is 2, so $a = 2$. Find the value of b by using the point (1, 0.5) and $a = 2$.

$y = ab^x$ **Write function.**

$0.5 = 2 \cdot b^1$ **Substitute.**

$0.25 = b$ **Solve for *b*.**

A function rule is $y = 2(0.25)^x$.

EXAMPLE 2

CAR VALUE A family purchases a car for \$11,000. The car depreciates (loses value) at a rate of about 16% annually. Write a function that models the value of the car over time. Find the approximate value of the car in 4 years.

Let V represent the value (in dollars) of the car, and let t represent the time (in years since the car was purchased). The initial value is 11,000, and the decay rate is 0.16.

$V = \boldsymbol{a}(1 - \boldsymbol{r})^t$ **Write exponential decay model.**

$= \mathbf{11{,}000}(1 - \mathbf{0.16})^t$ **Substitute 11,000 for *a* and 0.16 for *r*.**

$= 11{,}000(0.84)^t$ **Simplify.**

To find the approximate value of the car in 4 years, substitute 4 for t.

$V = 11{,}000(0.84)^t = 11{,}000(0.84)^4 \approx \$5477$

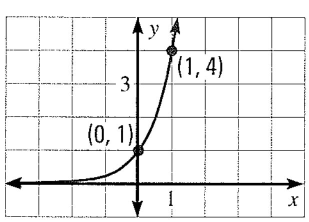
The approximate value of the car in 4 years is \$5477.

EXERCISES

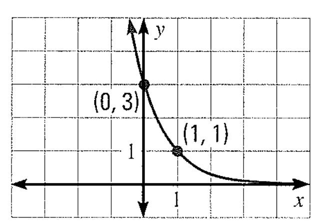
EXAMPLES 4 and 5 on pp. 533–534 for Exs. 40–42

Tell whether the graph represents *exponential growth* or *exponential decay*. Then write a rule for the function.

40.



41.



42. CAR VALUE The value of a car is \$13,000. The car depreciates (loses value) at a rate of about 15% annually. Write an exponential decay model for the value of the car. Find the approximate value of the car in 4 years.

8 CHAPTER TEST

Simplify the expression. Write your answer using exponents.

1. $(62 \cdot 17)^4$
2. $(-3)(-3)^6$
3. $\frac{8^4 \cdot 8^5}{8^3}$
4. $(8^4)^3$
5. $\frac{2^{15}}{2^8}$
6. $5^3 \cdot 5^0 \cdot 5^5$
7. $[(-4)^3]^2$
8. $\frac{(-5)^{10}}{(-5)^3}$

Simplify the expression.

9. $t^2 \cdot t^6$
10. $\left(\frac{s}{t}\right)^6$
11. $\frac{1}{9^{-2}}$
12. $-(6p)^2$
13. $(5xy)^2$
14. $\frac{1}{z^7} \cdot z^9$
15. $(x^5)^3$
16. $\left(-\frac{4}{c}\right)^2$

Simplify the expression. Write your answer using only positive exponents.

17. $\left(\frac{a^{-3}}{3b}\right)^4$
18. $\frac{3}{4d} \cdot \frac{(2d)^4}{c^3}$
19. $y^0 \cdot (8x^6y^{-3})^{-2}$
20. $(5r^5)^3 \cdot r^{-2}$

Write the number in scientific notation.

21. 423.6
22. 7,194,548
23. 500.32
24. 71.23884
25. 0.562
26. 0.0348
27. 0.000123
28. 0.5603002

Write the number in standard form.

29. 4.02×10^5
30. 5.3121×10^4
31. 9.354×10^8
32. 1.307×10^{19}
33. 1.3×10^{-3}
34. 3.32×10^{-4}
35. 7.506×10^{-5}
36. 9.3119×10^{-7}
37. Graph the function $y = 4^x$. Identify its domain and range.
38. Graph the function $y = \frac{1}{2} \cdot 4^x$. Compare the graph with the graph of $y = 4^x$.
39. **ANIMATION** About 1.2×10^7 bytes of data make up a single frame of an animated film. There are 24 frames in 1 second of a film. About how many bytes of data are there in 1 hour of an animated film?
40. **SALARY** A recent college graduate accepts a job at a law firm. The job has a salary of $32,000 per year. The law firm guarantees an annual pay increase of 3% of the employee's salary.
    a. Write a function that models the employee's salary over time. Assume that the employee receives only the guaranteed pay increase.
    b. Use the function to find the employee's salary after 5 years.
41. **SCIENCE** At sea level, Earth's atmosphere exerts a pressure of 1 atmosphere. Atmospheric pressure P (in atmospheres) decreases with altitude and can be modeled by $P = (0.99987)^a$ where a is the altitude (in meters).
    a. Identify the initial amount, decay factor, and decay rate.
    b. Use a graphing calculator to graph the function.
    c. Estimate the altitude at which the atmospheric pressure is about half of what it is at sea level.